

Cosmological constraints on quintessence scalar field models against the astronomical observations

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Scalar field models play an essential role in describing various phases of the universe evolution. A survey of literature reports that various potentials have been studied in the past couple of years. Now, with successive developments in the observational cosmology, it becomes essential to test their viabilities with the universe evolution. Thus, being motivated, here we consider a varieties of quintessence scalar field models in a homogeneous and isotropic geometry of the universe with zero spatial curvature aiming to provide stringent constraints using a series of astronomical data sets, namely, the cosmic microwave background (CMB) observations, baryon acoustic oscillations (BAO), redshift space distortions (RSD), joint light curve analysis (JLA) from Supernovae Type Ia and the Hubble parameter measurements from the cosmic chronometers (CC). From the qualitative evolution of the models, we find all of them are able to execute a fine transition from the past decelerating phase to the presently accelerating expansion where in addition, the equation of state of the scalar field (also the effective equation of state) might be close to that of the Λ CDM cosmology depending on its free parameters. From the statistical analysis, we find that within 68% CL, the observational data always allow a constant potential. We also find that for all the models a strong negative correlation between the parameters (H_0, Ω_{m0}) exists, while the parameters (H_0, σ_8) are not correlated. We also comment that the present models are unable to reconcile the tension on H_0 . Finally, we conclude our work with the Bayesian analyses which report that the non-interacting Λ CDM model is preferred over all the quintessence scalar field models.

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1. INTRODUCTION

Since the detection of accelerating universe by measuring the luminosity distances of the type-Ia supernovae [1, 2] – a new era of modern cosmology has began. Subsequent investigations by different groups [3, 4] conveyed that the current acceleration of our universe could be an effect of some hypothetical fluid with large negative pressure usually known as dark energy [5], which is completely unknown by its character and origin. Usually, two distinct approaches are considered to describe such dark energy fluids, one is the modifications of the matter sector of the universe in the context of the Einstein gravity [5, 6], and secondly, the modifications of the Einstein's gravitational theories [7, 8, 9, 10, 11] also lead to some extra geometrical terms (alternatively known as the geometrical dark energy fluids in order make a difference between the accelerating effects coming from the matter modifications or geometry modifications) can also explain this late accelerating expansion. Apart from the above two approaches, the readers might be interested to know that there is another alternative to describe this accelerating universe, namely the gravitational particle production mechanism, see [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] and the references therein. However, overall, the actual dynamics of these mysterious components, are unknown, but thanks to the recent observational evidences, we have an estimation of such dark fluids. According to the observational predictions, such dark energy fluids contribute nearly 68% of the total energy density of the universe [4]. Additionally, another bulk content of the matter sector, about 28% of the total energy density of the universe, is occupied by some non-luminous dark matter component. Thus, overall, almost 96% of the total energy content of the universe has been filled up by

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these dark fluids, namely, dark energy and dark matter and probing their nature, evolution and the origin is the most intriguing fact of modern cosmology.

In the present work we shall confine ourselves to Einstein gravity and thus incorporate the dark energy fluid through the matter sector in the field equations. The dark energy fluids may appear in many forms – from the simplest cosmological constant to more complicated ones. Although the cosmological constant characterized by the constant equation of state $w_\Lambda = -1$, is consistent with the present observations, but also allow slight variation from the constant equation of state. Moreover, the problem related to the magnitude of the cosmological constant further motivated to consider some dark energy fluids evolving with the cosmic time.

The scalar field models, arising in the context of particle physics theory, is a very natural choice for the dynamical dark energy models. The physics of scalar field models, which is mostly contained in the potential, $V(\phi(t))$ where $\phi(t)$ is the underlying field, have gained a considerable interest in the cosmological community due to explaining various stages of the universe evolution, see [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43] (also see [44, 45, 46, 47, 48]). Varieties of scalar field models are existing in the literature at present, such as, quintessence, phantom, k-essence, tachyon, ghost condensates and the dilatonic scalar field models, see [5, 6] for more details on them. However, in the present work we shall consider some specific quintessence models aiming to impose stringent cosmological constraints using the latest cosmological sources, namely, the cosmic microwave background temperature and polarization data [49, 50], baryon acoustic oscillations distance measurements [51, 52, 53], Redshift space distortion [54], Supernovae Type Ia [55], and finally the Hubble parameter measurements from the cosmic chronometers [56]. The underlying geometry has been chosen to be the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric where the matter fields of the universe is minimally coupled to the gravity – described by the usual Einstein gravity.

The work has been organized as follows. In section 2, we describe the evolution equations for any quintessence scalar field model at the level of background and perturbations in a spatially flat FLRW universe. The section 3 introduces a class of quintessence models and their qualitative behavior that we wish to study in this work. In section 4, we introduce the observational data, the statistical technique and the results of the models. In particular, the subsection 4.1 contains the observational constraints on the models; the subsection 4.2 includes a geometrical probe, namely, the Om diagnostic; the subsection 4.3 provides with the Bayesian model comparison and in subsection 4.4 we present an overall comparison between all the quintessence scalar field models. Finally, with section 5 we close the present work summarizing the main findings.

2. BASIC FRAMEWORK

As usual we consider a homogeneous and isotropic space-time of the universe characterized by the Friedmann-Lemaître-Robertson-Walker line element

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (1)$$

where $a(t)$ is the expansion scale factor of the universe and K is the curvature scalar which for 0, +1, -1, respectively represent the spatially flat, closed and open universe. In agreement with the observational evidences [4] we consider the spatial flatness of this universe, that means, we shall consider $K = 0$ throughout the analysis of the present work. Let us now consider the action in such a universe where the matter fields minimally coupled to gravity as follows

$$\mathcal{A} = \int \sqrt{-g} d^4x \left[\frac{R}{2\kappa} - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi) \right] + S_m + S_r \quad (2)$$

where $\kappa = 8\pi G$ is the Einstein's gravitational constant (G is the Newton's gravitational constant); S_m, S_r are respectively the actions representing the matter and radiation sectors; $V(\phi)$ is the potential of the scalar field ϕ . The corresponding field equations for flat FLRW universe are given by

$$H^2 = \frac{8\pi G}{3} \rho_{\text{eff}}, \quad (3)$$

$$2\dot{H} + 3H^2 = -8\pi G p_{\text{eff}}, \quad (4)$$

where an overhead dot represents the cosmic time differentiation; $H \equiv \dot{a}/a$, is the Hubble parameter; $\rho_{\text{eff}} = \rho_r + \rho_b + \rho_c + \rho_\phi$ is the total energy density of the universe. Here, ρ_i ($i = r, b, c, \phi$) represents the energy density of the i -th fluid where the symbol r, b, c, ϕ corresponds to the radiation, baryons, cold dark matter and the scalar field sector,

respectively. Similarly, one can define the total pressure by adding the pressure of each fluid, i.e., $p_{\text{eff}} = p_r + p_b + p_c + p_\phi$ in which p_i stands for the pressure term for the i -th fluid ($i = r, b, c, \phi$). For the flat FLRW universe, the energy density and the pressure of the scalar field model, respectively are, $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$, $p_\phi = \dot{\phi}^2/2 - V(\phi)$. Additionally, we assume that each fluid follows the barotropic equation of state $p_i = w_i \rho_i$, $i = r, b, c, \phi$ (w_i being the barotropic index for the i -th fluid) where $w_r = 1/3$, $w_b = 0$, $w_c = 0$, and $w_\phi = p_\phi/\rho_\phi = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}$. Since there is no such interaction between any two fluids, hence, each fluid satisfies their own conservation equation, i.e., $\dot{\rho}_i + 3H(p_i + \rho_i) = 0$. Following this conservation equation, one can see the evolutions of the component fluids as follows: $\rho_r \propto a^{-4}$, $\rho_b \propto a^{-3}$, $\rho_c \propto a^{-3}$. The evolution of the remaining fluid, i.e., the quintessence scalar field sector is governed by the following equation

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi} \quad (5)$$

which can also be obtained from equations (3) and (4). However, one can clearly understand that the dynamics of the scalar field can only be determined once the potential $V(\phi)$ is prescribed and there is no such specific rule to select the potential. In this work, we shall consider a few phenomenological forms for the potential and render stringent constraints from the latest observational data coming from different astronomical sources.

Let us define some important cosmological parameters exhibiting the qualitative evolution of the scalar field model. The effective equation of state for the total fluid characterized by ρ_{eff} , p_{eff} as

$$w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = -1 - \frac{2\dot{H}}{3H^2} \quad (6)$$

and the deceleration parameter q that quantifies the accelerating or decelerating phase of the universe, is given by

$$q = -1 - \frac{\dot{H}}{H^2}. \quad (7)$$

One may also recall the usual definitions of the density parameters for the fluids as follows: $\Omega_i = \rho_i/\rho_c$, where $\rho_c = 3H_0^2/8\pi G$, is the critical density of the universe.

Having the above set of equations in hand, in principle one can determine the evolution of the quintessence scalar field model at the background level but not at the level of perturbations. Since the observed structure formation of the universe is related to the evolution at the level of perturbations, thus, we now proceed to the investigations of the perturbation equations. In order to do so, we consider the perturbed FLRW metric in the following form

$$ds^2 = a^2(\tau) [-(1 + 2\Psi)d\tau^2 + (1 + 2\Phi)\delta_{ij}dx^i dx^j] \quad (8)$$

where τ is the conformal time; Ψ , Φ are the perturbation quantities. For the above perturbed FLRW metric (8), one may write down the Einstein's field equations as [57]:

$$\delta'_A + 3aH(c_{s,A}^2 - w_A)\delta_A = -(1 + w_A)(\theta_A + 3\Phi'), \quad (9)$$

$$\theta'_A + \left(aH(1 - 3w_A) + \frac{w'_A}{1 + w_A}\right)\theta_A = k^2 \left(\frac{c_{s,A}^2}{1 + w_A}\delta_A + \Psi + \sigma_A\right), \quad (10)$$

$$k^2\Phi = 4\pi G a^2 \rho_A \left(\delta_A + 3aH(1 + w_A)\frac{\theta_A}{k^2}\right), \quad (11)$$

and $\Psi = -\Phi$. Here, we recognize $\delta_A = \delta\rho_A/\rho_A$, as the density perturbation for the fluid A ; $\theta_A = i\vec{k}\cdot\vec{v}_A$, as the divergence of the peculiar velocity v_A of the component A . The prime attached to any quantity denotes the differentiation with respect to the conformal time τ . The quantity $c_{s,A}^2 = \delta p_A/\delta\rho_A$, is the sound speed of the fluid A which we consider to be non-negative in order to avoid any kind of instabilities.

3. SCALAR FIELD MODELS

As discussed, the scalar field models play an essential role to describe the dynamical phases of the universe. In this section we shall introduce some scalar field models belonging to the quintessence class and investigate their qualitative behavior.

3.1. Model 1

Let us first consider the following scalar field model characterized by the potential

$$V(\phi) = V_0 \cosh(\beta\phi^u) \quad (12)$$

where V_0 , u and β are real constants. The parameter β in (12) quantifies the deviation of the model from the constant potential $V(\phi) = V_0$. One can clearly notice that the potential (12) is nothing but the sum of two exponential potentials:

$$V(\phi) = \frac{V_0}{2} \left(e^{\beta\phi^u} + e^{-\beta\phi^u} \right),$$

and it is the generalized version of the potential $V = V_0 \cosh(\beta\phi)$ [58] which is obtained by setting $u = 1$ in (12). Now, in order to study the qualitative behavior of this potential, we choose a specific value of u , the simplest one namely, $u = 1$, and solve the conservation equation (5) numerically. One may wonder why we fix $u = 1$ since other values of u are equally favorable. Thus, we performed similar calculations for other values of u , namely, $u = 2, 3$, but we did not find any significant qualitative changes in the cosmological parameters sketching the evolution of the universe. However, concerning the observational constraints on this potential, indeed varying u is appealing, but this certainly increases the degeneracy between other parameters of the model. So, fixing u to some preassigned value reduces the degeneracy between the parameters. In fact, one can see how the cosmological constraints change for different positive or negative values of u leading to a class of quintessence potentials, such as $V = V_0 \cosh(\beta\phi^2)$, $V_0 \cosh(\beta\phi^3)$, ... $V_0 \cosh(\beta\phi^{-1})$, $V_0 \cosh(\beta\phi^{-2})$, ... $V_0 \cosh(\beta\phi^{1/2})$, $V_0 \cosh(\beta\phi^{1/3})$, etc. This could really be an interesting investigation in future. So, essentially, we are interested in the original potential $V = V_0 \cosh(\beta\phi)$ of [58] although we keep the original u -dependent model (12) in this work for a far reaching purpose just mentioned above. The qualitative evolution of this potential is presented in Fig. 1 in terms of various cosmological parameters. In the upper left plot of Fig. 1 we show the evolution of this potential which shows that the potential has an extremum and the parameter β plays the role of a scaling character. The extreme right plot of the upper panel of Fig. 1 presents the evolution of the scalar field itself. The middle plot of the upper panel of Fig. 1 depicts the evolution of the deceleration parameter from which a smooth transition from the past deceleration to present acceleration is clearly displayed and the transition occurs around $z = 0$. Moreover, from the evolution of the deceleration parameter, one can also notice that it is similar to that of the Λ CDM cosmology, however, for this case, the transition occurs after the transition for Λ CDM. The lower panel of Fig. 1 also displays various parameters, namely, the energy density of the fluids (lower left plot of Fig. 1); the equation of state for the scalar field as well as the total equation of state (lower middle plot of Fig. 1); and the density parameters for the fluids (lower right plot of Fig. 1).

