The kinematic dipole in galaxy redshift surveys

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In the concordance model of the Universe, the matter distribution – as observed in galaxy number counts or the intensity of line emission (such as the 21cm line of neutral hydrogen) – should have a kinematic dipole due to the Sun's motion relative to the CMB rest-frame. This dipole should be aligned with the kinematic dipole in the CMB temperature. Accurate measurement of the direction of the matter dipole will become possible with future galaxy surveys, and this will be a critical test of the foundations of the concordance model. The amplitude of the matter dipole is also a potential cosmological probe. We derive formulas for the amplitude of the kinematic dipole in galaxy redshift and intensity mapping surveys, taking into account the Doppler, aberration and other relativistic effects. The amplitude of the matter dipole can be significantly larger than that of the CMB dipole. Its redshift dependence encodes information on the evolution of the Universe and on the tracers, and we discuss possible ways to determine the amplitude.

I. INTRODUCTION

The concordance model of cosmology assumes statistical isotropy and homogeneity of the Universe. Since the Solar System has a nonzero velocity $\boldsymbol{v}_o$ relative to the cosmic frame defined by the cosmic microwave background (CMB), this assumption requires that the matter distribution must have a kinematic dipole which is aligned with that of the CMB[1]. This is a critical test of the foundations of the concordance model, as first pointed out and attempted in [2].

Up to now, attempts to measure the kinematic direction $\hat{\boldsymbol{v}}_o$ via the matter dipole are compromised by low median redshifts and low number densities (see [3–6] for recent work). Accurate measurement of $\hat{\boldsymbol{v}}_o$ from the matter distribution will become possible with next-generation galaxy surveys that will cover much of the sky out to high redshift and with high number densities [7–9].

In addition to the direction $\hat{\boldsymbol{v}}_o$, observations measure the amplitude of the matter dipole. This amplitude is determined not only by Doppler and aberration effects, but also by properties of the galaxy survey and by the evolution of the Universe. Here we focus on the amplitude of the kinematic dipole in galaxy redshift surveys, based on number counts or on intensity mapping of HI (or other line emission). This redshift-dependent kinematic dipole has not previously been presented, and measurements of the kinematic dipole up to now have used the projected, two-dimensional number counts.

We analyse the contributions to the matter kinematic dipole, using observational variables and taking care to include all relativistic effects. We start from the basic relativistic boost contribution to the number counts, and then include contributions from the expansion history of the Universe, the evolution of source number density and the magnification bias that arises from the magnitude limit of a galaxy redshift survey. Then we treat the case of HI intensity mapping. The redshift-dependent amplitude of the matter kinematic dipole encodes independent information about properties of the Universe and the tracer. We compare the amplitude of the matter kinematic dipole in the angular correlation function to that of the CMB kinematic dipole. We also discuss possible ways to determine the amplitude of the matter kinematic dipole.

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[1] Provided that the matter kinematic dipole is measured with high enough median redshift to beat down the influence of the local structure dipole (see [1] for a detailed discussion).

II.   KINEMATIC DIPOLE IN GALAXY NUMBER COUNTS

Assuming that the observer moves with velocity $\boldsymbol{v}_o$ as measured by CMB observations (with[2] $v_0 \approx 10^{-3}$), what is the resulting dipole in number counts for a galaxy redshift survey? We break the answer down into steps.

A.   Doppler and aberration effects

The boosted observer's 4-velocity $\tilde{u}_o^\mu$ is related to the CMB rest-frame observer's 4-velocity u_o^μ by

$$\tilde{u}_o^\mu = \gamma(v_o)\big[u_o^\mu + v_o^\mu\big] = u_o^\mu + v_o^\mu + O(v_o^2) \qquad \text{where} \qquad u_o^\mu v_{o\mu} = 0. \tag{1}$$

From now on, we neglect terms of $O(v_o^2)$ and second-order terms in metric perturbations. In the CMB rest-frame, $v_o^\mu = (0, \boldsymbol{v}_o)$. The boosted observer measures a different cosmological redshift $\tilde{z}$ to that measured in the CMB frame, z. For a source in the unit direction $n^\mu = (0, \boldsymbol{n})$ in the CMB frame, we have $u_o^\mu n_\mu = 0$, and the backward lightray vector is $k_o^\mu \propto -u_o^\mu + n^\mu$. It follows that

$$\frac{1 + \tilde{z}}{1 + z} = \frac{(u_\mu k^\mu)_o}{(\tilde{u}_\mu k^\mu)_o} = \frac{1}{1 + \boldsymbol{n} \cdot \boldsymbol{v}_o}, \tag{2}$$

since the contributions to $z, \tilde{z}$ at the source are the same and cancel. Thus the observed redshifts are Doppler related:

$$1 + \tilde{z} = (1 + z)(1 - \boldsymbol{n} \cdot \boldsymbol{v}_o). \tag{3}$$

The direction of sources is affected by aberration, $\boldsymbol{n} \to \tilde{\boldsymbol{n}}$. The photon wave vector k^μ is invariant under a boost, so that $-\tilde{u}_o^\mu + \tilde{n}^\mu = (1 - \boldsymbol{n} \cdot \boldsymbol{v}_o)[-u_o^\mu + n^\mu]$, where we used (3). Projecting into the rest-space and using (1), we obtain

$$\tilde{\boldsymbol{n}} = (1 - \boldsymbol{n} \cdot \boldsymbol{v}_o)\boldsymbol{n} + \boldsymbol{v}_o. \tag{4}$$

