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SUPERNOVA SHOCK ACCELERATION OF COSMIC RAYS IN THE GALAXY

R. D. BLANDFORD
California Institute of Technology

AND

JEREMIAH P. OSTRIKER
Princeton University Observatory
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ABSTRACT

The regeneration of galactic cosmic rays by first-order Fermi acceleration due to supernova shock waves traversing interstellar space is investigated. It is shown that if most of the volume of the interstellar medium comprises a low density "coronal" phase and supernova blast waves can propagate for more than 100 pc, then reacceleration of existing cosmic rays is important. Model Green's functions describing this redistribution in energy are calculated, and their properties are contrasted with stochastic and source function acceleration schemes. Shock wave acceleration is incorporated in a model containing simple treatments of escape, ionization, radiation, and spallation loss. The injection of low energy particles is treated in an *ad hoc* fashion. It is shown that it is possible to reproduce the observed cosmic ray spectra for protons, electrons, light, medium and heavy nucleons up to energies $\lesssim 1000$ GeV per nucleon with an effective galactic supernova rate that can be as low as 8 kpc^{-2} Myr^{-1}. The relative importance of direct acceleration of suprathermal particles produced in a shock front and reacceleration of existing cosmic rays may be best gauged by observing the energy dependence of the abundance ratio of light to medium nucleons and the low energy proton and electron spectra. Observational tests for the importance of this acceleration mechanism are proposed.

Subject headings: cosmic rays: general — shock waves — stars: supernovae

I. INTRODUCTION

A generally accepted explanation for the origin and acceleration of galactic cosmic rays has remained elusive despite decades of effort. As the observational foundations of the subject have improved, constraints on a self-consistent theory have become tighter, and many initially promising acceleration mechanisms are now known to be of only minor importance. There are two general types of mechanism: the acceleration may be associated with either supernova explosions (e.g., Baade and Zwicky 1934; Colgate and Johnson 1969; Gunn and Ostriker 1969; Kulsrud, Ostriker, and Gunn 1972; Scott and Chevalier 1975) or interstellar space (e.g., Fermi 1949; Jokipii 1977; Kulsrud 1979). There are inherent advantages to both classes, which exist regardless of the specific physical mechanism considered. The energy available in ordered motion ($\sim 10^{51}$ ergs) in supernova explosions and the observed existence of synchrotron-emitting relativistic electrons in their remnants are the most persuasive arguments for a supernova origin. The smooth power-law spectrum ($dN/dT \propto T^{-2.65}$ for $T \gtrsim 3$ GeV per nucleon) extending over many decades of energy, the isotropy ($\delta \approx 10^{-4}$ at $T \approx 100$ GeV per nucleon), the approximate equality of cosmic ray and interstellar pressures, and the apparent absence of strong isotopic anomalies are adduced as arguments for an interstellar origin.

However, there are also difficulties for both supernova and interstellar theories. On observational and on theoretical grounds it is believed that cosmic rays are well-coupled to the background medium through which they propagate. This implies that cosmic rays that have been produced comparatively soon after a supernova explosion will suffer catastrophic energy losses, thereby making unacceptable demands on the efficiency of the acceleration mechanism (e.g., Kulsrud and Zweibel 1975). Furthermore, in young supernova remnants (SNRs) the cosmic-ray proton pressure cannot exceed the relativistic electron pressure by the same factor (~ 100), as is appropriate for locally observed particles (e.g., Woltjer 1958, 1972; Bell 1977). Electrons either suffer unusually high losses relative to protons (at moderate energies) or the protons are accelerated elsewhere. There are three main arguments against an interstellar origin. First, no one has been able to find a mechanism capable of converting the observed interstellar turbulence into cosmic-ray energy with enough efficiency to account for the bulk of the cosmic-ray energy at ~ 1 GeV. Second, if particles are accelerated stochastically in interstellar space, then those at high energy ought to be oldest, but from measurements of the abundance ratio of light to medium nuclei, we know that high energy cosmic rays are retained by the galaxy for a shorter time than low energy cosmic rays. Finally, interstellar acceleration schemes have generally failed at subrelativistic energies where they were unable to overcome ionization losses in the interstellar medium.

Two recent developments have made possible a hybrid process which synthesizes these two viewpoints; it has, we believe, the advantages of both classes of theory, and avoids most of the above noted difficulties. First, our picture of the interstellar medium has changed dramatically. Observations of the soft X-ray background and of interstellar UV absorption lines have indicated that a large fraction of interstellar space is filled with a high temperature, low density "coronal" gas. Simultaneously, theoretical investigations by several authors including Cox and Smith (1974) and McKee and Ostriker (1977) have shown how this hot medium will be maintained by repeated supernova explosions. In the latter study the typical interstellar conditions were found to be as follows: the hot coronal component with $(n, T) = (10^{-2.5}\ \mathrm{cm}^{-3}, 10^{5.8}\ \mathrm{K})$ fills 70–80% of interstellar space. Rather fluffy warm clouds with $(n, T) = (0.25\ \mathrm{cm}^{-3}, 8000\ \mathrm{K})$ fill 15–30% of space, and these have cold cores corresponding to the "standard interstellar clouds" which have $(n, T) = (40\ \mathrm{cm}^{-3}, 80\ \mathrm{K})$ fill only 2–5% of space but contain most of the mass. For a comprehensive recent review of both observations and theory, see McCray and Snow (1979). Supernova blast waves propagate largely through the hot medium, bypassing, but compressing, the cold cloud cores. Since the volume transversed by a supernova shock wave during its adiabatic phase (i.e., before radiative losses are significant) goes as $\rho_0^{-5/4}$, where ρ_0 is the ambient density, each supernova explosion processes a volume of interstellar space much larger (by a factor of order 1000) than if the same mass of gas had been distributed uniformly with its mean density $(\langle\rho\rangle \approx 2 \times 10^{-24}\ \mathrm{g\ cm}^{-3})$. Indeed, shock waves from neighboring supernovae may touch or burst into the galactic halo before they become radiative. This idea has some direct observational support. For example, Cowie and York (1978) have argued from the absence of interstellar Si IV absorption lines that the radiative interstellar shocks which would produce these lines do not exist in the solar neighborhood; in any case, it now seems likely that the average SNR processes a volume of $\gtrsim 10^7\ \mathrm{pc}^3$ containing $\gtrsim 3 \times 10^{50}$ ergs in ambient cosmic rays.

