The Schwarzschild singularity: a semiclassical bounce?

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We discuss the opportunity that the singularity inside a Schwarzschild black hole could be replaced by a regular bounce, described as a regular minimum of the spherical radius (instead of zero) and a regular maximum of the longitudinal scale (instead of infinity) in the corresponding Kantowski-Sachs metric. Such a metric in a vicinity of the bounce is shown to be a solution to the Einstein equations with the stress-energy tensor representing vacuum polarization of quantum matter fields, described by a combination of curvature-quadratic terms in the effective action. The indefinite parameters of the model can be chosen in such a way that it remains a few orders of magnitude apart from the Planck scale (say, on the GUT scale), that is, in a semiclassical regime.

1 Introduction

The existence of cosmological and black hole singularities is well known as a natural, though undesirable feature of general relativity (GR) and many alternative theories of gravity. It still appears that most of the researcher do not believe that infinite values of curvature invariants and/or matter densities and temperatures, inherent to such singularities, really exist in nature. It seems to be much more plausible that a modified theory of gravity must replace GR at large curvatures (or at high energies and the corresponding small length and time scales), and that such a modification should be related to quantum phenomena.

In the extremely numerous attempts to avoid singularities in the description of nature, three basic trends may be singled out:[4]

(a) Various models of quantum gravity, which should be treated as tentative ones since a consistent and generally accepted theory of quantum gravity is so far lacking [1–3];

(b) Models of semiclassical gravity, treating gravity itself as a classical field and using the equations of GR or another classical theory of gravity to describe the geometry, but including certain averages of quantum matter fields as sources of gravity [4–9];

(c) Inclusion of various classical sources of gravity, violating the usual energy conditions, such as, for example, phantom scalar fields in GR, effective stress-energy tensors originating from additional geometric quantities like torsion, or those borrowed from extra space-time dimensions [10–17].

One can notice a deep similarity between the singularity problems in the Big Bang cosmology and those occurring in black hole interiors, at least those like the Schwarzschild singularity, located in a "T-region" of the black hole, where the metric describes a special case of Kantowski-Sachs anisotropic cosmology. It is therefore not surprising that the same tools are used by the researchers in order to regularize the cosmological and black hole singularities.

Models of quantum gravity [1–3] (or, more precisely, their effective representations written in the language of classical geometry) provide nonsingular models both in black hole physics and cosmology, but their general shortcoming is that they stop a collapse at curvatures and densities on the Planck scale or very close to it. Since quantum matter fields demonstrate quantum properties already at the atomic or even macroscopic scales (recall, e.g., lasers or the Casimir effect), there can be a hope that a cosmological collapse or black hole singularities may be prevented at scales not so far from the

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[4]We here do not even try to give a full list of references which would be enormously long; instead, we only mention some known reviews and some of the papers that we discussed in the course of this study.

conventional ones as is the Planck scale. This looks more attractive from an observational viewpoint as well as from the positions of theory since the corresponding results would look, at least to-date, more confident than those of quantum gravity. As to the third trend (c), although it has brought about a great number of studies of great interest such as models of cosmological bounce, regular black holes, wormholes, etc. (see, e.g., [10–17]), one should admit that, on one hand, their necessary "exotic" components require conjectures not yet confirmed by the experiment, and, on the other, their possible existence is usually treated as a phenomenological description of some underlying quantum effects.

In this paper, we try to take into account the effects of quantum fields at approach to the Schwarzschild singularity ($r = 0$, often incorrectly called the central singularity whereas it is actually cosmological in nature).

We consider a toy model in which the interior of the Schwarzschild space-time is studied within the scope of semiclassical approach for a possible bounce in its interior instead of the singularity. We try to take into account that any space-time always contains quantum oscillations of all physical fields, but here we do not make any assumptions on their particular composition, restricting ourselves to pure vacuum polarization effects supposed to support a sought-for bouncing solution. In this simplified statement of the problem it turns out that there is a wide choice of the free parameters of the model providing realization of such a scenario.

Unlike the studies trying to take into account the effects of Hawking radiation inside black holes, e.g., [7, 8, 18], we only discuss vacuum effects close to a would-be singularity. Such a description can probably be relevant to sufficiently large black holes, say, of stellar mass or those in galactic nuclei, for which the influence of Hawking radiation may be neglected. That is, it may be a possible answer to the following question: if a body (a particle, a planet, a spacecraft) falls into a large black hole, which geometry will it meet there?

