

Primordial Black Holes as Dark Matter

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Primordial black holes (PBHs) could provide the dark matter but a variety of constraints restrict the possible mass windows to $10^{16} - 10^{17}$ g, $10^{20} - 10^{24}$ g and $10 - 10^3 M_{\odot}$. The last possibility is contentious but of special interest in view of the recent detection of black-hole mergers by LIGO/Virgo. PBHs might have important consequences and resolve various cosmological conundra even if they have only a small fraction of the dark-matter density. In particular, those larger than $10^3 M_{\odot}$ could generate cosmological structures through the ‘seed’ or ‘Poisson’ effect, thereby alleviating some problems associated with the standard cold dark matter scenario, and sufficiently large PBHs might provide seeds for the supermassive black holes in galactic nuclei. More exotically, the Planck-mass relics of PBH evaporations or stupendously large black holes bigger than $10^{12} M_{\odot}$ could provide an interesting dark component.

I. INTRODUCTION

Primordial black holes (PBHs) have been a source of interest for nearly 50 years [1], despite the fact that there is still no evidence for them. One reason for this interest is that only PBHs could be small enough for Hawking radiation to be important [2]. This has not yet been confirmed experimentally and there remain major conceptual puzzles associated with the process. Nevertheless, this discovery is generally recognised as one of the key developments in 20th century physics because it beautifully unifies general relativity, quantum mechanics and thermodynamics. The fact that Hawking was only led to this discovery through contemplating the properties of PBHs illustrates that it can be useful to study something even if it does not exist! But, of course, the situation is much more interesting if PBHs do exist.

PBHs smaller than about 10^{15} g would have evaporated by now with many interesting cosmological consequences [3]. Studies of such consequences have placed useful constraints on models of the early Universe and, more positively, evaporating PBHs have been invoked to explain certain features: for example, the extragalactic [4] and Galactic [5] γ -ray backgrounds, antimatter in cosmic rays [6], the annihilation line radiation from the Galactic centre [7], the reionisation of the pregalactic medium [8, 9] and some short-period γ -ray bursts [10]. However, there are usually other possible explanations for these features, so there is no definitive evidence for evaporating PBHs. Only the original papers for each topic are cited here and a more comprehensive list of references can be found in Ref. [3].

Attention has therefore shifted to the PBHs larger than 10^{15} g, which are unaffected by Hawking radiation. Such PBHs might have various astrophysical consequences, such as providing seeds for the supermassive black holes in galactic nuclei [11], the generation of large-scale structure through Poisson fluctuations [12] and important effects on the thermal and ionisation history of the Universe [13, 14]. Again only the original papers are cited here. But perhaps the most exciting possibility — and the main focus of this review — is that they could provide the dark matter (DM) which comprises 25% of the critical density [15], an idea that goes back to the earliest days of PBH research [16]. Since PBHs formed in the radiation-dominated era, they are not subject to the well-known big bang nucleosynthesis (BBN) constraint that baryons can have at most 5% of the critical density [17]. They should therefore be classified as non-baryonic and behave like any other form of cold dark matter (CDM) [18]. It is sometimes assumed that they must form before BBN, corresponding to an upper limit of $10^5 M_{\odot}$, but this need not be true since the fraction of the Universe in PBHs at that time would be tiny, so the effect on BBN would be small.

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As with other CDM candidates, there is still no compelling evidence that PBHs provide the dark matter. There have been claims of evidence for PBH dark matter from dynamical and lensing effects. In particular, there was also a flurry of excitement in 1997, when the MACHO microlensing results [19] suggested that the dark matter could be in compact objects of mass $0.5 M_\odot$. Alternative dark-matter candidates could be excluded and PBHs of this mass might naturally form at the quark-hadron phase transition at 10^{-5} s [20]. Subsequently, however, it was shown that such objects could comprise only 20% of the dark matter and indeed the entire mass range $10^{-7} M_\odot$ to $10 M_\odot$ was later excluded from providing the dark matter [21]. In recent decades attention has focused on other mass ranges in which PBHs could provide the dark matter and numerous constraints allow only three possibilities: (A) the asteroid mass range ($10^{16} - 10^{17}$ g); (B) the sublunar mass range ($10^{20} - 10^{26}$ g); and (C) the intermediate mass black-hole range ($10 - 10^3 M_\odot$).

We discuss the constraints on $f(M)$, the fraction of the halo in PBHs of mass M , in Sec. III and this is a much reduced version of the recent review by Carr *et al.* [22]. The results are summarised in Fig. 1, all the limits assuming that the PBHs have a monochromatic mass function and cluster in the Galactic halo in the same way as other forms of CDM. Although included for completeness, we will not be interested in the evaporating PBHs in this article, except in so much as they may leave stable Planck-mass relics and these could be possible dark-matter candidates. However, it is worth stressing that there is still no direct evidence for Hawking radiation and, if it was wrong for some reason, PBHs could provide the dark matter all the way down to the Planck mass, with very few non-gravitational constraints.

At first sight, the implication of this figure is that PBHs are excluded from having an appreciable density in every mass range. However, the intention is not to put nails in the coffin of the PBH scenario because every constraint is a potential signature. In particular, there are still some mass ranges in which PBHs could provide *most* of the dark matter. The PBHs in either scenario A or B could be generated by inflation but theorists are split as to which window they favour. For example, Inomata *et al.* [23] argue that double inflation can produce a peak at around 10^{20} g, while Clesse and García-Bellido [24] argue that hybrid inflation can produce a peak at around $10 M_\odot$. A peak at this mass could also be produced by a reduction in the pressure at the quark-hadron phase transition [25], even if the primordial fluctuations have no feature on that scale. There is a parallel here with the search for particle dark matter, where there is also a split between groups searching for light and heavy candidates.

It should be stressed that non-evaporating PBHs are dark even if they do not solve *the* dark-matter problem, so the title of this review is deliberately ambiguous and we do not focus exclusively on the proposal that PBHs provide the dark matter. Many objects are dark, so it is not implausible that the dark matter comprises some mixture of PBHs, WIMPs and MACHOs. Indeed, we will see that a mixture of PBHs and WIMPs has interesting consequences for both of them. Even if PBHs provide only a small fraction of the dark matter, they may still be of great cosmological interest. For example, they could play a rôle in generating the supermassive black holes in galactic nuclei and these have obvious astrophysical significance even though they provide only 0.1% of the dark matter.

The constraints discussed above assume that the PBH mass function is monochromatic (i.e. with a width $\Delta M \sim M$). However, there are many scenarios in which one would expect the mass function to be extended. For example, inflation tends to produce a lognormal mass function [26] and critical collapse generates an extended low mass tail [27–29]. In the context of the dark-matter problem, this is a two-edged sword [15]. On the one hand, it means that the *total* PBH density may suffice to explain the dark matter, even if the density in any particular mass band is small and within the observational bounds discussed above. On the other hand, even if PBHs can provide all the dark matter at some mass scale, the extended mass function may still violate the constraints at some other scale. This issue has been addressed in a number of recent papers [30, 31], though with somewhat different conclusions.

The proposal that the dark matter could comprise PBHs in the intermediate mass range has attracted much attention recently as a result of the LIGO detections of merging binary black holes with mass around $30 M_\odot$ [32, 33]. Since the black holes are larger than initially expected, it has been suggested that they could represent a new population. One possibility is that they were of Population III origin (i.e. forming between decoupling and galaxy formation). The suggestion that LIGO might detect gravitational-waves (GWs) from coalescing intermediate mass Population III black holes was first made more than 30 years ago [34] and — rather remarkably — Kinugawa *et al.* predicted a Population III coalescence peak at $30 M_\odot$ shortly before the first LIGO detection [35]. Another possibility — more relevant to the present considerations — is that the LIGO black holes were primordial, as first discussed in Ref. [36]. This does not necessarily require the PBHs to provide *all* the dark matter. While several authors have made this connection [37, 38], the predicted merger rate depends on when the binaries form and uncertain astrophysical factors, so others argue that the dark-matter fraction could be small [39]. Indeed the LIGO

results have been used to constrain the PBH dark-matter fraction [40, 41]. Note that the PBH density should peak at a lower mass than the coalescence signal for an extended PBH mass function, since the gravitational-waves amplitude scales as the black-hole mass.

The plan of this review paper is as follows: In Sec. II we elaborate on several aspects of PBH formation, including a general discussion of their mass and density, a review of PBH formation scenarios, and a consideration of the effects of non-Gaussianity and non-sphericity. In Sec. III we review current constraints on the density of PBH with a monochromatic mass function, these being associated with a variety of lensing, dynamical, accretion and gravitational-wave effects. At first sight, these seem to exclude PBHs providing the dark matter in *any* mass range but this conclusion may be avoided for an extended mass function and most limits are subject to caveats anyway. More positively, in Sec. IV we overview various observational conundra which can be explained by PBHs, especially those associated with intermediate mass and supermassive black holes. In Sec. V we discuss how the thermal history of the Universe naturally provides peaks in the PBH mass function at the mass scales associated with these conundra, the bumpy mass function obviating some of the limits discussed in Sec. III. We also present a recently-developed mechanism which illuminates the long-standing fine-tuning problem associated with PBH formation. In Sec. VI we discuss scenarios which involve a mixture of PBHs and particle dark matter. In Sec. VII we draw some general conclusions about PBHs as dark matter.

II. PBH FORMATION

PBHs could have been produced during the early Universe due to various mechanisms. For all of these, the increased cosmological energy density at early times plays a major rôle [42, 43], yielding a rough connection between the PBH mass and the horizon mass at formation:

$$M \sim \frac{c^3 t}{G} \sim 10^{15} \left(\frac{t}{10^{-23} \text{ s}} \right) \text{ g}. \quad (\text{II.1})$$

Hence PBHs could span an enormous mass range: those formed at the Planck time (10^{-43} s) would have the Planck mass (10^{-5} g), whereas those formed at 1 s would be as large as $10^5 M_\odot$, comparable to the mass of the holes thought to reside in galactic nuclei. By contrast, black holes forming at the present epoch (eg. in the final stages of stellar evolution) could never be smaller than about $1 M_\odot$. In some circumstances PBHs may form over an extended period, corresponding to a wide range of masses. Even if they form at a single epoch, their mass spectrum could still extend much below the horizon mass due to “critical phenomena” [28, 29, 44–46], although most of the PBH density would still be in the most massive ones.

A. Mass and Density Fraction of PBHs

The fraction of the mass of the Universe in PBHs on some mass scale M is epoch-dependent but its value at the formation epoch of the PBHs is denoted by $\beta(M)$. For the standard Λ CDM model, in which the age of the Universe is $t_0 = 13.8$ Gyr, the Hubble parameter is $h = 0.68$ [47] and the time of photon decoupling is $t_{\text{dec}} = 380$ kyr [48]. If the PBHs have a monochromatic mass function, the fraction of the Universe’s mass in PBHs at their formation time t_i is related to their number density $n_{\text{PBH}}(t_i)$ by [3]

$$\beta(M) \equiv \frac{M n_{\text{PBH}}(t_i)}{\rho(t_i)} \approx 7.98 \times 10^{-29} \gamma^{-1/2} \left(\frac{g_{*i}}{106.75} \right)^{1/4} \left(\frac{M}{M_\odot} \right)^{3/2} \left(\frac{n_{\text{PBH}}(t_0)}{1 \text{ Gpc}^{-3}} \right), \quad (\text{II.2})$$

where $\rho(t_i)$ is the density at time t_i and γ is the ratio of the PBH mass to the horizon mass. g_{*i} is the number of relativistic degrees of freedom at PBH formation, normalised to its value at 10^{-5} s since it does not increase much before that in the Standard Model and this is the period in which most PBHs are likely to form.

The current density parameter for PBHs which have not yet evaporated is

$$\Omega_{\text{PBH}} = \frac{M n_{\text{PBH}}(t_0)}{\rho_{\text{crit}}} \approx \left(\frac{\beta(M)}{1.03 \times 10^{-8}} \right) \left(\frac{h}{0.68} \right)^{-2} \gamma^{1/2} \left(\frac{g_{*i}}{106.75} \right)^{-1/4} \left(\frac{M}{M_\odot} \right)^{-1/2}, \quad (\text{II.3})$$

where ρ_{crit} is critical density. Equation (II.3) can be expressed in terms of the ratio of the current PBH mass density to that of the CDM density:

$$f \equiv \frac{\Omega_{\text{PBH}}}{\Omega_{\text{CDM}}} \approx 3.8 \Omega_{\text{PBH}} \approx 2.4 \beta_{\text{eq}}, \quad (\text{II.4})$$

where β_{eq} is the PBH mass fraction at matter-radiation equality and we use the most recent value $\Omega_{\text{CDM}} = 0.26$ indicated by Planck [49]. The ratio of the energy densities of matter and radiation (all relativistic species) at any time is

$$\frac{\Omega_{\text{M}}}{\Omega_{\text{R}}} = \frac{\Omega_{\text{B}} + \Omega_{\text{CDM}}}{\Omega_{\text{R}}} \approx \frac{1700}{g_*(z)} \frac{1 + \chi}{1 + z}, \quad (\text{II.5})$$

where $\chi \equiv \Omega_{\text{CDM}}/\Omega_{\text{B}} \approx 5$ is the ratio of the dark matter and baryonic densities. At PBH formation, the fraction of domains that collapse is

$$\beta \equiv f_{\text{PBH}} \frac{\chi \Omega_{\text{B}}}{\Omega_{\text{R}}} \simeq f_{\text{PBH}} \frac{\chi \eta}{g_*(T)} \frac{0.7 \text{ GeV}}{T}, \quad (\text{II.6})$$

where $\eta = n_{\text{B}}/n_{\gamma} = 6 \times 10^{-10}$ is the observed baryon-to-photon ratio (i.e. the baryon asymmetry prior to 10^{-5}s). As discussed later, this relationship suggests a scenario in which baryogenesis is linked with PBH formation, with the smallness of the η reflecting the rarity of the Hubble domains that collapse [50]. The collapse fraction can also be expressed as

$$\beta \approx 0.5 f_{\text{tot}} \left[\chi \gamma^{-1/2} \eta g_*^{1/4} \right] \left(\frac{M}{M_{\odot}} \right)^{1/2}, \quad (\text{II.7})$$

where f_{tot} is the total dark-matter fraction and the square-bracketed term has a value of order 10^{-9} .

B. Formation Scenarios

We now review the large number of scenarios which have been proposed for PBH formation and the associated PBH mass functions. We have seen that PBHs generally have a mass of order the horizon mass at formation, so one might expect a monochromatic mass function (i.e. with a width $\Delta M \sim M$). However, in some scenarios PBHs form over a prolonged period and therefore have an extended mass function (eg. with its form of the mass function depending on the power spectrum of the primordial fluctuations). As we will see, even PBHs formed at a single epoch may have an extended mass function.