Taking the cross product with $\hat{\boldsymbol{v}}_o$, we have

$$\sin\tilde{\theta} = (1 - \boldsymbol{n} \cdot \boldsymbol{v}_o)\sin\theta, \tag{5}$$

where $\theta = \arccos(\boldsymbol{n} \cdot \hat{\boldsymbol{v}}_o)$ and similarly for $\tilde{\theta}$. By writing $\tilde{\theta} = \theta + \delta\theta$ and expanding (5), we find

$$\tilde{\theta} = \theta - v_o \sin\theta. \tag{6}$$

This shows that source images are aberrated towards the forward dipole direction. The azimuthal angle is unaffected by boosting, so that solid angles at the boosted and rest-frame observers are related by

$$\frac{d\tilde{\Omega}_o}{d\Omega_o} = \frac{\sin\tilde{\theta}}{\sin\theta}\frac{d\tilde{\theta}}{d\theta}\frac{d\varphi}{d\varphi} = 1 - 2\,\boldsymbol{n} \cdot \boldsymbol{v}_o, \tag{7}$$

where we used (5) and (6). Since the intrinsic area of a source is invariant, we have $\tilde{d}_A^2\, d\tilde{\Omega}_o = d_A^2\, d\Omega_o$, and hence the angular diameter distance obeys

$$\tilde{d}_A(\tilde{z}, \tilde{\boldsymbol{n}}) = d_A(z, \boldsymbol{n})\big[1 + \boldsymbol{n} \cdot \boldsymbol{v}_o\big]. \tag{8}$$

The reduction of the solid angle along the forward dipole direction corresponds to an increase in the area distance, since the object of given intrinsic size appears smaller and hence further away.

The total number of galaxies is invariant:

$$\tilde{N}\, d\tilde{z}\, d\tilde{\Omega}_o = N\, dz\, d\Omega_0, \tag{9}$$

where $N, \tilde{N}$ are the numbers observed per redshift interval and per solid angle. Then, using (3) and (7), the number count for the boosted observer is given by

$$\tilde{N}(\tilde{z}, \tilde{\boldsymbol{n}}) = N(z, \boldsymbol{n})\big[1 + 3\,\boldsymbol{n} \cdot \boldsymbol{v}_o\big]. \tag{10}$$

The kinematic dipole in the number counts is thus $3\,\boldsymbol{n} \cdot \boldsymbol{v}_o$, which can be thought of as the special relativistic approximation to the matter dipole. There are two corrections that are needed: a redshift correction and a magnitude correction. We deal with these in turn.

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[2] We choose units with $c = 1$ so that v_0 is dimensionless.

B. The redshift correction

Equation (10) contains implicitly an important correction arising from the difference between the observed $(z, \tilde{z})$ and background $(\bar{z})$ redshifts. This becomes apparent when we consider the fractional fluctuations in the number counts (at first order in perturbations):

$$\delta_{\tilde{N}}(\tilde{z}, \tilde{\boldsymbol{n}}) = \frac{\tilde{N}(\tilde{z}, \tilde{\boldsymbol{n}}) - \bar{N}(\tilde{z})}{\bar{N}(\tilde{z})} = \frac{N(z, \boldsymbol{n})\left[1 + 3\,\boldsymbol{n} \cdot \boldsymbol{v}_o\right] - \left[\bar{N}(\bar{z}) - (1+\bar{z})\boldsymbol{n} \cdot \boldsymbol{v}_o\, d\bar{N}(\bar{z})/d\bar{z}\right]}{\bar{N}(\bar{z}) - (1+\bar{z})\boldsymbol{n} \cdot \boldsymbol{v}_o\, d\bar{N}(\bar{z})/d\bar{z}}, \tag{11}$$

where we used (3). Thus

$$\delta_{\tilde{N}}(\tilde{z}, \tilde{\boldsymbol{n}}) = \delta_N(z, \boldsymbol{n}) + \left[3 + \frac{d\ln \bar{N}(\bar{z})}{d\ln(1+\bar{z})}\right] \boldsymbol{n} \cdot \boldsymbol{v}_o, \tag{12}$$

which shows that the evolution of $\bar{N}$ modifies the kinematic dipole. The contribution of $\tilde{\boldsymbol{n}} - \boldsymbol{n}$ to $\delta_{\tilde{N}}$ is second order in perturbations and we can rewrite (12) as

$$\delta_{\tilde{N}}(\tilde{z}, \boldsymbol{n}) = \delta_N(z, \boldsymbol{n}) + \delta_N^{\boldsymbol{v}_o}(z, \boldsymbol{n}), \qquad \delta_N^{\boldsymbol{v}_o}(z, \boldsymbol{n}) = \left[3 + \frac{d\ln \bar{N}(\bar{z})}{d\ln(1+\bar{z})}\right] \boldsymbol{n} \cdot \boldsymbol{v}_o. \tag{13}$$

We are assuming that the survey detects all galaxies, without any magnitude limit; the case of a magnitude limited survey is considered below.

The observed proper volume element in the background is $dV = dR\, dA = (a\, dr)\, (\bar{d}_A^2\, d\Omega_o) = (a/H)d\bar{z}\,(ar)^2 d\Omega_o$, where r is the line-of-sight comoving distance. Then the background number of sources per $\bar{z}$ per Ω_o is given by

$$\bar{N} = \left(\frac{r^2}{H}\right)\left[(1+\bar{z})^{-3}\bar{n}_s\right], \tag{14}$$

where $\bar{n}_s$ is the proper number density of sources. The term in round brackets is the comoving volume per redshift per solid angle, while the term in square brackets is the comoving number density. Thus the redshift correction in (13) encodes a volume and a number density contribution.