The second development is the discovery of an attractive feature of a specific acceleration mechanism that may operate under these conditions (Axford, Leer, and Skadron 1977; Bell 1978*a*, *b*; Blandford and Ostriker 1978; Blandford 1979). *A group of particles on passing through a strong collisionless shock will, in the presence of efficient Alfvén wave scattering, be accelerated via a first-order Fermi process to a power-law distribution function.* The slope of this power law depends solely on the shock compression and is fairly close to the slope directly observed in galactic cosmic rays.

These two developments lead naturally to a model in which the ordered energy in expanding SNRs propagating through the galaxy reaccelerates ambient cosmic-ray particles. In Blandford and Ostriker (1978) we argued on energetic and spectral grounds that this mechanism was capable of maintaining the galactic cosmic-ray reservoir. It is our purpose in the present paper to carry out a more detailed, quantitative analysis of this acceleration scheme, including the effects of escape, Coulomb and nuclear collisions, and radiative losses modeled at the lowest nontrivial level. We shall not address the problems posed by temporal and spatial inhomogeneity. Neither shall we discuss the injection of low energy cosmic rays necessary in a steady state to replace the escaping high energy particles and also to explain the observed primary cosmic ray abundance ratios.

In § II we summarize some of the inferences that have been drawn from existing cosmic-ray observations and outline the overall energetics and time scales of the acceleration scheme. In § III we develop a formalism for treating cosmic-ray acceleration by supernova shock waves. An integral operator allowing both large and small changes in particle energy replaces the differential operator familiar in typical stochastic acceleration models. We thus introduce the concept of redistribution which describes the rapid reacceleration by a shock of existing cosmic rays to energies which may be much greater than their preshock values. Redistribution may be thought of as combining the mathematical function of a power-law source (conventional in existing supernova models) with a statistical acceleration mechanism (appropriate for interstellar turbulence). This formalism will be added to a standard treatment of conventional physical processes in § IV. One important simplification that we introduce is to replace light, medium, and heavy nucleons by fictitious L, M, and H species that have the average properties of these elements. Numerical solutions for steady state cosmic-ray spectra are presented in § V, and these are compared, where possible, with approximate solutions and extant observations. Our results are collected in § VI, and some possible tests of this model are outlined.

II. ENERGETICS OF GALACTIC COSMIC RAYS

In the kinetic energy range 5 GeV per nucleon $< T < 100$ GeV per nucleon, all cosmic-ray nucleon species are observed to have power-law intensity spectra $J(T) \propto T^{-\eta}$. For H, He, and medium nuclei (e.g., C, N, O), $\eta \approx 2.7$. The spectra of light nuclei (Li, Be, B) are somewhat steeper ($\eta \approx 3.0$), and of heavy nuclei (e.g., Fe) somewhat flatter ($\eta \approx 2.4$). Recent observations have been reviewed by Ormes and Freier (1978). The light nuclei are almost exclusively of secondary origin, resulting from the spallation of medium cosmic rays on interstellar hydrogen. The inverse process of fragmentation of interstellar nuclei by cosmic-ray protons is less important because of the relative underabundance of hydrogen in cosmic rays and because, for a given cosmic-ray kinetic energy per nucleon, the secondaries will be produced with much lower energies. From the ratio of light-to-medium nuclei and the known spallation cross sections, it can be concluded that for $T \approx 3$ GeV per nucleon, cosmic rays have traversed a column density $\lambda \approx 5$ g cm^{-2} of normal interstellar material. For 3 GeV per nucleon $< T <$ 100 GeV per nucleon the column density is reduced to $\lambda \approx 3(T/10 \text{ GeV per nucleon})^{-0.5}$ g cm^{-2} (e.g., Juliusson 1974). In fact nearly all elements observed above a few GeV per nucleon have abundances that are compatible with a primary source spectrum $S(T) \propto T^{-\eta}$ with $\eta \approx 2.2$, and an exponential path length distribution whose mean value satisfies the above law (e.g., Ormes and Freier 1978; Orth *et al.* 1978; Cassé 1979). The conventional interpretation of these observations is that cosmic rays are continuously produced within the galaxy and escape with a fixed probability per unit time, this probability increasing proportional to $T^{0.5}$ at the higher energies—the so-called energy-dependent "leaky box" model.

As particle transport within the galaxy ought to be primarily a rigidity dependent phenomenon, the same escape law ought to apply (at a given rigidity) to protons. We can then estimate the cosmic-ray power radiated by the galaxy. The mean column density of gas through the galactic disk is $\lambda_D \approx 2 \times 10^{-3}$ g cm^{-2} locally (Spitzer 1978). Most of the cosmic-ray energy density $e_{\rm CR}$ is contained in the 1–5 GeV protons which travel at speeds $\sim c$. At present it is not possible to measure $e_{\rm CR}$ to better than a factor of 2, because of uncertainties due to solar modulation, although it can probably be bounded above by the sum of the ambient interstellar gas and magnetic pressures. We adopt a value 7×10^{-13} ergs cm^{-3}. It is straightforward to show that apart from a model-dependent numerical factor of order unity, the energy flux in cosmic rays radiated by the galaxy is

$$F_{\rm CR} \approx \frac{\lambda_D}{\lambda} e_{\rm CR} c \approx 3 \times 10^{51} \text{ ergs kpc}^{-2} \text{ Myr}^{-1} , \quad (1)$$

independent of the confinement time. The expected anisotropy is $\sim\frac{1}{2}(\lambda_D/\lambda) \approx 2 \times 10^{-4}$, just about compatible with existing upper limits.