1. Primordial Inhomogeneities

The most natural possibility is that PBHs form from primordial density fluctuations. Overdense regions will then stop expanding some time after they enter the particle horizon and collapse against the pressure if they are larger than the Jeans mass. If the horizon-scale fluctuations have a Gaussian distribution with dispersion σ , one expects for the fraction of horizon patches collapsing to a black hole to be [51]

$$\beta \approx \text{Erfc} \left[\frac{\delta_{\text{c}}}{\sqrt{2} \sigma} \right]. \quad (\text{II.8})$$

Here ‘Erfc’ is the complementary error function and δ_{c} is the density-contrast threshold for PBH formation. In a radiation-dominated era, a simple analytic argument [51] suggests $\delta_{\text{c}} \approx 1/3$, but more precise arguments — both numerical [28] and analytical [52] — suggest a somewhat larger value: $\delta_{\text{c}} = 0.45$. Note that this value is sensitive to any non-Gaussianity [53], the shape of the perturbation profile [54] and the equation of state of the medium (cf. Ref. [55]).

2. Collapse from Scale-Invariant Fluctuations

If the PBHs form from scale-invariant fluctuations (i.e. with constant amplitude at the horizon epoch), their mass spectrum should have the power-law form [51]

$$\frac{dn}{dM} \propto M^{-\alpha} \quad \text{with} \quad \alpha = \frac{2(1+2\gamma)}{1+\gamma}, \quad (\text{II.9})$$

where γ specifies the equation of state ($p = \gamma \rho c^2$) at PBH formation. The exponent arises because the background density and PBH density have different redshift dependencies. At one time it was argued that the primordial fluctuations would be *expected* to be scale-invariant [56, 57] but this does not apply in the inflationary scenario. Nevertheless, one would still expect the above equations to apply if the PBHs form from cosmic loops because the collapse probability is then scale-invariant. If the PBHs contain a fraction f_{DM} of the dark matter, this implies that the fraction of the dark matter in PBHs of mass larger than M is

$$f(M) \approx f_{\text{DM}} \left(\frac{M_{\text{DM}}}{M} \right)^{\alpha-2} \quad (M_{\text{min}} < M < M_{\text{max}}), \quad (\text{II.10})$$

where $2 < \alpha < 3$, and $M_{\text{DM}} \approx M_{\text{min}}$ is the mass scale which contains most of the dark matter. In a radiation-dominated era, the exponent in Eq. (II.10) becomes 1/2.

3. Collapse in a Matter-Dominated Era

PBHs form more easily if the Universe becomes pressureless (i.e. matter-dominated) for some period. For example, this may arise at a phase transition in which the mass is channeled into non-relativistic particles [58] or due to slow reheating after inflation [59, 60]. In a related context, Hidalgo *et al.* have recently studied [61] PBH formation in a dust-like scenario of an oscillating scalar field during an extended period of preheating. Since the value of α in the above analysis is 2 for $\gamma = 0$, one might expect $\rho(M)$ to increase logarithmically with M . However, the analysis breaks down in this case because the Jeans length is much smaller than the particle horizon, so pressure is not the main inhibitor of collapse. Instead, collapse is prevented by deviations from spherical symmetry and the probability of PBH formation can be shown to be [58]

$$\beta(M) = 0.02 \delta_{\text{H}}(M)^5. \quad (\text{II.11})$$

This is in agreement with the recent analysis of Harada *et al.* [62] and leads to a mass function

$$\frac{dn}{dM} \propto M^{-2} \delta_{\text{H}}(M)^5. \quad (\text{II.12})$$

The collapse fraction $\beta(M)$ is still small for $\delta_{\text{H}}(M) \ll 1$ but much larger than the exponentially suppressed fraction in the radiation-dominated case. If the matter-dominated phase extends from t_1 to t_2 , PBH formation is enhanced over the mass range

$$M_{\text{min}} \sim M_{\text{H}}(t_1) < M < M_{\text{max}} \sim M_{\text{H}}(t_2) \delta_{\text{H}}(M_{\text{max}})^{3/2}. \quad (\text{II.13})$$

The lower limit is the horizon mass at the start of matter-dominance and the upper limit is the horizon mass when the regions which bind at the end of matter-dominance enter the horizon. This scenario has recently been studied in Ref. [63].

4. Collapse from Inflationary Fluctuations

If the fluctuations generated by inflation have a blue spectrum (i.e. decrease with increasing scale) and the PBHs form from the high- σ tail of the fluctuation distribution, then the exponential factor in Eq. (II.8) might suggest that the PBH mass function should have an exponential upper cut-off at the horizon mass when inflation ends. This corresponds to the reheat time t_{R} , which the cosmic microwave background

(CMB) quadrupole anisotropy requires to exceed 10^{-35} s, so this places a lower limit of around 1 g on the mass of such PBHs. The first inflationary scenarios for PBH formation were proposed in Refs. [64] and subsequently there have been a huge number of papers on this topic. In some scenarios, the PBHs form from a smooth symmetric peak in the inflationary power spectrum, in which case the PBH mass function should have the lognormal form:

$$\frac{dn}{dM} \propto \frac{1}{M^2} \exp\left[-\frac{(\log M - \log M_c)^2}{2\sigma^2}\right]. \quad (\text{II.14})$$

This was first suggested by Dolgov & Silk [26] (see also Refs. [24, 65]) and has been demonstrated both numerically [30] and analytically [66] for the case in which the slow-roll approximation holds. It is therefore representative of a large class of inflationary scenarios, including the axion-curvaton and running-mass inflation models considered by Kühnel *et al.* [29]. Equation (II.14) implies that the mass function is symmetric about its peak at M_c and described by two parameters: the mass scale M_c itself and the width of the distribution σ . The integrated mass function is

$$f(M) = \int_M d\tilde{M} \tilde{M} \frac{dn}{d\tilde{M}} \approx \text{erfc}\left(\ln \frac{M}{\sigma}\right). \quad (\text{II.15})$$

However, not all inflationary scenarios produce the mass function (II.14). Inomata *et al.* [67] propose a scenario which combines a broad mass function at low M (to explain the dark matter) with a sharp one at high m (to explain the LIGO events).

5. Quantum Diffusion

Most of the relevant inflationary dynamics happens in regimes in which the classical inflaton-field evolution dominates over the field's quantum fluctuations. Under certain circumstances, however, the situation is reversed. There are two cases for which this happens. The first applies at larger values of the inflaton potential $V(\varphi)$, yielding eternally expanding patches of the Universe [68]. The second applies when the inflaton potential possesses one or more plateau-like features. Classically, using the slow-roll conditions, $|\dot{\varphi}| \ll 3H|\dot{\varphi}|$, $(\dot{\varphi})^2 \ll 2V(\varphi)$, where an overdot represents a derivative with regard to cosmic time t , $H \equiv \dot{a}/a$ is the Hubble parameter and a is the scale factor, the number of inflationary e-folds is $N = \int d\varphi H/\dot{\varphi}$, which implies $\delta\varphi_C = \dot{\varphi}/H$. On the other hand, the corresponding quantum fluctuations are $\delta\varphi_Q = H/2\pi$. Since the primordial metric perturbation is

$$\zeta = \frac{H}{\dot{\varphi}} \delta\varphi = \frac{\delta\varphi_Q}{\delta\varphi_C}, \quad (\text{II.16})$$

quantum effects are expected to be important whenever this quantity becomes of order one, i.e. $\zeta \sim \mathcal{O}(1)$. This is often the case for PBH formation, where recent investigations indicate an increase of the power spectrum and hence the PBH abundance [69]. This quantum diffusion is inherently non-perturbative and so Kühnel & Freese [70] have developed a dedicated resummation technique in order to incorporate all higher-order corrections [70]. Ezquiaga *et al.* have argued that quantum diffusion generically generates a high degree of non-Gaussianity [71].

6. Critical Collapse

It is well known that black-hole formation is associated with critical phenomena [72] and the application of this to PBH formation has been studied by various authors [29, 46, 73, 74]. The conclusion is that the mass function has an upper cut-off at around the horizon mass but there is also a low-mass tail [75]. If we assume for simplicity that the density fluctuations have a monochromatic power spectrum on some mass scale K and identify the amplitude of the density fluctuation when that scale crosses the horizon, δ , as the control parameter, then the black-hole mass is [72]

$$M = K (\delta - \delta_c)^\eta. \quad (\text{II.17})$$

Here K can be identified with a mass M_f of order the particle horizon mass, δ_c is the critical fluctuation required for PBH formation and the exponent η has a universal value for a given equation of state. For $\gamma = 1/3$, one has $\delta_c \approx 0.4$ and $\eta \approx 0.35$. Although the scaling relation (II.17) is expected to be valid only in the immediate neighborhood of δ_c , most black holes should form from fluctuations with this value because the probability distribution function declines exponentially beyond $\delta = \delta_c$ if the fluctuations are blue. Hence it is sensible to calculate the expected PBH mass function using Eq. (II.17). This allows us to estimate the mass function independently of the form of the probability distribution function of the primordial density fluctuations. A detailed calculation gives the mass function [75]

$$\frac{dn}{dM} \propto \left(\frac{M}{\xi M_f} \right)^{1/\eta-1} \exp \left[- (1-\eta) \left(\frac{M}{\eta M_f} \right)^{1/\eta} \right], \quad (\text{II.18})$$

where $\xi \equiv (1 - \eta/s)^\eta$, $s = \delta_c/\sigma$, $M_f = K$ and σ is the dispersion of δ . The above analysis depends on the assumption that the power spectrum of the primordial fluctuations is monochromatic. As shown by Kühnel *et al.* [29] for a variety of inflationary models, when a realistic model for the power spectrum is used, the inclusion of critical collapse can lead to a significant shift, lowering and broadening of the PBH mass spectra — in some cases by several orders of magnitude.

7. Collapse at QCD Phase Transition

At one stage it was thought that the QCD phase transition at 10^{-5} s might be first-order. This would mean that the quark-gluon plasma and hadron phases could coexist, with the cosmic expansion proceeding at constant temperature by converting the quark-gluon plasma to hadrons. The sound-speed would then vanish and the effective pressure would be reduced, significantly lowering the threshold δ_c for collapse. PBH production during a first-order QCD phase transitions was first suggested by Crawford & Schramm [76] and later revisited by Jedamzik [77]. The amplification of density perturbations due to the vanishing of the speed of sound during the QCD transition was also considered by Schmid and colleagues [78], while Cardall & Fuller developed a semi-analytic approach for PBH production during the transition [79]. It is now thought unlikely that the QCD transition is 1st order but one still expects some softening in the equation of state. Recently Byrnes *et al.* [25] have discussed how this softening – when combined with critical phenomena and the exponential sensitivity of $\beta(M)$ to the equation of state – could produce a significant change in the mass function. The mass of a PBH forming at the QCD epoch is

$$M = \frac{\gamma \xi^2}{g_*^{1/2}} \left(\frac{45}{16\pi^3} \right)^{1/2} \frac{M_p^3}{m_p^2} \approx 0.9 \left(\frac{\gamma}{0.2} \right) (g_*)^{-1/2} \left(\frac{\xi}{5} \right)^2 M_\odot, \quad (\text{II.19})$$

where M_p is the Planck mass, m_p is the proton mass, g_* is normalised appropriately and $\xi \equiv m_p/(k_b T) \approx 5$ is the ratio of the proton mass to the QCD transition temperature. This is necessarily close to the Chandrasekhar mass:

$$M_{\text{Ch}} = \frac{\omega}{\tilde{\mu}^2} \left(\frac{3\pi}{4} \right)^{1/2} \frac{M_p^3}{m_p^2} \simeq 5.6 \tilde{\mu}^{-2} M_\odot, \quad (\text{II.20})$$

where $\omega = 2.018$ is a constant that appears in the solution of the Lane-Emden equation and $\tilde{\mu}$ is the number of electrons per nucleon (1 for hydrogen, 2 for helium). The two masses are very close for the relevant parameter choices. Since all stars have a mass in the range $(0.1 - 10) M_{\text{Ch}}$, this has the interesting consequence that dark and visible objects have comparable masses. From Eq. (II.7) it is also interesting that the collapse fraction at the QCD epoch is

$$\beta \approx 0.4 f^{\text{tot}} \chi \eta \xi \approx 10 \eta, \quad (\text{II.21})$$

where we assumed $f^{\text{tot}} \approx 1$ and $\chi \approx 5.5$ at the last step. This result is easily understood since one necessarily has $\rho_b/\rho_\gamma \sim \eta$ at the QCD epoch. We exploit this result in Sec. V by suggesting that the collapse fraction determines the baryon-asymmetry.

8. Collapse of Cosmic Loops

In the cosmic string scenario, one expects some strings to self-intersect and form cosmic loops. A typical loop will be larger than its Schwarzschild radius by the factor $(G\mu)^{-1}$, where μ is the string mass per unit length. If strings play a rôle in generating large-scale structure, $G\mu$ must be of order 10^{-6} . However, as discussed by many authors [80, 81], there is always a small probability that a cosmic loop will get into a configuration in which every dimension lies within its Schwarzschild radius. This probability depends upon both μ and the string correlation scale. Note that the holes form with equal probability at every epoch, so they should have an extended mass spectrum with [81]

$$\beta \sim (G\mu)^{2x-4}, \tag{II.22}$$

where $x \equiv L/s$ is the ratio of the string length to the correlation scale. One expects $2 < x < 4$ and requires $G\tilde{\mu} < 10^{-7}$ to avoid overproduction of PBHs.

9. Collapse through Bubble Collisions

Bubbles of broken symmetry might arise at any spontaneously broken symmetry epoch and various people have suggested that PBHs could form as a result of bubble collisions [76, 82]. However, this happens only if the bubble-formation rate per Hubble volume is finely tuned: if it is much larger than the Hubble rate, the entire Universe undergoes the phase transition immediately and there is not time to form black holes; if it is much less than the Hubble rate, the bubbles are very rare and never collide. The holes should have a mass of order the horizon mass at the phase transition, so PBHs forming at the GUT epoch would have a mass of 10^3 g, those forming at the electroweak unification epoch would have a mass of 10^{28} g, and those forming at the QCD (quark-hadron) phase transition would have mass of around $1 M_\odot$. There could also be wormhole production at a 1st-order phase transition [83]. The production of PBHs from bubble collisions at the end of first-order inflation has been studied extensively by Khlopov and his colleagues [84].

10. Collapse of Domain Walls

The collapse of sufficiently large closed domain walls produced at a 2nd-order phase transition in the vacuum state of a scalar field, such as might be associated with inflation, could lead to PBH formation [85]. These PBHs would have a small mass for a thermal phase transition with the usual equilibrium conditions. However, they could be much larger if one invoked a non-equilibrium scenario [86]. Indeed Khlopov *et al.* argue that they could span a wide range of masses, with a fractal structure of smaller PBHs clustered around larger ones. Vilenkin and colleagues have argued that bubbles formed during inflation would (depending on their size) form either black holes or baby universes connected to our Universe by wormholes [87]. In this case, the PBH mass function would be very broad and extend to very high masses [88].

