On leaky-box approximation to GALPROP

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***Abstract*. The Galactic Propagation (GALPROP) numerical code is now accepted as an advanced tool for simulations of cosmic ray diffusion and interaction in the Galaxy. The code is used for the interpretation of a large body of cosmic ray data. In some cases, including in particular the case of stable primary and secondary nuclei, one can use a simple leaky-box model for handling of data on cosmic ray energy spectra and composition. We find an adequate leaky-box approximation to the basic GALPROP model and estimate its accuracy.**

***Keywords*: Cosmic rays; Propagation; Leaky box**

I. Introduction

Cosmic ray propagation in the Galaxy is commonly described in the diffusion approximation [1,2]. The galactic model with a flat extended cosmic ray halo and the distribution of cosmic-ray sources (supernova remnants) in the galactic disk is accepted as the most adequate. The comprehensive numerical realization of this model is the GALPROP code [3-6] that allows calculating the transport and interactions of relativistic protons, nuclei, electrons and positrons. The code incorporates detailed distributions of the interstellar gas, magnetic field and background radiation needed for such calculations. More simple approximate models are also used in the investigations of cosmic ray propagation. The most popular is the leaky-box model where the transport of energetic particles is described by introducing the mean escape time of cosmic rays from the Galaxy T_e that is function of particle energy. This simple model satisfactorily describes the set of data on stable primary and secondary nuclei in cosmic rays. The adequacy of the leaky-box model is confirmed by solutions of diffusion equations for uncomplicated models with flat halo. The purpose of the present work is to find the leaky-box model that most closely reproduces the GALPROP results on the calculations of primary and secondary stable nuclei at the solar system location in the Galaxy and to determine the precision of such approximation.

II. Cosmic-ray propagation equations

We consider the transport of cosmic ray protons and stable nuclei in the interstellar medium. In the diffusion approximation, the isotropic part of the cosmic-ray distribution function $f(t, \mathbf{r}, \mathrm{p})$ normalized as $N = 4\pi \int dpp^2 f$, where N is the total cosmic ray number density, obeys the equation for nucleus of type i of the form [1]:

$$\frac{\partial f_i}{\partial t} - \nabla\left(D_i \nabla f_i\right) + nv\sigma_i f_i + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\left(\frac{dp}{dt}\right)_i f_i\right] - \frac{1}{p^2}\left(\frac{\partial}{\partial p}p^2 k_i \frac{\partial}{\partial p} f_i\right) = q_i + n\sum_{j<i} 4\pi \int_p^\infty dp' p'^2 v' \frac{d\sigma_{ij}(p,p')}{dp'} f_j(t,\mathbf{r},\mathrm{p}'). \quad (1)$$

Here $D_i(\mathbf{r}, p)$ is the diffusion coefficient , $n(\mathbf{r})$ is the number density of interstellar gas nuclei, p is the particle momentum, $\sigma_i(p)$ is the total cross section of nuclear spallation in the interstellar gas, $\sigma_{ij}(p, p)$ is the production cross section of nuclei i by a more heavy nucleus j, $\kappa_i(\mathbf{r}, \mathrm{p})$ is the diffusion coefficient on momentum, $q_i(t, \mathbf{r}, \mathrm{p})$ is the source distribution.

The GALPROP code numerically solves the transport equation (1) rewritten for the function $\psi = 4\pi p^2 f$ for all cosmic ray species. The transport coefficients are determined by the data on B/C ratio. The spatial diffusion coefficient is taken as $D = k\beta\left(R/R_0\right)^a$, $k = const$, where $R = pc/Z$ is the particle magnetic rigidity and $\beta = v/c$, if necessary with a break. The source spectrum is assumed to be a power law in momentum, $q(p) \propto p^{-\gamma_s - 2}$ and may also have breaks.

If cosmic-ray transport is described in the leaky-box approximation, the diffusion term in Eq. (1) is substituted with the simple expression $f_i/T_{e,i}(E)$, where $T_{e,i}(E)$ has the meaning of the leakage time from the Galaxy. In the steady state we have for the cosmic ray intensity $I(E)dE = vf(p)p^2 dp$

$$\frac{I_i}{X_{e,i}} + \frac{\sigma_i(E)}{m} I_i + \frac{\partial}{\partial E}\left[\left(w_i(E) + \frac{2b_i(E)}{A_i^2 m\sqrt{E(E+2E_0}}\right) I_i\right] - \frac{\partial}{\partial E}\left(\frac{vb_i(E)}{A_i^2 m}\frac{\partial}{dE} I_i\right) = \frac{Q_i(E)}{\rho} + \sum_{j<i}\frac{\sigma_{ij}(E)}{m} I_i, \quad (2)$$

where $X_{e,i}(E) = v\rho T_{e,i}(E)$ is the escape length and all quantities including the cosmic-ray intensity I_i, the source density Q_i, and the gas density ρ do not depend on position **r**.