If the local supernova rate in the galactic disk is $100\Sigma_2$ supernovae kpc^{-2} Myr^{-1} (corresponding to a total rate of roughly Σ_2 per 12 years if this rate applied throughout the disk at a rate proportional to the pulsar number density), then the energy requirement per supernova is

$$\Delta E_{\rm CR} \approx 3 \times 10^{49} \Sigma_2{}^{-1} \text{ ergs per SNR} . \quad (2)$$

Now, suppose that each supernova remnant expands out to a radius r_c, multiplying the average energy of the swept up cosmic rays by a factor $1 + k$ in the process. If there are no other significant energy gains or losses for the protons, then in a steady state we must have

$$r_c \lesssim r^* = \left(\frac{4}{4\pi} \frac{\Delta E_{\rm CR}}{e_{\rm CR} k}\right)^{1/3} \approx 70 k^{-1/3} \Sigma_2{}^{-1/3} \text{ pc} . \quad (3)$$

If $r_c \ll r^*$, then the majority of the cosmic rays will have to be freshly injected as in the conventional source function formalism. If $r_c \approx r^*$, then most cosmic rays will have been accelerated more than once, and redistribution is important (Ostriker 1979). Adopting as conservative but illustrative numbers, $\Sigma_2 \approx 0.6$, $k \sim 0.2$, we obtain $r^* \approx 135$ pc. This is less than the radius at cooling of a remnant in McKee and Ostriker's (1977) model, though much larger than the cooling radius of a remnant ($\sim$20 pc) in a high density interstellar medium (ISM) (e.g., Woltjer 1972).

In recent years it has been possible to put a lower bound on the confinement time τ_e at least for $T \lesssim 300$ MeV per nucleon. This is because the isotope ^{10}Be, which is produced at an estimable rate as a secondary and β-decays with a half-life of $\sim$1.6 Myr, appears to have a low abundance. It has been concluded that cosmic rays of this energy are confined for times $\tau_e \gtrsim 20$ Myr (Garcia-Munoz *et al.* 1977). Combining this lower limit with the mean grammage, the mean gas density sampled by the cosmic rays is $\rho \lesssim 3 \times 10^{-25}$ g cm^{-3}, which is less than a fifth of the local interstellar gas density. Either cosmic rays have a scale height $\gtrsim$0.5 kpc and fill a flattened galactic halo, or they are excluded from denser gas clouds. There is some observational evidence for the former view (e.g., Webster 1978), and theoretical arguments can be advanced against the latter view (e.g., Cesarsky and Volk 1978). Theoretical arguments have been advanced for this type of limited halo by Jones (1978) and Owens and Jokipii (1977), and it has been suggested that at low energy, cosmic rays are effectively convected away from the galactic disk, and thus their escape time is energy-independent, whereas at high energy diffusive escape is quicker, and the escape time decreases with increasing energy. The energy where the mode of escape changes ($\sim$2 GeV) arises fairly naturally in these models and is quite consistent with the observed energy-dependence of the grammage $\lambda(T)$.

If acceleration in interstellar space is important, then secondaries such as ^{10}Be have a lower Lorentz factor γ averaged over their lifetime than when observed at a given energy. Thus, their lifetime is shorter and the interstellar density greater than calculated if reacceleration is ignored.

In summary, the observations seem to require a source spectrum $S(T) \propto T^{-2.2}$. This is just what is expected from the shock acceleration model when, on average, the compression is reduced from the strong shock value of 4 (Blandford and Ostriker 1978; Jokipii and Higdon 1979). The particles that are accelerated may either be freshly injected from the ISM or redistributed GeV cosmic rays. The relative volumes swept out by shocks of different strengths is determined by the SNR model, and it is this problem that we now address.

III. THE REDISTRIBUTION FUNCTION

a) Notation

We wish to determine the steady state spectra for various cosmic ray species, measured at an average point in the galactic plane ignoring spatial and temporal variation. As the distribution function transmitted by a shock is a power law in momentum p, it is useful to use the rigidity $R = p/Z$ $(e = c = 1)$ as the independent variable. Propagation within and escape from the galaxy should be similar for different particles of the same rigidity. A convenient dependent variable is $Y(R)$, which is related to the usually quoted intensity $J(T)$ [$\mathrm{m^{-2}\,sr^{-1}\,s^{-1}}$ (GeV per nucleon)$^{-1}$] as a function of kinetic energy per nucleon T (GeV per nucleon) by

$$Y(R) \equiv Z^2 J(T) R^2/A^2 \ \mathrm{m^{-2}\,sr^{-1}\,s^{-1}\,GeV}\,, \quad (4)$$

where

$$R = (A/Z)(T^2 + 2Tm_p)^{1/2} = p/Z \ \mathrm{GeV}\,, \quad (5)$$

with A the atomic weight, Z the atomic number and m_p the proton rest mass. We assume that the intensity is isotropic. If we designate the density of particles in momentum space by $f(p)$, where $dN \equiv 4\pi p^2 dp dV f(p)$ is the number of particles in volume element dV with momentum p in dp, then

$$f(p) = AZ^{-4}R^{-4}Y(R) \quad (6)$$

relates f and Y. For electrons we set $A = Z = 1$, and $m_p \rightarrow m_e$.

b) Shock Acceleration and Adiabatic Decompression

As shown in Bell (1978*b*) and Blandford and Ostriker (1978), shock acceleration takes a group of upstream particles at given momentum, p_0, and transforms them to a power law distribution p^{-q} $(p > p_0)$ downstream. The exponent q is given by

$$q = \frac{3s}{s-1} = \frac{3(\gamma+1)M^2}{2(M^2-1)}\left(= \frac{4M^2}{M^2-1}\,, \gamma = \tfrac{5}{3}\right), \quad (7)$$

where s is the compression ratio in the shock, M the Mach number, and γ the specific heat ratio, and we assume that the shock is strongly super-Alfvénic so that the scatterers can be considered to be at rest with respect to the background fluid. We assume that $\gamma = 5/3$ unless otherwise noted. The power law will hold so long as the scattering, dynamical, and diffusion times are ordered as follows: $\tau_{sc} < \tau_{dy} < \tau_{diff}$.