C. Non-Gaussianity and Non-Sphericity

As PBHs form from the extreme high-density tail of the spectrum of fluctuations, their abundance is acutely sensitive to non-Gaussianities in the density-perturbation profile [89]. For certain models — such as the hybrid waterfall or simple curvaton models [90] — it has even been shown that no truncation of non-Gaussian parameters can be made to the model without changing the estimated PBH abundance [89]. However, non-Gaussianity induced PBH production can have serious consequences for the viability of PBH dark matter. PBHs produced with non-Gaussianity lead to isocurvature modes that could be detected in the CMB [91]. With the current Planck exclusion limits [47], this implies that the non-Gaussianity parameters f_{NL} and g_{NL} for a PBH-producing theory are both less than $\mathcal{O}(10^{-3})$. For theories like the curvaton and hybrid inflation models [24, 92], this leads to the immediate exclusion of PBH dark matter, as the isocurvature effects would be too large.

Non-sphericity has not yet been subject to extensive numerical studies of the kind in Ref. [28] but Ref. [54] shows that non-zero ellipticity leads to possibly large effects on PBH mass spectra. It gives an approximate analytical approximation for the collapse threshold, which will be larger than in the spherical case,

$$\delta_{ec}/\delta_c \simeq 1 + \kappa \left(\frac{\sigma^2}{\delta_c^2} \right)^{\tilde{\gamma}}, \quad (\text{II.23})$$

with δ_c being the threshold value for spherical collapse, σ^2 the amplitude of the density power spectrum at the given scale, $\kappa = 9/\sqrt{10\pi}$ and $\tilde{\gamma} = 1/2$. Note that Ref. [93] had already obtained this result for a limited class of cosmologies but this did not include the case of ellipsoidal collapse in a radiation-dominated model. A thorough numerical investigation is still needed to precisely determine the change of the threshold for fully relativistic non-spherical collapse. Note also that the effect due to non-sphericities is partly degenerate with that of non-Gaussianities [54].

D. Multi-Spike Mass Functions

If one is interested in PBHs explaining several observational conundra, it is pertinent to consider the possibility that the PBH mass spectrum has several spikes. There are two known recent mechanisms for generating such spikes. The first has been proposed by Cai *et al.* [94], who have discussed a new type of resonance effect which leads to prolific PBH formation. This arises because the sound-speed can oscillate in some inflationary scenarios, leading to parametric amplification of the curvature perturbation and hence a significant peak in the power spectrum of the density perturbations on some critical scale. The resonances are in narrow bands around certain harmonic frequencies with one of the peaks dominating. It turns out, one can easily get a peak of order unity. Although most PBHs form at the first peak, a small number will also form at subsequent peaks. The second mechanism for generating multi-spiked PBH mass spectra has recently been proposed by Carr & Kühnel [95] and has been demonstrated for most of the well-studied models of PBH formation. This mechanism relies on the choice of non-Bunch-Davies vacua, leading to oscillatory features in the inflationary power spectrum, which in turn generates oscillations in the PBH mass function with exponentially enhanced spikes.

III. CONSTRAINTS AND CAVEATS

We now review the various constraints for PBHs which are too large to have evaporated completely by now, updating the equivalent discussion in Refs. [3] and [15]. All the limits assume that PBHs cluster in the Galactic halo in the same way as other forms of CDM, unless they are so large that there is less than one per galaxy. Throughout this section the PBHs are taken to have a monochromatic mass function, in the sense that they span a mass range $\Delta M \sim M$. In this case, the fraction $f(M)$ of the halo in PBHs is related to $\beta(M)$ by Eq. (II.4). Our limits on $f(M)$ are summarised in Fig. 1, which is based on Fig. 10 of Ref. [22]. The main constraints derive from PBH evaporations, various gravitational-lensing experiments, numerous dynamical effects and PBH accretion. Where there are several limits in the same mass range, we usually show only the most stringent one. It must be stressed that the constraints in Fig. 1 have varying degrees of certainty and they all come with caveats. For some, the observations are well understood but there are uncertainties in the black-hole physics. For others, the observations themselves are not fully understood or depend upon additional astrophysical assumptions. The constraints may also depend on other physical parameters which are not shown explicitly. It is important to stress that some of the constraints can be circumvented if the PBHs have an extended mass function. Indeed, as discussed in Sec. V, this may be *required* if PBHs are to provide most of the dark matter.

A. Evaporation Constraints

A PBH of initial mass M will evaporate through the emission of Hawking radiation on a timescale $\tau \propto M^3$ which is less than the present age of the Universe for M below $M_* \approx 5 \times 10^{14}$ g [98]. There is a strong constraint on $f(M_*)$ from observations of the extragalactic γ -ray background [4]. PBHs in the narrow band $M_* < M < 1.005 M_*$ have not yet completed their evaporation but their current mass is below

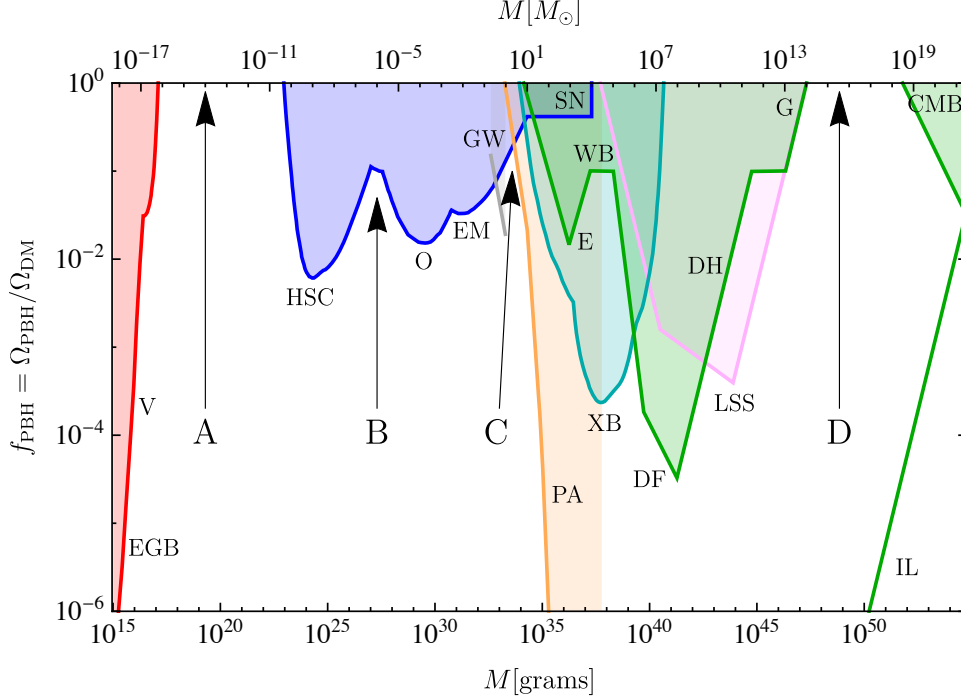


FIG. 1: Constraints on $f(M)$ for a monochromatic mass function, based in part on Ref. [22], from evaporations (red), lensing (blue), gravitational waves (GW) (gray), dynamical effects (green), accretion (light blue), CMB distortions (orange) and large-scale structure (purple). Evaporation limits come from the extragalactic gamma-ray background (EGB), the Voyager positron flux (V) and annihilation-line radiation from the Galactic centre (GC). Lensing limits come from microlensing of supernovae (SN) and of stars in M31 by Subaru (HSC), the Magellanic Clouds by EROS and MACHO (EM) and the Galactic bulge by OGLE (O). Dynamical limits come from wide binaries (WB), star clusters in Eridanus II (E), halo dynamical friction (DF), galaxy tidal distortions (G), heating of stars in the Galactic disk (DH) and the CMB dipole (CMB). Large-scale structure constraints derive from the requirement that various cosmological structures do not form earlier than observed (LSS). Accretion limits come from X-ray binaries (XB) and Planck measurements of CMB distortions (PA). The incredulity limits (IL) correspond to one PBH per relevant environment (galaxy, cluster, Universe). There are four mass windows (A, B, C, D) in which PBHs could have an appreciable density. Possible constraints in window D are discussed in the text but not in the past literature.

the mass $M_q \approx 0.4 M_*$ at which quark and gluon jets are emitted. For $M > 2 M_*$, one can neglect the change of mass altogether and the time-integrated spectrum of photons from each PBH is obtained by multiplying the instantaneous spectrum by the age of the Universe t_0 . The instantaneous spectrum is

$$\frac{d\dot{N}_\gamma^P}{dE}(M, E) \propto \frac{E^2 \sigma(M, E)}{e^{EM} - 1} \propto \begin{cases} E^3 M^3 & (E < M^{-1}) \\ E^2 M^2 e^{-EM} & (E > M^{-1}), \end{cases} \quad (\text{III.1})$$

where $\sigma(M, E)$ is the absorption cross-section for photons of energy E , so this gives an intensity

$$I(E) \propto f(M) \times \begin{cases} E^4 M^2 & (E < M^{-1}) \\ E^3 M e^{-EM} & (E > M^{-1}). \end{cases} \quad (\text{III.2})$$

This peaks at $E^{\max} \propto M^{-1}$ with a value $I^{\max}(M) \propto f(M) M^{-2}$, whereas the observed intensity is $I^{\text{obs}} \propto E^{-(1+\epsilon)}$ with ϵ between 0.1 and 0.4, so putting $I^{\max}(M) \leq I^{\text{obs}}(M(E))$ gives [3]

$$f(M) \lesssim 2 \times 10^{-8} \left(\frac{M}{M_*} \right)^{3+\epsilon} \quad (M > M_*). \quad (\text{III.3})$$

We plot this constraint in Fig. 1 for $\epsilon = 0.2$. The Galactic γ -ray background constraint could give a stronger limit [98] but this depends sensitively on the form of the PBH mass function, so we do not discuss it here.

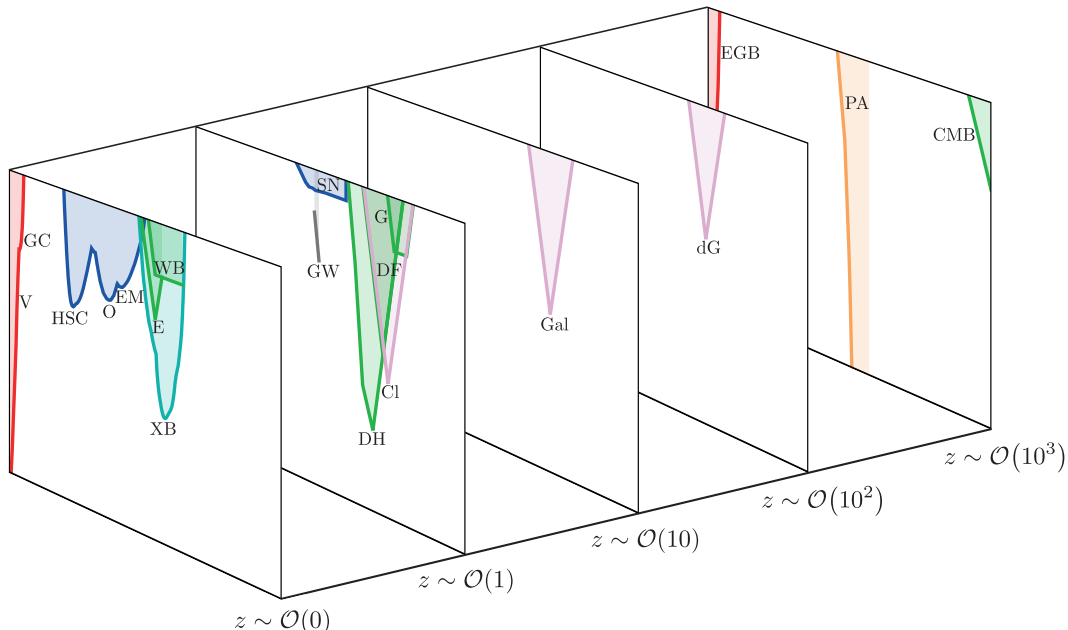


FIG. 2: Sketch of the limits shown in Fig. 1 for different redshifts, inspired by Ref. [96]. Here, we break down the large-scale structure limit into its individual components from clusters (Cl), Milky Way galaxies (Gal) and dwarf galaxies (dG), as these originate from different redshifts (cf. Ref.[97]).

There are various other evaporation constraints in this mass range. Boudad and Cirelli [99] use positron data from Voyager 1 in order to constrain evaporating PBHs of mass $M < 10^{16}$ g and obtain the bound $f < 0.001$. This complements the cosmological limit as it is based on local Galactic measurements. DeRocco and Graham [100] constrain $10^{16} - 10^{17}$ g PBHs using measurements of the 511 keV annihilation line radiation from the Galactic centre. The ionising effects of $10^{16} - 10^{17}$ g PBHs are also associated with interesting constraints [9].

B. Lensing Constraints

Constraints on MACHOs with very low M have been claimed from the femtolensing of γ -ray bursts. Assuming the bursts are at a redshift $z \sim 1$, early studies implied $f < 1$ in the mass range $10^{-16} - 10^{-13} M_{\odot}$ [101] and $f < 0.1$ in the range $10^{-17} - 10^{-14} M_{\odot}$ [102]. However, Katz *et al.* [103] argue that most GRBs are too large for these limits to apply, so we do not show them in Fig. 1.

Kepler data from observations of Galactic sources [104] imply a limit in the planetary mass range: $f(M) < 0.3$ for $2 \times 10^{-9} M_{\odot} < M < 10^{-7} M_{\odot}$. However, Niikura *et al.* [105] have carried out a seven-hour observation of M31 with the Subaru Hyper Suprime-Cam to search for microlensing of stars in M31 by PBHs lying in the halo regions of the Milky Way and M31 and obtain the much more stringent bound for $10^{-10} < M < 10^{-6} M_{\odot}$ which is shown in Fig. 1.

Microlensing observations of stars in the Large and Small Magellanic Clouds probe the fraction of the Galactic halo in MACHOs in a certain mass range [106]. The optical depth of the halo towards LMC and SMC is related to the fraction $f(M)$ by $\tau_{\text{L}}^{\text{(SMC)}} = 1.4 \tau_{\text{L}}^{\text{(LMC)}} = 6.6 \times 10^{-7} f(M)$ for the standard halo model [107]. The MACHO project detected lenses with $M \sim 0.5 M_{\odot}$ but concluded that their halo contribution could be at most 10% [108], while the EROS project excluded $6 \times 10^{-8} M_{\odot} < M < 15 M_{\odot}$ objects from dominating the halo. Since then further limits in the range $0.1 M_{\odot} < M < 20 M_{\odot}$ have

come from the OGLE experiment [109]. The combined results can be approximated by

$$f(M) < \begin{cases} 1 & (6 \times 10^{-8} M_{\odot} < M < 30 M_{\odot}) \\ 0.1 & (10^{-6} M_{\odot} < M < 1 M_{\odot}) \\ 0.05 & (10^{-3} M_{\odot} < M < 0.4 M_{\odot}) \\ 0.2 & (0.4 M_{\odot} < M < 20 M_{\odot}). \end{cases} \quad (\text{III.4})$$

Recently Niikura *et al.* [110] have used data from a five-year OGLE survey of the Galactic bulge to place much stronger limits in the range $10^{-6} M_{\odot} < M < 10^{-4} M_{\odot}$, although they also claim some positive detections. The precise form of the EROS and OGLE limits are shown in Fig. 1, while the possible detections are discussed further in Sec. IV.

