Mon. Not. R. astr. Soc. (1978) **182**, 147–156

The acceleration of cosmic rays in shock fronts – I

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Received 1977 June 23

Summary. It is shown that charged particles can be accelerated to high energies in astrophysical shock fronts. Fast particles are prevented from streaming away upstream of a shock front by scattering off Alfvén waves which they themselves generate. This scattering confines the particles to the region around the shock and results in first-order Fermi acceleration due to the particles crossing the shock many times. The consequent energy spectrum is a power law with an index close to that observed for galactic cosmic rays. The discussion relates to particles which are already relativistic, and their initial acceleration from thermal energies is not considered.

1 Introduction

Highly relativistic particles in the Universe are detected directly as cosmic rays and indirectly by such processes as synchrotron radio emission, and the energy distributions of these particles, for a wide range of situations, fit power laws with differential spectral indices of about -2.5. Many authors have noted the implication that the acceleration of particles is due to the same mechanism in each case, but there is as yet no agreement as to what it is. In this paper I discuss a mechanism which operates in the vicinity of a shock front and show that it leads to a power-law spectrum with a spectral index in good agreement with the observed values. In view of the widespread occurrence of shock fronts this mechanism may be important in many astrophysical objects.

Initially I consider a parallel shock, in which the direction of propagation is along the magnetic field lines, and discuss only those particles (electrons or protons) which have energies high enough for their gyroradii to be much larger than the thickness of the shock front. This thickness is usually thought to be of the order of, or less than, the gyroradius of a thermal proton (Boyd & Sanderson 1969), as is observed to be the case for the Earth's bow shock (Formisano 1974). The energetic particles are consequently able to pass freely between the regions upstream and downstream of the shock. The shock front itself is the

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region in which the mean plasma velocity changes. The kinetic streaming energy of the upstream plasma is randomized into turbulent and thermal energy in the shock front, and the turbulent motion is dissipated over a region downstream much larger in extent than the thickness of the shock itself. Such a turbulent region has been observed downstream of the Earth's bow shock (Formisano 1974).

Upstream of the shock there is turbulence in the form of Alfvén waves excited by energetic particles which pass through the shock and attempt to escape upstream. Since the shock velocity is super-Alfvénic the streaming velocity of the escaping particles with respect to the upstream plasma must be greater than the Alfvén speed, and consequently Alfvén waves are excited (Wentzel 1974). Such Alfvén waves have been detected upstream of the Earth's bow shock, perhaps excited by fast protons from the shock (Greenstadt 1975). These waves scatter the energetic particles, reduce their streaming to roughly the Alfvén speed and thus make it inevitable that they are overtaken by the shock. As a result, energetic particles can recross the shock front many times, being scattered from downstream to upstream by the turbulent wake and from upstream to downstream by the Alfvén waves. Each time a particle goes through this cycle its energy increases because of the velocity difference of the scattering centres upstream and downstream.

The mechanism is similar to those put forward by Jokipii (1966) and Fisk (1971) to account for the suprathermal particles observed in association with interplanetary shocks and the Earth's bow shock. It differs from them in that the fast particles themselves generate the waves responsible for their confinement near the shock, whereas Jokipii confines the particles between magnetic mirrors which convect with the thermal plasma, while Fisk confines them by scattering from magnetic irregularities of unspecified origin. The present discussion also goes beyond that of Fisk by deriving the particles energy spectrum resulting from the steady state in which mono-energetic particles are continuously injected into the region of the shock front; this derivation can then be generalized for an arbitrary energy distribution of the injected particles.

In Section 2 of this paper the energy gained at each crossing of the shock, and the probability of a particle making a given number of crossings, are calculated and from these the energy spectrum of the accelerated particles is obtained. In Section 3, the upstream distribution of the Alfvén waves and fast particles is discussed, taking into account two damping mechanisms which can prevent or restrict the growth of the Alfvén waves. In Section 4, the analysis is extended to include oblique shocks propagating at an angle to the magnetic field.