For adiabatic shocks $s < 4$, and thus $q > 4$. If the incident particles have momentum p_0, then the mean momentum $\langle p' \rangle$ of the downstream particles is

$$\frac{\langle p' \rangle}{p_0} = \frac{q-3}{q-4} = \frac{3}{4-s} = \frac{M^2+3}{4}\,, \quad (8)$$

and a similar expression can easily be derived for the second moment $\langle p'^2 \rangle / p_0^2$. Strong shocks $(M \gg 1)$ give the particles a large boost in momentum $\Delta p/p_0 \gg 1$. However, for weak shocks (with $M = 1 + m$; $m \ll 1$) the process acts like ordinary diffusion with

$$(\langle \Delta p \rangle / p_0) = \tfrac{1}{2}m + \tfrac{1}{4}m^2\,,$$

and

$$(\langle \Delta p^2 \rangle / p_0^2) = 2/(q-5)(q-4) = \tfrac{1}{2}m^2\,.$$

In this limit a narrow upstream momentum distribution is transformed to a slightly broadened and shifted downstream distribution.

The postshock density and pressure are above those in the ambient medium so the cosmic rays must be subsequently decompressed and will lose energy via adiabatic expansion before returning to the original average ambient state. If they reexpand by a factor F^{-3} in volume $(F < 1)$, then each particle's momentum is multiplied by F. Since we are interested in the net gain of energy around the cycle, we must allow for these losses. We have considered two prescriptions for specifying F and believe that conditions within an expanding SNR lie between the two limits.

a) Each mass element is returned on average from the postshock density $s\rho_0$ to its preshock density ρ_0. Then

$$F_d(q) = s^{-1/3} = \left(\frac{M^2+3}{4M^2}\right)^{1/3} = \left(\frac{q-3}{q}\right)^{1/3} \equiv \frac{p''}{p'}\,. \quad (9)$$

This would be appropriate if energy transport processes in the thermal gas were rapid $(\tau_{cool} < \tau_{dy})$.

b) In the other limit we imagine the thermal fluid to expand adiabatically (with $\gamma = 5/3$) from the postshock pressure P_s back to ambient pressure P_0. Then

$$F_p(q) = \left(\frac{P_s}{P_0}\right)^{-1/5} = \left(\frac{5M^2-1}{4}\right)^{-1/5} = \left(\frac{q-4}{q+1}\right)^{1/5} \equiv \frac{p''}{p'}\,. \quad (10)$$

As one expects there is a net energy gain over the cycle. If we let the final mean momentum of a group of particles initially at p_0 be $\langle p'' \rangle$, then one can show that

$$\frac{\langle p'' \rangle}{(p_0)_{d,p}} = \left(\frac{q-3}{q-4}\right) F_{d,p}(q) > 1 \quad \text{for } q > 4\,. \quad (11)$$

In the limit of weak shocks the Fokker-Planck coefficients, including decompression, are

$$\frac{\langle \Delta p \rangle}{p_0} = \frac{\langle p'' \rangle - p_0}{p_0} = \begin{Bmatrix} m^2/2 & (d) \\ 2m^2/5 & (p) \end{Bmatrix} \text{ for } m \ll 1\,.$$

$$\frac{\langle \Delta p^2 \rangle}{p_0^2} = \frac{\langle p''^2 \rangle}{p_0^2} - \frac{2\langle p'' \rangle}{p_0} + 1 = \frac{m^2}{4} \quad (d, p)\,. \quad (12)$$

The first order gain on passing through the shock is completely canceled by the adiabatic loss. But the relative second order energy gain for cosmic rays exceeds the third order energy gain for a particle in the background medium (e.g., Landau and Lifshitz 1959), and so more of the kinetic energy of a weak shock can go into cosmic rays than background thermal plasma. Also note that the two extreme limiting models for treating adiabatic losses give similar results for the weak shocks which process the largest volumes.

A given supernova explosion will, when it is young, have large M, and consequently $q \to 4$ (eq. [4]). Formally the ratio of downstream to upstream energy density tends to infinity in that limit because of the long tail produced in the power-law spectrum. This is of course fictional since, among other things, the criterion $\tau_{sc} < \tau_{dy}$ is not satisfied for very large energies. However, this does not cause us difficulties, since the volume of ambient cosmic rays $\to 0$ as $M \to \infty$, and early phases of supernova evolution do not give us spuriously large energy inputs. Similarly, at late phases when the volume processed becomes large, the mathematical formalism applied here becomes invalid since we no longer satisfy the criterion $\tau_{dy} < \tau_{diff}$. Again a spurious divergence is avoided since (see eq. [12]) the energy changes become very weak in the limit of weak shocks.

c) *The Redistribution Function*

In the general case the transmitted distribution functions is

$$f(p) = \frac{n_0(q-3)}{4\pi} F^{q-3} p_0^{q-3} p^{-q} H(p - Fp_0)\,, \tag{13}$$

for an assumed initial distribution function. Also,

$$f_0(p) = \frac{n_0}{4\pi p_0^2}\,\delta(p - p_0)\,. \tag{14}$$

Thus we have a Green's function for $f(p)$, and we can multiply it by the distribution of shock strengths encountered in interstellar space to determine the average rate of change of the distribution function.