PBHs cause most lines of sight to be demagnified relative to the mean, with a long tail of higher magnifications. Zumalacárregui and Seljak [111] have constrained the PBH model from the lack of lensing in type Ia supernovae (SNe), modelling the effects of large-scale structure and allowing a non-Gaussian model for the intrinsic SNe luminosity distribution. Using current JLA data, they derive a bound $f < 0.35$ for $10^{-2} M_{\odot} < M < 10^4 M_{\odot}$, the finite size of SNe providing the lower limit. García-Bellido & Clesse [112] argue that this limit can be weakened if the PBHs have an extended mass function or are clustered. There is some dispute about the extended case but Fig. 1 is only for a monochromatic mass function anyway.

The recent discovery of fast transient events in massive galaxy clusters is attributed to individual stars in giant arcs being highly magnified due to caustic crossing. Oguri *et al.* [113] argue that the particular event MACS J1149 excludes a high density of PBHs in the mass range $10^{-5} M_{\odot} < M < 10^2 M_{\odot}$ because this would predict too low magnifications. This corresponds to the ‘‘Icarus’’ line in Fig. 1.

Early studies of the microlensing of quasars [114] seemed to exclude all the dark matter being in objects with $10^{-3} M_{\odot} < M < 60 M_{\odot}$, although this limit preceded the Λ CDM picture. More recent studies of quasar microlensing suggest a limit [115] $f(M) < 1$ for $10^{-3} M_{\odot} < M < 60 M_{\odot}$, although we will see that these surveys may also provide positive evidence for PBHs. Millilensing of compact radio sources [116] gives a limit

$$f(M) < \begin{cases} (M/2 \times 10^4 M_{\odot})^{-2} & (M < 10^5 M_{\odot}) \\ 0.06 & (10^5 M_{\odot} < M < 10^8 M_{\odot}) \\ (M/4 \times 10^8 M_{\odot})^2 & (M > 10^8 M_{\odot}). \end{cases} \quad (\text{III.5})$$

Though weaker than the dynamical constraints in this mass range, we include it in Fig. 1 because it illustrates that lensing limits extend to very large values of M .

C. Dynamical Constraints

The effects of PBH collisions on astronomical objects have been a subject of long-standing interest, although we do not show these constraints in Fig. 1. Roncadelli *et al.* [117] have suggested that halo PBHs could be captured and swallowed by stars in the Galactic disc. The stars would eventually be accreted by the holes, producing radiation and a population of subsolar black holes which could only be of primordial origin and this leads to a constraint $f < (M/3 \times 10^{26} \text{ g})$, corresponding to a *lower* limit on the mass. Capela *et al.* have constrained PBH dark matter by considering their capture by white dwarfs [118] or neutron stars [119], while Pani and Loeb [120] have argued that this excludes PBHs from providing the dark matter throughout the sublunar window. However, these limits have been disputed [121] because the dark-matter density in globular clusters is now known to be much lower values than assumed in these analyses [122]. Graham *et al.* [123] argue that the transit of a PBH through a white dwarf (WD) causes localised heating through dynamical friction and initiates runaway thermonuclear fusion, causing the WD to explode as a supernova. The shape of the observed WD distribution rules out $10^{19} - 10^{20} \text{ g}$ PBHs from providing the dark matter and $10^{20} - 10^{22} \text{ g}$ ones are also constrained by the observed supernova rate.

A variety of dynamical constraints come into play at higher mass scales [124]. Many of them involve the destruction of various astronomical objects by the passage of nearby PBHs. If the PBHs have density ρ

and velocity dispersion V , while the objects have mass M_c , radius R_c , velocity dispersion V_c and survival time t_L , then the constraint has the form:

$$f(M) < \begin{cases} M_c V / (G M \rho t_L R_c) & (M < M_c (V/V_c)) \\ M_c / (\rho V_c t_L R_c^2) & (M_c (V/V_c) < M < M_c (V/V_c)^3) \\ M V_c^2 / (\rho R_c^2 V^3 t_L) \exp[(M/M_c)(V_c/V)^3] & (M > M_c (V/V_c)^3). \end{cases} \quad (\text{III.6})$$

The three limits correspond to disruption by multiple encounters, one-off encounters and non-impulsive encounters, respectively. The fraction is thus constrained over the mass range

$$\frac{M_c V}{G \rho_{\text{DM}} t_L R_c} < M < M_c \left(\frac{V}{V_c} \right)^3, \quad (\text{III.7})$$

the limits corresponding to the values of M for which $f = 1$. Various numerical factors are omitted in this discussion. These limits apply providing there is at least one PBH within the relevant environment. For an environment of mass M_E , this corresponds to the condition $f(M) > (M/M_E)$, where M_E is around $3 \times 10^{12} M_\odot$ for halos, $10^{14} M_\odot$ for clusters and $10^{22} M_\odot$ for the Universe.

One can apply this argument to wide binaries in the Galaxy, which are particularly vulnerable to disruption by PBHs [125]. This originally gave a constraint $f(M) < (M/500 M_\odot)^{-1}$ for $500 M_\odot < M < 10^3 M_\odot$ [126] but the upper limit on the mass which dominates the halo has been reduced to $7 - 12 M_\odot$ in more recent work [127], so the narrow window between the microlensing lower bound and the wide-binary upper bound is shrinking. A similar argument for the survival of globular clusters against tidal disruption by passing PBHs gives a limit $f(M) < (M/3 \times 10^4 M_\odot)^{-1}$ for $3 \times 10^4 M_\odot < M < 10^6 M_\odot$, although this depends sensitively on the mass and the radius of the cluster. The upper limit is consistent with the numerical calculations of Moore [128].

In a related argument, Brandt [129] infers an upper limit of $5 M_\odot$ from the fact that a star cluster near the centre of the dwarf galaxy Eridanus II has not been disrupted by halo objects. Koushiappas and Loeb [130] have also studied the effects of black holes on the dynamical evolution of dwarf galaxies. They find that mass segregation leads to a depletion of stars in the centers of such galaxies and the appearance of a ring in the projected stellar surface density profile. Using Segue 1 as an example, they exclude more than 4% of the dark matter being PBHs of a few tens of solar masses.

Halo objects will overheat the stars in the Galactic disc unless one has $f(M) < (M/3 \times 10^6 M_\odot)^{-1}$ for $M < 3 \times 10^9 M_\odot$ [131]. Another limit in this mass range arises because halo objects will be dragged into the nucleus of our own Galaxy by the dynamical friction of the spheroidal stars, this leading to excessive nuclear mass unless [124] $f(M) < (M/2 \times 10^4 M_\odot)^{-10/7} (r_c/2 \text{ kpc})^2$ for $M < 5 \times 10^5 M_\odot$, where r_c is the halo core radius. The limit is shown in Fig. 1 and bottoms out at $M \sim 10^7 M_\odot$ with a value $f \sim 10^{-5}$.

There are also interesting limits for black holes which are too large to reside in galactic halos. The survival of galaxies in clusters against tidal disruption by giant cluster PBHs gives a limit $f(M) < (M/7 \times 10^9 M_\odot)^{-1}$ for $7 \times 10^9 M_\odot < M < 10^{11} M_\odot$, although this depends sensitively on the mass and the radius of the cluster. If there were a population of huge intergalactic PBHs with density parameter $\Omega_D(M)$, each galaxy would have a peculiar velocity due to its gravitational interaction with the nearest one [132]. The typical distance to the nearest one should be $d \approx 30 \Omega_D(M)^{-1/3} (10^{16} M_\odot)^{1/3} \text{ Mpc}$, so this should induce a peculiar velocity $V_{\text{pec}} \approx GM t_0/d^2$ over the age of the Universe. Since the CMB dipole anisotropy shows that the peculiar velocity of our Galaxy is only 400 km s^{-1} , one infers $\Omega_D < (M/5 \times 10^{15} M_\odot)^{-1/2}$, so this gives the limit on the far right of Fig. 1.

Carr and Silk [97] point out that large PBHs could generate cosmic structures through the ‘seed’ or ‘Poisson’ effect even if f is small. If a region of mass \mathcal{M} contains PBHs of mass M , the initial fluctuation is M/\mathcal{M} for the seed effect and $(f M/\mathcal{M})^{1/2}$ for the Poisson effect, the fluctuation growing as z^{-1} from the redshift of CDM domination ($z_{\text{eq}} \approx 4000$). Even if PBHs do not play a rôle in generating cosmic structures (cf. Sec. IV), one can place interesting upper limits of the fraction of dark matter in them by requiring that various types of structure do not form too early. For example, if we apply this argument to Milky-Way-type galaxies, assuming these have a typical mass of $10^{12} M_\odot$ and must not bind before a redshift $z_B \sim 3$, we obtain

$$f(M) < \begin{cases} (M/10^6 M_\odot)^{-1} & (10^6 M_\odot < M \lesssim 10^9 M_\odot) \\ M/10^{12} M_\odot & (10^9 M_\odot \lesssim M < 10^{12} M_\odot), \end{cases} \quad (\text{III.8})$$

with the second expression corresponding to having one PBH per galaxy. This limit bottoms out at $M \sim 10^9 M_\odot$ with a value $f \sim 0.001$. Similar constraints apply for the first bound clouds, dwarf galaxies and clusters of galaxies and the limits for all the systems are collected together in Fig. 1.

D. Accretion Constraints

PBHs could have a large luminosity at early times due to accretion of the background gas and this imposes strong constraints on their number density. However, the analysis of the problem is complicated because the black hole luminosity will generally boost the matter temperature well above its Friedmann value even if the PBH density is small, thereby reducing the accretion. Thus there are two distinct but related PBH constraints: one associated with the effects on the Universe's thermal history and the other with the generation of background radiation. This problem was first studied in Ref. [13] and we briefly review that analysis here.

If the spectrum from the accreting PBHs is soft (not extending much into the UV), each one will be surrounded by an HII region of radius R_s , where the temperature is determined by the balance between photoionisation heating and inverse Compton cooling off the CMB photons. This implies a temperature $T \approx 10^4 (z/10^3)^{0.3}$ K, which is never much below 10^4 K. If the spectrum is hard (extending well beyond 10 eV), there is still an HII region but many photons escape from it unimpeded, so most of the black-hole luminosity goes into either global heating of the Universe or the background radiation.

Ref. [13] assumes that each PBH accretes at the Bondi rate [133]

$$\dot{M} \approx 10^{11} (M/M_\odot)^2 (n/\text{cm}^{-3}) (T/10^4 \text{ K})^{-3/2} \text{ g s}^{-1}, \quad (\text{III.9})$$

where a dot indicates differentiation with respect to cosmic time t and the appropriate values of n and T are those which pertain at the black-hole accretion radius:

$$R_a \approx 10^{14} (M/M_\odot) (T/10^4 \text{ K})^{-1} \text{ cm}. \quad (\text{III.10})$$

If $R_a > R_s$ or if the whole Universe is ionised (so that the individual HII regions merge), the appropriate values of n and T are those in the background Universe (\bar{n} and \bar{T}). In this case, after decoupling, \dot{M} is epoch-independent so long as \bar{T} has its usual Friedmann behaviour ($\bar{T} \propto z^2$). However, \dot{M} decreases if \bar{T} is boosted above the Friedmann value. If the individual HII regions have not merged and $R_a < R_s$, the appropriate values for n and T are those which pertain within the HII region. In this case, T is close to 10^4 K and pressure balance at the edge of the region implies $n \sim \bar{n} (\bar{T}/10^4 \text{ K})$. Thus $\dot{M} \propto z^5$ until \bar{T} deviates from Friedmann behaviour, so the accretion rate is smaller than in the $R_a > R_s$ situation.

If the accreted mass is converted into outgoing radiation with an epoch-independent efficiency ϵ , the associated luminosity is

$$L = \epsilon \dot{M} c^2. \quad (\text{III.11})$$

Ref. [13] takes the spectrum of emergent radiation to be constant, extending down to at least 10 eV and up to some energy $E_{\text{max}} = 10 \eta \text{ keV}$. This ensures that the black-hole luminosity heats the Universe through photoionisation when the background has low ionisation and Compton scattering off electrons when it has high ionisation. It also assumes that L cannot exceed the Eddington luminosity,

$$L_{\text{ED}} = 4\pi GM m_p / \sigma_T \approx 10^{38} (M/M_\odot) \text{ erg s}^{-1}, \quad (\text{III.12})$$

and a PBH will radiate at this limit for some period after decoupling providing

$$M > M_{\text{ED}} \approx 10^3 \epsilon^{-1} \Omega_g^{-1}, \quad (\text{III.13})$$

where Ω_g is the gas density parameter. This phase persists until a redshift z_{ED} which depends upon M , ϵ , Ω_{PBH} , and Ω_g and could be very late for large black holes.

The effect on the thermal history of the Universe for different (Ω_{PBH}, M) domains is indicated in Fig. 3. In domain (1), T is boosted above 10^4 K and the Eddington phase persists until after the redshift z_* at which most of the black-hole radiation goes into Compton heating. T attains the temperature of the hottest accretion-generated photons above the line in the top right-hand corner. In domain (2), T is also

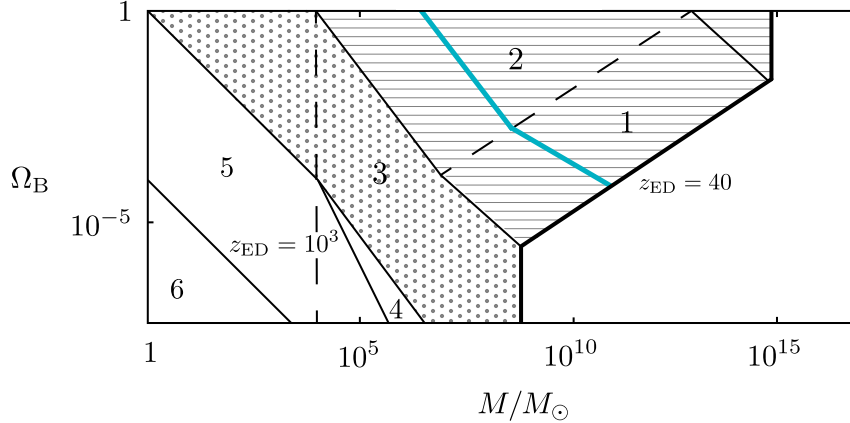


FIG. 3: This shows how the effect of PBH accretion on the evolution of the background matter temperature depends on the PBH mass and density, adapted from Ref. [13]. (We assume $\epsilon = 0.1$, $\Omega_g = 1$ and $E_{\max} = 10$ keV.) The Eddington phase ends at a redshift $z_{\text{ED}} = 10^3$, so the accretion rate exceeds the Eddington limit for some period after decoupling to the right of the line $z_{\text{ED}} = 10^3$ and it persists until after the Salpeter time to the right of the line $z_{\text{ED}} = 40$, the PBH mass increasing substantially in this region. The evolution of the matter temperature in the different domains is described in the text and the expression for z_{ED} in each domain is given by Eq. (III.14). Background light constraints imply the upper limit shown by the blue line and given by Eq. (III.15).

boosted above 10^4 K but the Eddington phase ends before z_* . In domain (3), T is boosted to 10^4 K but not above it because of the cooling of the CMB. In domain (4), T does not reach 10^4 K, so the Universe is not re-ionised, but there is a period in which it increases. In domain (5), T never increases but follows the CMB temperature, falling like z rather than z^2 throughout the Eddington phase. In domain (6), T never deviates from Friedmann behaviour. In domains 1 to 3, which are the most relevant ones for present considerations, the Eddington phase ends at [13]

$$z_{\text{ED}} \approx \begin{cases} 10^{3.8} (\Omega_{\text{PBH}} \eta)^{2/9} (M \epsilon \Omega_g / M_\odot)^{-4/27} & (1) \\ 10^{3.3} \Omega_{\text{PBH}}^{1/6} (M \epsilon / M_\odot)^{-1/9} \Omega_g^{-5/18} & (2) \\ 10^{4.0} (M \epsilon \Omega_g / M_\odot)^{-1/3} & (3). \end{cases} \quad (\text{III.14})$$

In the top right-hand corner $z_{\text{ED}} \approx 10^6 (M \epsilon \Omega_g / M_\odot)^{-1/3} \eta^{1/2}$. The above analysis assumes that the black hole mass M is constant. However, during the Eddington phase, each black hole doubles its mass on the Salpeter timescale, $t_S \approx 4 \times 10^8 \epsilon$ where ϵ is the luminosity efficiency [134]. Therefore M can only be regarded as constant if $t_{\text{ED}} < t_S$ and this implies the condition $z_{\text{ED}} > 40 (\epsilon/0.1)^{-2/3}$. From Eq. (III.14), this corresponds to (Ω_{PBH}, M) values to the left of the red line in Fig. 3. To the right of this line, one expects the mass of the PBH to increase by a factor $\exp[(40/z_{\text{ED}})^{3/2}]$.