2 The energy spectrum

The situation is pictured in Fig. 1 in the rest frame of the shock, which is treated as a discontinuity. The mean velocities of the scattering centres upstream and downstream of the shock are u_1 and u_2 respectively which are taken to be constant throughout the respective regions and non-relativistic. As was discussed in Section 1, all of the particles which penetrate into the upstream region return across the shock, but this is not the case for those travelling downstream, since the motion of the downstream scattering centres away from the shock tends to convect the particles away, whereas diffusion gives them a certain probability, $(1-\eta)$, of recrossing the shock into the upstream region. η can be calculated by applying the diffusion equation to particles in a given small energy range which enter the downstream region and finding the proportion of them which diffuse back across the shock:

$$\frac{\partial n}{\partial t} + u_2 \frac{\partial n}{\partial x} = \frac{\partial}{\partial x}\left(D(x)\frac{\partial n}{\partial x}\right) \tag{1}$$

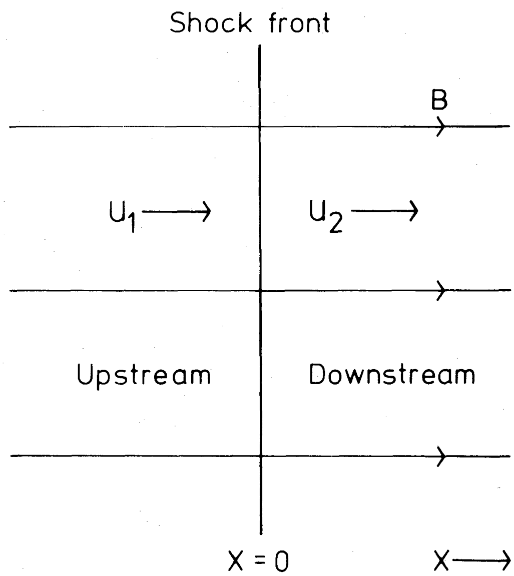

Figure 1. A parallel shock viewed in the frame in which it is stationary.

where $n(x, t)$ is the particle density. By injecting particles at the shock into the downstream region at the same rate as particles are convected away at a large distance downstream, the problem is made time-independent and the solution is

$$n(x, t) = A + B \exp\left[\int^{x} \frac{u_2}{D(x')}\, dx'\right]. \tag{2}$$

The region of scattering is taken to stretch to infinity, which is effectively the case if its extent is $\gg v/u_2$ mean free paths, where v is the particle velocity. $D(x)$ is thus bounded above, so $n(x, t)$ is unbounded for positive x unless B is zero. Hence a physical solution requires that B is zero and that $n(x, t)$ is constant.

The flow of particles away from the shock is $u_2 n - D(\partial n/\partial x)$ which is equal to $u_2 n(0, t)$. This is then the rate at which particles are escaping from the shock at a large distance downstream. The rate at which particles are crossing and recrossing the shock is $\frac{1}{4}n(0, t)\, v$, with an error of order u_2/v due to anisotropy in the velocity distribution. Of the $\frac{1}{4}n(0, t)\, v$ particles which pass from the upstream to the downstream region per unit time, $n(0, t)\, u_2$ escape and the rest diffuse back to the shock to recross into the upstream region. The probability of escape is therefore

$$\eta = 4\frac{u_2}{v}. \tag{3}$$

For very energetic particles ($v \simeq c$), this probability is independent of energy.

While a particle is in the upstream or downstream regions, its energy is constant when viewed in the rest frame of the relevant scattering centres. When it crosses from one region to the other, a Lorentz transformation gives its energy in the rest frame of the scattering centres in the new region. A particle with energy E_k which has performed k cycles, passing from upstream to downstream and back to upstream, performs a further cycle and has its energy increased to

$$E_{k+1} = E_k \left(\frac{1 + v_{k1}(u_1 - u_2) \cos \theta_{k1}/c^2}{1 + v_{k2}(u_1 - u_2) \cos \theta_{k2}/c^2}\right), \tag{4}$$

where v_{k1} is the velocity at which the particle crosses from upstream to downstream, its motion making an angle θ_{k1} with the shock normal; and v_{k2} and θ_{k2} are the respective quantities for the return crossing, all measured in the rest frame of the upstream scattering centres. If the particle is injected into the system with energy E_0, much greater than its rest mass energy, then $v_{k1} \simeq v_{k2} \simeq c$ and

$$\ln\left(\frac{E_l}{E_0}\right) = \sum_{k=0}^{l-1} \ln\left[1 + \frac{u_1 - u_2}{c}\cos\theta_{k1}\right] - \sum_{k=0}^{l-1} \ln\left[1 + \frac{u_1 - u_2}{c}\cos\theta_{k2}\right]. \tag{5}$$