3.2. Model 2

We now consider the model characterized by the following potential

$$V(\phi) = V_0 \left[1 + \epsilon \operatorname{sech}(\alpha\phi) \right] \quad (13)$$

where $\epsilon = \pm 1$, V_0 is a constant and α is the only free parameter of this model that quantifies the deviation of this potential from the constant potential $V = V_0$. The above potential was introduced in [59] and subsequently studied in [60, 61] where the authors discussed its cosmological features for $\epsilon = -1$. However, concerning the cosmological importance of this model, we do not find any strong reason to exclude the model with $\epsilon = +1$. Thus, we are intended to investigate this potential for both the values of ϵ in the context of late-time acceleration of the universe. For convenience, the model with $\epsilon = 1$ is denoted by Model 2a while the model with $\epsilon = -1$ is named as Model 2b. In the following we shall show that the sign of ϵ plays an interesting role in the evolution of the universe.

In order to understand the nature of the potential and its impact on the evolution of the universe, in Fig. 2 (for $\epsilon = 1$) and Fig. 3 (for $\epsilon = -1$), we have displayed the variations of $V(\phi)$ as well as the energy densities of different matter sources of the universe for different values of the free parameter α . In the top panels of Fig. 2 and Fig. 3, we have showed the evolution of the potential (13) for $\epsilon = +1$ (top left of Fig. 2) and for $\epsilon = -1$ (top left of Fig. 3). One can clearly notice that here α does not play any significant role except that its scaling character. The evolutions of the potentials are reversed due to $\epsilon = +1, -1$. But, both the potentials allow extremum values during its evolution. For this potential, we have numerically solved the conservation equation (5) and displayed the evolution of the scalar field in the top right plots of both Fig. 2 (for $\epsilon = +1$) and Fig. 3 (for $\epsilon = -1$). The energy density for this potential has been shown in the bottom left plot of Fig. 2 (for $\epsilon = +1$) and Fig. 3 (for $\epsilon = -1$). Here, we have some characteristic

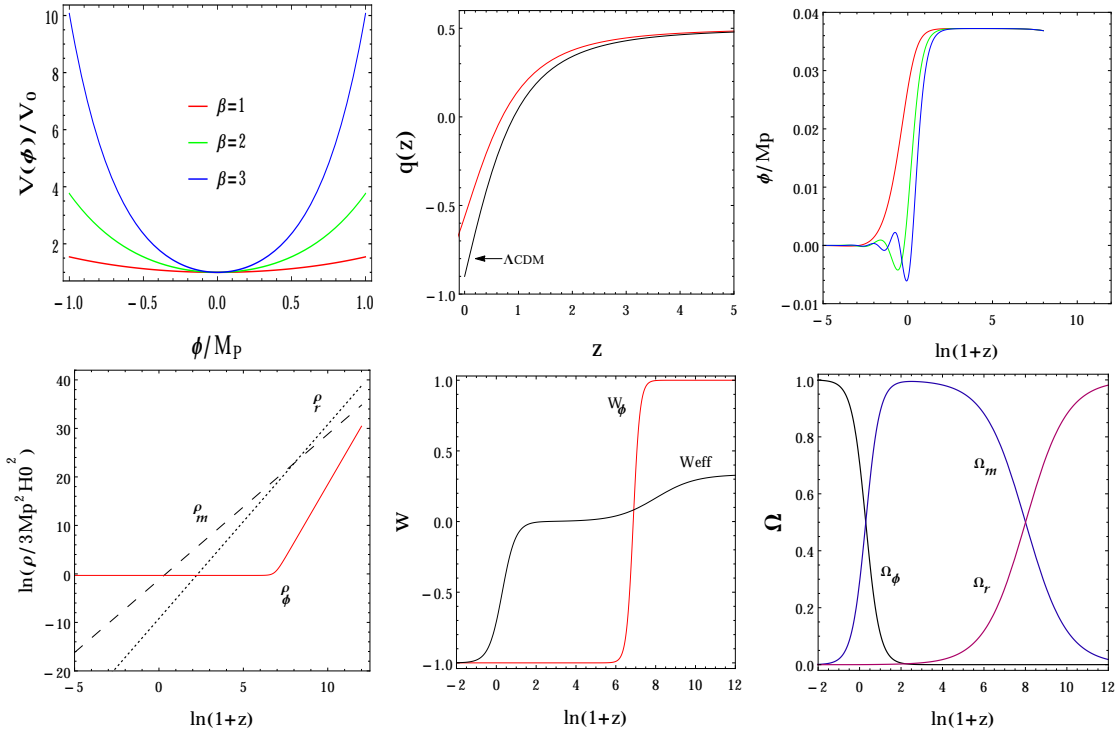


FIG. 1: This figure corresponds to the potential (12). We show different cosmological parameters for $u = 1$ taking various values of β as $\beta = 1$ (red), 2 (green), 3 (blue). Let us mention that in the plots of q , ρ and w , the trajectories for $\beta = 1, 2$ and 3 cannot be distinguished from one another. We note that during the analysis we have taken a specific value of V_0 as $V_0=2.2$.

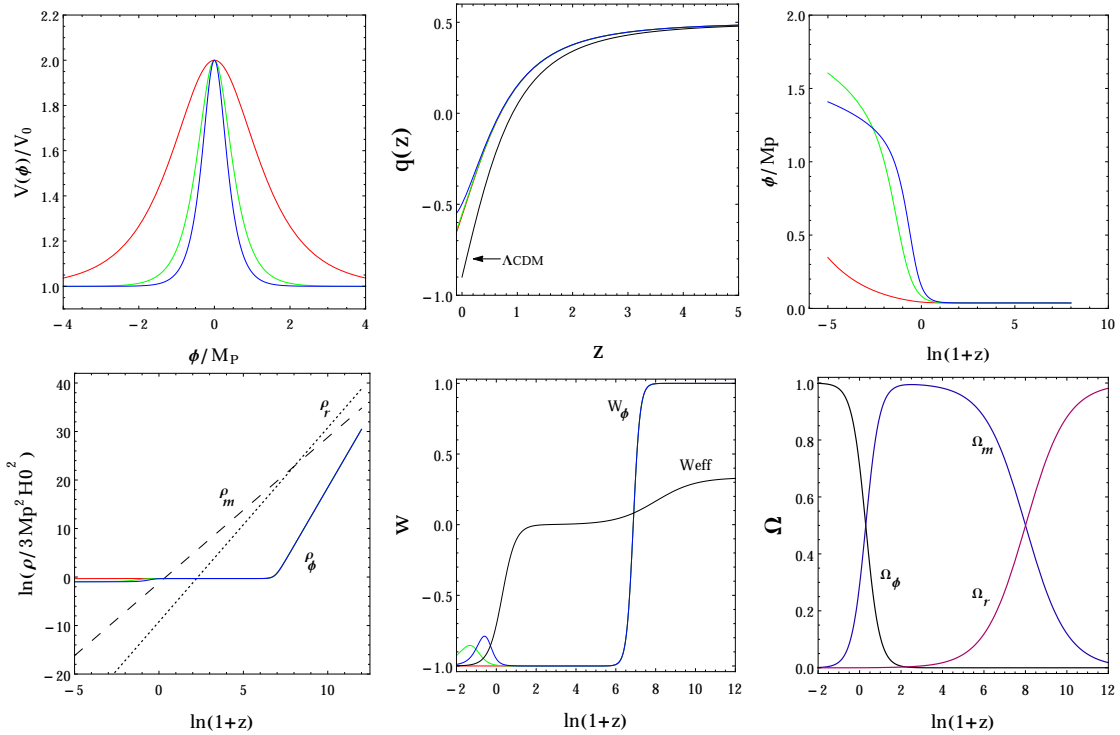


FIG. 2: This figure corresponds to the potential (13) with $\epsilon = +1$. We choose the model parameters as $V_0 = 1.1$, $\alpha = 1$ (red), 2.5 (green) and 3.5 (blue). From the plots one can realize that α works as a scaling parameter while the qualitative evolution of the model seems to be independent of α .

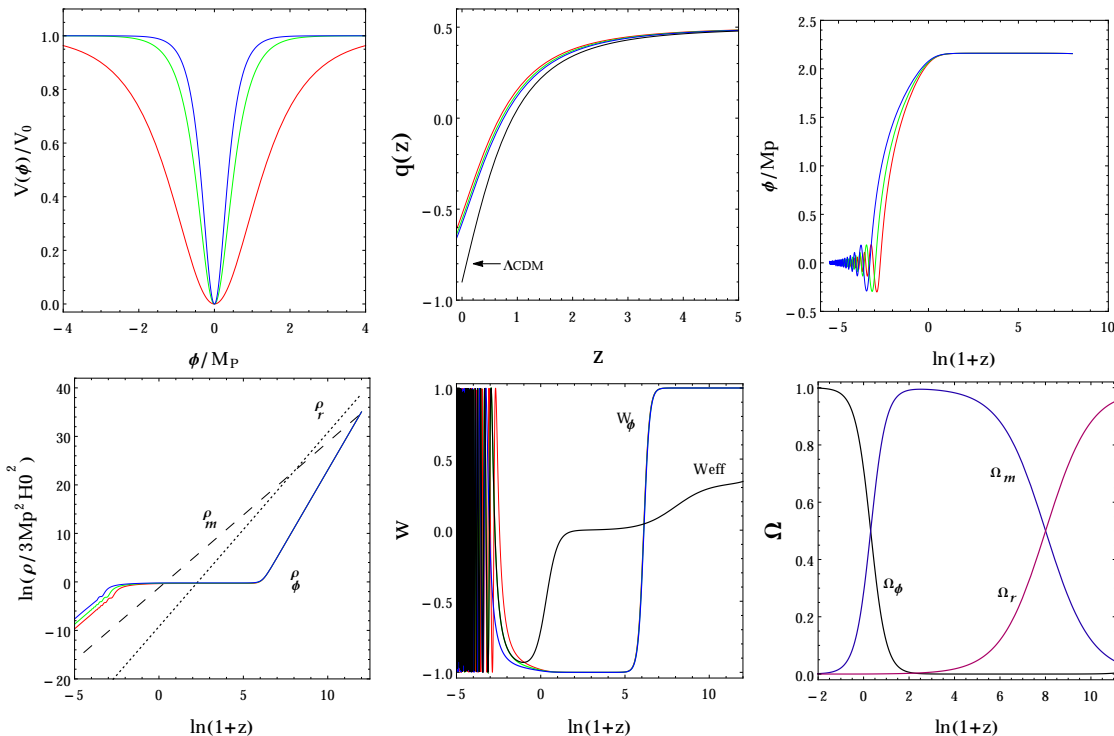


FIG. 3: This figure represents the potential (13) with $\epsilon = -1$. We show different cosmological parameters, namely the deceleration parameter q , equation-of-state, w_ϕ and w_{eff} for different values of the characteristic parameter α such as $\alpha = 0.9$ (red), 1 (green), 1.1 (blue) and $V_0=3$ whereas the evolution of the density parameter (right) corresponds to $\alpha = 1$. By looking the evolution of w_ϕ , one may conclude that the w_ϕ exhibits thawing behavior at present epoch, and oscillations in the future.

changes between these potentials as reflected from the bottom left plots of Fig. 2 and Fig. 3. For $\epsilon = +1$, from the evolution of the energy densities, one can see that at late time, the energy density of the scalar field, is actually constant which corresponds to the cosmological constant, and this seems to be independent of α . While on the other hand, a sharp fall of the scalar field energy density is observed which is also independent of α .

We have also presented the evolution of the deceleration parameter, q (top middle plots of both Fig. 2 (for $\epsilon = +1$) and Fig. 3 (for $\epsilon = -1$)), equation of state for scalar field (also the effective equation of state) in the bottom middle plots of both Fig. 2 (for $\epsilon = +1$) and Fig. 3 (for $\epsilon = -1$)), and the evolution of the density parameters (bottom right plots of both Fig. 2 (for $\epsilon = +1$) and Fig. 3 (for $\epsilon = -1$)). We find that for both the models a clear transition from the past decelerating phase to the current accelerating phase is suggested and α , the quantifying parameter of the model does not play any significant role in this picture. The evolution of the deceleration parameter is found to be very close to its evolution for the Λ CDM model. Since α does not play any crucial role for the evolution of the scalar field model, thus, pick up $\alpha = 1$ and displays the evolution of the density parameters for the matter fluids in the top right (for $\epsilon = +1$) and bottom right (for $\epsilon = -1$) panels. However, we observe some different behavior when the equation of state of the scalar field model (w_ϕ) and the effective equation of state of the models, w_{eff} .

For the potential with $\epsilon = +1$, we display the evolutions of w_ϕ and w_{eff} in the middle plot of the top panel of Fig. 2. The evolutions show that at late time, the scalar field behaves like a cosmological constant and moreover, one can further notice that the effective equation of state also mimics a cosmological constant fluid. For $\epsilon = -1$, similar quantities are displayed in the middle plot of the bottom panel of Fig. 3. We find that although at present (that corresponds to $z = 0$), the equation of state for the scalar field, w_ϕ and the effective equation of state, w_{eff} are very close to the cosmological constant boundary, however, slight different characteristics in both w_ϕ and w_{eff} are observed for $z < 0$, corresponding to the evolution in the future. We find that for $z < 0$, oscillating features of both w_ϕ and w_{eff} are allowed. So, qualitatively, Model 2b is slightly different compared to Model 2a (concerning the behaviour of the equation of state).