Constraints on the PBH density are derived by comparing the time-integrated emission from the PBHs with the observed background intensity in the appropriate waveband [135]. In domain (1) the biggest contribution comes from the end of the Eddington phase, which is after the epoch $z = z_*$; in domains (2) and (3) the biggest contribution comes from the epoch $z = z_*$, somewhat after the Eddington phase. If $\eta \sim 1$, corresponding to $E_{\max} \sim 10$ keV, then in domain (1) the radiation would currently reside in the 0.1 – 1 keV range where $\Omega_R \sim 10^{-7}$; in domain (2), it would reside at ~ 100 eV where $\Omega_R \sim 10^{-6.5}$. The associated limit on the PBH density parameter is then

$$\Omega_{\text{PBH}} < \begin{cases} (10 \epsilon)^{-5/6} (M/10^6 M_\odot)^{-5/6} \eta^{5/4} & (1) \\ (10 \epsilon)^{-1} (M/10^6 M_\odot)^{-1} \eta^{11/6} & (2). \end{cases} \quad (\text{III.15})$$

In principle, this analysis applies for arbitrarily large masses, thereby providing the only constraint for stupendously large PBHs in the range above $10^{12} M_\odot$. However, the extension of this constraint to such large masses was not included in Ref. [135] itself, so it is not shown in Fig. 1, this being a summary of previous literature. Also constraint (III.15) is very old and needs to be updated.

Later the black-hole accretion limits were investigated in more detail by Ricotti *et al.* [14], who showed that the emitted X-rays would produce measurable effects in the spectrum and anisotropies of the CMB.

Using FIRAS data to constrain the first, they obtained a limit $f(M) < (M/1 M_\odot)^{-2}$ for $1 M_\odot < M \lesssim 10^3 M_\odot$; using WMAP data to constrain the second, they obtained a limit $f(M) < (M/30 M_\odot)^{-2}$ for $30 M_\odot < M \lesssim 10^4 M_\odot$. Although these limits appear to exclude $f = 1$ down to masses as low as $1 M_\odot$, they are very model-dependent and not as secure as the dynamical ones.

This problem has been reconsidered by several groups, who argue that the limits are weaker than indicated in Ref. [14]. Ali-Haïmoud and Kamionkowski [136] calculate the accretion on the assumption that it is suppressed by Compton drag and Compton cooling from CMB photons and allowing for the PBH velocity relative to the background gas. They find the spectral distortions are too small to be detected, while the anisotropy constraints only exclude $f = 1$ above $10^2 M_\odot$. Horowitz [137] performs a similar analysis and gets an upper limit of $30 M_\odot$. Neither of these analyses includes the super-Eddington effects expected above some mass and this should lead to a flattening of the constraint. Poulin *et al.* [138] argue that the spherical accretion approximation likely breaks down and that an accretion disk should form. Accretion onto such objects leads to the emission of high-energy photons. This in turn affects the statistical properties of the CMB anisotropies. Provided that disks form early on, these constraints exclude a monochromatic distribution of PBH with masses above $2 M_\odot$ as the dominant form of dark matter.

More direct constraints can be obtained by considering the emission of PBHs at the present epoch. For example, Gaggero *et al.* [139] model the accretion of gas onto a population of massive PBHs in the Milky Way and compare the predicted radio and X-ray emission with observational data. The possibility that $\mathcal{O}(10) M_\odot$ PBHs can provide all of the dark matter is excluded at 5σ level by a comparison with a VLA radio catalog at 1.4 GHz and the Chandra X-ray catalog. Similar arguments have been made by Manshanden *et al.* [140]. PBH interactions with the interstellar medium should result in a significant X-ray flux, contributing to the observed number density of compact X-ray objects in galaxies, and Inoue and Kusenko [141] use the data to constrain the PBH number density in the mass range from a few to $2 \times 10^7 M_\odot$. Their limit is shown in Fig. 1. However, De Luca *et al.* [142] have stressed that the change in the mass of PBHs due to accretion may modify the interpretation of the observational bounds on $f(M)$ at the present epoch. In the mass range $10 - 100 M_\odot$ this can raise existing upper limits by several orders of magnitude.

E. CMB Constraints

If PBHs form from the high- σ tail of Gaussian density fluctuations, as in the simplest scenario [51], then another interesting limit comes from the dissipation of these density fluctuations by Silk damping at a much later time. This leads to a μ -distortion in the CMB spectrum [143] for $7 \times 10^6 < t < 3 \times 10^9$ s, leading to an upper limit $\delta(M) < \sqrt{\mu} \sim 10^{-2}$ over the mass range $10^3 < M/M_\odot < 10^{12}$. This limit was first given in Ref. [64], based on a result in Ref. [144], but the limit on μ is now much stronger. There is also a y distortion for $3 \times 10^9 < t < 3 \times 10^{12}$ s (the time of decoupling).

This argument gives a very strong constraint on $f(M)$ in the range $10^3 < M/M_\odot < 10^{12}$ [145] but the assumption that the fluctuations are Gaussian may be incorrect. For example, Nakama *et al.* [146] have proposed a “patch” model, in which the relationship between the background inhomogeneities and the overdensity in the tiny fraction of the volume which collapses to PBHs is modified, so that the μ -distortion constraint becomes much weaker. Recently, Nakama *et al.* [147] have used a phenomenological description of non-Gaussianity to calculate the μ -distortion constraints on $f(M)$, using the current FIRAS limit and the projected upper limit from PIXIE [148]. However, one would need huge non-Gaussianity to avoid the constraints in the mass range of $10^6 M_\odot < M < 10^{10} M_\odot$. Another way out is to assume that the PBHs are initially smaller than the lower limit but undergo substantial accretion between the μ -distortion era and the time of matter-radiation equality.

F. Gravitational-Wave Constraints

Interest in PBHs has intensified recently because of the detection of gravitational waves from coalescing black-hole binaries by LIGO/Virgo [33, 149–151]. Even if these are not of primordial origin, the observations place important constraints on the number of PBHs. For example, LIGO/Virgo searches find no compact binary systems with component masses in the range $0.2 - 1.0 M_\odot$ [152]. Neither black holes nor neutron stars are expected to form through normal stellar evolution below $1 M_\odot$ and one can infer

$f < 0.3$ for $M < 0.2 M_\odot$ and $f < 0.05$ for $M < 1 M_\odot$. A similar search from the second LIGO/Virgo run [153] found constraints on the mergers of $0.2 M_\odot$ and $1.0 M_\odot$ binaries corresponding to at most 16% or 2% of the dark matter, respectively.

A population of massive PBHs would be expected to generate a gravitational-wave background (GWB) [154] and this would be especially interesting if there were a population of binary black holes coalescing at the present epoch due to gravitational-radiation losses. Conversely, the non-observation of a GWB gives constraints on the fraction of dark matter in PBHs and these could be stronger than any other current constraint in the range $0.5 - 30 M_\odot$. Raidal *et al.* [40] show that the predicted GWB could be observed, with non-observation indicating that the observed events are not of primordial origin. This constraint has been updated in their more recent work [155, 156]. Bartolo *et al.* [157] calculate the anisotropies and non-Gaussianity of such a stochastic GWB and conclude that PBHs could not provide all the dark matter if these were large.

A different type of gravitational-wave constraint on $f(M)$ arises because of the large second-order tensor perturbations generated by the scalar perturbations which produce the PBHs [158]. The associated frequency was originally given as $10^{-8} (M/10^3 M_\odot)$ Hz but this estimate contained a numerical error [159] and was later reduced by a factor of 10^3 [160]. The limit on $f(M)$ just relates to the amplitude of the density fluctuations at the horizon epoch and is of order 10^{-52} . This effect has subsequently been studied by several other authors [161]. In particular, the limit from pulsar timing data already excludes PBHs with $0.03 M_\odot < M < 10 M_\odot$ from providing an appreciable amount of dark matter [162] and limits from LIGO, VIRGO and BBO could potentially cover the mass range down to 10^{20} g. Conversely, one can use PBH limits to constrain a background of primordial gravitational waves [163].

The robustness of the LIGO/Virgo bounds on $\mathcal{O}(10) M_\odot$ PBHs depends on the accuracy with which the formation of PBH binaries in the early Universe can be described. Ballesteros *et al.* [164] revisit the standard estimate of the merger rate, focusing on the spatial distribution of nearest neighbours and the expected initial PBH clustering. They confirm the robustness of the previous results in the case of a narrow mass function, which constrains the PBH fraction of dark matter to be $f \sim 0.001 - 0.01$.

Kühnel *et al.* [165] investigate gravitational-wave production by PBHs in the mass range $10^{-13} - 1 M_\odot$ orbiting a supermassive black hole. While an individual object would be undetectable, the extended stochastic emission from a large number of such objects might be. In particular, LISA could detect the extended emission from objects orbiting Sgr A* at the centre of the Milky Way if a dark-matter spike, analogous to the WIMP-spike predicted by Gondolo and Silk [166], forms there.

G. Interesting Mass Windows and Extended PBH Mass Functions

Figure 1 shows the three mass windows (A, B, C, D) in which PBHs could have an “appreciable” density, which we somewhat arbitrarily take to mean $f > 0.1$, although this does not mean there is positive evidence for this. A special comment is required about window D (i.e. the mass range $10^{14} < M/M_\odot < 10^{18}$), which seems to have been completely neglected in previous literature. Obviously such stupendously large black holes (which we term “SLABs”) could not provide the dark matter in galactic halos, since they are too large to fit inside them (i.e. they violate the galactic incredulity limit). However, they might in principle provide an *intergalactic* dark-matter component, and the lack of constraints in this mass range probably just reflects the fact that nobody has previously considered this possibility. Indeed, we know there are black holes with masses up to nearly $10^{11} M_\odot$ in galactic nuclei [167], so it is not inconceivable that SLABs could represent the high-mass tail of such a population, and PBHs with mass up to $10^{17} M_\odot$ could conceivably form in the radiation era. Although PBHs are unlikely to be this large at formation, they might increase their mass enormously through accretion of gas before galaxy formation, so they could certainly seed SLABs. This has motivated the study of SLABs in Ref. [168] and we have included a discussion of the limits from accretion and WIMP annihilation above. These constraints are not shown in Fig. 1 since this represents a summary of *previous* literature.

The constraints shown in Fig. 1 assume that the PBH mass function is quasi-monochromatic (i.e. with a width $\Delta M \sim M$). This is unrealistic and in most scenarios one would expect it to be extended. In the context of the dark-matter problem, this is a two-edged sword [15]. On the one hand, it means that the *total* PBH density may suffice to explain the dark matter, even if the density in any particular mass band is small and within the observational bounds discussed in Sec. III. On the other hand, even if PBHs can provide all the dark matter at some mass scale without violating the constraints there, the extended mass function may still violate the constraints at some other scale. This problem is particularly pertinent if the

mass function extends over many decades. A detailed assessment of this problem requires a knowledge of the expected PBH mass fraction, $f_{\text{exp}}(M)$, and the maximum fraction allowed by the monochromatic constraint, $f_{\text{max}}(M)$. However, one cannot just plot $f_{\text{exp}}(M)$ for a given model in Fig. 1 and infer that the model is allowed because it does not intersect $f_{\text{max}}(M)$.

Reference [15] assumes that each constraint can be treated as a sequence of flat constraints by breaking it up into narrow mass bins but this is a complicated procedure and has been criticised by Green [30]. A more elegant methodology — similar to Green’s — was taken in Ref. [169]. In this, one introduces the function

$$\psi(M) \propto M \frac{dn}{dM}, \quad (\text{III.16})$$

normalised so that the *total* fraction of the dark matter in PBHs is

$$f_{\text{PBH}} \equiv \frac{\Omega_{\text{PBH}}}{\Omega_{\text{CDM}}} = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \psi(M). \quad (\text{III.17})$$

The mass function is specified by the mean and variance of the $\log M$ distribution:

$$\log M_c \equiv \langle \log M \rangle_\psi, \quad \sigma^2 \equiv \langle \log^2 M \rangle_\psi - \langle \log M \rangle_\psi^2, \quad (\text{III.18})$$

where

$$\langle X \rangle_\psi \equiv f_{\text{PBH}}^{-1} \int dM \psi(M) X(M). \quad (\text{III.19})$$

Two parameters should generally suffice *locally* (i.e. close to a peak), since this just corresponds to the first two terms in a Taylor expansion. Any astrophysical observable $A[\psi(M)]$, depending on the PBH abundance, can generally be expanded as

$$A[\psi(M)] = A_0 + \int dM \psi(M) K_1(M) + \int dM_1 dM_2 \psi(M_1) \psi(M_2) K_2(M_1, M_2) + \dots, \quad (\text{III.20})$$

where A_0 is the background contribution and the functions K_j depend on the details of the underlying physics and the nature of the observation. If PBHs with different mass contribute independently to the observable, only the first two terms in Eq. (III.20) need be considered. If a measurement puts an upper bound on the observable,

$$A[\psi(M)] \leq A_{\text{exp}}, \quad (\text{III.21})$$

then for a monochromatic mass function with $M = M_c$ we have

$$\psi_{\text{mon}}(M) \equiv f_{\text{PBH}}(M_c) \delta(M - M_c). \quad (\text{III.22})$$

The maximum allowed fraction of dark matter in the PBHs is then

$$f_{\text{PBH}}(M_c) \leq \frac{A_{\text{exp}} - A_0}{K_1(M_c)} \equiv f_{\text{max}}(M_c). \quad (\text{III.23})$$

Combining Eqs. (III.20)–(III.23) then yields

$$\int dM \frac{\psi(M)}{f_{\text{max}}(M)} \leq 1. \quad (\text{III.24})$$

Once f_{max} is known, it is possible to apply Eq. (III.24) for an arbitrary mass function to obtain the constraints equivalent to those for a monochromatic mass function. One first integrates Eq. (III.24) over the mass range (M_1, M_2) for which the constraint applies, assuming a particular function $\psi(M; f_{\text{PBH}}, M_c, \sigma)$. Once M_1 and M_2 are specified, this constrains f_{PBH} as a function of M_c and σ . This procedure must be implemented separately for each observable and each mass function.