Using (14) in (13), we can display the 3 types of contribution to the kinematic dipole:

$$\delta_N^{\boldsymbol{v}_o} = \left\{3 + \left[\frac{\dot{H}}{H^2} + \frac{2(1+\bar{z})}{rH}\right] + \frac{d\ln\left[(1+\bar{z})^{-3}\bar{n}_s\right]}{d\ln(1+\bar{z})}\right\} \boldsymbol{n} \cdot \boldsymbol{v}_o. \tag{15}$$

The first term in braces is the special relativistic Doppler + aberration contribution; the second term, in square brackets, is the volume part of the redshift correction; the final term is the number density part of the redshift correction. This last term is given by the 'evolution bias' of the tracer [10, 11]:

$$b_e = -\frac{d\ln\left[(1+\bar{z})^{-3}\bar{n}_s\right]}{d\ln(1+\bar{z})}, \tag{16}$$

which measures the deviation from the idealized case of conservation of comoving number density, $\bar{n}_s \propto a^{-3}$, for which $b_e = 0$.

In summary, the kinematic dipole in the number count contrast, when we assume that there is no magnitude limit, is given by

$$\delta_N^{\boldsymbol{v}_o}(z, \boldsymbol{n}) = \mathcal{D}(z)\, \boldsymbol{n} \cdot \hat{\boldsymbol{v}}_o \quad \text{where} \quad \mathcal{D} = \left[3 + \frac{\dot{H}}{H^2} + \frac{2(1+z)}{rH} - b_e\right] v_o. \tag{17}$$

Here and below we drop the overbar on the background redshift where there is no ambiguity. Equation (17) holds for any perturbed Friedmann spacetime. For the concordance model, we have

$$\mathcal{D}(z) = \left[3 - \frac{3}{2}\Omega_m(z) + \frac{2(1+z)}{r(z)H(z)} - b_e(z)\right] v_o \qquad \text{LCDM.} \tag{18}$$

At low redshifts, the term $2(1+z)/rH$ dominates: e.g., for $z = 0.1$, it is ~ 20.

C.   Including the magnitude limit

Now we take account of the fact that only sources with observed magnitude $m < m_*$ are detected. Fluctuations in the observed magnitude, which can be given by fluctuations in the luminosity distance d_L, affect the number counts, so that $\delta_{\tilde{N}}(\tilde{z}, \boldsymbol{n})$ is replaced by [10–12]

$$\delta_{\tilde{N}}(\tilde{z}, \boldsymbol{n}, m < m_*) = \delta_{\tilde{N}}(\tilde{z}, \boldsymbol{n}) - 5s(\tilde{z}, m_*) \frac{\delta d_L(\tilde{z}, \boldsymbol{n})}{\bar{d}_L(\tilde{z})}. \tag{19}$$

Here,

$$s(\bar{z}, m_*) = \frac{\partial \log \bar{\mathcal{N}}_s(\bar{z}, m < m_*)}{\partial m_*} \tag{20}$$

is the magnification bias, where $\bar{\mathcal{N}}_s$ is the cumulative luminosity function:

$$\bar{\mathcal{N}}_s(\bar{z}, m < m_*) = \frac{2}{5} \ln 10 \int_{-\infty}^{m_*} dm \, \bar{n}_s(\bar{z}, m). \tag{21}$$

We are interested in the kinematic dipole of (19), which requires the kinematic dipole of the luminosity distance fractional fluctuation. Using the general reciprocity relation $\tilde{d}_L = (1 + \tilde{z})^2 \tilde{d}_A$, together with (3) and (8), we find that

$$\tilde{d}_L(\tilde{z}, \boldsymbol{n}) = d_L(z, \boldsymbol{n})\big[1 - \boldsymbol{n} \cdot \boldsymbol{v}_o\big]. \tag{22}$$

The luminosity distance in the direction of the boost is decreased since the observed flux $L/(4\pi \tilde{d}_L^2)$ increases while the intrinsic luminosity L is invariant.

Taking account of the difference between observed and background redshifts, we find from (3) and (22) that the fractional fluctuation is given by

$$\delta_{\tilde{d}_L}(\tilde{z}, \tilde{\boldsymbol{n}}) = \delta_{d_L}(z, \boldsymbol{n}) + \left[-1 + \frac{d \ln \bar{d}_L(\bar{z})}{d \ln(1 + \bar{z})}\right] \boldsymbol{n} \cdot \boldsymbol{v}_o. \tag{23}$$

In the background, $\bar{d}_L = (1 + \bar{z})r$, so that the kinematic dipole in the fractional luminosity distance fluctuation is[3]

$$\delta_{d_L}^{\boldsymbol{v}_o}(z, \boldsymbol{n}) = \frac{(1 + z)}{r(z)H(z)} \boldsymbol{n} \cdot \boldsymbol{v}_o. \tag{24}$$