For a significant increase in energy, l is $0[c/(u_1 - u_2)]$ and the distribution of $\ln(E_l/E_0)$ is strongly concentrated around the mean (central limit theorem). As a result all particles which go through l cycles can be treated as having their energy increased by the same amount, given by

$$\ln\left(\frac{E_l}{E_0}\right) = l\left[\left\langle \ln\left(1 + \frac{u_1 - u_2}{c}\cos\theta_{k1}\right)\right\rangle - \left\langle \ln\left(1 + \frac{u_1 - u_2}{c}\cos\theta_{k2}\right)\right\rangle\right]. \tag{6}$$

Since $(u_1 - u_2) \ll c$ the velocity distribution of the particles is nearly isotropic, so the number of particles crossing unit area of the shock front at an angle in the range θ to $\theta + d\theta$ is proportional to $(2\pi \sin\theta \, d\theta) \cos\theta$, and averaging over θ gives

$$\ln\left(\frac{E_l}{E_0}\right) = \frac{4}{3} l \frac{u_1 - u_2}{c}\left(1 + 0\left(\frac{u_1 - u_2}{c}\right)\right). \tag{7}$$

The probability P_l of a particle completing at least l cycles, and therefore of reaching an energy E_l, is given by

$$\ln P_l = l \ln\left(1 - \frac{4u_2}{c}\right) = -\left(\frac{3u_2}{u_1 - u_2} + 0\left(\frac{u_1 - u_2}{c}\right)\right)\ln\left(\frac{E_l}{E_0}\right), \tag{8}$$

so the differential energy spectrum is

$$N(E)\,dE = \frac{\mu - 1}{E_0}\left(\frac{E}{E_0}\right)^{-\mu} dE \tag{9}$$

where

$$\mu = \frac{2u_2 + u_1}{u_1 - u_2} + 0\left(\frac{u_1 - u_2}{c}\right). \tag{10}$$

A numerical factor, close to unity, is needed in equation (9) to correct for the assumption that all particles have the same energy after l cycles.

If v_s is the shock velocity and χ the factor by which gas is compressed at the shock then

$$u_1 = v_s - v_A, \quad u_2 = \frac{v_s}{\chi} + v_w, \tag{11}$$

where v_A is the Alfvén speed and v_w is the mean velocity in the x direction of the scattering centres in the downstream region when viewed in the rest frame of the downstream gas. It is assumed that the upstream Alfvén waves consist entirely of waves which are moving away from the shock, although this may not be true if, for example, the waves are decaying via the sound cascade (Skilling 1975b). In such cases the effective wave velocity is less than v_A, usually by a relatively small amount (Skilling, private communication). Substitution of equations (11) into equation (10) gives the spectral index of the differential energy spectrum of the accelerated particles

$$\mu = \frac{(2+\chi) + \chi(2v_w/v_s - 1/M_A)}{(\chi - 1) - \chi(v_w/v_s + 1/M_A)}, \tag{12}$$

where M_A is the Alfvén Mach number of the shock. The sound speed downstream of a strong shock is about half the shock velocity, so it might be thought that the terms in v_w/v_s are important, but if the turbulent waves interact with each other they will tend to isotropise and then $v_w \ll v_s$. For strong shocks, the Rankine–Hugoniot conditions give $\chi = 4$ and the assumption $v_w \ll v_s$ leads to a value $\mu = 2$, whereas the observed value for cosmic rays is close to 2.5, also typical of the energetic electrons in radio sources. Equation (12) gives $\mu = 2.5$ if $\chi = 4$ and $v_w = v_s/12$ or, alternatively, if $v_w = 0$ and the Alfvén Mach number is close to 4. Satellite observations of the Earth's bow shock have shown that shocks can be more complicated than is assumed in applying the Rankine–Hugoniot relations, e.g. the ratio of specific heats in the upstream and downstream regions can be different (Shen 1971). The values of χ actually measured are centred on 3 rather than 4 (Formisano 1974), and $\chi = 3$ implies $\mu = 2.5$ with $v_w = 0$ and $M_A \gg 1$.