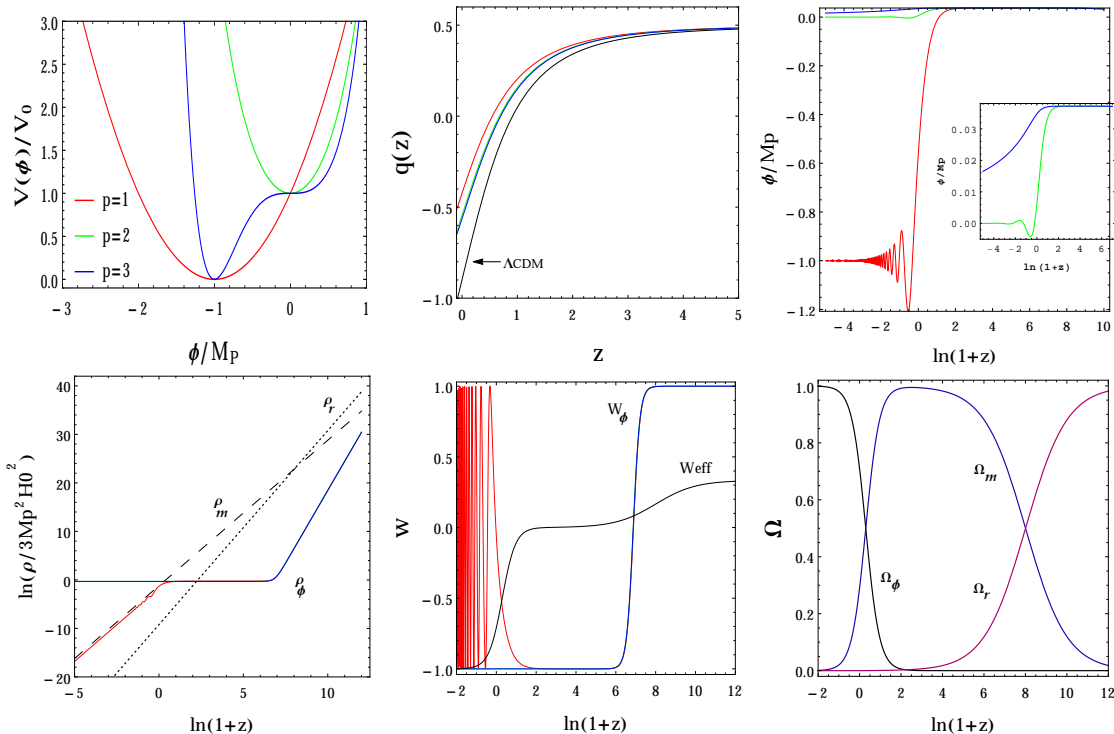


FIG. 4: The figure displays the evolutions of various cosmological parameters associated for the potential (14) with $\delta = +1$. We have analyzed the model for different values of $p = 1$ (red curves), 2 (green curves), 3 (blue curves). Let us note that for the trajectories of w (middle plot of the lower panel) and the energy density for the scalar field (left plot of the lower panel), the blue curve is completely overlapped with the green curve. Let us further mention that throughout the analysis we have fixed $V_0=2.2$.

3.3. Model 3

Finally, we consider the last model in this series:

$$V(\phi) = V_0 \left[1 + \delta \left(\frac{\phi}{M_P} \right)^p \right]^2, \quad (14)$$

that can be found in [62]. Here, in δ , p , V_0 are the real constants and $M_P = (8\pi G)^{-1/2}$. Let us note that the parameter δ quantifies the deviation of the model from the constant potential $V = V_0$. Here we classify Model 3 in the following manner. We denote Model 3 with $\delta \geq 0$ as Model 3a while by Model 3b we mean Model 3 with $\delta \leq 0$. Now, following the similar trend, we have numerically solved the conservation equation (5) for this potential and discuss various quantities graphically in Fig. 4 and Fig. 5. Let us describe the qualitative behaviour of Model 3a and Model 3b.

The Fig. 4 completely describes the qualitative evolution of Model 3a. The Fig. 4 has two panels, namely, the upper and lower panels where each panel contains three different plots. In the upper panel of Fig. 4, we show the evolution of the potential with respect to the scalar field ϕ (upper left panel of Fig. 4), the deceleration parameter as a function of the redshift (upper middle panel of Fig. 4) and the scalar field itself with respect to $\ln(1+z)$ (upper right panel of Fig. 4). In all three plots, we have used three different values of p , namely, $p = 1$ (red curves), $p = 2$ (green curves) and $p = 3$ (blue curves). From the evolution of the potential we find that that model allows an extremum during the evolution of the universe. From the upper middle panel (Fig. 4), a clear transition of the deceleration parameter from its past decelerating phase to the present accelerating phase of the universe is clearly seen and this evolution is quite similar to that of the Λ CDM cosmology. The scalar field appears to be sharply decrease (upper right panel of Fig. 4) with the evolution of the universe. The lower panel of Fig. 4 has three different plots, namely, the evolution of the energy densities of the component fluids (lower left panel of Fig. 4), the equation of state of the scalar field model and the effective equation of state (lower middle panel of Fig. 4), the evolution of the density parameters of the component fluids (lower left panel of Fig. 4). From the evolution of different energy densities, we

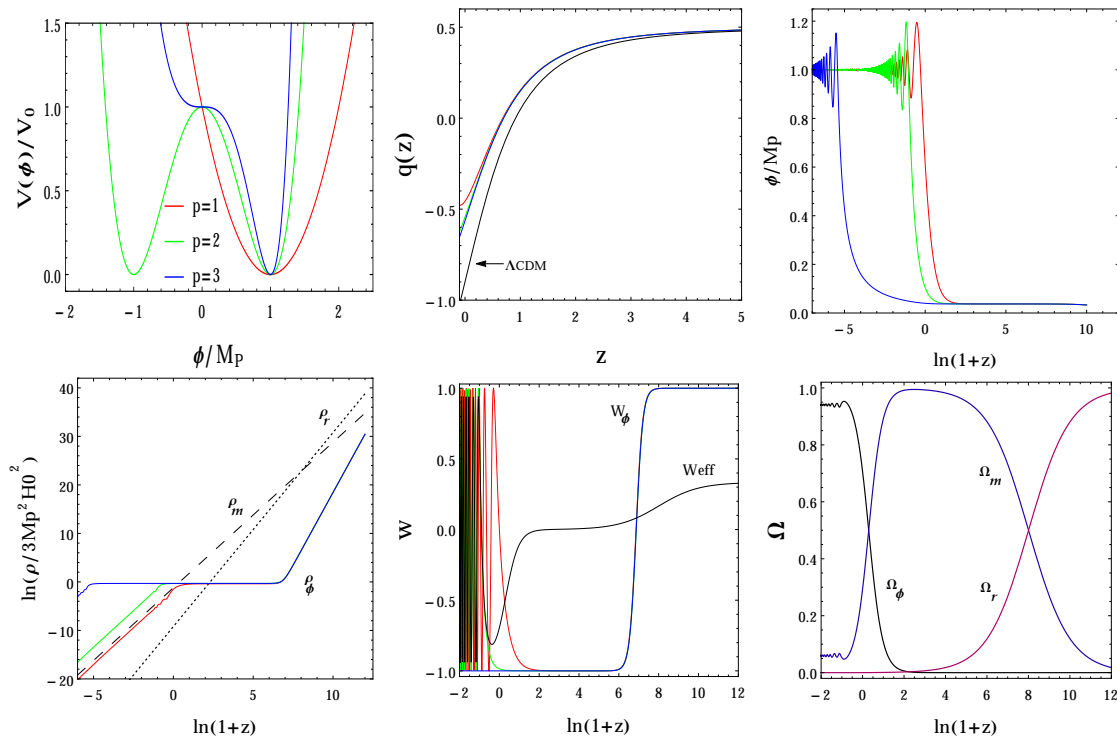


FIG. 5: The figure displays the evolutions of various cosmological parameters associated for the potential (14) with $\delta = -1$. We have analyzed the model for different values of $p = 1$ (red curves), 2 (green curves), 3 (blue curves). We note that throughout the analysis we have fixed $V_0=2.2$. Similarly, in the plot for the deceleration parameter, q (middle panel of the upper plot) the blue and green curves are almost indistinguishable from one another.

see that at late time, ρ_ϕ does not seem to evolve with the cosmic time, so effectively, ρ_ϕ behaves like a cosmological constant fluid. From the equation of state of both scalar field as well as the effective fluid, we see that at late time, both of them has a converging nature to -1 .

For Model 3b, we have described various cosmological parameters in Fig. 5. Similarly, we have made two panels – the upper panel containing three distinct plots, namely, the evolution of the potential (upper left panel of Fig. 5), deceleration parameter (upper middle panel of Fig. 5), evolution of the scalar field (upper right panel of Fig. 5) – and one the lower panel containing another three distinct plots, namely, the evolution of the energy densities of different fluids (lower left panel of Fig. 5), equation of state for the scalar field as well as for the effective fluid (lower middle panel of Fig. 5) and the density parameters of the fluids (lower right panel of Fig. 5). The evolution of this model (i.e., Model 3b) has similarities to Model 3a.

4. OBSERVATIONAL DATA, FITTING METHODOLOGY AND THE RESULTS

In order to constrain the model, we use several astronomical data with latest origin. In the following we describe the observational data and then we describe the analysis of the model.

- Cosmic microwave background observations: The data from cosmic microwave background (CMB) radiation are very effective for the analyses with dark energy models. Here, in this work, we employ the CMB temperature and polarization anisotropies together with their cross-correlations from the Planck’s team [49]. In particular, we include the combinations of high- and low- ℓ TT likelihoods in the overall multiple range $2 \leq \ell \leq 2508$ plus the combinations of the high- and low- ℓ polarization likelihoods [50]. To constrain the models, we use the freely available Planck likelihood [50], which marginalizes over several nuisance parameters related to the measurements.
- Baryon acoustic oscillations data: We have used four particular measurements of the baryon acoustic oscillations (BAO) data, namely, from the 6dF Galaxy Survey (6dFGS) (effective redshift at $z_{\text{eff}} = 0.106$) [51], the Main Galaxy Sample of Data Release 7 of Sloan Digital Sky Survey (SDSS-MGS) (at $z_{\text{eff}} = 0.15$) [52], CMASS and

LOWZ samples from the latest Data Release 12 (DR12) of the Baryon Oscillation Spectroscopic Survey (BOSS) (CMASS at $z_{\text{eff}} = 0.57$) [53] and (LOWZ at $z_{\text{eff}} = 0.32$) [53]. Let us describe how we use the BAO data in our analysis. Here we use the ratio of r_s/D_V acting as a “standard ruler” where r_s is the co-moving sound horizon at the baryon drag epoch and D_V is the effective distance determined by D_A where the relation between D_A and D_V is given by following [63]

$$D_V(z) = \left[(1+z)^2 D_A(z)^2 \frac{z}{H(z)} \right]^{1/3}. \quad (15)$$

In this work, we include four distinct measurements of r_s/D_V at four different redshifts, namely, from the 6dF Galaxy Survey (6dFGS) measurement [51]; from the Main Galaxy Sample of Data Release 7 of Sloan Digital Sky Survey (SDSS-MGS) [52]; from the CMASS and LOWZ samples from the latest Data Release 12 (DR12) of the Baryon Oscillation Spectroscopic Survey (BOSS) [53].

The likelihood function for the BAO data is given by

$$\chi_{\text{BAO}}^2 = \sum_i \frac{[r_{\text{BAO},i}^{\text{obs}} - r_{\text{BAO},i}^{\text{th}}]^2}{\sigma_i^2}, \quad (16)$$

where $r_{\text{BAO}} = r_s(z_d)/D_V$ and σ_i is the uncertainty in the measurement for each data point ($i = 1, 2, 3, 4$), where the measurements are given at $z_{\text{eff}} = 0.106$, $z_{\text{eff}} = 0.15$, $z_{\text{eff}} = 0.57$ and $z_{\text{eff}} = 0.32$.

- Redshift space distortion data: We also consider the redshift space distortion (RSD) data in our analysis. In particular, we consider the RSD data from the CMASS sample ($z_{\text{eff}} = 0.57$) [54] and the LOWZ sample ($z_{\text{eff}} = 0.32$) [54]. Let us describe the underlying methodology to analyze this dataset for the present analysis. The data-vector that contains the cosmological parameters, namely, $f(z)\sigma_8(z)$, $H(z)r_s(z_d)$ (in 10^3kms^{-1} units) and $D_A(z)/r_s(z_d)$, is given by

$$D_{\text{data}}(z) = \begin{pmatrix} f(z)\sigma_8(z) \\ H(z)r_s(z_d) \\ D_A(z)/r_s(z_d) \end{pmatrix}. \quad (17)$$

The data-vectors for the samples LOWZ and CMASS mentioned above can be formed respectively as (see Table 3 of [54] for a better understanding),

$$D_{\text{data}}(z_{\text{eff}} = 0.32) = \begin{pmatrix} 0.45960 \\ 11.753 \\ 6.7443 \end{pmatrix}, \quad (18)$$

from the LOWZ sample (at $k_{\text{max}} = 0.18 h\text{Mpc}^{-1}$) and

$$D_{\text{data}}(z_{\text{eff}} = 0.57) = \begin{pmatrix} 0.41750 \\ 13.781 \\ 9.3276 \end{pmatrix}, \quad (19)$$

from the CMASS sample (at $k_{\text{max}} = 0.22 h\text{Mpc}^{-1}$). The covariance matrix for the LOWZ sample (at $k_{\text{max}} = 0.18 h\text{Mpc}^{-1}$) is given by [54]

$$C^{\text{LOWZ}} = 10^{-3} \begin{pmatrix} 5.0837 & 23.818 & 10.490 \\ & 300.30 & 73.448 \\ & & 47.493 \end{pmatrix}, \quad (20)$$

while for the CMASS sample (at $k_{\text{max}} = 0.22 h\text{Mpc}^{-1}$) the covariance matrix is [54]

$$C^{\text{CMASS}} = 10^{-3} \begin{pmatrix} 1.3046 & 4.6434 & 3.5329 \\ & 77.713 & 22.773 \\ & & 21.700 \end{pmatrix}. \quad (21)$$

The likelihood for this particular data is given by

$$\mathcal{L} \propto e^{[-(D_{\text{data}} - D_{\text{model}})^T C^{-1} (D_{\text{data}} - D_{\text{model}})]}, \quad (22)$$

where D_{model} represents the vector with the model prediction for the same cosmological parameters as D_{data} ; C^{-1} is the inverse of the covariance matrix under consideration. Let us note that when we use these two RSD data in the analysis, the BOSS DR12 results will not be entertained.