We define a shock time τ_{sh} such that

$$dP = \tau_{sh}^{-1} Q(q) dq \tag{15}$$

is the probability per unit time of an average volume element in interstellar space being passed by a shock with strength $q \to q + dq$, subject to the normalization

$$\int_0^\infty Q(q) dq = 1\,. \tag{16}$$

Let the cosmic rays be confined within a thin disk of total width $H = \lambda_D/\bar{\rho}$. The average supernova rate per unit volume is then Σ/H, and so

$$d\dot{P} = 4\pi r^2 (\Sigma/H)(dr/dq) dq\,, \tag{17}$$

where r is an implicit function of q the exact nature of which depends on the models adopted for SNRs and for the ISM. As an illustration of how the shock time is determined by the supernova model, consider the case where $r \propto t^\eta$, $M \propto t^{\eta-1}$ which includes as special cases the Sedov solution $\eta = 2/5$ (e.g., Woltjer 1972) and the evaporation solution $\eta = 3/5$ of McKee and Ostriker (1977). Normalizing to the radius r_c where $M = M_c$, $q = q_c$ at which interior cooling occurs, and where we assume that acceleration ceases, we obtain, by equating (15) and (17) for the assumed dependence $r(t)$,

$$\tau_{sh}^{-1} = \frac{4\pi}{3} r_c^3 (\Sigma/H)\,, \tag{18}$$

$$Q(q)dq = \left(\frac{6\eta}{1-\eta}\right)\left(1 - \frac{4}{q}\right)^{(5\eta-2)/2(1-\eta)} \frac{dq}{q^2}\,,$$
$$4 < q < q_c = 4M_c^2/(M_c^2 - 1)\,, \tag{19}$$

and zero otherwise.

Returning to the formal development and combining the distribution in shocks, equation (5) with the Green's function (13), we can write an equivalent of the Boltzmann collision operator for the change in the distribution function due to shocks

$$\left(\frac{\partial f}{\partial t}\right)_{sh} = -\frac{f}{\tau_{sh}} + \frac{1}{\tau_{sh}} \int_4^{q_c} dq Q(q) \times \int_0^{p/F(q)} dp' f(p') p'^{q-1} p^{-q} F^{q-3} (q-3)\,, \tag{20a}$$

in terms of the function $Y \propto (p^4 f)$ defined by equation (4),

$$\left(\frac{\partial Y}{\partial t}\right)_{sh} = -\frac{Y}{\tau_{sh}} + \frac{1}{\tau_{sh}} \int_0^{p/F(4)} \frac{dp'}{p'} Y(p') \times \int_4^{q'} dq (q-3) Q(q) \left(\frac{p}{p'}\right)^{4-q} F^{q-3}\,, \tag{20b}$$

where we have reversed the order of integration. We define q' by

$$F(q') \equiv p/p' \quad \text{for } p'/p \geq F(q_c)\,,$$
$$q' = q_c \quad \text{for } p'/p \leq F(q_c)\,. \tag{21}$$

Now, defining a logarithmic variable for rigidity

$$x \equiv \ln R\,, \quad z \equiv (x - x') = \ln (R/R') \tag{22}$$

and the redistribution function (rdf)

$$\phi(z) \equiv \int_4^{q'} dq (q-3) Q(q) F(q)^{q-3} \exp[(4-q)z]\,, \tag{23}$$

the shock operator (20b) is simply

$$\left[\frac{\partial Y(x)}{\partial t}\right]_{sh} = \frac{1}{\tau_{sh}}\left[-Y(x) + \int_0^\infty \phi(x - x') Y(x') dx'\right]. \tag{24}$$

Conservation of particles implies the normalization

$$\int_0^\infty \phi(z) e^{-z} dz = 1\,. \tag{25}$$

A useful measure of the efficiency of a given rdf is the mean fractional increase in rigidity per shock time.

$$k = \frac{\tau_{\rm sh}}{R}\left\langle\frac{dR}{dt}\right\rangle = \int_0^\infty \phi(z)dz - 1 \,. \quad (26)$$

For relativistic particles, k agrees with the quantity introduced in § II. An estimate of the effective acceleration time is then $\sim \tau_{\rm sh}/k$ which, on energetic grounds should be comparable with the escape time at $R \approx 3$ GeV.

We have computed $\phi(z)$ for three dynamical models of an expanding SNR.

A) For an adiabatic blast wave in a medium with constant density, the Sedov-Taylor solution, equation (9), applies with $\eta = 2/5$. The solution follows basically from energy conservation, and thus the same exponent applies if thermal transport such as conduction (see Solinger *et al.* 1975) modifies the interior structure

$$Q(q)dq = \frac{4M_c^{\,2}}{q^2}\,dq\,, \quad 4 < q < q_c\,. \quad (27)$$

Figure 1 shows the redistribution function $\phi(z)$ defined by equation (23), for two models for adiabatic losses and various values of the Mach number at cooling, M_c.

B) McKee and Ostriker (1977) have argued that evaporation of clouds within the hot interior of an SNR raises the mean interior density to a value over the ambient density. In our numerical work we adopted the following prescription for the dynamics of this model prior to cooling. For radius $r < r_s = 50$ pc we assumed that evaporation was saturated and ineffective, so that the mean interior density equaled the exterior intercloud density ρ_0. Then the interior density increased sharply by a factor of 8 and declined smoothly as $r^{-5/3}$ reaching a value of $2\rho_0$ at $r = r_c$. Essentially this gives an $\eta = 2/5$ solution for $r < r_s$ and $\eta = 3/5$ for $r_s < r < r_c$. Results for this model are shown in Figure 2.

C) We follow Bell (1978*a*) in assuming that the scattering ahead of the shock is caused by self-excited Alfvén waves propagating away from the discontinuity and that the scatterers behind the shock are at rest with respect to the background medium. If the Alfvén speed significantly exceeds the sound speed and cooling is unimportant, then it is straightforward to show that for $\eta = 2/5$,

$$Q(q) = 2(q_c - 3)(q_c - 4)^{-2}(q - 4)(q - 3)^{-3}\,, \quad 4 \le q \le q_c\,. \quad (28)$$

The function $\phi(z)$ which is not especially sensitive to q_c (set equal to 10) is plotted in Figure 2. Note that it is very similar to the low M_c rdf's of models A and B.