Finally, (19) and (24) lead to:

$$\delta_N^{\boldsymbol{v}_o}(z, \boldsymbol{n}, m < m_*) = \delta_N^{\boldsymbol{v}_o}(z, \boldsymbol{n}) - \frac{5(1 + z)s(z, m_*)}{r(z)H(z)} \boldsymbol{n} \cdot \boldsymbol{v}_o = \mathcal{D}_{\text{gal}}(z, m_*) \boldsymbol{n} \cdot \hat{\boldsymbol{v}}_o, \tag{25}$$

where the amplitude of the dipole is

$$\mathcal{D}_{\text{gal}} = \left[3 + \frac{\dot{H}}{H^2} + (2 - 5s)\frac{(1 + z)}{rH} - b_e\right] v_o. \tag{26}$$

Since the number density is a function of m, (16) should now be replaced by

$$b_e(\bar{z}, m_*) = -\frac{\partial \ln\left[(1 + \bar{z})^{-3}\bar{\mathcal{N}}_s(\bar{z}, m < m_*)\right]}{\partial \ln(1 + \bar{z})}. \tag{27}$$

Equation (26) generalizes (17) and gives the theoretical prediction for the amplitude of the kinematic dipole in the number count contrast, for a magnitude-limited galaxy redshift survey.

To our knowledge, this expression is not given in previous literature on the cosmic kinematic dipole,[4] which has focused on the projected two-dimensional number counts. It shows how the redshift-dependent amplitude of the kinematic dipole in the matter distribution is modified from the naive special relativistic approximation not only by the expansion of the Universe, but also by the astrophysical characteristics (b_e and s) of the tracer used to map the matter distribution.

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[3] This is in agreement with [13], noting that their $\boldsymbol{n}$ is minus ours. Note also that the dipole for the area distance is the same as (24).

[4] $\mathcal{D}_{\text{gal}}\boldsymbol{n} \cdot \hat{\boldsymbol{v}}_o$ is part of the general equation for the number count contrast observed on the past lightcone, but terms at the observer are typically neglected in the final expression (e.g. [10, 11, 14]). There is also a monopole in the number count contrast, determined by the metric gravitational potentials at the observer, but that is not relevant here.

D. HI intensity mapping

The fractional fluctuations of brightness temperature for intensity mapping of the neutral hydrogen 21cm line emission after reionization,[5] can be obtained at first order from the number count contrast of HI atoms (see [12, 14] for details). Perturbations in the brightness temperature are determined by those in the number count and in the luminosity distance: $\delta_{T_{\rm IM}} = \delta_{N_{\rm IM}} - 2\delta d_L/\bar{d}_L$. Comparing this with (19), it follows that the map from number count to temperature contrast is given by setting $s = 2/5$ in (26):

$$\mathcal{D}_{\rm IM} = \left[3 + \frac{\dot{H}}{H^2} - b_e\right] v_o \qquad \left(s_{\rm IM} = \frac{2}{5}\right), \tag{28}$$

where

$$b_e = -\frac{d\ln\left[(1+\bar{z})^{-3}\bar{n}_{\rm IM}\right]}{d\ln(1+\bar{z})} = -\frac{d\ln\left[(1+\bar{z})^{-2}H\bar{T}_{\rm IM}\right]}{d\ln(1+\bar{z})}. \tag{29}$$

III. KINEMATIC DIPOLE IN THE ANGULAR CORRELATION FUNCTION

If we do not remove the kinematic dipole by a boost at the observer, then observations will measure the total fluctuation, i.e.,

$$\Delta_{\tilde{N}} = \Delta_N + \delta_N^{\boldsymbol{v}_o}, \tag{30}$$

where Δ_N is the observed fluctuation at the source:

$$\Delta_N = \delta_N - \frac{(1+z)}{H}\partial_r(\boldsymbol{v}\cdot\boldsymbol{n}) + (5s - 2)\kappa + \Delta^{\rm uls}. \tag{31}$$

The second term on the right is the observational correction from redshift space distortions and the third term is the lensing magnification correction. The last term represents the ultra-large scale relativistic observational corrections (see [10–12]). The angular correlation function for a redshift survey is defined by

$$\left\langle\Delta_{\tilde{N}}(z, \boldsymbol{n}, m < m_*)\Delta_{\tilde{N}}(z', \boldsymbol{n}', m' < m_*)\right\rangle = \sum_\ell \frac{(2\ell+1)}{4\pi}\tilde{C}_\ell(z, z', m_*)P_\ell(\boldsymbol{n}\cdot\boldsymbol{n}'). \tag{32}$$

Since the kinematic dipole term $\delta_N^{\boldsymbol{v}_o}$ does not correlate with the terms at the source, Δ_N, we have

$$\begin{aligned}
\left\langle\Delta_{\tilde{N}}(z, \boldsymbol{n}, m < m_*)\Delta_{\tilde{N}}(z', \boldsymbol{n}', m' < m_*)\right\rangle &= \left\langle\Delta_N(z, \boldsymbol{n}, m < m_*)\Delta_N(z', \boldsymbol{n}', m' < m_*)\right\rangle \\
&\quad + \left\langle\delta_N^{\boldsymbol{v}_o}(z, \boldsymbol{n}, m < m_*)\delta_N^{\boldsymbol{v}_o}(z', \boldsymbol{n}', m' < m_*)\right\rangle \\
&= \sum_\ell \frac{(2\ell+1)}{4\pi}C_\ell(z, z', m_*)P_\ell(\boldsymbol{n}\cdot\boldsymbol{n}') + \frac{3}{4\pi}C_1^{\boldsymbol{v}_o}(z, z', m_*)\,\boldsymbol{n}\cdot\boldsymbol{n}'.
\end{aligned} \tag{33}$$