- **Supernovae Type Ia:** The Supernovae Type Ia (SNIa) data are the first observational indicators for an accelerating universe, that means, for the existence of dark energy in the universe sector. In this work we have used the joint light curve analysis (JLA) sample [55], an updated list of SNIa data, distributed over the redshift range $z \in [0.01, 1.30]$ with 740 number of data points. The χ^2 function for this data is given by

$$\chi_{\text{JLA}}^2 = (\hat{\mu} - \hat{\mu}^m)^T C^{-1} (\hat{\mu} - \hat{\mu}^m). \quad (23)$$

where $\hat{\mu}$ denotes the vector of effective absolute magnitudes; C is the covariance matrix of $\hat{\mu}$ that quantifies the statistical and systematic errors, see [55]; and $\hat{\mu}^m(z) = 5 \log_{10} \left(\frac{D_L(z)}{10 \text{pc}} \right)$ is the distance modulus at the redshift z for the model where $D_L(z)$ is the luminosity distance.

- **Cosmic chronometers:** Finally, our analyses also include the Hubble parameter measurements from the most old and passively evolving galaxies, known as cosmic chronometers (CC). The measurements from the cosmic chronometers are believed to be the most potential and model independent measurements and hence, they could able to provide the most important information about the expansion history of the universe. The total number of data points spanned in $0 < z < 2$ are 30 (see Table 4 of [56]), see the details of the data and more discussions in [56]. Finally, we give the χ^2 function for this dataset:

$$\chi_{\text{CC}}^2 = \sum_{i=1}^{30} \frac{(H(z_i) - H_{th}(z_i))^2}{\sigma_i^2}, \quad (24)$$

where z_i is the measured redshift from the observations and σ_i is the corresponding error in the measurements.

Our analysis follows the likelihood $\mathcal{L} \propto \exp(-\chi^2/2)$, where

$$\chi^2 = \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{JLA}}^2 + \chi_{\text{RSD}}^2 + \chi_{\text{CC}}^2. \quad (25)$$

Now, in order to constrain the cosmological parameters, we use the markov chain monte carlo package *cosmomc* [64], an efficient sampling method. The convergence of the MCMC chains are assessed through the Gelman-Rubin statistics $R - 1$ [65]. In addition to that we also include the Bayesian model comparison which provides a goodness of fit of the model under consideration compared to some reference model, usually taken to be the Λ CDM cosmological model.

4.1. Observational Constraints

Here we present the observational constraints of the scalar field models using the following astronomical data.

- CMB (Planck TTTEEE+lowTEB)
- CMB+BAO
- CMB+BAO+JLA
- CMB+RSD+JLA
- CMB+BAO+JLA+RSD+CC

For the above scalar field models we run the *cosmomc* chains of different observational datasets until the convergence of each chain achieves the Gelman-Rubin criteria $R - 1 < 0.1$. In what follows, we separately describe the observational constraints imposed on the scalar field models.

4.1.1. Model 1

As mentioned earlier, for this model we focus on a specific value for u , namely, $u = 1$. The observational summary for this model has been shown in Table I where we present the constraints on the model parameters at 68% and 95% CL. In Fig. 6 we display the 1D marginalized posterior distributions for some selected parameters of the model as well as 2D contour plots considering several combinations of the model parameters. Our analyses clearly show that the parameter β is weakly constrained for all the observational datasets, see Fig. 6 which shows that β does not attain any upper or lower bound. In fact, one can note that the inclusion of any external dataset to CMB ($\beta = 4.320^{+1.553}_{-4.320}$ at 68% CL) does not reduce the error bars on β , rather it is found to increase for the dataset CMB+BAO: $\beta = 4.820^{+5.180}_{-4.820}$ (68% CL). This is true for other three datasets, namely, CMB+RSD+JLA and CMB+BAO+JLA+RSD+CC. But, interestingly, all the datasets allow $\beta = 0$ within 68% CL (see Table I), that means, the constant potential is also allowed by all the observational datasets as well. The constraints on Ω_{m0} are similar to the Planck's Λ CDM based estimation [4] while the estimated values of σ_8 are slightly large compared to the Planck's Λ CDM based estimation [4]. Here, looking at Fig. 6, one can see that a strong negative correlation between Ω_{m0} and H_0 exists, while the remaining parameters shown in Fig. 6 do not have any correlation amongst them. In particular, the parameters H_0 and σ_8 are not correlated. Finally, we can see that for almost all analyses, the Hubble parameter at present H_0 takes similar values to the Λ CDM based Planck's estimation [4]. However, one can crucially notice that the 68% error bars on H_0 for the scalar field model (considering all the data sets) are lower compared to its error from Planck [4]. However, it might be important to mention that the estimated values of H_0 from different observational datasets are not similar to its local estimation by Riess et al. [66], and thus, the tension on H_0 is not released for this quintessence scalar field model.

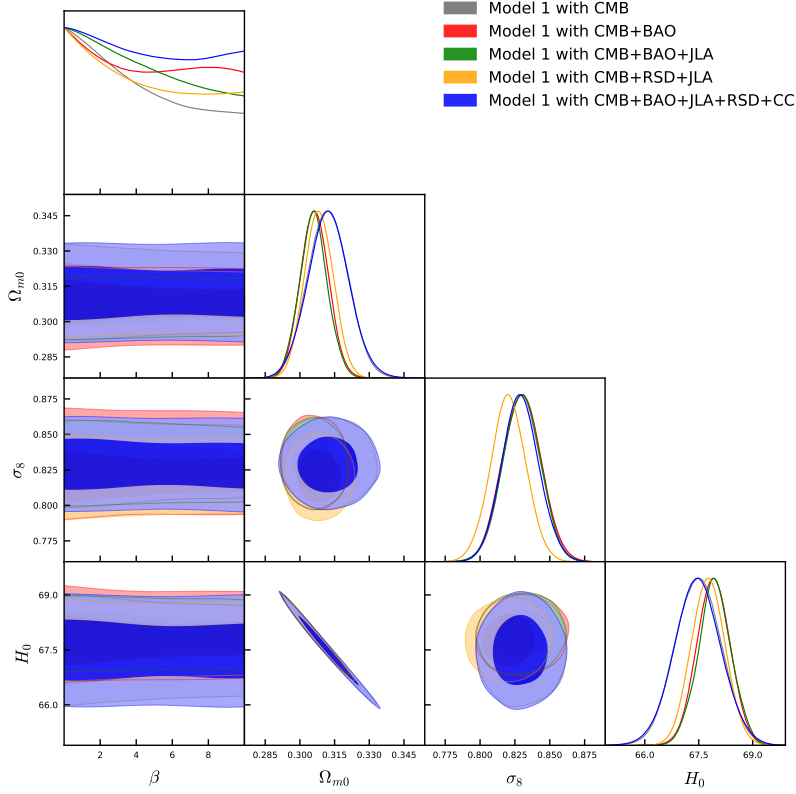


FIG. 6: 1D marginalized posterior distributions for $(\beta, \Omega_{m0}, \sigma_8, H_0)$ and 2D contour plots of several combinations of the above model parameters have been presented for Model 1: $V(\phi) = V_0 \cosh(\beta\phi^u)$ with $u = 1$, using various astronomical datasets CMB, CMB+BAO, CMB+BAO+JLA, CMB+RSD+JLA, CMB+BAO+JLA+RSD+CC.

Parameters	CMB	CB	CBJ	CRJ	CBJRC
$\Omega_c h^2$	$0.1192^{+0.0014+0.0027}_{-0.0014-0.0027}$	$0.1183^{+0.0010+0.0020}_{-0.0010-0.0020}$	$0.1182^{+0.0010+0.0019}_{-0.0010-0.0020}$	$0.1186^{+0.0010+0.0020}_{-0.0010-0.0020}$	$0.1193^{+0.0014+0.0028}_{-0.0014-0.0028}$
$\Omega_b h^2$	$0.02226^{+0.00015+0.00031}_{-0.00015-0.00031}$	$0.02233^{+0.00014+0.00027}_{-0.00014-0.00027}$	$0.02233^{+0.00014+0.00027}_{-0.00014-0.00028}$	$0.02229^{+0.00014+0.00028}_{-0.00014-0.00027}$	$0.02225^{+0.00015+0.00030}_{-0.00016-0.00030}$
$100\theta_{MC}$	$1.04076^{+0.00033+0.00063}_{-0.00032-0.00065}$	$1.04089^{+0.00030+0.00061}_{-0.00030-0.00057}$	$1.04089^{+0.00030+0.00060}_{-0.00030-0.00059}$	$1.04082^{+0.00030+0.00058}_{-0.00030-0.00057}$	$1.04078^{+0.00032+0.00063}_{-0.00033-0.00064}$
τ	$0.081^{+0.017+0.034}_{-0.017-0.033}$	$0.085^{+0.017+0.033}_{-0.017-0.032}$	$0.085^{+0.016+0.031}_{-0.016-0.034}$	$0.072^{+0.016+0.031}_{-0.016-0.031}$	$0.079^{+0.017+0.033}_{-0.017-0.032}$
n_s	$0.9661^{+0.0045+0.0092}_{-0.0045-0.0088}$	$0.9686^{+0.0038+0.0076}_{-0.0038-0.0076}$	$0.9688^{+0.0038+0.0076}_{-0.0038-0.0076}$	$0.9673^{+0.0037+0.0076}_{-0.0037-0.0072}$	$0.9660^{+0.0045+0.0092}_{-0.0047-0.0092}$
$\ln(10^{10} A_s)$	$3.095^{+0.034+0.065}_{-0.034-0.065}$	$3.101^{+0.033+0.066}_{-0.034-0.065}$	$3.101^{+0.032+0.061}_{-0.032-0.066}$	$3.076^{+0.031+0.062}_{-0.031-0.063}$	$3.092^{+0.032+0.065}_{-0.033-0.064}$
β	$4.320^{+1.553+5.680}_{-4.320-4.320}$	$4.820^{+5.180+5.180}_{-4.820-4.820}$	$4.522^{+1.677+5.478}_{-4.522-4.522}$	$4.580^{+5.420+5.420}_{-4.580-4.580}$	$4.843^{+5.157+5.157}_{-4.843-4.843}$
Ω_{m0}	$0.312^{+0.009+0.017}_{-0.009-0.016}$	$0.306^{+0.006+0.012}_{-0.006-0.012}$	$0.306^{+0.006+0.012}_{-0.006-0.012}$	$0.308^{+0.006+0.012}_{-0.006-0.012}$	$0.312^{+0.009+0.018}_{-0.009-0.017}$
σ_8	$0.830^{+0.014+0.026}_{-0.013-0.026}$	$0.830^{+0.014+0.027}_{-0.014-0.026}$	$0.830^{+0.013+0.025}_{-0.013-0.025}$	$0.820^{+0.013+0.025}_{-0.012-0.025}$	$0.829^{+0.013+0.026}_{-0.013-0.025}$
H_0	$67.48^{+0.63+1.22}_{-0.63-1.21}$	$67.91^{+0.46+0.89}_{-0.46-0.89}$	$67.94^{+0.46+0.90}_{-0.44-0.90}$	$67.76^{+0.47+0.93}_{-0.47-0.90}$	$67.47^{+0.64+1.28}_{-0.63-1.26}$

TABLE I: Summary of the 68% and 95% CL constraints on various model parameters of the scalar field model for Model 1: $V(\phi) = V_0 \cosh(\beta\phi^u)$ with $u = 1$ using different observational datasets. We note that Ω_{m0} is the present value of $\Omega_m = \Omega_r + \Omega_b$ and H_0 is in the units of km/s/Mpc. Here, we have shortened the notations as follows: CB = CMB+BAO, CBJ = CMB+BAO+JLA, CRJ = CMB+RSD+JLA, and CBJRC = CMB+BAO+JLA+RSD+CC.

4.1.2. Model 2

In order to extract the cosmological constraints of the scalar field model (13), we have separately analyzed two cases, one with $\epsilon = 1$ (Model 2a) and the other with $\epsilon = -1$ (Model 2b).

Let us now summarize the observational constraints for Model 2a, that means for the scalar field model: $V(\phi) = V_0[1 + \text{sech}(\alpha\phi)]$. In Table II we have summarized the observational constraints on various free parameters of this model for different astronomical datasets at 68% CL and 95% CL. Additionally, in Fig. 7, we present the 1D marginalized posterior distributions for some selected free parameters and 2D contour plots at 68% and 95% CL considering various combinations of the free parameters. Let us first concentrate on the free parameter α that quantifies the deviation of the potential from its constant value. From Table II, one can clearly see that the error bars on α are almost similar to its mean values obtained from different combinations of the observational datasets. For instance, for CMB alone, $\alpha = 4.770^{+5.230}_{-4.770}$ at 68% CL. For CMB+BAO, we can see that the error bars on α are similar to its mean value both at 68% lower and 68% upper CL ($\alpha = 4.608^{+5.392}_{-4.608}$), that means, when BAO data are added to CMB, we do not find any improvement of α . But in contrary to CMB and CMB+BAO, for the remaining three datasets, namely, CMB+BAO+JLA, CMB+RSD+JLA and CMB+BAO+JLA+RSD+CC, the error bars on α are reduced at 68% upper CL, but remain same for 68% lower CL. One can see that $\alpha = 3.845^{+1.361}_{-3.845}$ (68% CL, CMB+BAO+JLA), $\alpha = 3.896^{+1.626}_{-3.896}$ (68%, CMB+RSD+JLA), and $\alpha = 4.107^{+1.509}_{-4.107}$ (68% CL, CMB+BAO+JLA+RSD+CC). So, we see that the inclusion of JLA to CMB+BAO became successful to decrease the one sided error bar on α . Similarly, the inclusion of RSD+JLA to CMB also works well to decrease the one sided error bar on α and this remains valid for the final combination CMB+BAO+JLA+RSD+CC. We further add that the addition of any external dataset to CMB reduces the mean value of α , but from Fig. 7, one can also see that α does not attain any lower and upper bound, so this parameter is very weakly constrained by the observational data employed in the work. However, strikingly, one may notice that for all the datasets, $\alpha = 0$ is marginally accepted, that means, the constant potential is not excluded from the picture. We finally note that in all analyses, the Hubble parameter takes almost similar values as obtained in the Λ CDM based Planck's estimation, see [4] that means the tension on H_0 is not released by this scalar field model. Similar conclusion has already been found for Model 1.