In general, the rdf's show two components; a peak near $z = 0$ describing the essentially stochastic motion of the particles in momentum space attributable to many weak shocks, and a high energy tail representing redistribution by the strong shocks. This high energy tail falls off as only a power of the logarithm of the momentum increase, and so it behaves as a source term with a spectrum in relativistic kinetic energy slightly steeper than $S(T) \propto T^{-2}$. For large momentum gains in models A and B, $Q(q \to 4) = 1/4$, and taking $F_d \to 4^{-1/3}$, $F_p \to 5^{-1/5}$, we obtain asymptotic forms

$$\phi_d(z) \approx 0.16z^{-1}\,, \quad \phi_p(z) \approx 0.17z^{-6/5}\,.$$

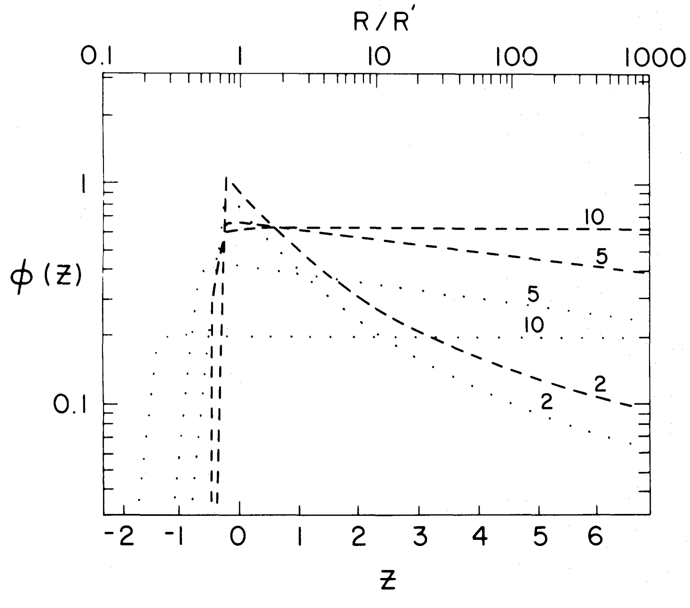

FIG. 1.—Redistribution functions $\phi(z)$ calculated for Sedov solution supernova models as a function of $z = \ln(R/R')$. The acceleration is presumed to cease when the Mach number attains the value labeling the curves. Dashed curves represent models in which the particles expand back to ambient density and dotted curves represent models in which the expansion is to the ambient pressure.

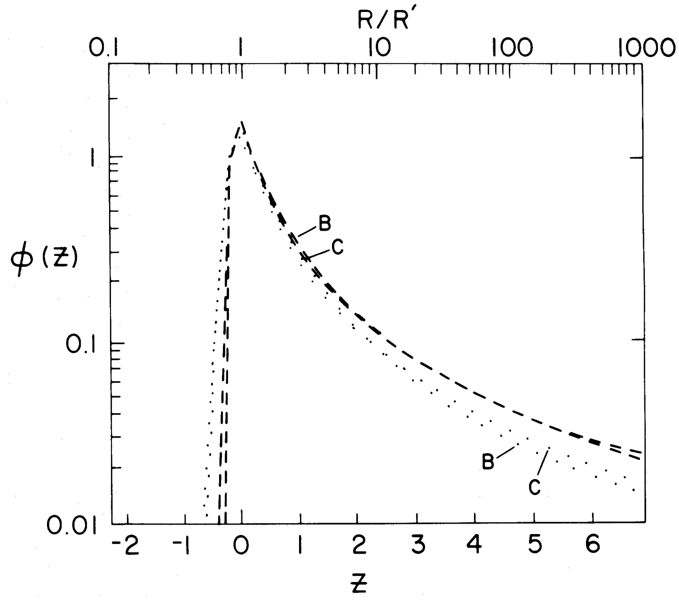

FIG. 2.—Redistribution functions $\phi(z)$ calculated for supernova models B and C as described in the text. Dashed curves represent models in which the particles expand back to the ambient density and dotted curves represent models in which the expansion is to the ambient pressure.

In model C,

$$\phi_d(z) \approx 1.26z^{-2}\,, \quad \phi_p(z) \approx 0.98z^{-11/5}\,.$$

Model C is different because for strong shocks, $q - 4$ is linear in M^{-1} rather than quadratic, which is the case when sonic effects dominate.

What distinguishes the different shock models is the relative importance of these two components. For Sedov models with high intercloud density and $M_c \gtrsim 5$ we see that $\phi(z)$ is fairly flat and the diffusive component is very small, whereas for coronal models of the ISM when $M_c \lesssim 2$ it is, in energetic terms, about as important as the high energy tail. When the shock operator acts on the pool of galactic cosmic rays with $R \approx 1$ GeV, the central peak in $\phi(z)$ will produce many small energy gains in particles of this rigidity. Occasionally these particles will be accelerated by a strong shock up to much higher energies. Strong shocks provide the effective source term for high energy cosmic rays.

In Table 1 we give the shock times and interstellar densities appropriate to supernova models parametrized by different values of M_c and indicate the relevant scalings. In § II we showed that redistribution is important if $r_c \approx r^* \approx 100$ pc. We expect that $E_{51} \lesssim 1$, $\Sigma_2 \lesssim 1$ and so from the table we confirm that redistribution can only be important in a low density ($\rho_{0\text{-}24} \approx 0.5$) ISM, because only then is the diffusive part of the shock operator generally significant.

d) *Effect of Shock Accelerated Particles on the Supernova Remnant*

The pressure in the cosmic-ray component of the gas increases substantially when a shock passes a given fluid element. One can show from equations (7) and (44a) that the ratio is $(P_s/P_0)_{\rm CR} = M^2$ for relativistic particles if the cosmic-ray pressure is much less than the background pressure and we can ignore Alfvén effects. For nonrelativistic particles one must treat weak and strong shocks separately, in the weak shock ($M < \sqrt{5}$) case which is most relevant $(P_s/P_0)_{\rm CR} = 4M^2/(5 - M^2)$, which is slightly larger than in the relativistic case. For the background medium, the ratio of pressure is $(P_s/P_0)_{\rm th} = (5M^2 - 1)/4$, about the same for $M^2 \approx 1$. In the ambient interstellar medium we estimate that $P_{0\rm th} \approx 5 \times 10^{-13}$ dynes cm^{-2} and $P_{0\rm CR} \approx 3 \times 10^{-13}$ dynes cm^{-2}. Here