The dipole in the boosted frame is related the intrinsic dipole in the rest-frame and the kinematic dipole as:

$$\tilde{C}_1 = C_1 + C_1^{\boldsymbol{v}_o}. \tag{34}$$

By (25), using

$$\left\langle(\boldsymbol{n}\cdot\hat{\boldsymbol{v}}_o)(\boldsymbol{n}'\cdot\hat{\boldsymbol{v}}_o)\right\rangle = \frac{1}{3}\,\boldsymbol{n}\cdot\boldsymbol{n}', \tag{35}$$

we find that the kinematic dipole is

$$C_1^{\boldsymbol{v}_o}(z, z', m_*) = \frac{4\pi}{9}\,\mathcal{D}(z, m_*)\mathcal{D}(z', m_*), \tag{36}$$

where for galaxies $\mathcal{D} = \mathcal{D}_{\rm gal}$ is given by (26), and for intensity mapping, $\mathcal{D} = \mathcal{D}_{\rm IM}$ is given by (28).

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[5] The dipole in the eopoch of reionization is considered in [15].
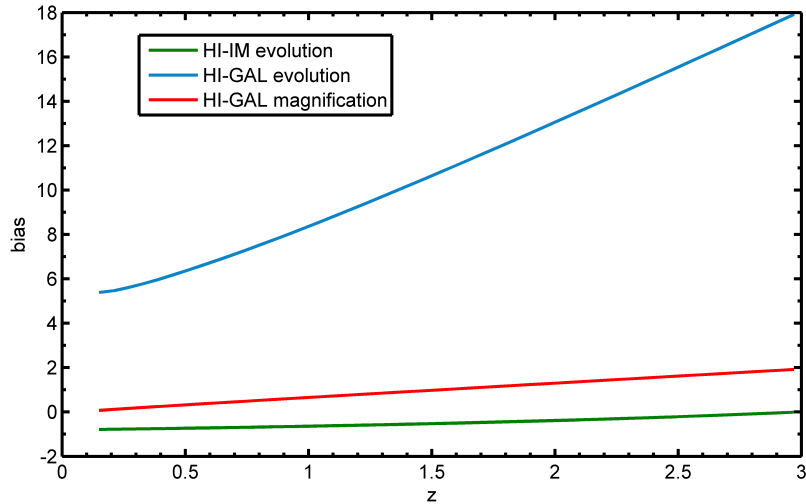
FIG. 1: The evolution and magnification biases in SKA galaxy ('gal') and intensity mapping ('IM') surveys. (The effective magnification bias for IM, $s_{\mathrm{IM}} = 2/5$, is not shown.)

A. Comparison with the amplitude of the CMB kinematic dipole

In the concordance model, the matter kinematic dipole is aligned with that of the CMB. However the amplitudes of the dipoles are different. The matter dipole depends on the expansion rate, and also on the astrophysical parameters b_e and s. In particular, the amplitude is redshift-dependent. By contrast the CMB temperature kinematic dipole is given simply by [16]

$$\mathcal{D}_{\mathrm{CMB}} = v_o. \tag{37}$$

The amplitude of the matter kinematic dipole in number counts/ intensity, relative to that of the CMB, is thus

$$r_{\mathcal{D}}(z, m_*) = \frac{\mathcal{D}(z, m_*)}{\mathcal{D}_{\mathrm{CMB}}} = \frac{\mathcal{D}(z, m_*)}{v_0} = 3 - \frac{3}{2}\Omega_m(z) + [2 - 5s(z, m_*)]\frac{(1+z)}{r(z)H(z)} - b_e(z, m_*). \tag{38}$$

In terms of the observable angular correlations, the matter kinematic dipole amplitude relative to that of the CMB is

$$r_C(z, z', m_*) \equiv \frac{C_1^{\boldsymbol{v}_o}(z, z', m_*)}{C_1^{\mathrm{CMB}}} = \frac{\mathcal{D}(z, m_*)\mathcal{D}(z', m_*)}{v_o^2}, \tag{39}$$

by (36) and (37).

We compute the ratios (38) and (39) for two redshift surveys that are planned for the Square Kilometre Array (SKA) [17]: an HI galaxy survey in Phase 2, and an HI intensity mapping survey in Phase 1. In order to predict the amplitude of the kinematic dipole in these surveys, we need only the evolution and magnification biases. For the redshift evolution of the Hubble rate, we use the concordance model parameters $\Omega_{m0} = 0.3 = 1 - \Omega_{\Lambda 0}, h = 0.68$.

$\underline{b_e \text{ and } s}$:

Fitting formulas for the evolution and magnification biases of the SKA2 HI galaxy survey are given in [18] for a range of possible flux cuts. We choose a flux cut of $3\,\mu$Jy. For the SKA1 HI intensity mapping survey, the effective magnification bias parameter is $2/5$ [see (28)]. The brightness temperature is given in [19], and then the evolution bias is computed via (29) (see also [12]). The results are shown in Fig. 1.