We now discuss the observational constraints of Model 2b: $V(\phi) = V_0[1 - \text{sech}(\alpha\phi)]$. The observational summary for this model has been displayed in Table III where we have shown 68% and 95% CL constraints on each free parameter. And in Fig. 8, we show the 1D marginalized posterior distributions for some selected free parameters and 2D contour plots at 68% and 95% CL considering various combinations of the free parameters. From the analysis summarized in Table III, we notice that this model returns almost similar fit to Model 2a, and in a similar fashion, the parameter α is again found to be weakly constrained. And interestingly, for all the observational data set, we see that $\alpha = 0$ is allowed within 68% CL which means that within this confidence-level, the potential mimics the constant potential $V = V_0$. Similar to the previous case (i.e., the potential (13) with $\epsilon = +1$), here too, we observe that the estimated values of the Hubble constant for all the combinations are very close to the Λ CDM based Planck's estimation [4]. So, both the models, as one can see, cannot solve the H_0 -tension. This agrees with two earlier conclusions, namely for Model 1 and Model 2a.

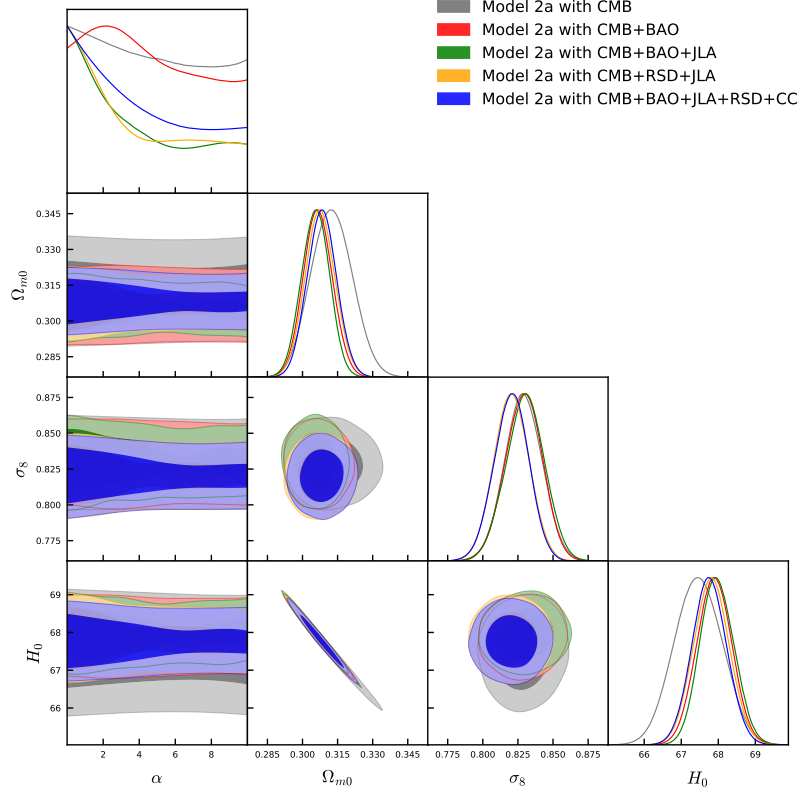


FIG. 7: 1D marginalized posterior distributions for $(\alpha, \Omega_{m0}, \sigma_8, H_0)$ and 2D contour plots of several combinations of the above model parameters have been presented for Model 2a: $V(\phi) = V_0 [1 + \text{sech}(\alpha\phi)]$ ($\alpha \geq 0$), using various astronomical datasets CMB, CMB+BAO, CMB+BAO+JLA, CMB+RSD+JLA, CMB+BAO+JLA+RSD+CC.

Parameters	CMB	CB	CBJ	CRJ	CBJRC
$\Omega_c h^2$	$0.1193^{+0.0014+0.0028}_{-0.0014-0.0027}$	$0.11832720^{+0.0010+0.0020}_{-0.0010-0.0020}$	$0.1182^{+0.0010+0.0019}_{-0.0010-0.0019}$	$0.1185^{+0.0011+0.0021}_{-0.0010-0.0021}$	$0.1186^{+0.0010+0.0020}_{-0.0010-0.0020}$
$\Omega_b h^2$	$0.02225^{+0.00016+0.00031}_{-0.00016-0.00030}$	$0.02232^{+0.00014+0.00027}_{-0.00015-0.00026}$	$0.02233^{+0.00014+0.00028}_{-0.00014-0.00027}$	$0.02230^{+0.00014+0.00027}_{-0.00014-0.00027}$	$0.02229^{+0.00014+0.00027}_{-0.00014-0.00027}$
$100\theta_{MC}$	$1.04076^{+0.00032+0.00065}_{-0.00033-0.00065}$	$1.04089^{+0.00031+0.00058}_{-0.00030-0.00060}$	$1.04090^{+0.00030+0.00059}_{-0.00030-0.00059}$	$1.04085^{+0.00030+0.00058}_{-0.00030-0.00060}$	$1.04084^{+0.00031+0.00059}_{-0.00031-0.00061}$
n_s	$0.9660^{+0.0046+0.0089}_{-0.0046-0.0089}$	$0.9685^{+0.0039+0.0080}_{-0.0040-0.0077}$	$0.9691^{+0.0038+0.0076}_{-0.0041-0.0074}$	$0.9677^{+0.0038+0.0074}_{-0.0039-0.0077}$	$0.9673^{+0.0037+0.0074}_{-0.0037-0.0073}$
τ	$0.079^{+0.017+0.034}_{-0.017-0.033}$	$0.0838^{+0.0162+0.0307}_{-0.0164-0.0324}$	$0.086^{+0.018+0.032}_{-0.016-0.033}$	$0.073^{+0.015+0.030}_{-0.015-0.030}$	$0.072^{+0.015+0.029}_{-0.015-0.030}$
$\ln(10^{10} A_s)$	$3.092^{+0.034+0.065}_{-0.033-0.064}$	$3.0986^{+0.0322+0.0609}_{-0.0319-0.0636}$	$3.102^{+0.036+0.063}_{-0.032-0.065}$	$3.077^{+0.031+0.059}_{-0.030-0.060}$	$3.076^{+0.030+0.058}_{-0.030-0.060}$
α	$4.770^{+5.230+5.230}_{-4.770-4.770}$	$4.608^{+5.392+5.392}_{-4.608-4.608}$	$3.845^{+1.361+6.155}_{-3.845-3.845}$	$3.896^{+1.626+6.104}_{-3.896-3.896}$	$4.107^{+1.509+5.893}_{-4.107-4.107}$
Ω_{m0}	$0.313^{+0.009+0.017}_{-0.009-0.016}$	$0.307^{+0.006+0.012}_{-0.006-0.012}$	$0.306^{+0.006+0.011}_{-0.006-0.012}$	$0.308^{+0.006+0.013}_{-0.006-0.013}$	$0.308^{+0.006+0.012}_{-0.006-0.012}$
σ_8	$0.829^{+0.013+0.026}_{-0.013-0.026}$	$0.829^{+0.013+0.025}_{-0.013-0.026}$	$0.830^{+0.014+0.026}_{-0.014-0.026}$	$0.820^{+0.012+0.025}_{-0.012-0.024}$	$0.820^{+0.012+0.024}_{-0.012-0.024}$
H_0	$67.46^{+0.65+1.23}_{-0.64-1.22}$	$67.89^{+0.46+0.92}_{-0.47-0.91}$	$67.97^{+0.44+0.90}_{-0.49-0.86}$	$67.81^{+0.48+0.94}_{-0.48-0.93}$	$67.76^{+0.46+0.92}_{-0.46-0.90}$

TABLE II: Summary of the 68% and 95% CL constraints on various model parameters of Model 2a: $V(\phi) = V_0 [1 + \text{sech}(\alpha\phi)]$ ($\alpha \geq 0$) using different observational datasets. We note that Ω_{m0} is the present value of $\Omega_m = \Omega_r + \Omega_b$ and H_0 is in the units of km/s/Mpc. Here, we have shortened the notations as follows: CB = CMB+BAO, CBJ = CMB+BAO+JLA, CRJ = CMB+RSD+JLA, and CBJRC = CMB+BAO+JLA+RSD+CC.

4.1.3. Model 3

Let us now present the observational constraints for the scalar field model (14). We have analyzed the potential (14) for two different regions of the parameter δ , namely for $\delta \geq 0$ (identified as Model 3a) and for $\delta \leq 0$ (identified as Model 3b). In both the cases, we have analyzed the model for a fixed value of p , namely, $p = 1$ and we further note that the parameter δ quantifies the deviation of the potential from its constant value $V = V_0$. In what follows we describe the observational constraints for each model separately.

Table IV summarizes the observational constraints for Model 3a (i.e. when $\delta \geq 0$ in eqn. (14)) presented at 68% and 95% CL using different observational datasets. Similarly, in Fig. 9 we display the 1D marginalized posterior

Parameters	CMB	CB	CBJ	CRJ	CBJRC
$\Omega_c h^2$	$0.1193^{+0.0013+0.0026}_{-0.0014-0.0027}$	$0.1183^{+0.0010+0.0019}_{-0.0010-0.0020}$	$0.1182^{+0.0010+0.0020}_{-0.0010-0.0019}$	$0.1184^{+0.0011+0.0021}_{-0.0010-0.0020}$	$0.1186^{+0.0010+0.0020}_{-0.0011-0.0020}$
$\Omega_b h^2$	$0.02225^{+0.00015+0.00031}_{-0.00015-0.00030}$	$0.02232^{+0.00014+0.00027}_{-0.00014-0.00028}$	$0.02233^{+0.00014+0.00028}_{-0.00014-0.00027}$	$0.02231^{+0.00014+0.00028}_{-0.00015-0.00028}$	$0.02229^{+0.00014+0.00027}_{-0.00014-0.00028}$
$100\theta_{MC}$	$1.04077^{+0.00031+0.00063}_{-0.00032-0.00063}$	$1.04090^{+0.00030+0.00059}_{-0.00030-0.00059}$	$1.04089^{+0.00030+0.00057}_{-0.00030-0.00060}$	$1.04086^{+0.00029+0.00058}_{-0.00029-0.00060}$	$1.04083^{+0.00030+0.00060}_{-0.00030-0.00059}$
n_s	$0.9660^{+0.0044+0.0087}_{-0.0044-0.0088}$	$0.9686^{+0.0037+0.0074}_{-0.0037-0.0075}$	$0.9688^{+0.0040+0.0075}_{-0.0038-0.0076}$	$0.9678^{+0.0038+0.0076}_{-0.0039-0.0074}$	$0.9672^{+0.0038+0.0074}_{-0.0037-0.0074}$
τ	$0.080^{+0.017+0.032}_{-0.017-0.032}$	$0.085^{+0.016+0.031}_{-0.016-0.032}$	$0.085^{+0.017+0.032}_{-0.017-0.032}$	$0.074^{+0.015+0.031}_{-0.016-0.030}$	$0.073^{+0.016+0.031}_{-0.016-0.031}$
$\ln(10^{10} A_s)$	$3.092^{+0.032+0.063}_{-0.033-0.063}$	$3.101^{+0.032+0.062}_{-0.032-0.065}$	$3.1008^{+0.0324+0.0636}_{-0.0324-0.0645}$	$3.077^{+0.030+0.060}_{-0.030-0.059}$	$3.076^{+0.031+0.062}_{-0.031-0.062}$
α	$4.758^{+5.242+5.242}_{-4.758-4.758}$	$4.697^{+5.303+5.303}_{-4.697-4.697}$	$4.048^{+1.482+5.952}_{-4.048-4.048}$	$4.631^{+5.369+5.369}_{-4.631-4.631}$	$4.346^{+1.765+5.654}_{-4.346-4.346}$
Ω_{m0}	$0.312^{+0.008+0.017}_{-0.008-0.016}$	$0.306^{+0.006+0.012}_{-0.006-0.012}$	$0.306^{+0.006+0.012}_{-0.006-0.011}$	$0.307^{+0.006+0.013}_{-0.006-0.012}$	$0.308^{+0.006+0.012}_{-0.006-0.012}$
σ_8	$0.829^{+0.013+0.026}_{-0.013-0.026}$	$0.830^{+0.013+0.026}_{-0.013-0.026}$	$0.830^{+0.013+0.026}_{-0.013-0.026}$	$0.820^{+0.012+0.024}_{-0.012-0.024}$	$0.820^{+0.012+0.025}_{-0.013-0.025}$
H_0	$67.46^{+0.60+1.21}_{-0.59-1.21}$	$67.91^{+0.45+0.91}_{-0.45-0.86}$	$67.93^{+0.45+0.89}_{-0.45-0.91}$	$67.86^{+0.48+0.94}_{-0.48-0.94}$	$67.77^{+0.47+0.94}_{-0.48-0.89}$

TABLE III: Summary of the 68% and 95% CL constraints on various model parameters of the scalar field model with potential $V(\phi) = V_0 [1 - \text{sech}(\alpha\phi)]$ ($\alpha \geq 0$) using different observational datasets. We note that Ω_{m0} is the present value of $\Omega_m = \Omega_r + \Omega_b$ and H_0 is in the units of km/s/Mpc. Here, we have shortened the notations as follows: CB = CMB+BAO, CBJ = CMB+BAO+JLA, CRJ = CMB+RSD+JLA, and CBJRC = CMB+BAO+JLA+RSD+CC.

