Particle Accelerators inside Spinning Black Holes

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On the basis of the Kerr metric as a model for a spinning black hole accreting test particles from rest at infinity, I show that the center-of-mass energy for a pair of colliding particles is generically divergent at the inner horizon. This shows that not only are classical black holes internally unstable, but also that Planck-scale physics is a characteristic feature within black holes at scales much larger than the Planck length.

Recently, Bambi, Silk and West [2] (BSW) suggested that rotating black holes could serve as particle colliders with arbitrarily high center-of-mass energies. This suggestion was soon criticized. Berti et al [3] pointed out that the BSW mechanism requires fine tuning (a degenerate black hole and a critical angular momentum for one of the particles). Further, they pointed out that in the real world one would obtain only modest center-of-mass energies. To prove the general divergence at \( r_- \) first note that the denominator of \( E_{cm}^{Kerr} \) obviously vanishes there. For the numerator we note that \( N \) evaluates to

\[
N_- = -2a(l_1 + l_2) + l_1l_2r_+ + 4r_-
- \sqrt{(l_1^2r_+ + 4(r_- - al_2))(l_2^2r_+ + 4(r_- - al_2))}
\]

at \( r = r_- \). Whereas the detailed properties of geodesics in the Kerr metric are, let us say, involved; it is adequate for our purposes here to note the following:

- over the range \( 0 < a < 1 \) particles with \(-4 < l_1 < 0 \) have no turning points
- particles with \( A < l_2 < 2 \), where

\[
A = \frac{2a}{1 + \sqrt{(1 - a)(1 + a)}},
\]

have no turning points.

Here I show that the BSW mechanism requires fine tuning (a degenerate black hole and a critical angular momentum for one of the particles). Moreover, as they showed in some detail, the effects of gravitational radiation are not ignorable. At about the same time, Jacobson and Sotiriou [5] carefully analyzed the fine tuning required by the BSW mechanism, also pointed out the consequences of the Thorne limit, and showed how the redshift further lowers realizable energies.

As shown in [2 and 5], for a pair of particles of mass \( m \) that fall from rest at infinity, the center-of-mass energy in the Kerr metric is given by

\[
\left( E_{cm}^{Kerr} \right)^2 = \frac{2m^2N}{r(r^2 - 2r + a^2)}
\]

where

\[
N = 2a^2(1 + r) - 2a(l_1 + l_2) - l_1l_2(r - 2) + 2(r - 1)r^2 - \sqrt{(2(a - l_1)^2 - l_1^2r + 2r^2)(2(a - l_2)^2 - l_2^2r + 2r^2)}
\]

the black hole is given unit mass, the angular momentum per unit mass of the black hole is given by \( a \) and the particles have orbital angular momenta of \( l_1 \) and \( l_2 \). Here we consider black holes in the range \( 0 < a < 1 \).

The horizons are given by \( r_\pm = 1 \pm \sqrt{1 - a^2} \) and here we are concerned only with the inner horizons \( r_- \). To prove the general divergence at \( r_- \) first note that the denominator of \( E_{cm}^{Kerr} \) obviously vanishes there. For the numerator we note that \( N \) evaluates to

\[
N_- = -2a(l_1 + l_2) + l_1l_2r_+ + 4r_-
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at \( r = r_- \). Whereas the detailed properties of geodesics in the Kerr metric are, let us say, involved; it is adequate for our purposes here to note the following:

- over the range \( 0 < a < 1 \) particles with \(-4 < l_1 < 0 \) have no turning points
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\[
A = \frac{2a}{1 + \sqrt{(1 - a)(1 + a)}},
\]

have no turning points.

Whereas there is a critical turning point (inflection in \( r \)) at \( r = r_- \) for \( l_2 = A [7] \), and whereas \( N_- = 0 \) for \( l_2 = A \), this is of no consequence here. Over the stated ranges in \( l_1 \) and \( l_2 \), \( N_- \) does not evaluate to zero. This is demonstrated in Fig. [1].

Because of the generic nature of the divergence discussed above, a divergence that suffers none of the limitations of the BSW mechanism, it is reasonable to conclude that the use of the Kerr metric and the point-particle geodesic approximation has not given rise to a fictitious result. Given this, the principal conclusion here is that Planck-scale physics is a characteristic feature of black hole interiors at scales much larger than the Planck length.

Acknowledgments

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FIG. 1: A plot of $N_-$, given by (3), for $a = 1/2$ within the stated ranges for $l_1$ and $l_2$. Other plots in the range $0 < a < 1$ are qualitatively similar.

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[7] The essential content of the BSW mechanism is the choice $a = 1$ and $l_2 = A$. 