$\underline{r_{\mathcal{D}} \text{ and } r_C}$:

Using the predicted b_e and s, we compute $r_{\mathcal{D}}$ and r_C for the two SKA surveys. The results are displayed in Fig. 2:
The upper left panel shows $r_{\mathcal{D}}(z, m_*)$.
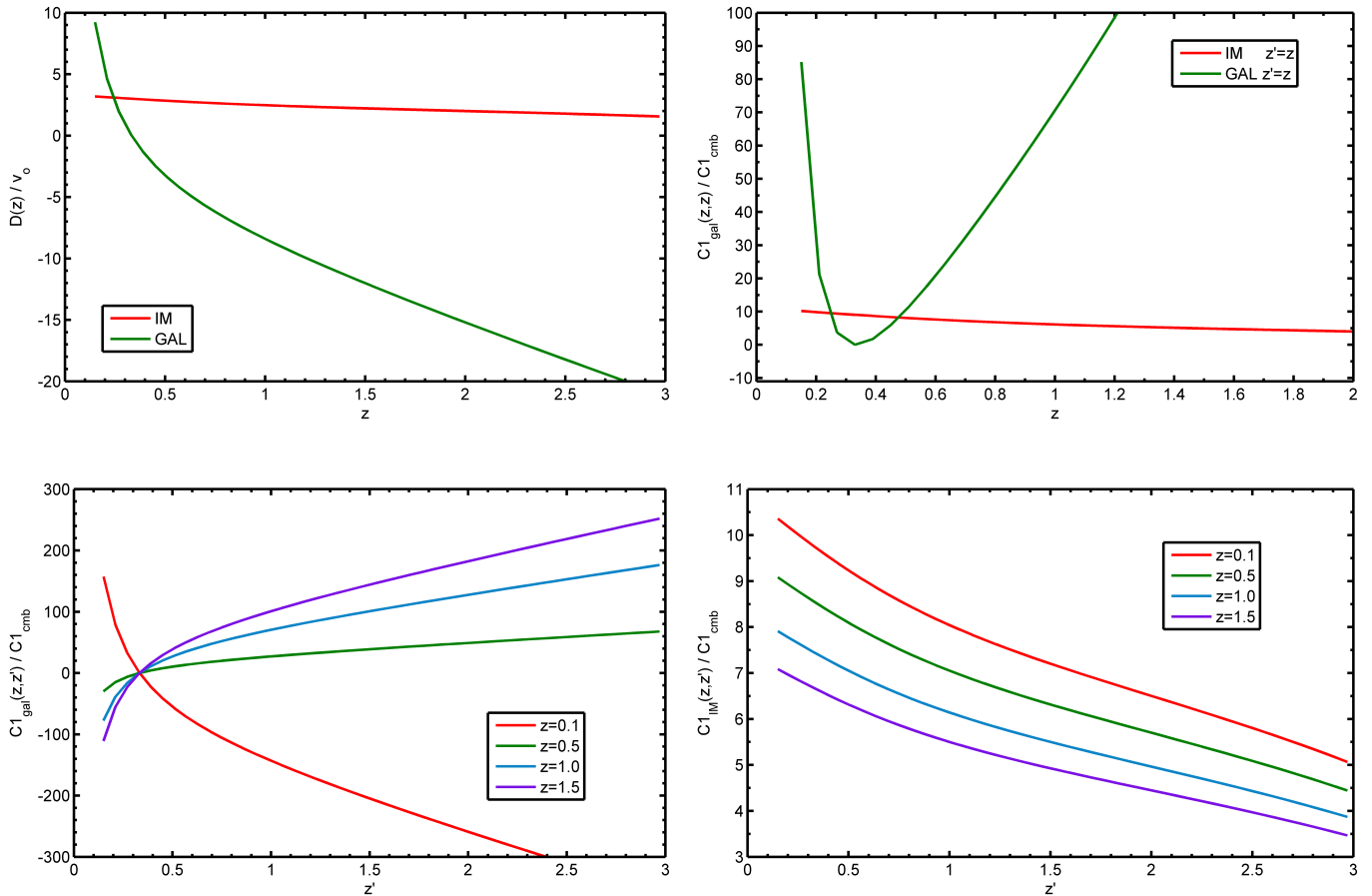The upper right panel shows $r_C(z, z, m_*) = [r_{\mathcal{D}}(z, m_*)]^2$ for both surveys.

FIG. 2: The kinematic matter dipole in SKA galaxy ('gal') and intensity mapping ('IM') surveys, relative to the CMB.
Upper: The dipole in HI number counts/ intensity relative to the CMB [see (38)] (*left*); the dipole in the angular correlations at the same redshift, $z' = z$, relative to the CMB [see (39)] (*right*).
Lower: The dipole in the angular correlations at different redshifts, in the galaxy (*left*) and intensity mapping (*right*) surveys, relative to the CMB.

The lower panels show $r_C(z, z', m_*) = r_{\mathcal{D}}(z, m_*) r_{\mathcal{D}}(z', m_*)$, for the galaxy (left panel) and intensity mapping (right panel) surveys.

Figure 2 demonstrates that the theoretical amplitude of the matter kinematic dipole can be much larger than that of the CMB, especially for the galaxy survey:

- The $r_{\mathcal{D}}(z, m_*)$ curves in the upper left panel explain the behaviour of $r_C(z, z, m_*)$ in the upper right panel:
  Galaxies – $r_{\mathcal{D}}$ decreases monotonically and goes through zero at a low redshift, $z \sim 0.3$, so that r_C rises after the zero-point.
  Intensity mapping – $r_{\mathcal{D}}$ also decreases monotonically but does not go through zero; as a result, r_C decreases monotonically.