$$\frac{P_{s\rm CR}}{P_{s\rm th}} \gtrsim 0.6\,\frac{4M^2}{5M^2 - 1} \gtrsim 0.5\,, \qquad (29)$$

independent of Mach number. We therefore conclude that under these assumptions, at least a third of the postshock pressure is contributed by the cosmic rays independent of the Mach number. Adiabatic losses after this point will increase the importance of the cosmic-ray fluid in the total pressure since it will decompress with a lower specific-heat ratio than the background medium. For a power-law rigidity spectrum $f(p) \propto p^{-q}$, the effective specific heat ratio is $\gamma_{\rm CR} = 1 + P_{\rm CR}/e_{\rm CR} = 2/3$, which equals 1.55 if $q = 4.65$. For the steady state cosmic ray spectra computed below, we obtain $1.4 \lesssim \gamma_{\rm CR} \lesssim 1.5$. We adopt a value of 1·5 for $\gamma_{\rm CR}$, significantly lower than the specific heat ratio of 1.67 appropriate to the background medium.

We can use $\gamma_{\rm CR}$ to make a rough estimate of the increase in the total cosmic ray energy density relative to thermal energy density after decompression as follows. A volume element dV'/s of shocked fluid will expand by a factor of $[P_s(V')/\bar{P}(V)]^{1/\gamma}$ where $\bar{P}(V)$ is

TABLE 1

REDISTRIBUTION FUNCTIONS

Model	Decompression	M_c	q_c	$\rho_{0,-24}$ g cm^{-3} [a]	R_c pc [b]	R_* pc [c]	k	τ_{sh} My [d]	$(\langle\Delta\dot{P}\rangle/P)$ Myr^{-1}
A	d	1.5	7.2	0.02	140	74	0.8	1.8	0.88
	...	2	5.3	0.03	110	63	1.2	3.6	0.67
	...	5	4.2	0.15	62	49	2.9	20	0.29
	...	10	4.0	0.45	39	45	3.5	80	0.088
	...	20	4.0	1.4	24	45	3.6	340	0.021
	...	30	4.0	2.7	19	45	3.7	700	0.011
	p	1.5	7.2	0.02	140	84	0.6	1.8	0.67
	...	2	5.3	0.03	110	72	0.9	3.6	0.50
	...	5	4.2	0.15	62	59	1.6	20	0.16
	...	10	4.0	0.45	39	62	1.4	80	0.035
	...	20	4.0	1.4	24	69	1.0	340	0.0059
	...	30	4.0	2.7	19	78	0.8	700	0.0023
B	d	1.5	6.7	0.007	180	88	0.5	0.8	1.25
	p	1.5	6.7	0.007	180	94	0.4	0.8	1.00
C	d	...	10	...	...	88	0.5	...	...
	p	...	10	...	...	94	0.4	...	...

[a] Values $\propto E_{51}{}^{0.1} p_{0,-12}{}^{0.8}$.
[b] Values $\propto E_{51}{}^{0.3} p_{0,-12}{}^{-0.3}$.
[c] For $\Sigma_2 = 0.5$, scales as $\Sigma_2{}^{-0.3}$.
[d] For $\Sigma_2 = 0.5$, scales as $E_{51}{}^{-1} p_{0,-12} \Sigma_2{}^{-1} H$.

an average pressure in the interior of the remnant at its present volume V. However, the sum of all these expanded volume elements must equal V. If we assume that s is constant and that $P_s(V) \propto \bar{P}(V) \propto V^{-1}$, as is appropriate during the Sedov phase at least, then we obtain

$$\bar{P}(V) = [\gamma/s(\gamma - 1)]^{\gamma} P_s(V) . \tag{30}$$

(This prescription ensures conservation of energy.) For $\gamma = 5/3$, $s = 4$, the coefficient is 0.46 which is in good agreement with the Sedov value of 0.47. For $\gamma_{CR} = 1.5$, the coefficient is 0.65 and so the ratio of the cosmic-ray energy to the thermal energy in the interior of the remnant is a factor ~ 1.5 larger than the postshock value. Using equation (29), we obtain $P_{CR}/P_{th} \gtrsim 0.7$. In other words, cosmic rays supply a large fraction of the internal energy within an expanding SNR.

A result similar to this has been proposed on quite different grounds by Eichler (1979). Eichler assumes that the energy input into the cosmic-ray fluid is dominated by freshly accelerated particles and that there is no restriction on the rate at which they are injected from the thermal pool into the shock acceleration process. Both derivations can be criticized on several grounds. Firstly, when the postshock cosmic-ray and thermal pressures are comparable, it is no longer safe to assume that the shock jump conditions are those appropriate to the background medium alone. In fact, the effect of the cosmic-ray component will be to increase s and thus flatten the transmitted spectrum. The opposite effect, however, will occur if there is a deceleration of the background medium (and hence the scattering centers) ahead of the shock due to interaction with the cosmic ray precursor (see Eichler 1979). Finally, the magnetic pressure in the interstellar medium is conventionally estimated as 4×10^{-13} dynes cm^{-2}, comparable with both the thermal and ambient cosmic-ray pressures. This leads to magnetic corrections to s and also raises the possibility that the field not be able to support the cosmic-ray gradient if the postshock cosmic-ray pressure greatly exceeds the preshock magnetic pressure. In particular the presence of the cosmic rays will increase the effective Alfvén velocity (Kulsrud and Zweibel 1975). The efficiency of the acceleration will again be impaired.

These are clearly quite subtle issues and there seems little prospect at present of understanding them well from a purely theoretical standpoint. The formalism developed above and used below may well exaggerate the importance of redistribution, especially when low Mach number shocks are involved. We proceed with this caveat in mind.