2 The metric at bounce

The interior (T-region) of a generic spherically symmetric black hole space-time in the appropriate coordinate system can be described by the metric

$$ds^2 = d\tau^2 - e^{2\gamma(\tau)}dx^2 - e^{2\beta(\tau)}d\Omega^2, \qquad (1)$$

where τ is the "cosmologiical" time coordinate, and x is a spatial coordinate that appears beyond the horizon replacing the time coordinate of the static region, $d\Omega^2$ is the metric on a unit sphere $\mathbb{S}^2$, while β and γ are smooth functions of time. This metric describes a homogenious anisotropic cosmology of Kantowski—Sachs type, with spatial section topology $\mathbb{R} \times \mathbb{S}^2$.

Let us assume that sufficiently far from a singularity this space-time corresponds to the classical Schwarzschild solution, which in its T-region ($r < 2m$) has the form ($m = GM$, where M is the black hole mass, and we are using the units convention $\hbar = c = 1$).

$$ds^2 = \Big(\frac{2m}{T} - 1\Big)^{-1} dT^2 - \Big(\frac{2m}{T} - 1\Big) dx^2 - T^2 d\Omega^2, \qquad (2)$$

where, as compared to the usual expression, we have re-denoted $r \to T$ because in the T-region the coordinate r is temporal. At small T, substituting $\sqrt{T/(2m)}dT = d\tau$, we adjust the asymptotic form of the metric at small T to the form (1):

$$ds^2 = d\tau^2 - \Big(\frac{4}{3}m\Big)^{2/3} \tau^{-2/3} dx^2 - \Big(\frac{9}{2}m\Big)^{2/3} \tau^{4/3} d\Omega^2, \qquad (3)$$

valid at $\tau/m \ll 1$. As $\tau \to 0$, the scale along the x axis infinitely stretches while the coordinate spheres shrink to zero.

Our basic assumption in this study will be that the effect of quantum fields do not allow the limit $r = e^\beta \to 0$ but provide, instead, a regular minimum at some value $r = a > 0$, while the scale along the x axis has a regular maximum at $\tau = 0$. Then at sufficiently small τ, in accord with (2) and (3), the metric acquires the form

$$ds^2\Big|_{\rm bounce} \simeq d\tau^2 - \frac{2m}{a}(1 - c^2\tau^2)dx^2 - a^2(1 + b^2\tau^2)d\Omega^2 \qquad (4)$$

with some positive constants a, b, c of appropriate dimensions.[5]

Under this assumption, let us also assume symmetry of the metric with respect to the bouncing time $\tau = 0$ and, using the form (1) of the metric, represent the functions $\beta(\tau)$ and $\gamma(\tau)$ as Taylor series with even powers and constants β_i, γ_i ($i =$

[5]Please do not confuse this c with the speed of light.

$0, 2, 4, 6, ...)$:

$$\beta(\tau) = \beta_0 + \frac{1}{2}\beta_2\tau^2 + \frac{1}{24}\beta_4\tau^4 + \frac{1}{720}\beta_6\tau^6 + ...$$
$$\gamma(\tau) = \gamma_0 + \frac{1}{2}\gamma_2\tau^2 + \frac{1}{24}\gamma_4\tau^4 + \frac{1}{720}\gamma_6\tau^6 + ..., \quad (5)$$

so that, in particular, according to (4),

$$a = \mathrm{e}^{\beta_0}, \qquad 2m/a = \mathrm{e}^{2\gamma_0},$$
$$2b^2 = \beta_2/\beta_0, \qquad 2c^2 = -\gamma_2/\gamma_0. \quad (6)$$

Now, in the semiclassical approach, we will seek the appropriate solution to the Einstein equations

$$G_\mu^\nu = -\varkappa \langle T_\mu^\nu \rangle, \qquad \varkappa = 8\pi G, \quad (7)$$

with the renormalized quantum stress-energy tensor $\langle T_\mu^\nu \rangle$, containing a contribution from vacuum polarization.