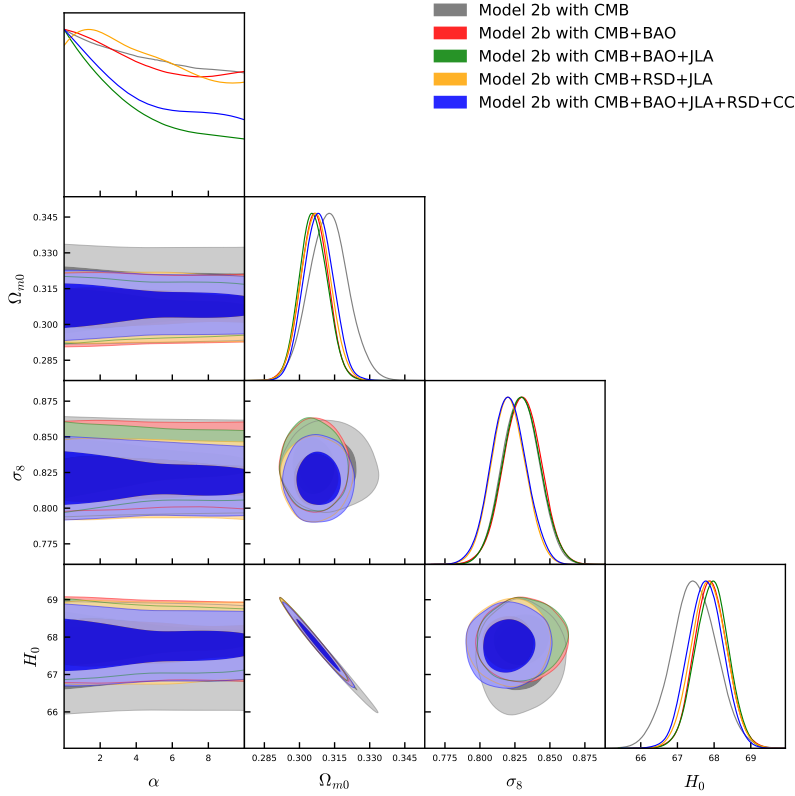


FIG. 8: 1D marginalized posterior distributions for $(\alpha, \Omega_{m0}, \sigma_8, H_0)$ and 2D contour plots of several combinations of the above model parameters have been presented for Model 2b using various astronomical datasets CMB, CMB+BAO, CMB+BAO+JLA, CMB+RSD+JLA, CMB+BAO+JLA+RSD+CC.

distributions for some selected parameters of this model plus the 2D contour plots at 68% and 95% CL considering various combinations of the model parameters. Now, following the similar trends as in Model 1 and Model 2, we focus on the observational bounds on δ that quantifies this model from the constant potential $V = V_0$. From the Table IV we conclude that the parameter δ is weakly constrained. As one can see that the mean values and the error bars of δ are almost similar, for instance, one can see the constraints from CMB alone, $\delta = 4.843^{+5.157}_{-4.843}$ (68% CL). The inclusion of BAO to CMB slightly improves the parameters space by lowering the one sided error bars on $\delta = 4.630^{+1.819}_{-4.630}$ (68% CL), but for other datasets, namely, CMB+BAO+JLA, CMB+RSD+JLA and CMB+BAO+JLA+RSD+CC, we find similar constraints on δ as obtained for the CMB data alone. The evidence for δ to be weakly constrained is clear from the 2D contour plots of (δ, H_0) , (δ, σ_8) , (δ, Ω_{m0}) in Fig. 9 showing no upper or lower bound of δ parameter. This

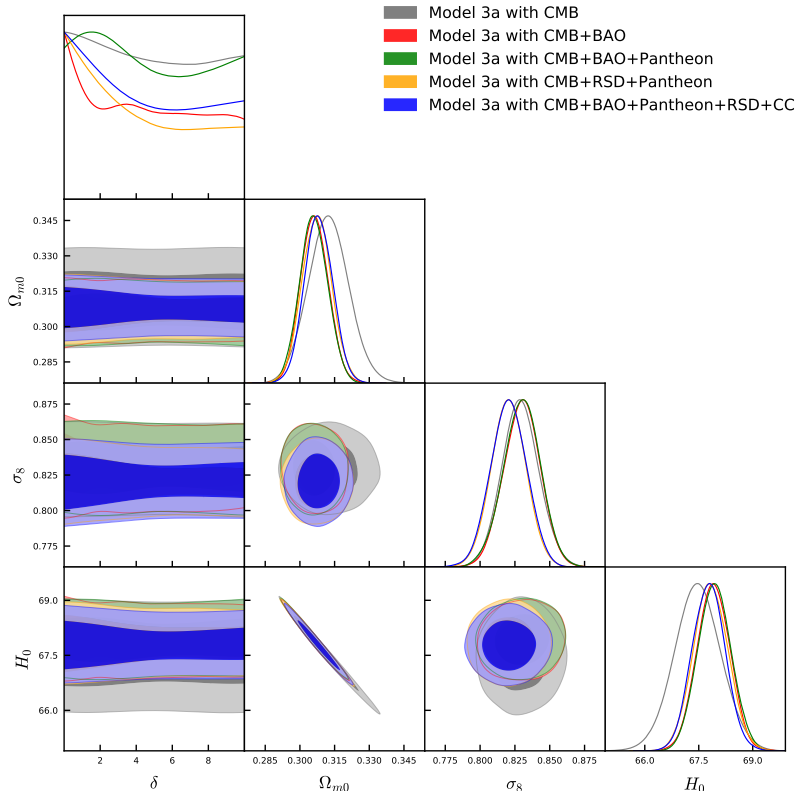


FIG. 9: 1D marginalized posterior distributions for $(\delta, \Omega_{m0}, \sigma_8, H_0)$ and 2D contour plots of several combinations of the above model parameters have been presented for Model 3a using various astronomical datasets CMB, CMB+BAO, CMB+BAO+JLA, CMB+RSD+JLA, CMB+BAO+JLA+RSD+CC.

is also clear from the 1D marginalized posterior distributions shown in Fig. 9. However, one may note that $\delta = 0$ is always allowed by all the datasets within the 68% CL, and thus, the constant potential is allowed too. We also note that for some combinations, namely, CMB+BAO and CMB+BAO+JLA, σ_8 assumes higher values in compared to the Λ CDM based Planck's estimation [4]. Concerning the estimated values of H_0 from different data, we again come up with the same conclusion as found in previous scalar field models, that means, the tension in H_0 still exists.

We perform similar analyses for Model 3b (i.e., when $\delta \leq 0$ in eqn. (14)) and summarize the results in Table V. Also, in Fig. 10 we show the 1D marginalized posterior distributions for some selected parameters as well as the 2D contour plots for some combinations of the model parameters. From the analyses, we again find that, δ is weakly constrained for all the observational data (see Fig. 10) and interestingly, $\delta = 0$ is allowed within 68% CL which means that within 68% CL, the constant potential is still allowed by the data. But, here we would like to point out one more thing that perhaps will not so interesting but also cannot be ruled out too. For all the analyses performed here, we see that for no single analysis, the error bars on δ decrease unlike to Model 3a where the dataset CMB+BAO decreased the error bars on δ . Finally, we also note that the conclusion on H_0 as already observed in other scalar field models, remains same for this model too.

4.2. Geometrical test: The Om diagnostic

According to the current literature, a large number of cosmological models are introduced to understand the late-time accelerated phase of the universe. Amongst them, sometimes from the statistical ground, finding the differences between the cosmological models becomes very difficult and not sound. Thus, a geometrical way that enables us to distinguish the cosmological models is welcome and very appealing. One of such geometrical tests that we are

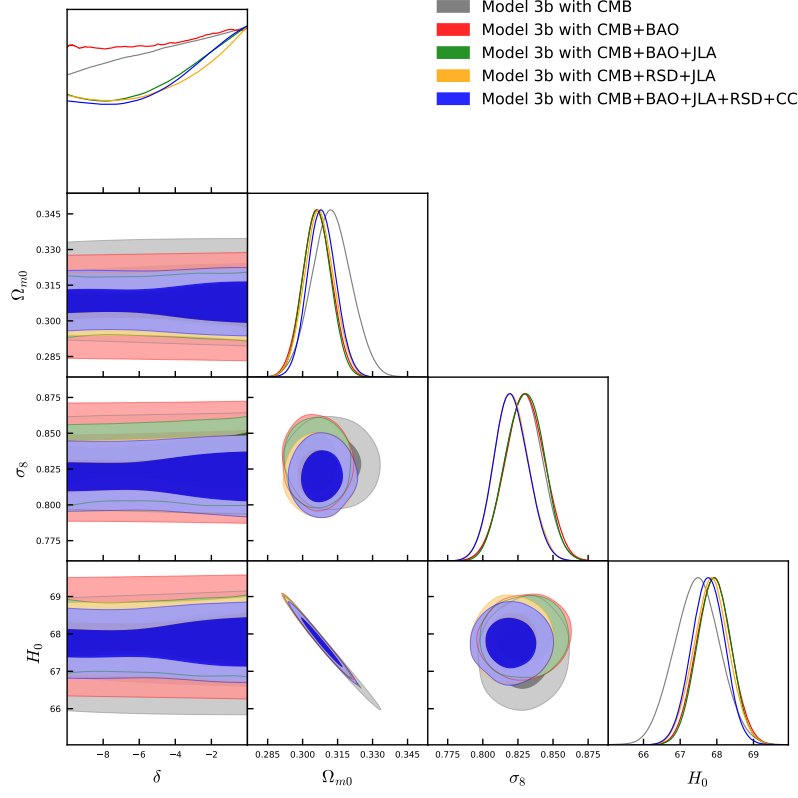


FIG. 10: 1D marginalized posterior distributions for $(\delta, \Omega_{m0}, \sigma_8, H_0)$ and 2D contour plots of several combinations of the above model parameters have been presented for Model 3b using various astronomical datasets CMB, CMB+BAO, CMB+BAO+JLA, CMB+RSD+JLA, CMB+BAO+JLA+RSD+CC.

Parameters	CMB	CB	CBJ	CRJ	CBJRC
$\Omega_c h^2$	$0.1193^{+0.0014+0.0028}_{-0.0014-0.0028}$	$0.1183^{+0.0010+0.0020}_{-0.0010-0.0019}$	$0.1182^{+0.0010+0.0020}_{-0.0010-0.0020}$	$0.1184^{+0.0010+0.0021}_{-0.0011-0.0021}$	$0.1185^{+0.0010+0.0020}_{-0.0010-0.0020}$
$\Omega_b h^2$	$0.02225^{+0.00015+0.00030}_{-0.00016-0.00030}$	$0.02231877^{+0.00014+0.00027}_{-0.00014-0.00027}$	$0.02233^{+0.00014+0.00027}_{-0.00015-0.00026}$	$0.02230^{+0.00013+0.00027}_{-0.00014-0.00027}$	$0.02229^{+0.00013+0.00027}_{-0.00013-0.00027}$
$100\theta_{MC}$	$1.04078^{+0.00032+0.00063}_{-0.00033-0.00064}$	$1.04089^{+0.00031+0.00058}_{-0.00030-0.00059}$	$1.04091^{+0.00030+0.00058}_{-0.00030-0.00061}$	$1.04085^{+0.00031+0.00060}_{-0.00031-0.00059}$	$1.04085^{+0.00031+0.00058}_{-0.00030-0.00059}$
τ	$0.079^{+0.017+0.033}_{-0.017-0.032}$	$0.085^{+0.017+0.031}_{-0.016-0.033}$	$0.085^{+0.017+0.032}_{-0.016-0.032}$	$0.073^{+0.015+0.030}_{-0.015-0.031}$	$0.073^{+0.016+0.030}_{-0.016-0.031}$
n_s	$0.9660^{+0.0045+0.0092}_{-0.0047-0.0092}$	$0.9686^{+0.0038+0.0073}_{-0.0038-0.0075}$	$0.9688^{+0.0039+0.0076}_{-0.0039-0.0079}$	$0.9679^{+0.0040+0.0075}_{-0.0039-0.0075}$	$0.9674^{+0.0037+0.0074}_{-0.0038-0.0073}$
$\ln(10^{10} A_s)$	$3.092^{+0.032+0.065}_{-0.033-0.064}$	$3.101^{+0.034+0.061}_{-0.031-0.064}$	$3.101^{+0.032+0.063}_{-0.032-0.065}$	$3.077^{+0.030}_{-0.030-0.061}$	$3.077^{+0.032+0.061}_{-0.031-0.062}$
δ	$4.843^{+5.157+5.157}_{-4.843-4.843}$	$4.630^{+1.819+5.370}_{-4.630-4.630}$	$4.764^{+5.236+5.236}_{-4.764-4.764}$	$4.242^{+5.758+5.758}_{-4.242-4.242}$	$4.516^{+5.484+5.484}_{-4.516-4.516}$
Ω_{m0}	$0.312^{+0.009+0.018}_{-0.009-0.017}$	$0.306^{+0.006+0.012}_{-0.006-0.011}$	$0.306^{+0.006+0.012}_{-0.006-0.012}$	$0.307^{+0.006+0.013}_{-0.006-0.012}$	$0.308^{+0.006+0.012}_{-0.006-0.012}$
σ_8	$0.829^{+0.013+0.026}_{-0.013-0.025}$	$0.830^{+0.013+0.025}_{-0.013-0.025}$	$0.830^{+0.013+0.025}_{-0.013-0.027}$	$0.821^{+0.012+0.024}_{-0.012-0.024}$	$0.821^{+0.013+0.025}_{-0.013-0.025}$
H_0	$67.47^{+0.64+1.28}_{-0.63-1.26}$	$67.91^{+0.45+0.87}_{-0.46-0.87}$	$67.94^{+0.46+0.91}_{-0.46-0.88}$	$67.83^{+0.48+0.96}_{-0.47-0.94}$	$67.78^{+0.45+0.88}_{-0.46-0.87}$