Generally the allowed mass range for fixed f_{PBH} decreases with increasing width σ , thus ruling out the possibility of evading the constraints by simply extending the mass function. Reference [31] performs a more comprehensive analysis, covering the mass range $10^{-18} - 10^4 M_\odot$. However, we stress that the

situation could be more complicated than assumed above, with more than two parameters being required to describe the PBH mass function. For example, Hasegawa *et al.* [170] have proposed an inflationary scenario in the minimally supersymmetric standard model which generates both intermediate-mass PBHs to explain the LIGO detections and lunar-mass PBHs to explain the dark matter. Section V considers a scenario in which the PBH mass function has four peaks, each associated with a particular cosmological conundrum.

IV. CLAIMED SIGNATURES

Most of the PBH literature has focussed on constraints on their contribution to the dark matter, as discussed in the last section. However, a number of observations have been claimed as positive evidence for them, the masses ranging over 16 orders of magnitude, from $10^{-10} M_{\odot}$ to $10^6 M_{\odot}$. In particular, Ref. [55] summarises seven current observational conundra which may be explained by PBHs. The first three are associated with lensing effects: (1) microlensing events towards the Galactic bulge generated by planetary-mass objects [110], well above the expectations for free-floating planets; (2) microlensing of quasars [171], including ones that are so misaligned with the lensing galaxy that the probability of lensing by a star is very low; (3) the unexpected high number of microlensing events towards the Galactic bulge by dark objects in the ‘mass gap’ between 2 and $5 M_{\odot}$ [172] where stellar evolution models fail to form black holes [173]. The next three are associated with accretion and dynamical effects: (4) unexplained correlations in the source-subtracted X-ray and cosmic infrared background fluctuations [174]; (5) the non-observation of ultra-faint dwarf galaxies (UFDGs) below the critical radius of dynamical heating by PBHs [175]; (6) the unexplained correlation between the masses of galaxies and their central SMBHs. The last one is associated with gravitational-wave effects: (7) the observed mass, spin and coalescence distributions for the black holes found by LIGO/Virgo [151]. Silk has also given arguments for why PBHs in the intermediate mass range may solve various observational problems [176]. We now discuss this evidence in more detail, discussing the conundra in order of increasing mass. In Sec. V we discuss how these conundra may have a unified explanation in the scenario proposed in Ref. [55].

A. Lensing

Observations of M31 by Niikura *et al.* [105] with the HSC/Subaru telescope have identified a single candidate microlensing event with mass in the range $10^{-10} < M < 10^{-6} M_{\odot}$. Kusenko *et al.* [177] have argued that nucleation of false vacuum bubbles during inflation could produce PBHs with this mass. Niikura *et al.* also claim that data from the five-year OGLE survey of 2622 microlensing events in the Galactic bulge [110] have revealed six ultra-short ones attributable to planetary-mass objects between 10^{-6} and $10^{-4} M_{\odot}$. These would contribute about 1% of the CDM, much more than expected for free-floating planets [178], and compatible with the bump associated with the electro-weak phase transition in the best-fit PBH mass function of Ref. [55].

The MACHO collaboration originally reported 17 LMC microlensing events and claimed that these were consistent with compact objects of $M \sim 0.5 M_{\odot}$, compatible with PBHs formed at the QCD phase transition [107]. Although they concluded that such objects could contribute only 20% of the halo mass, the origin of these events is still a mystery and this limit is subject to several caveats. Calcino *et al.* [179] argue that the usual semi-isothermal sphere for our halo is no longer consistent with the Milky Way rotation curve. When the uncertainties in the shape of the halo are taken into account, they claim that the LMC microlensing constraints weaken for $M \sim 10 M_{\odot}$ but tighten at lower masses. Hawkins [180] makes a similar point, arguing that low-mass Galactic halo models would relax the constraints and allow 100% of the dark matter to be solar-mass PBHs. Several authors have claimed that PBHs could form in tight clusters, giving a local overdensity well in excess of that provided by the halo concentration alone [85, 181], and this may remove the microlensing constraint at $M \sim 1 - 10 M_{\odot}$ altogether for certain MACHO populations, especially if the PBHs have a wide mass distribution.

OGLE has detected around 60 long-duration microlensing events in the Galactic bulge, of which around 20 have GAIA parallax measurements. This breaks the mass-distance degeneracy and implies that they are probably black holes [172]. The event distribution implies a mass function peaking between 0.8 and $5 M_{\odot}$, which overlaps the gap from 2 to $5 M_{\odot}$ in which black holes are not expected to form as the endpoint of stellar evolution [173]. This is also consistent with the peak originating from the reduction of pressure at the QCD epoch [55].

Hawkins [182] originally claimed evidence for a critical density of Jupiter-mass PBHs objects from observations of quasar microlensing. However, his later analysis yielded a lower density (dark matter rather than critical) and a larger mass of around $1 M_\odot$ [183]. Mediavilla *et al.* [171] have also claimed positive evidence for quasar microlensing, these indicating that 20% of the total mass is in compact objects in the mass range $0.05 - 0.45 M_\odot$. These events might be explained by intervening stars but in several cases the stellar region of the lensing galaxy is not aligned with the quasar, which suggests a different population of subsolar halo objects. Indeed, Hawkins [184] has argued that the individual quasar images are best explained as microlensing by PBHs distributed along the lines of sight to the quasars. The best-fit PBH mass function of Ref. [55] is consistent with these findings and requires $f_{\text{PBH}} \simeq 0.05$ in this mass range.

Recently Vedantham *et al.* [185] claim to have detected long-term radio variability in the light-curves of active galactic nuclei (AGN). This is associated with a pair of opposed and strongly skewed peaks in the radio flux density observed over a broad frequency range. They propose that this arises from gravitational millilensing of features in AGN jets due to relativistically moving features in jets move through gravitational lensing caustics created by $10^3 - 10^6 M_\odot$ subhalo condensates or black holes located within intervening galaxies.

B. Dynamical

Lacey & Ostriker once argued that the observed puffing of the Galactic disc could be due to black holes of around $10^6 M_\odot$ [131], older stars being heated more than younger ones. They claimed that this could explain the scaling of the velocity dispersion of the disk stars, the relative velocity dispersions in the radial, azimuthal, and vertical directions, as well as the existence of a high-energy tail of stars with large velocity [186]. However, later measurements gave different velocity dispersions for older stars [187] and it is now thought that heating by a combination of spiral density waves and giant molecular clouds may better fit the data [188].

If there were an appreciable number of PBHs in galactic halos mass distributions, CDM-dominated UFDGs would be dynamically unstable if they were smaller than some critical radius, which also depends on the mass of a possible central black hole. The non-detection of galaxies smaller than $r_c \sim 10 - 20$ parsecs, despite their magnitude being above the detection limit, suggests compact halo objects in the solar-mass range. Moreover, rapid accretion in the densest PBH haloes could explain the extreme UFDG mass-to-light ratios observed [175].

Fuller *et al.* [189] show that some *r*-process elements can be produced by the interaction of PBHs with neutron stars if those with masses $10^{-14} - 10^{-8} M_\odot$ have $f > 0.01$. When the PBH is captured by a rotating millisecond neutron star, the resulting spin-up ejects $\sim 0.1 - 0.5 M_\odot$ of relatively cold neutron-rich material. This can also produce a kilonova-type afterglow and a fast radio burst. Abramowicz and Bejger [190] argue that collisions of neutron stars with PBHs of mass 10^{23} g may explain the millisecond durations and large luminosities of fast radio bursts.

As discussed in Sec. III, PBHs larger than this could generate cosmic structures through the ‘seed’ or ‘Poisson’ effect even if f is small [97]. If a region of mass \mathcal{M} contains PBHs of mass M , the initial fluctuation is M/\mathcal{M} for the seed effect and $(fM/\mathcal{M})^{1/2}$ for the Poisson effect. If $f = 1$, Poisson dominates; if $f \ll 1$, the seed dominates for $\mathcal{M} < M/f$. This fluctuation grows as z^{-1} from the redshift of CDM domination ($z_{\text{eq}} \approx 4000$), so the mass binding at redshift z_{B} is

$$\mathcal{M} \approx \begin{cases} 4000 M z_{\text{B}}^{-1} & (\text{seed}) \\ 10^7 f M z_{\text{B}}^{-2} & (\text{Poisson}). \end{cases} \quad (\text{IV.1})$$

Having $f = 1$ requires $M < 10^3 M_\odot$ and so the Poisson effect could only bind a scale $\mathcal{M} < 10^{11} z_{\text{B}}^{-2} M_\odot$, which is necessarily subgalactic. However, this would still allow the dwarf galaxies to form earlier than in the standard scenario, which would have interesting observational consequences.

Having $f \ll 1$ allows the seed effect to be important and raises the possibility that the $10^6 - 10^{10} M_\odot$ black holes in AGN are primordial in origin and actually generate the galaxies. Most quasars contain $10^8 M_\odot$, so it is interesting that this suffices to bind a region of mass $10^{11} M_\odot$ at the epoch of galaxy formation. It is sometimes argued that BBN requires PBHs to form before 1 s, corresponding to a limit $M < 10^5 M_\odot$. There is no reason in principle why such PBHs should not form. For example, the fraction of the Universe in PBHs at time t is only $10^{-6} (t/\text{s})^{1/2}$, so the effect on BBN should be tiny.

The softening of the pressure at e^+e^- annihilation at 10s naturally produces a peak at $10^6 M_\odot$. This does not correspond to any feature in the data. However, as discussed in Sec. III D, such large PBHs would inevitably increase their mass through accretion, so one must distinguish between their initial and current mass. This raises the question of whether a $10^6 M_\odot$ PBH would naturally grow to $10^8 M_\odot$, this entailing a corresponding increase in the value of f .

C. X-Ray/Infrared Background

As shown by Kashlinsky [174, 191], the spatial coherence of the X-ray and infrared source-subtracted backgrounds implies that black holes are required. Although these need not be primordial, the level of the infrared background suggests an overabundance of high-redshift haloes and this could be explained if a significant fraction of the CDM comprises solar-mass PBHs, the Poisson fluctuations in their number density then growing all the way from matter-radiation equality. In these haloes, a few stars form and emit infrared radiation, while PBHs emit X-rays due to accretion. It is challenging to find other scenarios that naturally produce such features.

D. LIGO/Virgo

A population of massive PBHs would be expected to generate a gravitational-wave background (GWB) [154]. This would be especially interesting if there were a population of binary PBHs, since some of them might be coalescing at the present epoch due to gravitational-radiation losses, thereby generating potentially detectable individual coalescences. This was first discussed in Ref. [34] in the context of Population III black holes and later in Refs. [36, 192] in the context of PBHs. However, the precise formation epoch of the holes is not crucial since the coalescence occurs much later. In either case, the black holes would be expected to cluster inside galactic halos and so the detection of the gravitational waves would provide a probe of the halo distribution [193]. Indeed, the LIGO data had already placed weak constraints on such scenarios a decade ago [194].

The suggestion that the dark matter could comprise PBHs has attracted much attention in recent years as a result of the LIGO/Virgo detections [33, 149]. To date, they have observed 10 events with component masses in the range $8 - 51 M_\odot$. Using slightly different approaches, Refs. [37] and [38] derive merger rates for possible PBH populations and find them to be compatible with the range $9 - 240 \text{ Gpc}^{-3} \text{ y}^{-1}$ obtained by the LIGO analysis. Reference [39] points out that the lower limit on the merger rate may be in tension with the CMB distortion constraints [14] for PBHs in the intermediate mass range if they provide the dark matter. However, the accretion and merger of smaller PBHs after decoupling might still provide the required PBH population without violating the CMB constraints [38].

Most of the observed coalesced black holes have effective spins compatible with zero. Although the statistical significance of this result is low [195], it goes against a stellar binary origin [196] but is a prediction of the PBH scenario [197]. Whether the binaries formed early or late, the expected merger rate is comparable to that observed if PBHs provide a significant fraction of the CDM [37, 38, 41, 155]. For the mass distribution of Ref. [55], the PBHs have $f_{\text{PBH}}^{\text{tot}} = 1$ but $f_{\text{PBH}}(M) \sim 0.01$ in the range $10 - 100 M_\odot$.

Raidal *et al.* [40] have studied the production and merging of PBH binaries for an extended mass function and possible PBH clustering. They show that it is possible to satisfy all current PBH constraints for a lognormal mass function. However, the limit on the fraction of dark matter in PBHs from the non-observation of a GWB is stronger than any other current constraint in the mass range $0.5 - 30 M_\odot$. In subsequent work [155, 156] they have studied the formation and disruption of PBH binaries in more detail, using both analytical and numerical calculations for a general mass function. If PBHs make up just 10% of the dark matter, the analytic estimates are reliable and indicate that the constraint from the observed mergers is strongest in the mass range $2 - 160 M_\odot$. However, their general conclusion is that the PBH constraints are weakened because of the suppression of mergers.

Ali-Haïmoud *et al.* [41] have computed the probability distribution of orbital parameters for PBH binaries. Their analytic estimates indicate that the tidal field of halos and interactions with other PBHs, as well as dynamical friction by unbound standard dark-matter particles, do not provide a significant torque on PBH binaries. They also calculate the binary merger rate from gravitational capture in present-day halos. If binaries formed in the early Universe survive to the present time, as expected, they dominate

the total PBH merger rate. Moreover, this merger rate would be above the current LIGO upper limit unless $f(M) < 0.01$ for $10 - 300 M_\odot$ PBHs.

One of the few mass ranges in which PBHs could provide the dark matter is around $10^{-12} M_\odot$. If these PBHs are due to enhanced scalar perturbations produced during inflation, their formation is inevitably accompanied by the generation of non-Gaussian gravitational waves with frequency peaked in the mHz range (the maximum sensitivity of LISA). Bartolo *et al.* [198] show that LISA will be able to detect not only the GW power spectrum but also the non-Gaussian three-point GW correlator, thus allowing this scenario to be thoroughly tested. If PBHs with masses of $10^{20} - 10^{22}$ g are the dark matter, the corresponding GWs will be detectable by LISA, irrespective of the value of f_{NL} .