It should be noted that μ is independent of v_s if $v_s \ll c$.

3 The upstream Alfvén waves and their damping

3.1 GENERATION OF ALFVÉN WAVES

Whenever energetic particles stream faster than the Alfvén speed, they generate Alfvén waves with wavelengths close to the gyroradius of the particles. These waves grow in response to a concentration gradient of particles (Skilling 1975c) and scatter the particles with a 'collision frequency' proportional to the wave intensity. The equations relating the growth of waves to the particle distribution (equation 15) and the development of the particle distribution under the influence of the waves (equation 13) are derived by Skilling (1975a, b and c), and the reader is referred to these papers and that by Wentzel (1974) for a full discussion of the subject. The equations are:

$$\frac{\partial f}{\partial t} + u_1 \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(D(x)\frac{\partial f}{\partial x}\right), \tag{13}$$

where

$$D(x) = \frac{4}{3\pi}\frac{pv}{eB\mathscr{I}(x,p)}, \tag{14}$$

p is the particle momentum,

$\mathscr{I}(x,p)$ is the energy density of Alfvén waves per unit logarithmic bandwidth (which are resonant with particles of momentum p) relative to the ambient magnetic energy density (U_M),

B is the magnetic field upstream of the shock,

$f(x,p,t)$ is the particle distribution in phase space and includes both electrons and protons. It is a function spatially of x only, and in momentum space a function of the magnitude of the momentum,

and

$$\frac{\partial \mathscr{I}}{\partial t} + u_1 \frac{\partial \mathscr{I}}{\partial x} - \sigma\mathscr{I} + \Gamma\mathscr{I} = 0, \tag{15}$$

where

$$\sigma = \frac{4\pi}{3} \frac{v_A P^4 v}{U_M} \frac{\partial f}{\partial x}. \tag{16}$$

Γ is the damping rate which is assumed to be negligible, an assumption examined in Section 3.2. These equations can be solved in the time-independent case for relativistic particles subject to the following boundary conditions:

(1) $\mathscr{I} \to 0$ as $x \to -\infty$, which results from assuming that all the Alfvén waves in the system are generated by backstreaming of particles from the shock and none pre-existed in the undisturbed medium.

(2) The particle distribution $f(0, p)$ at the shock is given by equation (9).

(3) As $x \to -\infty$ the particle distribution tends to the background distribution $f_0(p)$ in the undisturbed medium into which the shock is propagating. The solution is then

$$f - f_0 = \frac{a}{x_0 - x}, \quad \mathscr{I} = \frac{b}{x_0 - x}, \tag{17}$$

where

$$a = \frac{1}{\pi^2} \frac{U_M}{eBp^3 v_A}, \tag{18}$$

$$b = \frac{4}{3\pi} \frac{pv}{eBu_1}, \tag{19}$$

and

$$x_0 = \frac{1}{\pi^2} \frac{U_M}{eBv_A} \frac{1}{p^3[f(0, p) - f_0(p)]}. \tag{20}$$

x_0 gives the length scale and is inversely dependent on the particle concentration at the shock. Substitution of a typical distribution for the galactic cosmic ray background, namely $p^3 f_{gal}(p) = 8.5 \times 10^{-33} p^{-1.5}\,\mathrm{m}^{-3}$, in place of $p^3[f(0, p) - f_0(p)]$, gives

$$x_0 = 5 \times 10^{14} \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{E}{\mathrm{GeV}}\right)^{1.5} \mathrm{m},$$

where n_e is the electron density in the pre-shocked gas and the magnetic field does not appear. However, particles are being accelerated and so the appropriate $f(0, p)$ will in fact be many times its background value.

An example of an object in which shock waves are important and in which cosmic rays are probably being accelerated is the supernova remnant Cas A, and in this instance the density of cosmic ray particles exceeds that typical of the Galaxy as a whole by at least 10^4, making x_0 correspondingly smaller by the same factor. For particles with an energy of 1 GeV, x_0 is smaller than the radius of Cas A by a factor of $\sim 10^6$. x_0 nevertheless increases with particle energy and this places a restriction on the energies to which particles can be accelerated.