TABLE IV: Summary of the 68% and 95% CL constraints on various model parameters of Model 3a: $V(\phi) = V_0 \left[1 + \delta \left(\frac{\phi}{M_P}\right)^p\right]^2$ (where $\delta \geq 0$) with $p = 1$, using different observational datasets. We note that Ω_{m0} is the present value of $\Omega_m = \Omega_r + \Omega_b$ and H_0 is in the units of km/s/Mpc. Here, we have shortened the notations as follows: CB = CMB+BAO, CBJ = CMB+BAO+JLA, CRJ = CMB+RSD+JLA, and CBJRC = CMB+BAO+JLA+RSD+CC.

interested in this work is the Om diagnostic [67, 68]. This diagnostic is also considered to be the simplest one in compared to the statefinder and cosmographic parameters since the only geometrical parameter involved in this method is the Hubble parameter, in other words, only the first order derivative of the cosmic time appears, while the cosmographic constructions involve the higher order derivatives of the cosmic time. We refer to some works on cosmographic parameters used to distinguish the dark energy models in the literature [69, 70, 71, 72, 73, 74]. We begin our analysis with the definition of Om as follows [67, 68]

Parameters	CMB	CB	CBJ	CRJ	CBJRC
$\Omega_c h^2$	$0.1193^{+0.0014+0.0026}_{-0.0013-0.0027}$	$0.1183^{+0.0010+0.0021}_{-0.0010-0.0021}$	$0.1183^{+0.0010+0.0019}_{-0.0010-0.0019}$	$0.1184^{+0.0011+0.0021}_{-0.0011-0.0021}$	$0.1186^{+0.0010+0.0020}_{-0.0010-0.0019}$
$\Omega_b h^2$	$0.02225^{+0.00015+0.00030}_{-0.00016-0.00030}$	$0.02232^{+0.00014+0.00028}_{-0.00014-0.00027}$	$0.02232^{+0.00014+0.00027}_{-0.00014-0.00028}$	$0.02231^{+0.00014+0.00028}_{-0.00014-0.00028}$	$0.02229^{+0.00013+0.00026}_{-0.00013-0.00026}$
$100\theta_{MC}$	$1.04077^{+0.00033+0.00066}_{-0.00033-0.00066}$	$1.04089^{+0.00031+0.00060}_{-0.00030-0.00061}$	$1.04091^{+0.00030+0.00060}_{-0.00030-0.00060}$	$1.04086^{+0.00031+0.00060}_{-0.00030-0.00060}$	$1.04084^{+0.00030+0.00058}_{-0.00030-0.00059}$
τ	$0.080^{+0.017+0.033}_{-0.017-0.033}$	$0.085^{+0.017+0.033}_{-0.017-0.034}$	$0.085^{+0.017+0.031}_{-0.016-0.032}$	$0.073^{+0.015+0.031}_{-0.015-0.030}$	$0.072^{+0.015+0.030}_{-0.016-0.029}$
n_s	$0.9661^{+0.0045+0.0088}_{-0.0045-0.0087}$	$0.9688^{+0.0039+0.0077}_{-0.0039-0.0074}$	$0.9686^{+0.0039+0.0076}_{-0.0039-0.0074}$	$0.9679^{+0.0039+0.0076}_{-0.0039-0.0074}$	$0.9673^{+0.0038+0.0073}_{-0.0038-0.0074}$
$\ln(10^{10} A_s)$	$3.092^{+0.034+0.065}_{-0.034-0.066}$	$3.101^{+0.034+0.065}_{-0.034-0.067}$	$3.101^{+0.035+0.060}_{-0.032-0.063}$	$3.077^{+0.030+0.060}_{-0.030-0.058}$	$3.076^{+0.030+0.059}_{-0.029-0.057}$
δ	$-4.728^{+4.728+4.728}_{-1.687-5.272}$	$-4.878^{+4.878+4.878}_{-5.122-5.122}$	$-4.452^{+4.452+4.452}_{-5.548-5.548}$	$-4.498^{+4.498+4.498}_{-5.502-5.502}$	$-4.390^{+4.390+4.390}_{-1.674-5.610}$
Ω_{m0}	$0.312^{+0.008+0.017}_{-0.008-0.016}$	$0.306^{+0.006+0.013}_{-0.006-0.012}$	$0.306^{+0.006+0.012}_{-0.006-0.011}$	$0.307^{+0.006+0.013}_{-0.006-0.012}$	$0.308^{+0.006+0.012}_{-0.006-0.012}$
σ_8	$0.829^{+0.013+0.026}_{-0.013-0.026}$	$0.830^{+0.014+0.027}_{-0.014-0.027}$	$0.830^{+0.014+0.025}_{-0.013-0.026}$	$0.820^{+0.012+0.024}_{-0.013-0.023}$	$0.820^{+0.012+0.024}_{-0.013-0.023}$
H_0	$67.47^{+0.61+1.23}_{-0.62-1.20}$	$67.92^{+0.47+0.94}_{-0.47-0.92}$	$67.92^{+0.45+0.89}_{-0.46-0.89}$	$67.85^{+0.48+0.96}_{-0.48-0.95}$	$67.76^{+0.45+0.89}_{-0.45-0.88}$

TABLE V: Summary of the 68% and 95% CL constraints on various model parameters of Model 3b: $V(\phi) = V_0 \left[1 + \delta \left(\frac{\phi}{M_P}\right)^p\right]^2$ (where $\delta \leq 0$) with $p = 1$, using different observational datasets. We note that Ω_{m0} is the present value of $\Omega_m = \Omega_r + \Omega_b$ and H_0 is in the units of km/s/Mpc. Here, we have shortened the notations as follows: CB = CMB+BAO, CBJ = CMB+BAO+JLA, CRJ = CMB+RSD+JLA, and CBJRC = CMB+BAO+JLA+RSD+CC.

$$Om(z) = \frac{E(z)^2 - 1}{(1+z)^3 - 1} \quad (26)$$

where $E(z) = H(z)/H_0$. Looking at eqn. (26), One can easily see that in a spatially flat Λ CDM driven universe, where $E(z)^2 = \Omega_{m0}(1+z)^3 + (1-\Omega_{m0})$, the definition for $Om(z)$ returns, $Om(z) = \Omega_{m0}$, that means, for the Λ CDM universe, $Om(z)$ is time independent. Conversely, if for any cosmological model, we are given that, $Om(z) = \Omega_{m0}$, during the evolution of the universe, then using equation (26), one may conclude that, the model is basically Λ CDM where the background is described by the usual flat FLRW universe, so mathematically, one can say that, $Om(z) = \Omega_{m0}$, *iff* the model is the Λ CDM. This statement actually works to distinguish the cosmological models from the Λ CDM and it has been used extensively in the literature. Thus, for any cosmological model, any deviation of $Om(z)$ from Ω_{m0} actually signals the difference of the toy model from the base Λ CDM cosmological model. In addition to that, the Om diagnostic also offers a characterization of the dark energy models whether they belong to the quintessence class or the phantom class [75, 76, 77, 78]. Now, in order to understand the evolutions of $Om(z)$ for the scalar field models, we have numerically solved the Hubble function for each scalar field model and present the variations of $Om(z)$ in Fig. 11. From Fig. 11, one can easily find that the models actually are very close to each other and at large redshifts, the evolution of $Om(z)$ for the scalar field models is same to that of the Λ CDM model (represented by the solid horizontal line), while for $z \lesssim 2$, the deviation of $Om(z)$ for each scalar field model compared to the Λ CDM are clearly pronounced.

4.3. Bayesian Evidence: A statistical tool for model comparison

In this section we present a statistical comparison of the scalar field models using the Bayesian analysis, an analysis that quantifies the support of the cosmological model with the observational data. The analysis needs a reference model with respect to which the comparison should be made, and without any doubt, Λ CDM model will be the best choice based on its performance with the observational evidences. Let us first describe how the Bayesian evidence is calculated. In the Bayesian analysis we need the posterior probability of the model parameters (denoted by the symbol θ) of any preassigned model M , given a particular data set x which is employed to analyze the model, and any prior information. Now, recalling the Bayes theorem, one may easily write:

$$p(\theta|x, M) = \frac{p(x|\theta, M) \pi(\theta|M)}{p(x|M)} \quad (27)$$

where $p(x|\theta, M)$ is the likelihood function that entirely depends on the model parameters (θ) with the fixed data set (x); and $\pi(\theta|M)$ is the prior information that is supplied during the analysis. The quantity $p(x|M)$ placed in the denominator in the right hand side of eqn. (27) is the Bayesian evidence for the model comparison and this is nothing but the integral over the unnormalised posterior $\tilde{p}(\theta|x, M) \equiv p(x|\theta, M) \pi(\theta|M)$ taking the following expression

$$E \equiv p(x|M) = \int d\theta p(x|\theta, M) \pi(\theta|M), \quad (28)$$

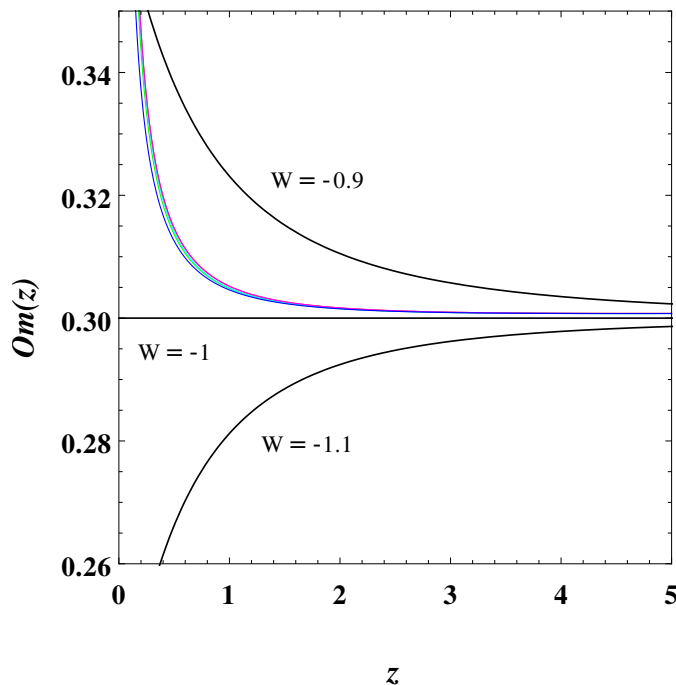


FIG. 11: Qualitative evolution of the $Om(z)$ function for the scalar field models (12), (13) and (14). The different trajectories of $Om(z)$ correspond to Model 1 with $\beta = 2$ and $u = 1$ (red curve), Model 2 with $\alpha = 2$ (green curve for $\epsilon = +1$ and cyan curve for $\epsilon = -1$), and Model 3 with $p = 2$ (magenta curve for $\delta = +1$ and blue curve for $\delta = -1$). We note that the curves (red, green, cyan, magenta, and blue) describing the $Om(z)$ function for different scalar field models are very close to each other and hence it is very hard to distinguish from one another. The horizontal solid black line depicts the $Om(z)$ function (which is constant as shown in the text) for the spatially flat Λ CDM model. The remaining black lines exhibit the evolution of $Om(z)$ for the dark energy models with constant equation of state $w = -0.9, -1$ and -1.1 , respectively from the top to bottom. The dark energy models having $w > -1$ (quintessence) show negative curvature whereas the model with $w < -1$ (phantom) designates positive curvature which are generic features of quintessence and phantom models, respectively.

which is also referred to as the marginal likelihood. Now, for any particular model M_i and the reference model M_j (this is the Λ CDM model under consideration), the posterior probability is given by

$$\frac{p(M_i|x)}{p(M_j|x)} = \frac{\pi(M_i)}{\pi(M_j)} \frac{p(x|M_i)}{p(x|M_j)} = \frac{\pi(M_i)}{\pi(M_j)} B_{ij}. \quad (29)$$

where the quantity $B_{ij} = \frac{p(x|M_i)}{p(x|M_j)}$, is the Bayes factor of the model M_i relative to the reference model M_j (here it is Λ CDM). This factor essentially tells us how the observational data support the cosmological model under consideration. For $B_{ij} > 1$, we say that the observational data support the model M_i more strongly than the model M_j . For different values of B_{ij} (or alternatively, $\ln B_{ij}$) we quantify the models. The quantification is generally adopts the widely accepted revised Jeffreys scales [79] (see Table VI for the details). If B_{ij} (or $\ln B_{ij}$) assumes negative values, the result is reversed, that means, the negative values of $\ln B_{ij}$ indicate that the model M_j is preferred over the model M_i .

The computation of the Bayesian evidence is now easy since one can directly use the MCMC chains that are already found to extract the parameters space for different observational data sets. For a detailed explanation and implementation of the Bayesian evidence for any cosmological model we refer to the original works [80, 81]. Here we use the code `MCEvidence`¹ for the computation of the Bayesian evidence of the models. In Table VII, we present the

¹ This code is freely available at github.com/yabebalFantaye/MCEvidence.

$\ln B_{ij}$	Strength of evidence for model M_i
$0 \leq \ln B_{ij} < 1$	Weak
$1 \leq \ln B_{ij} < 3$	Definite/Positive
$3 \leq \ln B_{ij} < 5$	Strong
$\ln B_{ij} \geq 5$	Very strong

TABLE VI: Revised Jeffreys scale used to quantify the observational support of any model M_i with respect to one another model (reference model) M_j .