The nonzero components of the Einstein tensor for the metric (1) are

$$-G_0^0 = \dot\beta(\dot\beta + 2\dot\gamma) + \mathrm{e}^{-2\beta},$$
$$-G_1^1 = 2\ddot\beta + 3\dot\beta^2 + \mathrm{e}^{-2\beta},$$
$$-G_2^2 = G_3^3 = \ddot\gamma + \ddot\beta + \dot\gamma^2 + \dot\beta^2 + \dot\beta\dot\gamma. \quad (8)$$

Using the Taylor expansion (5), one can write these components explicitly up to $O(\tau^2)$ as follows:

$$-G_0^0 = \frac{1}{a^2}\left(1 - \frac{\beta_2}{2\beta_0}\tau^2\right) + \beta_2(\beta_2 + 2\gamma_2)\tau^2,$$
$$-G_1^1 = \frac{1}{a^2}\left(1 - \frac{\beta_2}{2\beta_0}\tau^2\right) + 2\beta_2 + \beta_4\tau^2 + 3\beta_2^2\tau^2,$$
$$-G_2^2 = \beta_2+\gamma_2+\frac{1}{2}(\beta_4+\gamma_4)\tau^2+(\beta_2^2+\gamma_2^2+\beta_2\gamma_2)\tau^2, \quad (9)$$

while the r.h.s. of (7) will be discussed in the next section.

3 Stress-energy tensor

Following numerous papers on quantum field theory in curved spaces, in particular, the books [19, 20], we can present the renormalized vacuum stress-energy tensor T_ν^μ in terms of a linear combination of certain geometric quantities ${}^{(i)}H_{\mu\nu}$ $(i = 1, 2)$ (obtainable by variation of actions quadratic in the Ricci tensor and scalar), with phenomenological constants N_1, N_2, and two more contributions, ${}^{(c)}H_\nu^\mu$ and P_ν^μ:

$$\langle T_\nu^\mu \rangle = N_1 {}^{(1)}H_\nu^\mu + N_2 {}^{(2)}H_\nu^\mu + {}^{(c)}H_\nu^\mu + P_\nu^\mu \quad (10)$$

where

$${}^{(1)}H_\nu^\mu \equiv 2RR_\nu^\mu - \frac{1}{2}\delta_\nu^\mu R^2 + 2\delta_\nu^\mu \Box R - 2\nabla_\nu\nabla^\mu R,$$
$${}^{(2)}H_\nu^\mu \equiv -2\nabla_\alpha\nabla_\nu R^{\alpha\mu} + \Box R_\nu^\mu + \frac{1}{2}\delta_\nu^\mu\Box R$$
$$+ 2R^{\mu\alpha}R_{\alpha\nu} - \frac{1}{2}\delta_\nu^\mu R^{\alpha\beta}R_{\alpha\beta}, \quad (11)$$

and $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$. The tensor ${}^{(c)}H_\nu^\mu$ is a local contibution depending on space-time topology or special boundary conditions (the Casimir effect [21, 22]), while P_ν^μ depends on the choice of the quantum state and describes, in particular, particle production due to a nonstationary nature of the metric. The tensor P_μ^ν is nonlocal in the sense that it is not a function of a point in space-time but depends on the whole previous history. Its calculation is rather complicated and, moreover, depends on additional assumptions on the quantum states of the constituent fields. We will not consider it in the present paper, assuming that its contribution can be insignificant at least for some admissible choices of quantum states.

The components of the tensors ${}^{(i)}H_\nu^\mu$ (which turns out to be diagonal) are easily found using the ansatz (1) and the Taylor series (5). At $\tau = 0$, that is, at bounce (higher orders in τ will not be necessary in our calculations) they are

$${}^{(1)}H_0^0 = -\frac{2}{a^4} + 8\beta_2^2 + 8\beta_2\gamma_2 + 2\gamma_2^2,$$
$${}^{(1)}H_1^1 = -\frac{2}{a^4} - 32\beta_2^2 - 16\beta_2\gamma_2 - 6\gamma_2^2 - 8\beta_4 - 4\gamma_4,$$
$${}^{(1)}H_2^2 = \frac{2}{a^4} + \frac{12\beta_2}{a^2} - 24\beta_2^2 - 20\beta_2\gamma_2 - 10\gamma_2^2$$
$$- 8\beta_4 - 4\gamma_4,$$
$${}^{(2)}H_0^0 = -\frac{1}{a^4} + 3\beta_2^2 + 2\beta_2\gamma_2 + \gamma_2^2,$$
$${}^{(2)}H_1^1 = -\frac{1}{a^4} - 9\beta_2^2 - 6\beta_2\gamma_2 - 3\gamma_2^2 - 2\beta_4 - 2\gamma_4,$$
$${}^{(2)}H_2^2 = \frac{1}{a^4} + \frac{4\beta_2}{a^2} - 9\beta_2^2 - 6\beta_2\gamma_2 - 3\gamma_2^2$$
$$- 3\beta_4 - \gamma_4. \quad (12)$$

The numerical coefficients N_1 and N_2 in (10) are unknown and should be determined from experiment or observations. One can qualitatively estimate the order of magnitude for these coefficients recalling that they actually appear as coefficients at curvature-squared terms in higher-derivative gravity models with action of the form $S \sim \int d^4x\sqrt{-g}(R/(2\varkappa) + N_1R^2 + N_2R_{\mu\nu}^2 + ...)$ since the tensors ${}^{(1,2)}H_{\mu\nu}$ result from variation of terms with R^2 and $R^2_{\mu\nu} \equiv R_{\mu\nu}R^{\mu\nu}$ with respect to the metric [19, 20].