If $\mathscr{I}$ is anywhere greater than unity, the amplitude of the variations in the magnetic field which constitute the Alfvén waves exceeds the mean magnetic field and the equations do not apply. The maximum value of $\mathscr{I}$ for waves resonant with particles of energy E occurs at the shock and the condition $\mathscr{I}_{max} \gg 1$ is

$$\frac{f(0,p)-f_0(p)}{f_{gal}(p)} < 1.1 \times 10^3 \left(\frac{100u_1}{c}\right)\left(\frac{B}{3\mu G}\right)\left(\frac{n_e}{cm^{-3}}\right)^{1/2}\left(\frac{E}{GeV}\right)^{1/2}. \tag{21}$$

The particle density $f_{gal}(p)$ is introduced only as a quantity with which to compare $f(0,p)-f_0(p)$. For a shock front propagating into undisturbed interstellar gas, $f_0(p) = f_{gal}(p)$. In some astrophysical cases such as Cas A this condition may not be met, and the waves grow until $\mathscr{I}$ reaches a value at which a damping mechanism becomes important. The confinement of particles is not impaired though, and their acceleration is not hindered.

3.2 DAMPING OF THE UPSTREAM WAVES

The solution obtained above assumed that damping is unimportant and I now consider the effect of two damping mechanisms which are believed to be important for Alfvén waves. The first of these is damping due to collisions between charged and neutral particles, and Kulsrud & Cesarsky (1971) have computed the damping rate, Γ_n, for different temperatures. They give $\Gamma_n = 3.29 \times 10^{-9}(n_H/cm^{-3})\,s^{-1}$ at a temperature of 10^3 K, where n_H is the neutral hydrogen density. The values at 10^2 and 10^4 K are about two times smaller and larger respectively. These damping rates may be compared with the growth rate of the Alfvén waves

$$\sigma(x,p) = 6.24 \times 10^{-9}\left(\frac{100u_1}{c}\right)\left(\frac{f(x,p)-f_0(p)}{f_{gal}(p)}\right)\left(\frac{n_e}{cm^{-3}}\right)^{-1/2}\left(\frac{E}{GeV}\right)^{-3/2} s^{-1} \tag{22}$$

which is derived from equations (13), (14) and (16). The damping is unimportant for particle densities greater than that given by equating Γ_n and $\sigma(x,p)$ but, at a sufficient distance upstream of the shock, $[f(x,p)-f_0(p)]$ and hence the growth rate of the waves becomes small, and consequently the damping term dominates. Particles which reach that point are allowed to escape freely upstream. The acceleration of particles hence becomes less efficient when $[f(x,p)-f_0(p)]$ at that point becomes comparable with $(u_1/c)[f(0,p)-f_0(p)]$, i.e. u_1/c times its value at the shock front. The rate of escape of particles upstream is then comparable with the rate of escape downstream. The critical energy above which this effect is important is

$$\frac{E_{crit}}{GeV} = 0.07\left(\frac{100u_1}{c}\right)^{4/3}\left(\frac{n_e}{cm^{-3}}\right)^{-1/3}\left(\frac{n_H}{cm^{-3}}\right)^{-2/3}\left(\frac{f(0,p)-f_0(p)}{f_{gal}(p)}\right)^{2/3}. \tag{23}$$

The spectral index given by equation (12) can be expected to hold up to this energy, above which the spectrum steepens. As mentioned above, $f(0,p)$ is in fact many times the average galactic value, and the region into which shocks such as those in supernova remnants propagate is probably almost completely ionized, i.e. n_H is small. Parameters which may be typical of a young supernova remnant are $n_e = 1\,cm^{-3}$, $n_H = 10^{-2}\,cm^{-3}$, $[f(0,p)-f_0(p)] = 10^4 f_{gal}(p)$ and $u_1 = c/30$, in which case $E_{crit} = 3.5 \times 10^{12}$ eV.