- Similarly, for $r_C(z, z', m_*)$ in the lower panels:
  Galaxies – $r_C(z, z', m_*)$ is negative when $r_{\mathcal{D}}$ has opposite signs at z and z'. For the $z = 0.1$ curve (red), this occurs when $z' \gtrsim 0.3$, while for the other curves ($z = 0.5, 1, 1.5$), it occurs when $z' \lesssim 0.3$.
  Intensity mapping – $r_C(z, z')$ is always positive since $r_{\mathcal{D}} > 0$ at all z, z'.

The main driver for the difference between the two surveys is evolution bias: in the interval $0.1 < z < 3$, we have $5 < b_e < 19$ for galaxies,[6] and $-1 < b_e < 0$ for intensity mapping. The b_e for galaxies is large enough that $-b_e$ in (26) can produce $\mathcal{D}_{\text{gal}} = 0$ at low z, and then push $\mathcal{D}_{\text{gal}}$ to large negative values at high z. Both kinematic dipoles contain

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[6] Note that b_e for galaxies is sensitive to the flux cut. We chose $3\,\mu$Jy – for higher values (lower sensitivity), b_e is larger.

the same positive expansion term $3 - 3\Omega_m/2$. The magnification bias contribution is zero for intensity mapping. For galaxies, $0 < s < 2$, so that the term $(2 - 5s)(1 + z)/rH$ is positive and large at low z, but negative and small in magnitude for higher z.

B. Determining the amplitude of the kinematic dipole

We briefly discuss theoretical options for the determination of the dipole amplitude (we do not consider here the issue of optimal estimators and uncertainties). We assume that $\boldsymbol{v}_o$ is known from the CMB.

The amplitude of the kinematic dipole as a function of redshift is given by a weighted integral of the boosted correlation function:

$$C_1^{\boldsymbol{v}_o}(z, z', m_*) + C_1(z, z', m_*) = 2\pi \int_{-1}^{1} d\mu \, \mu \, \langle \Delta_{\tilde{N}}(z, \boldsymbol{n}, m < m_*) \, \Delta_{\tilde{N}}(z', \boldsymbol{n}', m' < m_*) \rangle, \qquad \mu = \boldsymbol{n} \cdot \boldsymbol{n}', \qquad (40)$$

which follows from (33), (34). The intrinsic dipole is a combination of the dipole from primordial perturbations, which is of the same order as the perturbative $C_\ell, \ell > 1$, and the dipole from local structure. The perturbative dipole can be neglected relative to the kinematic dipole. However, the local structure dipole dominates over the kinematic dipole at low redshift, $z, z' \lesssim 0.7$ [1]:

$$\begin{aligned} \tilde{C}_1 &= C_1^{\boldsymbol{v}_o} + C_1^{\text{loc}} + C_1^{\text{prim}} \approx C_1^{\boldsymbol{v}_o} + C_1^{\text{loc}} \\ &\approx C_1^{\boldsymbol{v}_o} \qquad \text{for} \quad z, z' \gtrsim 1. \end{aligned} \qquad (41)$$

In the case $z' = z$, the upper right panel of Fig. 2 gives examples of $C_1^{\boldsymbol{v}_o}$, in units of the CMB dipole amplitude. At low redshift, $C_1^{\boldsymbol{v}_o}$ is not a good approximation to $\tilde{C}_1$.

The contribution of $\langle \Delta_N \Delta_N' \rangle$, i.e. the rest-frame correlation function, in (33) can be suppressed if $z' \gg z$ (e.g. $z = 0.1, z' = 2.5$). The reason is that for widely separated redshifts, the only non-negligible correlations from Δ_N are those involving the lensing convergence contribution $(5s - 2)\kappa$ to Δ_N [see (31)]. From (33),

$$\langle \Delta_{\tilde{N}} \Delta_{\tilde{N}}' \rangle \approx (5s' - 2)\langle \Delta_N \kappa' \rangle + \langle \delta_N^{\boldsymbol{v}_o} \delta_N^{\boldsymbol{v}_o\prime} \rangle \approx \langle \delta_N^{\boldsymbol{v}_o} \delta_N^{\boldsymbol{v}_o\prime} \rangle \qquad \text{for} \quad z' \gg z. \qquad (42)$$

The local structure dipole in Δ_N at redshift z dominates over $\delta_N^{\boldsymbol{v}_o}$ at z – but it does not correlate with any of the terms at z' and thus we can neglect it. Therefore

$$\langle \Delta_{\tilde{N}}(z, \boldsymbol{n}, m < m_*) \, \Delta_{\tilde{N}}(z', \boldsymbol{n}', m' < m_*) \rangle \approx \frac{3}{4\pi} C_1^{\boldsymbol{v}_o}(z, z', m_*) \, \boldsymbol{n} \cdot \boldsymbol{n}' \qquad \text{for} \quad z' \gg z. \qquad (43)$$

The red curves ($z = 0.1$) at $z' = 2.5$ in the lower panels of Fig. 2 give examples of $C_1^{\boldsymbol{v}_o}$ (in units of the CMB kinematic dipole amplitude) with $z' \gg z$. For intensity mapping, the lensing term in (31), and hence in (42), vanishes at first order.

IV. CONCLUSION

Next-generation galaxy surveys over huge volumes of the Universe will deliver the accuracy required to perform a key consistency test of the standard model of cosmology: that the kinematic dipole in the matter distribution should be aligned with that of the CMB. In addition to the direction of the matter kinematic dipole, future surveys will also be able to measure the amplitude of the dipole, which has not previously been investigated for galaxy redshift surveys.