The current empirical upper bound for these parameters is $N_{1,2} \lesssim 10^{60}$ (see [27]). This restriction follows from observations performed at very small curvatures, therefore possible effects of terms $\sim R^2$ are hardly noticeable. The estimates of $N_{1,2}$ may be different if such theories of gravity are applied to the early (inflationary) Universe where curvatures are much larger, for example, $N_1 \sim 10^{10}$ [23–25]. In any case, for our purposes we can feel more or less free in the choice of these parameters.

As to the Casimir contribution, there are reasons to believe that it should be small as compared to that of ${}^{(i)}H_{\mu\nu}$. Consider, for example, the static case of the metric (1) with $\mathrm{e}^{\beta} = r = a$, actually describing what can be called an infinitely long wormhole throat. A calculation performed in [26] for a massless conformally coupled scalar field gives for this geometry

$$
{}^{(c)}H^{\mu}_{\nu} = \frac{1}{2880\pi^2 a^4}\Big[2\,\mathrm{diag}(-1,-1,1,1)\ln(a/a_0) + \mathrm{diag}(0,0,-1,-1)\Big], \tag{13}
$$

where a_0 is a fixed length that can only be determined by experiment. The quantity (13) corresponds to a single massless scalar, while the total Casimir contribution must include all fields with different spins and masses.

On the other hand, for the same geometry,

$$
{}^{(1)}H^{\nu}_{\mu} = 2\,{}^{(2)}H^{\nu}_{\mu} = \frac{2}{a^4}\,\mathrm{diag}(-1,-1,1,1). \tag{14}
$$

Thus, if N_1 and/or N_2 are of the order of unity or larger (as we shall finally need), the tensors ${}^{(i)}H_{\mu\nu}$ will evidently much stronger contribute to $\langle\, T^{\nu}_{\mu}\,\rangle$ than ${}^{(c)}H_{\mu\nu}$, unless the uncertain quantity a_0 in (13) is unnaturally high, or the total number of fields is so great as to overcome the denominator of about 10^4. In our further estimates we will assume that ${}^{(c)}H^{\mu}_{\nu}$ can also be neglected in our more general geometry and thus take into account only ${}^{(i)}H^{\nu}_{\mu}$.

4 Semiclassical bounce

Consider the Einstein equations (7) with the stress-energy tensor (10), taking there into account only the first two terms, and let us find out whether there are solutions consistent with our assumption on the bouncing metric (4), and if yes, what are restrictions on the free parameters of the model providing their semiclassical scale.

To do that, we should express G^{μ}_{ν} and ${}^{(i)}H_{\mu\nu}$ in terms of the Taylor series (5) and equate coefficients at equal powers of τ on the two sides of resulting equations. For convenience, we introduce the dimensionless parameters

$$
\begin{aligned}
&A = \varkappa a^{-2}, \quad B_2 = \varkappa\beta_2, \quad C_2 = \varkappa\gamma_2,\\
&B_4 = \varkappa^2\beta_4, \quad C_4 = \varkappa^2\gamma_4, \quad \text{etc.}
\end{aligned} \tag{15}
$$

Since $\varkappa \approx l^2_{\rm pl}$ (the Planck length squared), it is clear that our system will remain on the semiclassical scale as long as all parameters (15) are much smaller than unity. This means, in particular, that the minimum radius $r = a$, reached at bounce, will be much larger than the Planck length. Other parameters are the values of the derivatives $\ddot\beta, \ddot\gamma$, etc. at bounce.

It turns out that in the approximation used it is sufficient to check only the order $O(1)$ in the $\binom{0}{0}$ component of the equations. This yields

$$
\begin{aligned}
A = {}& N_1[-2A^2 + 2(2B_2 + C_2)^2]\\
&+ N_2[-A^2 + (B_2 + C_2)^2 + 2B_2^2].
\end{aligned} \tag{16}
$$

Other equations will only express the constants B_4, C_4, etc. in terms of A, B_2, C_2. So we have only one equation for the latter three parameters and also N_1, N_2 and have a large space of possible solutions.