A second important damping mechanism is the loss of energy to sound waves (Chin & Wentzel 1972; Skilling 1975b). This takes place only when the sound speed is less than the Alfvén speed, which requires that

$$T < 3100\left(\frac{B}{3\mu G}\right)^2\left(\frac{n_e}{cm^{-3}}\right)^{-1} K.$$

An upper limit to the effective damping rate introduced by the sound cascade can be obtained by assuming that the interaction of the Alfvén waves with the sound waves makes the wave intensities of the Alfvén waves equal in each direction along the magnetic field. Equation (36) of Chin & Wentzel (1972) then gives the damping rate as

$$\Gamma_s = 7.41 \times 10^{-6} \left(\frac{E}{\mathrm{GeV}}\right)^{-1} \left(\frac{B}{3\mu G}\right)^{2} \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{-1/2} \left(p\frac{\partial \mathscr{I}}{\partial p}\right) \mathrm{s}^{-1}. \tag{24}$$

Γ_s can exceed the growth rate σ and so this damping mechanism can play an important role. Γ_s is proportional to $\partial\mathscr{I}/\partial p$, and σ is independent of $\mathscr{I}$ for a given particle density. Consequently damping is dominant at large wave intensities and growth at small intensities. Hence the effect of this damping is not to make the waves disappear completely, but to limit their wave intensity to the value

$$\mathscr{I} = 7.67 \times 10^{-4} \left(\frac{p}{\mathscr{I}}\frac{\partial \mathscr{I}}{\partial p}\right)^{-1} \left(\frac{100 u_1}{c}\right) \left(\frac{B}{3\mu G}\right)^{-2} \left(\frac{E}{\mathrm{GeV}}\right)^{-1/2} \left(\frac{f(x,p) - f_0(p)}{f_{\mathrm{gal}}(p)}\right) \tag{25}$$

below which the damping rate is less than the growth rate. In cases where this damping is important, it extends the length scales in front of the shock, allowing the particles to travel further away from the shock before being overtaken by it. Because this damping mechanism does not remove the waves completely, it does not impair the efficiency of particle acceleration. In extreme cases, however, damping via the sound cascade can reduce E_{crit}, the energy at which damping by neutral particles steepens the spectrum.

4 Generalization to oblique shocks

The general description of an oblique shock, one in which the direction of propagation is neither parallel nor perpendicular to the magnetic field, involves an electric as well as a magnetic field to enable the plasma to drift across the magnetic field lines. Hudson (1965) has shown that oblique shocks can be transformed into an inertial frame in which the shock front is stationary and there is no electric field either upstream or downstream. In this frame all bulk plasma motions are along the magnetic field lines. The acceleration of particles is considered in this frame and the resulting particle energy distribution must then be Lorentz-transformed to obtain the energy distribution in the original frame. The transformation back to the original frame involves only a small correction to the energy distribution unless the transformation velocity is close to the velocity of light. The transformation velocity is the velocity at which the point of intersection of the shock with a field line moves along the field line in the original frame, and this is less than the velocity of light as long as the angle α between the normal to the shock and the magnetic field is less than $\cos^{-1}(v_s/c)$. If α is greater than this, the shock cannot be transformed into a suitable frame and the discussion below does not apply. In such a case, the particles are convected through the shock front with the magnetic field but cannot return from downstream and hence cannot be accelerated to high energies.

The velocities of the waves along the field lines relative to the shock are $\mathbf{w}_1$ and $\mathbf{w}_2$ upstream and downstream of the shock respectively (Fig. 2). Hudson analysed the motion of an energetic particle with a gyroradius much greater than the shock thickness, showing that it can cross the discontinuity many times in progressing from one side of the shock to the other. A particle which has passed clear of the shock into the downstream region is constrained to move along a magnetic field line, and so the probability of it not returning is

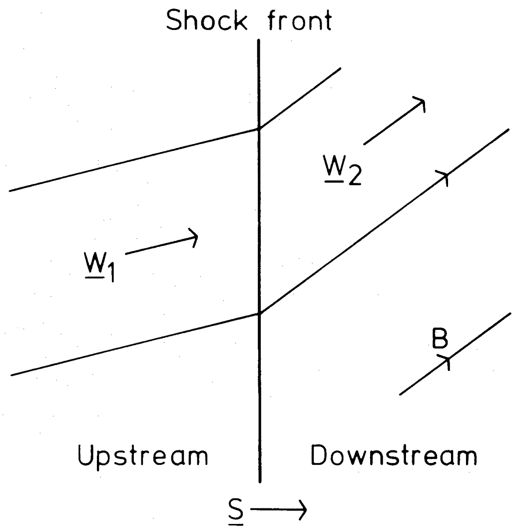

Figure 2. An oblique shock viewed in the frame in which there is no electric field and the shock is stationary. The plasma flow is along the magnetic field lines in both the upstream and downstream regions.