III. LEAKY-BOX APPROXIMATION TO THE GALPROP DIFFUSION MODEL

It is instructive to introduce the following function of cross section:

$$X_{ef} = \frac{m}{\sigma}\left(\frac{I(\sigma=0)}{I(\sigma)} - 1\right), \quad (3)$$

where the intensity $I(\sigma = 0)$ is taken at zero cross section.

The intrinsic property of the leaky-box model is the independence of X_e on cross section σ which is now considered as a variable. To find how close is the diffusion model to the leaky box model, we calculated the effective escape length X_{ef} defined by Eq. (3) in a wide range of cross sections with the help of the GALPROP code. The calculations of intensity $I(\sigma)$ were made for primary nuclei without contribution of spallation from other nuclei and without account of any energy change and radioactive decays in plain diffusion model, reacceleration model and model with damping. The rigidity dependence of the diffusion coefficient for these models can be found in [7].

The results of calculations using the plain diffusion model are presented in Figure 1. It is evident that $X_{ef}(\sigma)$ is nearly constant and slightly rises starting from very large cross sections σ about 1000 mb. It is significant that the typical value of the total cross section for iron group nuclei is of the order of 700 mb and therefore the leaky box approximation works well for all nuclei included in the GALPROP code. The deviation of X_{ef} from its asymptotic constant value at small cross sections does not exceed 2% for Fe nuclei at a few GeV/nucleon.

The found effective escape length that fits the GALPROP plain diffusion model is
$X_{ef} = 19\beta^3 \mathrm{g/cm}^2$ at $R \leq 3$ GV,
$X_{ef} = 19\beta^3 \left(R/3\mathrm{GV}\right)^{-0.6} \mathrm{g/cm}^2$ at $R>3$ GV.

The effective escape length for the reacceleration model is
$X_{ef} = 7.2 \left(R/3\mathrm{GV}\right)^{-0.34} \mathrm{g/cm}^2$
applied at $R>40$ GV, and
$X_{ef} = 13 \left(R/3\mathrm{GV}\right)^{-0.5} \mathrm{g/cm}^2$ applied at $R > 10$ GV for the model with damping.

IV. INTERPRETATION OF OBTAINED RESULTS

To understand why the leaky-box is a good approximation to the solution of the set of diffusion equations that describe propagation and spallation of stable nuclei in a flat-halo galaxy, let us consider a simple one-dimension case with the z-axis directed perpendicular to the central galactic plain. The diffusion equation for one type of primary nuclei with no energy loss is

$$-D\frac{dI}{dz} + n(z)v\sigma I = q(z). \quad (4)$$

Let us assume that the cosmic ray sources are uniformly distributed with the density q in the disk $|z| \leq h_s$; the interstellar gas with density n is uniformly distributed in the disk $|z| \leq h_g$; the cosmic rays fill the halo $|z| \leq H$ and it is assumed that $H > h_s > h_g$ (the corresponding numerical values are H = 4 kpc, $h_s = h_g = 100$ pc, $n = 1$ cm^{-3}.

Solving Eq.(4) under the continuity conditions for intensity I and the diffusion flux $-D\frac{dI}{dz}$ at the boundaries of the source and gas disks, and under the condition I = 0 at the halo boundaries, and using (3) one can find [7]:

$$X_{ef} = \frac{m}{\sigma}[\lambda^2 h_s \left(H - \frac{h_s}{2}\right)$$
$$\left(1 - \frac{1 - \lambda^2(h_s - h_g)\left(H - \frac{h_s+h_g}{2}\right)}{ch(\lambda h_g) + \lambda(H-h_g)sh(\lambda h_g)}\right)^{-1} - 1]. \quad (5)$$

The dependence of X_{ef} on cross section (5) is illustrated in Figure 2. The value of X_{ef} is approximately constant at small cross sections and increases at large cross sections. Thus the probable reason for increase of X_{ef} with σ in Figure 2 lies in the difference in the widths of gas and source distributions in the GALPROP models. In the case, when the gas disk is wider than the source disk, X_{ef} decreases at large cross sections.