According to our purposes, we assume that a is much larger than the Planck length $l_{\rm pl} \sim \sqrt{\varkappa}$, which means that $A \ll 1$, or $A = O(\varepsilon)$, ε being a small parameter. Let us also make a natural assumption that $B_2, C_2 = O(\varepsilon)$, that is, $\ddot\beta$ and $\ddot\gamma$ are of the same order of magnitude as $1/a^2$. Then, since the r.h.s. of Eq. (16) is $O(\varepsilon^2)$ while the l.h.s. is $O(\varepsilon)$, to maintain the equality we must have N_1 and/or N_2 being large enough, of order $O(1/\varepsilon)$.

From the remaining Einstein equations $\binom{1}{1}$ and $\binom{2}{2}$ at $\tau = 0$ it then follows that B_4 and C_4 are of order $O(\varepsilon^2)$ (see (11)), so that the 4th order derivatives of β and γ are of a correct order of smallness relative to the Planck scale (see (15)). Similar inferences follow for B_6, C_6, etc. if we consider the equations in the order $O(\tau^2)$, and so on. One can also verify that the curvature invariants R, $R^2_{\mu\nu}$ and $\mathcal{K} \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ are small at bounce ($\tau = 0$)

as compared to the Planck scale:

$$R = \frac{2}{a^2} + 4\beta_2 + 2\gamma_2 = O\Big(\frac{\varepsilon}{\varkappa}\Big),$$
$$R^2_{\mu\nu} = \frac{2}{a^4} + \frac{4\beta_2}{a^2} + 6\beta_2^2 + 4\beta_2\gamma_2 + 2\gamma_2^2 = O\Big(\frac{\varepsilon^2}{\varkappa^2}\Big),$$
$$\mathcal{K} = \frac{4}{a^4} + 8\beta_2^2 + 4\gamma_2^2 = O\Big(\frac{\varepsilon^2}{\varkappa^2}\Big). \qquad (17)$$

Let us illustrate the situation with a numerical example. Assume $N_1 = 0$, $N_2 = 10^{10}$, and $A = 10^{-10}$ (hence a minimal radius a of 10^5 Planck lengths). Furthermore, since by construction (see (6) and (15)) $B_2 > 0$ and $C_2 < 0$, we can safely assume $B_2 + C_2 = 0$. Under these assumptions, from Eq. (16) we find

$$B_2 = -C_2 = 10^{-10}.$$

Substituting this into the $\binom{1}{1}$ and $\binom{2}{2}$ components of the Einstein equations at $\tau = 0$, using the expressions (8) and (12), we obtain the values of B_4 and C_4:

$$B_4 = 3.5 \times 10^{-20}, \qquad C_4 = -8.5 \times 10^{-20}.$$

From equations of order $O(\tau^2)$ one can then find B_6, C_6, and so on.

It is known that at a regular minimum of the spherical radius $e^\beta = r$, be it a wormhole throat in an R-region or a bounce in a T-region, the stress-energy-tensor must satisfy the requirement $T^0_0 - T^1_1 < 0$ thus violating the Null Energy Condition, see, e.g., [11, 28]. In our case, since we suppose a bounce at $\tau = 0$, the inequality $T^0_0 - T^1_1 < 0$ automatically holds as long as we use the metric (4).

5 Concluding remarks

We can conclude that under our assumptions a semiclassical bounce instead of a Schwarzschild singularity is possible and can occur at semiclassical scales, without need for quantum gravity effects.

As already mentioned, this result can tell us which kind of geometry may be seen by an observer falling into a sufficiently large black hole for which Hawking radiation is negligible due to its extremely low temperature. Though, for a more complete and convincing answer to the same question it would be necessary to take into consideration nonlocal effects (above all, particle creation) depending on the choice of quantum states of different physical fields, also making clear which quantum states can be physically relevant.

Of even greater interest can be similar studies for charge and rotating black holes where the singularities are not so simple as in the Schwarzschild case. We hope to address these problems in our future studies.

There seems to appear an attractive opportunity that the terms in the effective action quadratic in the Ricci tensor, with $N_{1,2} \sim 10^{10}$ or so (that is, on the GUT rather than Planck scale), being able to drive inflation in the early Universe [23–25], are also able to prevent black hole singularities but are quite unnoticeable under usual physical conditions.

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