$4w_2/c$, from equation (3). The energy increase due to one crossing is

$$E_{k+1} = E_k \left(\frac{1 + \mathbf{v}_{k1} \cdot (\mathbf{w}_1 - \mathbf{w}_2)}{1 + \mathbf{v}_{k2} \cdot (\mathbf{w}_1 - \mathbf{w}_2)} \right) \tag{26}$$

and this gives a smaller increase than the comparable equation (equation (4)) for a parallel shock because $\mathbf{w}_1$ and $\mathbf{w}_2$ are not parallel. However, since the particle will be gyrating about a field line, on each occasion that it returns from downstream its complete passage across the shock may consist of many individual crossings, each of which increases the energy. These effects of the reduced energy increase, and the increased number of crossings on each occasion that the particle returns cancel out, and the same energy spectrum results as for the parallel shock.

The energy spectrum can be derived by considering the mean probability of return for any particle which crosses from upstream to downstream, whether or not it reaches the stage of gyrating freely in the downstream region. Once again, the time-independent diffusion equation requires that the particle density at the shock is the same as that downstream, and equation (5.7) of Hudson (1965) shows that the particle distribution at the shock is isotropic. Hence the rate at which particles are entering the downstream region through unit area of the shock is ¼nc, and the rate at which particles are escaping downstream is $n\mathbf{w}_2 \cdot \mathbf{s}$, where $\mathbf{s}$ is the unit shock normal. The probability of return is therefore

$$\eta = 4 \frac{\mathbf{w}_2 \cdot \mathbf{s}}{c}. \tag{27}$$

This probability is the properly weighted average for all the angles at which particles cross the shock front. Since a particle needs to make a large number of crossings to be accelerated significantly, the probability P_l of a particle making at least l crossings is given by

$$\ln P_l = l \ln \left(1 - 4 \frac{\mathbf{w}_2 \cdot \mathbf{s}}{c} \right). \tag{28}$$

A combination of this and the expression for the energy gained in l crossings (deduced from equation (26)), leads to a power law for the differential energy spectrum with a spectral index of

$$\mu = 1 + \frac{|\mathbf{w}_1 - \mathbf{w}_2|}{\mathbf{w}_2 \cdot \mathbf{s}}. \tag{29}$$

Since $|\mathbf{w}_1 - \mathbf{w}_2| = \mathbf{w}_1 \cdot \mathbf{s} - \mathbf{w}_2 \cdot \mathbf{s}$ and the compression, χ, is $(\mathbf{w}_1 \cdot \mathbf{s})/(\mathbf{w}_2 \cdot \mathbf{s})$ if the upstream and downstream waves are stationary, this reduces to equation (12). Hence the energy spectrum for an oblique shock is the same as for a parallel shock. As remarked upon above, this argument does not apply to shocks which intersect the upstream magnetic field at velocities close to or greater than the velocity of light, i.e. $v_s \sim c \cos \alpha$, where α is the angle between the magnetic field and the shock normal. If $v_s \ll c$, then the range of α over which this mechanism does not apply is very small.

5 Conclusions

This paper shows that particles which are already sufficiently energetic to see the shock front as a discontinuity are accelerated by bouncing back and forth across the shock. The resulting energy spectrum is a power law with an index similar to that observed for cosmic rays. The highest energy to which particles are accelerated is limited either by damping of the Alfvén waves, or by the scale length $x_0(\propto E^{1.5})$ being comparable or greater than the radius of curvature of the shock front. This mechanism works for all shocks except those which propagate nearly normal to the magnetic field. It does, however, assume that there is another mechanism by which particles are accelerated from thermal energies to the energy necessary for the mechanism described above to take over. The manner of the injection of particles at suitable energies, and possible applications of the acceleration mechanism, will be discussed in a further paper.

Acknowledgments

I am indebted to Dr P. A. G. Scheuer, Dr J. Skilling and Mr S. J. Schwartz for their valuable help and guidance, and to Dr J. R. Shakeshaft for help with the manuscript. I acknowledge the support of a Science Research Council studentship.

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