E. Arguments for Intermediate-Mass PBHs

Silk has argued that intermediate-mass PBHs (IMPBHs) could be ubiquitous in early dwarf galaxies [176], being mostly passive today but active in their gas-rich past. This would be allowed by current AGN observations [199] and early feedback from IMPBHs could provide a unified explanation for many dwarf galaxy anomalies. Besides providing a phase of early galaxy formation and seeds for SMBHs at high z (discussed above), they could: (1) suppress the number of luminous dwarfs; (2) generate cores in dwarfs by dynamical heating; (3) resolve the “too big to fail” problem; (4) create bulgeless disks; (5) form ultra-faint dwarfs and ultra-diffuse galaxies; (6) reduce the baryon fraction in Milky-Way-type galaxies; (7) explain ultra-luminous X-ray sources in the outskirts of galaxies; (8) trigger star formation in dwarfs via AGN. As we will see, IMPBH production could be naturally triggered by the thermal history of the Universe [55]. This could also lead to several observational signatures: they would generate extreme-mass-ratio inspiral (EMRI) merger events detectable by LISA; they would tidally disrupt white dwarfs much more rapidly than main-sequence stars, leading to luminous flares and short time-scale nuclear transients [200]; they would induce microlensing of extended radio sources [201].

V. UNIFIED PBH SCENARIO

In this section we describe a particular scenario in which PBHs naturally form with an extended mass function and provide a unified explanation of some of the conundra discussed above. The scenario is discussed in detail Ref. [55] and based on the idea that the thermal history of the Universe leads to dips in the sound-speed and therefore enhanced PBH formation at scales corresponding to the electroweak phase transition ($10^{-6} M_\odot$), the QCD phase transition ($1 M_\odot$), the pion-plateau ($10 M_\odot$) and $e^+ e^-$ annihilation ($10^6 M_\odot$). This scenario requires that most of the dark matter is in PBHs formed at the QCD peak and is marginally consistent with the constraints discussed in Sec. III, even though this suggested that there is no mass window where PBHs with a monochromatic mass function can provide the dark matter. This illustrates the importance of considering an extended mass function.

A. Thermal History of the Universe

Reheating at the end of inflation fills the Universe with radiation. In the absence of extensions beyond the Standard Model, it remains dominated by relativistic particles with an energy density decreasing as the fourth power of the temperature. As time increases, the number of relativistic degrees of freedom remains constant until around 200 GeV, when the temperature of the Universe falls to the mass thresholds of the Standard Model particles. The first particle to become non-relativistic is the top quark at $T \simeq M_t = 172$ GeV, followed by the Higgs boson at 125 GeV, the Z boson at 92 GeV and the W boson at 81 GeV. At the QCD transition at around 200 MeV, protons, neutrons and pions condense out of the free light quarks and gluons. A little later the pions become non-relativistic and then the muons, with $e^+ e^-$ annihilation and neutrino decoupling occur at around 1 MeV.

Whenever the number of relativistic degrees of freedom suddenly drops, it changes the effective equation of state parameter w . As shown Fig. 4 (right panel), there are thus four periods in the thermal history of the Universe when w decreases. After each of these, w resumes its relativistic value of $1/3$ but because the threshold δ_c is sensitive to the equation-of-state parameter $w(T)$, the sudden drop modifies the probability of gravitational collapse of any large curvature fluctuations. This results in pronounced

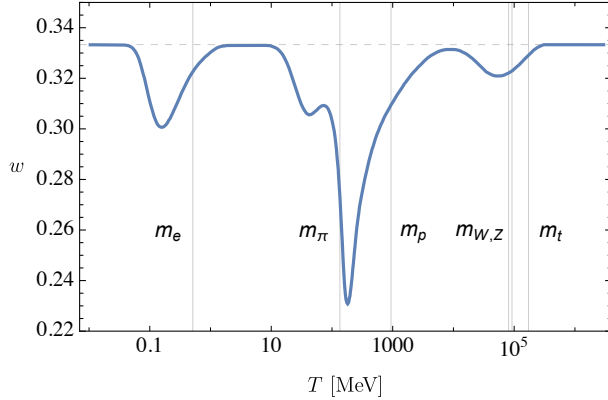


FIG. 4: Equation-of-state parameter w (right panel) as a function of temperature T , from Ref. [55]. The grey vertical lines correspond to the masses of the electron, pion, proton/neutron, W/Z bosons and top quark, respectively. The grey dashed horizontal lines correspond to $g_* = 100$ and $w = 1/3$.

features in the PBH mass function even for a uniform power spectrum. If the PBHs form from Gaussian inhomogeneities with root-mean-square amplitude δ_{rms} , then Eq. (II.8) implies that the fraction of horizon patches undergoing collapse to PBHs when the temperature of the Universe is T should be [51]

$$\beta(M) \approx \text{Erfc} \left[\frac{\delta_c(w[T(M)])}{\sqrt{2} \delta_{\text{rms}}(M)} \right], \quad (\text{V.1})$$

where the value δ_c comes from Ref. [28] and the temperature is related to the PBH mass by

$$T \approx 200 \sqrt{M_\odot/M} \text{ MeV}. \quad (\text{V.2})$$

Thus $\beta(M)$ is exponentially sensitive to $w(M)$ and the present CDM fraction for PBHs of mass M is

$$f_{\text{PBH}}(M) \equiv \frac{1}{\rho_{\text{CDM}}} \frac{d\rho_{\text{PBH}}(M)}{d \ln M} \approx 2.4 \beta(M) \sqrt{\frac{M_{\text{eq}}}{M}}, \quad (\text{V.3})$$

where $M_{\text{eq}} = 2.8 \times 10^{17} M_\odot$ is the horizon mass at matter-radiation equality. The numerical factor is $2(1 + \Omega_{\text{B}}/\Omega_{\text{CDM}})$ with $\Omega_{\text{CDM}} = 0.245$ and $\Omega_{\text{B}} = 0.0456$ [49].

There are many inflationary models and these predict a variety of shapes for $\delta_{\text{rms}}(M)$. Some of them, including single-field models like Higgs inflation [202] or two-field models like hybrid inflation [24], produce an extended plateau or dome-like feature in the power spectrum. Instead of focussing on any specific scenario, Ref. [55] assumes a quasi-scale-invariant spectrum,

$$\delta_{\text{rms}}(M) = A \left(\frac{M}{M_\odot} \right)^{(1-n_s)/4}, \quad (\text{V.4})$$

where the spectral index n_s and amplitude A are treated as free phenomenological parameters. This could represent any spectrum with a broad peak, such as might be generically produced by a second phase of slow-roll inflation. The amplitude is chosen to be $A = 0.0661$ for $n_s = 0.97$ in order to get an integrated abundance $f_{\text{PBH}}^{\text{tot}} = 1$. The ratio of the PBH mass and the horizon mass at re-entry is denoted by γ and we assume $\gamma = 0.8$, following Refs. [50, 203]. The resulting mass function is represented in Fig. 8, together with the relevant constraints from Sec. III. It exhibits a dominant peak at $M \simeq 2 M_\odot$ and three additional bumps at $10^{-5} M_\odot$, $30 M_\odot$ and $10^6 M_\odot$. Reference [55] discusses how these bumps relate to the observational conundra discussed in Sec. IV. Observations of the mass ratios in coalescing binaries provide another useful probe of the PBH scenario, the distribution predicted in the unified model of Ref. [55] being shown in Fig. 6. The regions in red are not occupied by stellar black-hole mergers in the standard scenario. The distinctive prediction of the unified PBH proposal is the merger of objects with $1 M_\odot$ and $10 M_\odot$, corresponding to region (5).

For example, for a given PBH mass distribution, one can calculate the number of supermassive PBHs

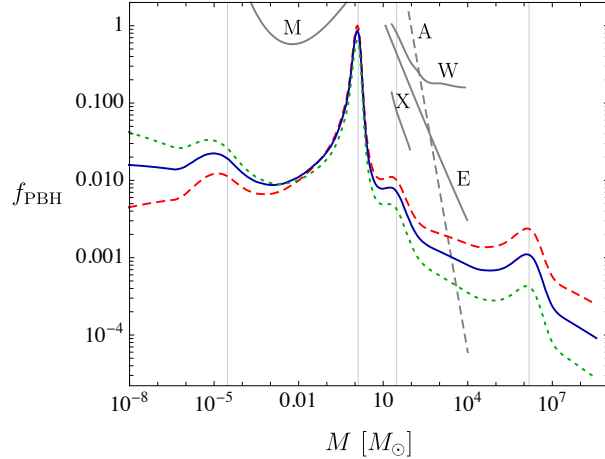


FIG. 5: The mass spectrum of PBHs with spectral index $n_s = 0.965$ (red, dashed), 0.97 (blue, solid), 0.975 (green, dotted), from Ref. [55]. The grey vertical lines corresponds to the EW and QCD phase transitions and e^+e^- annihilation. Also shown are the constraints associated with microlensing (M), wide-binaries (W), accretion (A), Eridanus (E) and X-ray observations (X).

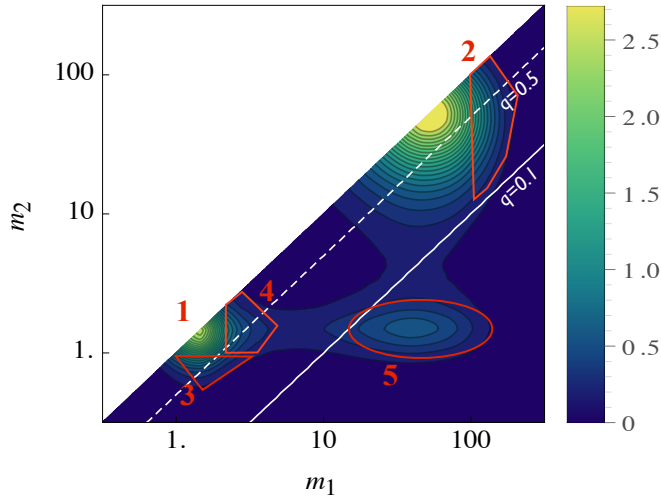


FIG. 6: Expected probability distribution of PBH merger detections with masses m_1 and m_2 (in solar units), assuming a PBH mass function with $n_s = 0.97$, and based on the LIGO sensitivity for the O2 run. The solid and dashed white lines correspond to mass ratios $q \equiv m_2/m_1$ of 0.1 and 0.5, respectively. Within the red regions stellar black-hole mergers are not expected to be detected. These regions are: (1) the peak of the distribution, which would be taken to be neutron-star mergers without electromagnetic counterparts; (2) events above $60 M_\odot$; (3) mergers with subsolar m_2 which might be taken for a neutron star with m_1 at the peak of the black-hole distribution; (4) mergers with m_1 in the mass gap; (5) a subdominant population of mergers with low mass ratios. From Ref. [55].

for each halo. It is found that there is one $10^8 M_\odot$ PBH per $10^{12} M_\odot$ halo, with 10 times as many smaller ones, possibly seeding a comparable number of dwarf satellites and faint CDM haloes. If one assumes a standard Press-Schechter halo mass function and identifies the PBH mass that has the same number density, one obtains the relation $M_h \approx M_{\text{PBH}}/f_{\text{PBH}}$, corresponding to roughly one SMBH per halo of mass $10^3 M_{\text{PBH}}$, in agreement with observations. The mass distribution given Eq. (V.4) with $n_s \approx 0.97$ reproduces the observed relation between the central black-hole mass and halo mass [204] if $f_{\text{PBH}}^{\text{tot}} \simeq 1$.

B. Resolving the Fine-Tuning Problem

The origin of the baryon asymmetry of the Universe (BAU) and the nature of dark matter are two of the most challenging problems in cosmology. The usual assumption is that high-energy physics generates the baryon asymmetry everywhere simultaneously via out-of-equilibrium particle decays or first-order phase transitions at very early times. García-Bellido *et al.* [205] propose a scenario in which the gravitational collapse of large inhomogeneities at the QCD epoch (already invoked above) can resolve both these problems. The collapse to a PBH is induced by fluctuations of a light spectator scalar field and accompanied by the violent expulsion of surrounding material. This might be regarded as a sort of “primordial supernova” and provides the ingredients for efficient baryogenesis around the collapsing regions, with the baryons subsequently propagating to the rest of the Universe. This scenario naturally explains why the observed BAU is of order the PBH collapse fraction, as required by Eq. (II.7), and why the baryons and dark matter have comparable densities.

We now discuss this proposal in more detail. The gravitational collapse of the mass within the QCD Hubble horizon can be extremely violent [28] with particles being driven out as a relativistically expanding shock-wave and acquiring energies a thousand times their rest mass from the gravitational potential energy released by the collapse. Such high density *hot spots* provide the out-of-equilibrium conditions required to generate a baryon asymmetry [206] through the well-known electroweak sphaleron transitions responsible for Higgs windings around the electroweak (EW) vacuum [207]. In this process, the charge-parity (CP) symmetry violation of the Standard Model suffices to generate a local baryon-to-photon ratio of order one or larger. The hot spots are separated by many horizon scales but — since the outgoing baryons are relativistic — they propagate away from the hot spots at the speed of light and become homogeneously distributed well before big bang nucleosynthesis. The large initial local baryon asymmetry is thus diluted to the tiny observed global BAU.

The energy available for hot spot electroweak baryogenesis can be estimated as follows. Energy conservation implies that the change in kinetic energy due to the collapse of matter within the Hubble radius to the Schwarzschild radius of the PBH is

$$\Delta K \simeq \left(\frac{1}{\gamma} - 1\right) M_{\text{H}} = \left(\frac{1-\gamma}{\gamma^2}\right) M_{\text{PBH}}. \quad (\text{V.5})$$

The energy acquired per proton in the expanding shell is $E_0 = \Delta K / (n_{\text{p}} \Delta V)$, where $\Delta V \equiv V_{\text{H}} - V_{\text{PBH}} = (1 - \gamma^3) V_{\text{H}}$ is the difference between the Hubble and PBH volumes, so E_0 scales as $(\gamma + \gamma^2 + \gamma^3)^{-1}$. For a PBH formed at $T \approx \Lambda_{\text{QCD}} \approx 140 \text{ MeV}$, the effective temperature is $T_{\text{eff}} = 2 E_0 / 3 \approx 5 \text{ TeV}$, which is well above the sphaleron barrier and induces a CP violation parameter is [208]

$$\delta_{\text{CP}}(T) = 3 \times 10^{-5} \left(\frac{20.4 \text{ GeV}}{T}\right)^{12}. \quad (\text{V.6})$$

The production of baryons can be very efficient, giving $n_{\text{B}} \gtrsim n_{\gamma}$ or $\eta \gtrsim 1$ locally.