We derived the expression for the kinematic dipole, due to the velocity $\boldsymbol{v}_o$ of the Solar System observer relative to the rest frame of the CMB, in the number count contrast of galaxy redshift surveys, including the special case of HI intensity mapping surveys. The kinematic dipole is determined not only by Doppler and aberration effects, but also by redshift and magnitude perturbations. The general expression is given by (25) and (26):

$$\delta_{\tilde{N}} = \delta_N + \delta_N^{\boldsymbol{v}_o}, \qquad \delta_N^{\boldsymbol{v}_o}(z, \boldsymbol{n}, m < m_*) = \mathcal{D}(z, m_*) \, \boldsymbol{n} \cdot \hat{\boldsymbol{v}}_o, \qquad \mathcal{D} = \left[3 + \frac{\dot{H}}{H^2} + (2 - 5s)\frac{(1 + z)}{rH} - b_e \right] v_o. \qquad (44)$$

For HI intensity mapping, the magnification bias is effectively $s = 2/5$ and b_e is determined by the background brightness temperature.

The number count contrast observed by the rest-frame observer, Δ_N, is given by the real-space contrast δ_N plus observational corrections from redshift-space distortions, lensing convergence and other relativistic effects on ultra-large scales. For the boosted observer, $\Delta_{\tilde{N}} = \Delta_N + \delta_N^{\boldsymbol{v}_o}$, and the boosted correlation function is $\langle \Delta_{\tilde{N}} \Delta'_{\tilde{N}} \rangle$. The kinematic dipole term $\delta_N^{\boldsymbol{v}_o}$ does not correlate with the terms at the source, Δ_N, and its contribution is much larger than the intrinsic dipole, $\tilde{C}_1 = C_1 + C_1^{\boldsymbol{v}_o} \approx C_1^{\boldsymbol{v}_o}$, provided that we auto-correlate at high enough redshift, or cross-correlate with $z' \gg z$, to beat down the effect of the local structure dipole. The boosted correlation function and its kinematic dipole are given by (33)–(36):

$$\langle \Delta_{\tilde{N}} \Delta'_{\tilde{N}} \rangle \;=\; \langle \Delta_N \Delta'_N \rangle + \langle \delta_N^{\boldsymbol{v}_o} \delta_N^{\boldsymbol{v}_o\prime} \rangle \approx \sum_{\ell=2} \frac{(2\ell+1)}{4\pi} C_\ell P_\ell + \frac{3}{4\pi} C_1^{\boldsymbol{v}_o} \, \boldsymbol{n} \cdot \boldsymbol{n}', \tag{45}$$

$$C_1^{\boldsymbol{v}_o}(z, z', m_*) \;=\; \frac{4\pi}{9} \, \mathcal{D}(z, m_*)\mathcal{D}(z', m_*) = 2\pi \int_{-1}^{1} d\mu \, \mu \, , m < m_* \langle \Delta_{\tilde{N}}(z, \boldsymbol{n}, m < m_*) \, \Delta_{\tilde{N}}(z', \boldsymbol{n}', m' < m_*) \rangle. \tag{46}$$

We presented a theoretical computation of the amplitude of the matter kinematic dipole, using future SKA galaxy and intensity mapping surveys as two examples. The evolution bias and magnification bias for these surveys, based on simulations, are shown in Fig. 1. The amplitudes $\mathcal{D}$ and $C_1^{\boldsymbol{v}_o}$ of the kinematic dipoles in these surveys are shown in Fig. 2, in units of the corresponding CMB quantity, and as functions of redshift.

The matter dipole amplitude is larger than that of the CMB for most redshifts – especially in the case of the HI galaxy survey. In addition, $\mathcal{D}_{\mathrm{gal}}(z, m_*)$ goes through zero and thus $C_{1\,\mathrm{gal}}^{\boldsymbol{v}_o}(z, z, m_*)$ has a minimum of zero, at $z \sim 0.3$. The main driver of this behaviour is the large b_e for galaxies. For angular correlations across different redshifts, $C_{1\,\mathrm{gal}}^{\boldsymbol{v}_o}(z, z', m_*)$ may decrease or increase monotonically, depending on z. For intensity mapping, it is always monotonically decreasing in the redshift range $0 < z < 3$.

The amplitude of the matter kinematic dipole can be measured by correlations averaged over directions. When $z' \gg z$, the rest-frame correlations $\langle \Delta_N \Delta'_N \rangle$ in (45) are strongly suppressed, with only a small lensing contribution surviving – which can be neglected relative to the kinematic dipole.

In principle, measurements of the amplitude give information on the Hubble rate $H(z)$, similarly to the kinematic dipole in luminosity distance [20], shown in (23). The more complicated kinematic dipole of spectroscopic number counts/ intensity mapping, (44), also contains the tracer astrophysical parameters b_e and s, which makes extraction of H more difficult – but the advantage is the high number density. Future work will investigate estimators to measure the amplitude of the kinematic dipole, and the signal-to-noise for various surveys.

Acknowledgments:

We thank Carlos Bengaly and Camille Bonvin for very helpful comments. All authors are funded in part by the South African SKA Project and the NRF (South Africa). RM and CC are also supported by the UK STFC, Grants ST/N000668/1 (RM) and ST/P000592/1 (CC), and SC is also supported by the Claude Leon Foundation.

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