Eq. (5) allows finding the analytical expression for the constant part of effective escape length

$$X_{ef}(\sigma=0) = \frac{\rho v h_g H}{D}[\left(1 - \frac{h_g}{2H}\right) -$$
$$\frac{(h_g)^2}{2h_s H}\left(1/3 - \frac{h_g}{4H}\right)\left(1 - \frac{h_s}{2H}\right)^{-1}] \quad (6)$$

and to estimate the dependent on σ relative deviation of $X_{ef}(\sigma)$ from $X_{ef}(\sigma = 0)$:

$$\frac{\delta X_{ef}}{X_{ef}(\sigma=0)} \approx \frac{\sigma n v {h_g}^2}{6D}\left(1 - \frac{h_g}{h_s}\right) \approx$$
$$0.05\beta\left(\frac{\sigma}{1000\mathrm{mb}}\right)\left(\frac{10^{28}\mathrm{cm}^2/\mathrm{s}}{D}\right)\left(1 - \frac{h_g}{h_s}\right), (7)$$

the sign of this deviation has an evident dependence on the ratio h_g/h_s (we neglect small values h_s/H, $h_g \ll 1$ in the last equation for δX_{ef}, and assume that $\delta X_{ef} \ll X_{ef}(r = 0)$.

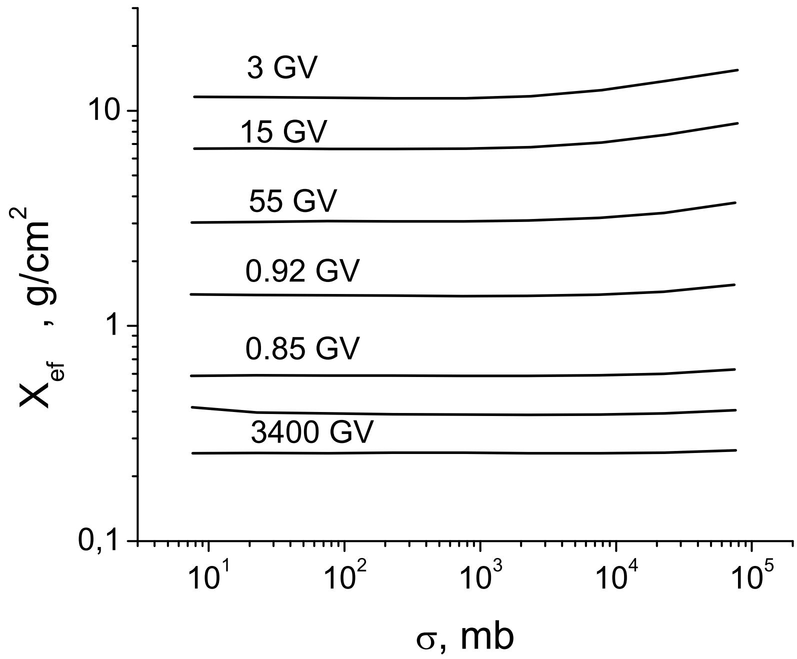

Fig. 1: Effective escape length determined from Eq. (3) where $I(\sigma)$ is calculated with GALPROP code in the plain diffusion model at different rigidities.

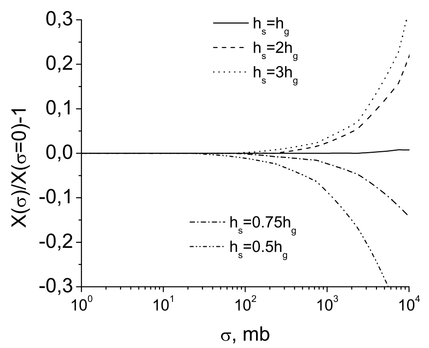

Fig. 2: Deviation (in percents) of effective escape length from its value at zero cross section for two versions of the one-dimensional diffusion model the source disk is more thick than the gas disk, $h_s = 2h_g$, and the source disk is more thin than the gas disk, $h_s = h_g/2$.

V. Conclusions

In this work we showed that the leaky box model is a good approximation to the basic GALPROP model for all stable nuclei included in the GALPROP code. The deviation of $X_{\rm ef}$ from its asymptotic constant value at small cross sections does not exceed 2% for Fe nuclei at a few GeV/nucleon. Therefore all propagation can be described by some escape length of cosmic rays from the Galaxy $X_{\rm e}$ (measured in g/cm^2) that is a function of particle energy. The efficiency of such approximation can be explained by the concentration of cosmic rays sources and the interstellar gas in a relatively thin galactic disk immersed in the flat but fat cosmic ray halo [1, 8]. We found that the small deviation between results obtained by calculation using the diffusion model and the leaky-box model for very heavy nuclei which depends on relation between widths of source and gas disks. Thus the probable reason for increase of $X_{\rm ef}$ with σ in Figure 2 lies in the difference in the widths of gas and source distributions in the GALPROP models.We also found effective escape length that fits the GALPROP plain diffusion model, reacceleration model and model with damping.

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