This scenario naturally links the PBH abundance to the baryon abundance and the BAU to the PBH collapse fraction ($\eta \sim \beta$). The spectator field mechanism for producing the required curvature fluctuations also avoids the need for a fine-tuned peak in the power spectrum, which has long been considered a major drawback of PBH scenarios. One still needs fine-tuning of the mean field value to produce the observed values of η and β (i.e. $\sim 10^{-9}$). However, the stochasticity of the field during inflation (if it lasted for more than 60 e-folds) ensures that Hubble volumes exist with all possible field values and this means that one can explain the fine-tuning by invoking a single anthropic selection argument. The argument is discussed in Ref. [50] and depends on the fact that only a small fraction of patches will have the PBH and baryon abundance required for galaxies to form. In most others, the field is too far from the slow-roll region to produce either PBHs or baryons. Such patches lead to radiation universes without any dark matter or matter-antimatter asymmetry. In other (much rarer) patches, PBHs are produced too copiously, leading to rapid accretion of most of the baryons, as might have happened in ultra-faint dwarf galaxies. This anthropic selection effect may therefore explain the observed values of η and β .

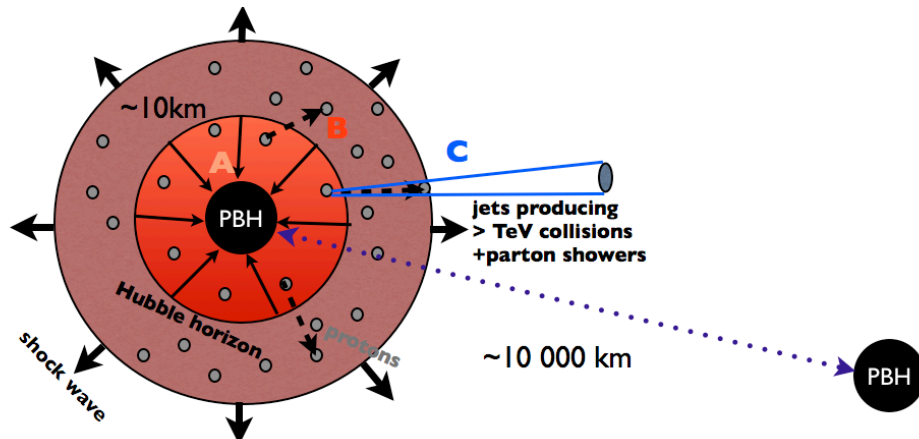


FIG. 7: Qualitative representation of the three steps in the discussed scenario, from Ref. [205]. (A) Gravitational collapse to a PBH of the curvature fluctuation at horizon re-entry. (B) Sphaleron transition in hot spot around the PBH, producing $\eta \sim \mathcal{O}(1)$ locally through EW baryogenesis. (C) Propagation of baryons to rest of Universe through jets, resulting in the observed BAU with $\eta \sim 10^{-9}$.

VI. PBH VERSUS PARTICLE DARK MATTER

Presumably most particle physicists would prefer the dark matter to be elementary particles rather than PBHs, although there is still no direct evidence for this. However, even if this transpires to be the case, we have seen that PBHs could still play an important cosmological rôle, so we must distinguish between PBHs providing *some* dark matter and *all* of it. This also applies for the particle candidates. Nobody would now argue that neutrinos provide the dark matter but they still play a hugely important rôle in astrophysics. Therefore one should not necessarily regard PBHs and particles as rival dark-matter candidates. Both could exist and we end by discussing two scenarios in this spirit. The first assumes that particles dominate the dark matter but that PBHs still provide an interesting interaction with this. The second involves the notion that evaporating black holes leave stable Planck-mass (or even sub-Planck-mass) relics, although such relics are in some sense more like particles than black holes.

A. Combined PBH and Particle Dark Matter

If most of the dark matter is in the form of elementary particles, these will be accreted around any small admixture of PBHs. This can already happen during the radiation-dominated era. In the case of WIMPs, Eroshenko [209] has shown that a low-velocity subset of these will accumulate around PBHs as density spikes shortly after the WIMPs kinetically decouple from the background plasma. Their annihilation will give rise to bright gamma-ray sources and comparison of the expected signal with Fermi-LAT data then severely constrains Ω_{PBH} for $M > 10^{-8} M_{\odot}$. These constraints are several orders of magnitude more stringent than other ones if one assumes a WIMP mass of $m_{\chi} \sim \mathcal{O}(100)$ GeV and the standard value of $\langle\sigma v\rangle = 3 \times 10^{-26} \text{ cm s}^{-1}$ for the velocity-averaged annihilation cross-section. Boucenna *et al.* [210] have investigated this scenario for a larger range of values for $\langle\sigma v\rangle$ and m_{χ} and reach similar conclusions.

Apart from this early formation of the density spikes for PBHs which are light enough to form well before matter-radiation equality, WIMP accretion around heavier PBHs can also occur by secondary infall [211]. This leads to a different halo profile, yielding a constraint $f_{\text{PBH}} \lesssim \mathcal{O}(10^{-9})$ for $\langle\sigma v\rangle = 3 \times 10^{-26} \text{ cm s}^{-1}$ and $m_{\chi} \sim \mathcal{O}(100)$ GeV. While Adamek *et al.* [212] had shown this for solar-mass PBHs, Carr *et al.* [168] have recently extended this argument to much bigger masses, even up to $10^{15} M_{\odot}$ for stupendously large black holes. The constraint at intermediate M comes from the integrated effect of a population of such objects and is flat:

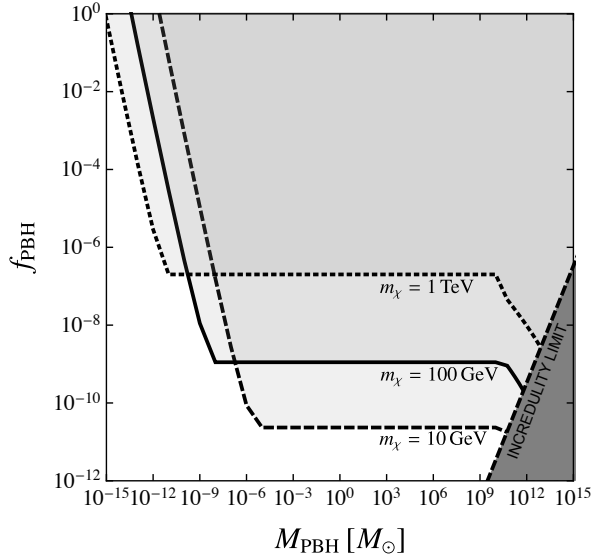


FIG. 8: Constraints on f_{PBH} as a function of PBH mass, from Ref. [168]. Results are shown for $m_\chi = 10$ GeV (dashed lines), $m_\chi = 100$ GeV (solid lines) and $m_\chi = 1$ TeV (dotted lines), setting $\langle\sigma v\rangle = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. Also shown is the incredulity limit (black dashed line).

$$f_{\text{PBH}} < \frac{16}{3} \frac{\Phi_{100 \text{ MeV}}^{\text{Fermi}} H_0}{\rho_{\text{DM}} \tilde{N}_\gamma(m_\chi)} \left(\frac{2 m_\chi^4 t_0^2}{\langle\sigma v\rangle \rho_{\text{eq}}} \right)^{1/3}, \quad (\text{VI.1})$$

where $\Phi_{100 \text{ MeV}}^{\text{Fermi}} = 6 \times 10^{-9} \text{ cm}^{-2} \text{ s}^{-1}$ is the Fermi point-source sensitivity above the threshold energy $E_{\text{th}} = 100 \text{ MeV}$, t_0 is the age of the Universe, $\rho_{\text{eq}} = 2.1 \times 10^{-19} \text{ g cm}^{-3}$ is its average energy density at matter-radiation equality, and \tilde{N}_γ is the average number of photons produced,

$$\tilde{N}_\gamma(m_\chi) \equiv \int_{E_{\text{th}}}^{m_\chi} dE \frac{dN_\gamma}{dE} \int_0^\infty dz \frac{H_0}{H(z)} e^{-\tau_E(z, E)}, \quad (\text{VI.2})$$

where dN_γ/dE is the number of gamma-rays emitted from the annihilations occurring per unit time and energy. The optical depth τ_E is the result of processes such as [213] (i) photon-matter pair production, (ii) photon-photon scattering, (iii) photon-photon pair production. The Limit (VI.1) is derived in Ref. [168] using the numerical packages from Ref. [214] to obtain the optical depth and spectrum of by-products from WIMP annihilations.

Figure 8 shows constraints on f_{PBH} for the different WIMP masses $m_\chi = 10$ GeV (dashed lines), $m_\chi = 100$ GeV (solid lines) and $m_\chi = 1$ TeV (dotted lines). There are deviations from the flat constraint (VI.1) at both the lower and upper mass ends. The first arises because the WIMP kinetic energy plays an essential rôle [209], the second because the constraint from the nearest individual system dominates [168]. In the latter regime, the intersect with the incredulity limit leads to an upper limit on the mass of a detectable PBH [168]:

$$M_{\text{IL}} = 2 \times 10^{12} M_\odot \left(\frac{10}{N_\gamma} \right) \left(\frac{m_\chi}{100 \text{ GeV}} \right)^{4/3} \left(\frac{\langle\sigma v\rangle}{\langle\sigma v\rangle_{\text{F}}} \right)^{-2/3}. \quad (\text{VI.3})$$

For axion-like particles or sterile neutrinos, there is a similar density profile but one has decays rather than annihilations. Kühnel & Ohlsson [215] have derived bounds on the axion-like particle (ALP) and found that the detection prospects for combined dark-matter scenarios with ALP masses below $\mathcal{O}(1)$ keV and halos heavier than $10^{-5} M_\odot$ are far better than for the pure ALP scenario. For sterile-neutrino halos, there are good detection prospects through X-ray, gamma-ray and neutrino telescopes [216].

B. Planck-Mass Relics

If PBH evaporations leave stable Planck-mass relics, these might also contribute to the dark matter. This was first pointed out by MacGibbon [217] and subsequently explored in the context of inflationary scenarios by other authors [218]. If the relics have a mass κM_{Pl} and reheating occurs at a temperature T_{R} , then the requirement that they have less than the critical density implies [60]

$$\beta'(M) < 2 \times 10^{-28} \kappa^{-1} \left(\frac{M}{M_{\text{Pl}}} \right)^{3/2} \quad (\text{VI.4})$$

for the mass range

$$\left(\frac{T_{\text{Pl}}}{T_{\text{R}}} \right)^2 < \frac{M}{M_{\text{Pl}}} < 10^{11} \kappa^{2/5}. \quad (\text{VI.5})$$

One would now require the density to be less than $\Omega_{\text{CDM}} \approx 0.26$, which strengthens the original limit by about a factor of 4. The lower mass limit arises because PBHs generated before reheating are diluted exponentially. The upper mass limit arises because PBHs larger than this dominate the total density before they evaporate, in which case the final cosmological baryon-to-photon ratio is determined by the baryon-asymmetry associated with their emission. Limit (VI.4) still applies even if there is no inflationary period but then extends all the way down to the Planck mass.

It is usually assumed that such relics would be undetectable apart from their gravitational effects. However, Lehmann *et al.* [219] point out that they may carry electric charge, making them visible to terrestrial detectors. They evaluate constraints and detection prospects and show that this scenario, if not already ruled out by monopole searches, can be explored within the next decade using existing or planned experimental equipment.

In some scenarios PBHs could leave stable relics whose mass is very different from the Planck mass. For example, if one maintains the Schwarzschild expression but adopts the Generalised Uncertainty Principle, in which $\Delta x \sim 1/\Delta p + \alpha \Delta p$, then evaporation stops at a mass $\sqrt{\alpha} M_{\text{Pl}}$ [220]. On the other hand, if one adopts the Black Hole Uncertainty Principle correspondence [221], in which one has a unified expression for the Schwarzschild and Compton scales, $R_{\text{CS}} = 2GM + \beta M_{\text{Pl}}^2/M$, the mass can fall into the sub-Planckian regime in which $T \propto M$. In this case, evaporation stops when the black hole becomes cooler than the CMB at $M \sim 10^{-36} \text{ g}$ [222]. One motivation for this correspondence is Dvali's framework for black holes as graviton Bose-Einstein condensates [223]. Kühnel and Sandstad [224] have studied PBH formation in this context and argue that evaporation may stop because the gravitons are quantum-depleted much faster than the baryons or leptons, which are "caught" in the black-hole condensate. So at some point the balance of the strong and gravitational forces lead to stable relics. Recently, Dvali *et al.* [225] have shown that the decay of a black hole is substantially suppressed due to its high capacity for memory storage, leading to another mechanism for extremely long-lived or even stable relics.

VII. CONCLUSIONS

While the study of PBHs has been a minority interest for most of the last 50 years, they have become the focus of increasing attention recently. This is strikingly reflected in the annual publication rate on the topic, which has now risen to several hundred. While the evidence for PBHs is far from conclusive, there is a growing appreciation of their many potential rôles in cosmology and astrophysics. This is why we have stressed the possible evidence for PBHs in this review rather than just the constraints.

PBHs have been invoked for three main purposes: (1) to explain the dark matter; (2) to generate the observed LIGO/Virgo coalescences; (3) to provide seeds for the SMBHs in galactic nuclei. The discussion in Sec. V suggests that they could also explain several other observational conundra, as well as alleviating some of the well-known problems of the CDM scenario. This illustrates that PBHs could still play an important cosmological rôle even if most of the dark matter transpires to be elementary particles.

As regards (1), there are only a few mass ranges in which PBHs could provide the dark matter. We have focused on the intermediate mass range $10 M_{\odot} < M < 10^2 M_{\odot}$, since this may be relevant to (2), but the sublunar range $10^{20} - 10^{24}$ g and the asteroid range $10^{16} - 10^{17}$ g have also been suggested. If the PBHs have a monochromatic mass function, the discussion in Sec. III suggests that only the lowest mass range is viable. However, the discussion in Sec. V indicates that this Conclusion may not apply if they have an extended mass function.

As regards (2), while the possibility that the LIGO/Virgo sources could be PBHs is acknowledged by the gravitational-wave community, this is not the mainstream view. Indeed, the next LIGO/Virgo runs should be able to test the PBH proposal and possibly eliminate it. Note that proposal (2) does not require the PBHs to provide *all* the dark matter. If the PBHs have an extended mass function, the mass where the density peaks would be less than the mass which dominates the gravitational-wave signal.

As regards (3), this raises the issue of the maximum mass of a PBH. There is no reason in principle why this should not be in the supermassive range, in which case it is almost inevitable that they could seed SMBHs and perhaps even galaxies themselves. The main issue is whether there are enough PBHs but one only requires them to have a very low cosmological density. While the mainstream assumption is that galaxies form first, with the SMBHs forming in their nuclei through dynamical processes, this scenario is not fully understood. A crucial question concerns the growth of such large black holes and this applies whether or not they are primordial.

Section V described a very optimistic scenario in which PBHs form with a bumpy mass function as a result of naturally occurring dips in the sound-speed at various cosmological epochs. We argued that this may simultaneously explain all of the cosmological conundra. This scenario also suggests that the cosmological baryon asymmetry may be generated by PBH formation at the QCD epoch, this naturally explaining the fine-tuning in the collapse fraction. This is not the mainstream view for the origin of the baryon asymmetry and some aspects of this proposal require further investigation but this at least addresses the much-neglected PBH fine-tuning problem. The possibility that evaporating PBHs leave stable relics opens up some of the mass range below 10^{15} g as a new world of compact dark-matter candidates waiting to be explored.

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