Dataset	Model	$\ln B_{ij}$	Strength of evidence for model Λ CDM
CMB	Model 1	-1.9	Definite/Positive
CB	Model 1	-5.9	Very Strong
CBJ	Model 1	-6.2	Very Strong
CJR	Model 1	-3.2	Strong
CBJRC	Model 1	-2.7	Definite/Positive
CMB	Model 2a	-3.1	Strong
CB	Model 2a	-5.1	Very Strong
CBJ	Model 2a	-4.9	Strong
CJR	Model 2a	-1.6	Definite/Positive
CBJRC	Model 2a	-1.1	Definite/Positive
CMB	Model 2b	-2.6	Definite/Positive
CB	Model 2b	-7.0	Very Strong
CBJ	Model 2b	-6.1	Very Strong
CJR	Model 2b	-1.8	Definite/Positive
CBJRC	Model 2b	-1.9	Definite/Positive
CMB	Model 3a	-3.3	Strong
CB	Model 3a	-4.0	Strong
CBJ	Model 3a	-5.2	Very Strong
CJR	Model 3a	-3.1	Strong
CBJRC	Model 3a	-2.3	Strong
CMB	Model 3b	-2.5	Definite/Positive
CB	Model 3b	-3.9	Strong
CBJ	Model 3b	-5.4	Very Strong
CJR	Model 3b	-1.7	Definite/Positive
CBJRC	Model 3b	-1.5	Definite/Positive

TABLE VII: The values of $\ln B_{ij}$, the logarithm of the Bayes factor for different scalar field models in this work with respect to the base Λ CDM model for different observational data sets as well as the corresponding strength of evidence for Λ CDM quantified based on the modified Jeffreys scale (see Tab. VI). From the Bayesian point of view, the negative values of $\ln B_{ij}$ indicate that the Λ CDM model is certainly preferred in respect to the scalar field models.

$\ln B_{ij}$ values calculated for all the scalar field models with respect to the base cosmological model Λ CDM obtained for different observational data sets employed in this work. The negative values of $\ln B_{ij}$ for any scalar field model obtained for any observational data set actually indicate that the Λ CDM model is preferred over the scalar field models. From Table VII one can see that Λ CDM model is clearly favored over all the scalar field models. In particular, for Model 3a, this becomes much clear while one may note that, depending on different data sets, a slight changes appear in the conclusion, but however, the overall results favor Λ CDM without any bias.

4.4. A comparative study of the scalar field models

In the previous subsections, we have presented the observational constraints on the scalar field models and also performed their Bayesian evidences from which it has been found that Λ CDM is always preferred over the present

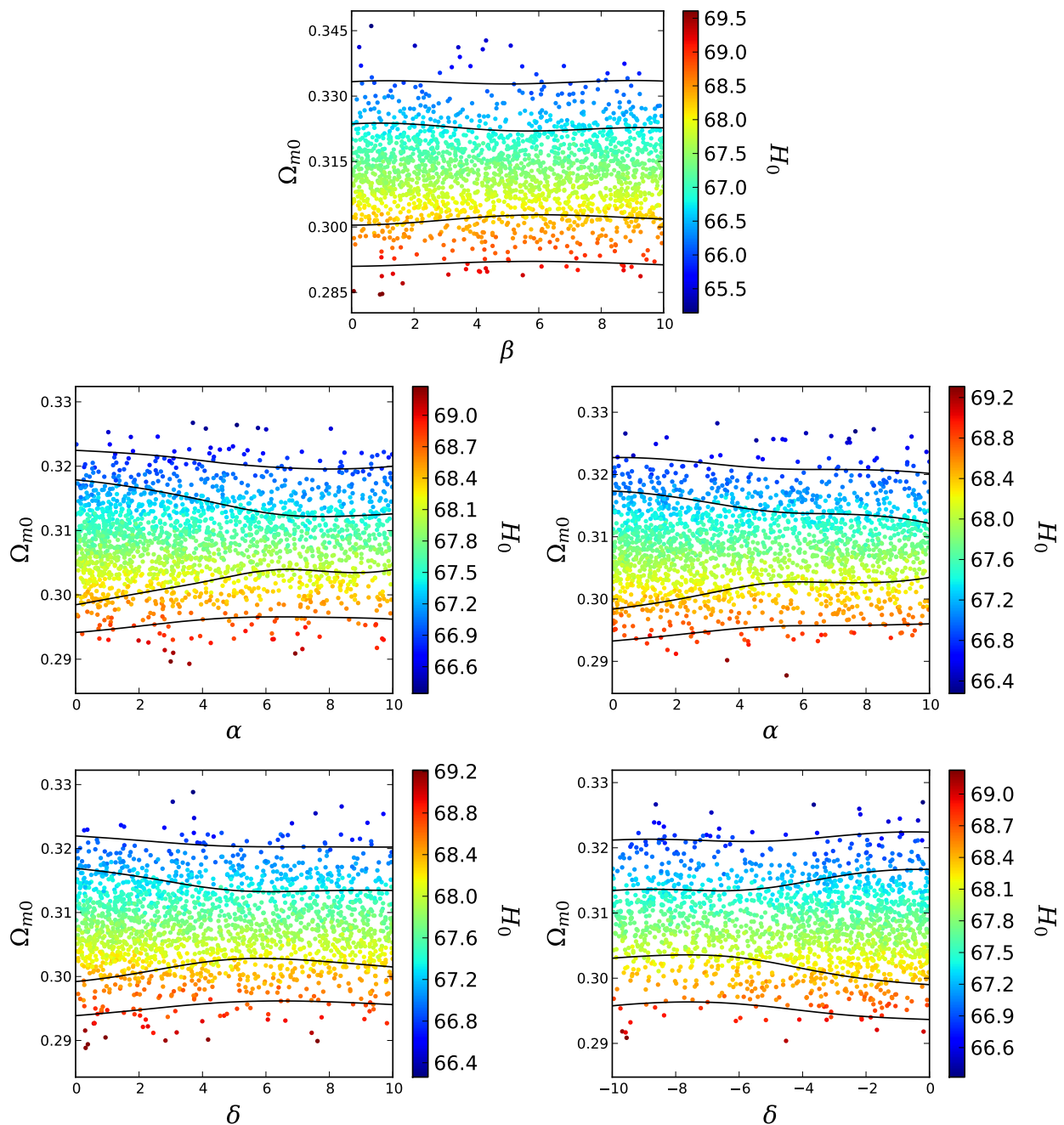


FIG. 12: MCMC samples in the (\tilde{x}, Ω_{m0}) plane where $\tilde{x} = \beta$ (for Model 1), α (for Model 2a and Model 2b) and δ (for Model 3a and Model 3b) colored by the values of H_0 for the combined analysis CMB+BAO+JLA+RSD+CC. The upper panel stands for Model 1; the middle panels correspond to Model 2 (Model 2a in the left side and Model 2b in the right side); the lower panels are for Model 3 (Model 3a in the left side and Model 3b in the right side).

scalar field models, at least according to the observational data employed in this work. In this section, we aim to offer an overall comparison between the present scalar field models (12), (13) and (14) focusing on some generic properties of the models independent of the observational datasets. In all scalar field models, we find that the parameter which quantifies the model from the constant potential, that means, β for Model 1 of eqn. (12), α for Model 2 of eqn. (13) and Model 3 of eqn. (14) are very weakly constrained and within 68% CL, the models allow the constant potential $V(\phi) = V_0$. This is not a new result in the scalar field theory. One might recall similar investigations with Peebles-Ratra potential $V(\phi) \propto \phi^{-\alpha}$ ($\alpha \geq 0$) [24, 25] where the parameter α quantifying the deviation of the model from the constant potential cannot be constrained well even with the latest astronomical datasets, see the results

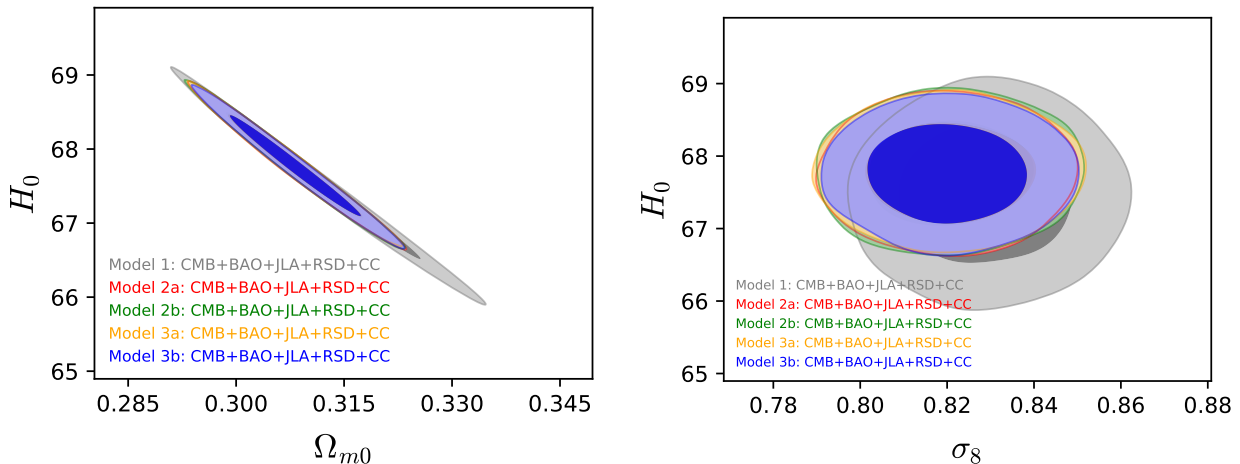


FIG. 13: The left panel shows a strong negative correlation between the parameters (Ω_{m0}, H_0) for the scalar field models and the right panel shows that the parameters (σ_8, H_0) for all the scalar field models are uncorrelated. We have shown both the results for the combined analysis CMB+BAO+JLA+RSD+CC, however, this result is valid for the remaining datasets.

in [82]. For the exponential potential [76], a similar conclusion finding that the constant potential is allowed by the data was achieved but again the quantifying parameter was weakly constrained. Some of the models studied in this work are closely related to the exponential potential since the first two models, Model 1, Model 2 can be expressed in terms of the exponential potential and Model 3 has similar structure to the power law type potential (although it looks similar to the power law potential [82] for negative values of p , but strictly not). However, to understand this scenario much better, in Fig. 12, we have shown the 2D contour plots between the parameters (\tilde{x}, Ω_{m0}) where $\tilde{x} = \beta$ (for Model 1), α (for Model 2a and Model 2b) and δ (for Model 3a and Model 3b) colored by the H_0 values. The points in all the sub plots are the samples from the chains of the markov chain monte carlo (mcmc) analysis using the combined dataset CMB+BAO+JLA+RSD+CC (we only show the final analysis because the others give similar conclusions). From this figure (Fig. 12), one can now clearly see that for higher or lower values of H_0 (used different color to distinguish), the parameter \tilde{x} ($= \beta, \alpha, \delta$) mentioned above, does not offer any specific changes with different (higher/lower) values of Ω_{m0} . Apart from that, the scalar field models are very similar predicting a smooth transition from the past decelerating era to the present accelerating phase. Also, the equation of state w_ϕ , of the scalar field model and the effective equation of state w_{eff} in eqn. (6) of the cosmological model are close to ‘-1’, the cosmological constant limit. Concerning the statistical analysis, we find that there is a strong negative correlation between the parameters Ω_{m0} and H_0 (see the left panel of Fig. 13) while there is no such correlation found for the parameters (σ_8, H_0) . Both the results are independent of all the datasets considered. We close this subsection with two possible far-reaching possibilities in scalar field models concerning the presence and absence of correlations between the above three parameters $\{\Omega_{m0}, H_0, \sigma_8\}$. One may recall that the correlations between Ω_{m0} and H_0 exist for some other well known potentials [76, 82]. While the absence of correlation between the parameters σ_8 and H_0 is another interesting feature of the present scalar field models since the correlations between these two parameters are often present in other cosmological models explored recently [83, 84, 85, 86, 87, 88]. Thus, clearly, the investigations toward this direction might be continued in order to see whether the presence or absence of the correlations between the above three parameters are independent of the scalar field potentials or not. We hope to address some of them in our forthcoming works.

5. SUMMARY AND CONCLUSIONS

Scalar field models are well known due to their diversities in explaining various phases of the universe evolution. They have been found to explain both early- and late-accelerating phases of the universe in a satisfactory way [22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Interestingly, a connection between these two distinct accelerating phases is found via inflationary-quintessential models, see for instance [29, 36, 37, 38, 39, 40, 41]. Thus, without any doubt, the scalar field models might be thought to furnish a viable description for the universe evolution and consequently, this triggered the investigations in this direction that resulted in numerous scalar field models [5]. In the present work we focus on a specific scalar field model, namely, the quintessence scalar field model. Since the potential plays the key

role in determining the nature of the model, thus, it is natural to ask the observational viabilities of the models. Thus, remembering this issue, in the present work we consider a varieties of quintessence scalar field models in order to test their observational viabilities and to provide stringent constraints using the latest astronomical data from various sources, namely, the observations from cosmic microwave background (CMB), baryon acoustic oscillations (BAO), redshift space distortions (RSD), joint light curve analysis (JLA) from Supernovae Type Ia and the Hubble parameter measurements from the cosmic chronometers (CC). The analyses of the scalar field models have been performed using markov chain monte carlo package `cosmomc` [64], a fast converging algorithm equipped with a convergence criteria by Gelman-Rubin [65]. The analyses of the results for different scalar field models have been presented in Table I (Model 1), Table II (Model 2a), Table III (Model 2b) Table IV (Model 3a), Table V (Model 3b) with the corresponding 1D marginalized posterior distributions as well as the 2D contour plots between several combinations of the model parameters. Our analyses show that the free parameters quantifying the deviations of the models from the constant potential cannot be constrained well, at least from the observational data we have used in this work. However, within 68% CL, all the observational datasets allow the constant potential $V = V_0$. It is interesting to note that in all scalar field models, a strong negative correlation between the parameters (H_0, Ω_{m0}) exists, whilst on the other hand, the parameters (H_0, σ_8) are not correlated. Perhaps it will be important to continue the investigations in order to see whether the presence of correlations between (H_0, Ω_{m0}) and the absence of correlations between (H_0, σ_8) in the scalar field models are generic or not. The models have been further analyzed using the Bayesian analysis with respect to the base Λ CDM model for all the astronomical data sets employed in the work. The Bayesian analysis provides a clear picture (summarized in Table VII) of the scalar field models over the Λ CDM model. This shows that Λ CDM model is always preferred over all the scalar field models where in addition one can also notice that Model 3a has a large difference from Λ CDM.

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