IV. COMPUTATION OF THE STEADY STATE SPECTRUM

a) *Other Physical Processes*

In addition to the redistribution term, we require terms from all of the conventional processes to add to the right-hand side of the kinetic equation (24).

i) *Ionization*

Low energy particles will lose energy systematically as a consequence of Coulomb collisions with interstellar matter. If the particles experience a mean density of $\bar{\rho} = 10^{-24}\bar{\rho}_{-24}$ g cm^{-3} and the composition of the ISM is 90% H and 10% He by number and predominantly atomic, then the deceleration of nucleons can be expressed as

$$\frac{dR}{dt}=\frac{A}{Z}\frac{dT}{dl}=-Z\omega(u)\rho_{-24}\ \text{GeV Myr}^{-1}\,,\quad u\equiv\frac{ZR}{Am_p}\,, \tag{31a}$$

where

$$\omega(u)=4.9\times10^{-4}\left[\frac{(1+u^2)}{u^2}\ln(232u)-0.5\right]. \tag{31b}$$

For electrons,

$$\omega(u)=3.7\times10^{-4}(\ln u+6.7) \tag{31c}$$

(e.g., Reames 1974).

ii) *Radiative Losses*

Electrons also suffer bremsstrahlung, synchrotron, and inverse Compton losses given collectively by

$$\frac{dR}{dt}=-(0.020R\rho_{-24}+3.2\times10^{-3}R^2U)\ \text{GeV Myr}^{-1}\,, \tag{32}$$

where U eV cm^{-3} is the combined average energy density associated with magnetic field in the stellar radiation field and the microwave background. ($U\approx0.9$ for a rms field of 3 μG.)

iii) *Escape from the Galaxy*

We shall use the standard "leaky box" model to deal with escape of the cosmic rays from the galaxy in which the probability per unit time of escape $\tau_e{}^{-1}$ is a simple function of the particle rigidity. In a simple one-component escape law, appropriate if the particles are diffusing out of the galaxy, we have

$$\tau_e{}^{-1}(R)=\beta R^{\mu}\tau_{e0}{}^{-1}\,, \tag{33a}$$

$$\tau_e{}^{-1}(R)=\beta R^{\mu}\tau_{e0}{}^{-1}+\tau_0{}^{-1}\,, \tag{33b}$$

where $\beta=R(R^2+A^2m_p{}^2/Z^2)^{-1/2}$ is the particle velocity. The mean free path should be expressed in terms of a power law in rigidity if the scattering is due to a power law spectrum of hydromagnetic turbulence. For a "Kolmogorov" spectrum, the exponent $\mu=1/3$ and for a "Kraichnan" spectrum, $\mu=1/2$ (Cesarsky 1971). As discussed in § II, the observed reduction in the ratio of secondaries to primaries leads one to expect that μ is in the range 0.3–0.6. At low energies, where difffusion is slow, equation (33a) is probably inapplicable because the dominant means of escape may be by convective motion in a wind or through a Parker-type bubble-like instability, whereby all particles in a given region of space escape. In this case, the escape time is energy-independent at low energies and a *two-component* escape law (33b) containing an additional constant term $\tau_0{}^{-1}$ is more appropriate. Note that the equivalent escape grammage at low energy is then $\propto\beta$ rather than constant as in the treatment of Ormes and Freier (1978).

iv) *Particle Injection*

One traditional difficulty with the Fermi mechanism is that protons injected with energies $\lesssim 10$ MeV per nucleon suffer such large ionization losses that, in order to supply enough particles for the comparatively slow acceleration, there must be an intolerably large heating of the interstellar medium associated with these losses. It seems unlikely that much more energy be associated with the injection than the acceleration and one natural way to avoid this in the current context is to inject particles from the suprathermal (but nonrelativistic particles) accelerated behind strong shock fronts by normal collisionless processes (see Bell 1978*b*; Eichler 1979). If we designate a fixed injection rigidity $R_0\ll1$ GV and a constant injection rate, then we can introduce a source term of the form

$$V(R)=V_0\phi(\ln R/R_0)\approx V_1R^{-\delta}\,, \tag{34}$$

where the exponent δ depends on the redistribution function and is in the range 0–0.3. In practice, the injection rigidity and rate will not be constant, but provided that $R_0\ll1$ GV, the power-law form is adequate for present purposes.

v) *Fragmentation and Spallation*

Medium and heavy nuclei undergo important nuclear reactions mainly with interstellar hydrogen. A fairly good approximation (except for H and He) that we shall adopt is to give the daughter nuclides the same energy per nucleon (or speed) as the heavier primary particles. Our kinetic equation must contain a term of the form

$$-\frac{\rho\sigma_i\beta}{m_p}Y_i(R)+\sum_j Y_j(RZ_iA_j/Z_jA_i)\frac{\rho\sigma_{ji}\beta}{m_p}\,, \tag{35}$$

where σ_i is the total inelastic cross section for species i per interstellar medium nucleon and σ_{ji} is the partial cross section for production of species i from species j. Measuring σ in millibars, henceforth equation (35) becomes

$$5.7\times10^{-4}\rho_{-24}\beta\left[-\sigma_iY_i+\sum_j\sigma_{ji}Y_i\right]. \tag{36}$$

For elements that can undergo β decay with a half-life τ_D, there is an extra term

$$-\frac{Am_p\beta Y_i}{ZR\tau_D}=-0.94\frac{A\beta Y_i}{ZR\tau_D}\,. \tag{37}$$

There is a corresponding source term for the daughter nuclide. For simplicity we make the approximation that $A=2Z$ for all nucleons heavier than hydrogen.

vi) *Light and Medium Nuclides*

One of the most sensitive diagnostics of cosmic-ray propagation is the ratio of the various spallation-produced light nuclide isotopes of (Li, Be, B) to their predominantly medium progenitors (C, N, O). A thorough analysis of the predicted spectra of all of these species is beyond the scope of the present treatment, but the main consequences of reacceleration over and above the results of conventional propagation calculations are, however, conveniently illustrated by performing a more modest calculation. We consider the evolution of a hypothetical medium nuclide $M(A = 14, Z = 7)$, injected with the same rigidity spectrum as the protons and subject to a total spallation cross section $\sigma_M = 200e^{-1/2R}$. It is the sole source of two light nuclides $L(A = 8, Z = 4)$ and $L'(A = 10, Z = 5)$, for both of which the partial production cross sections are $\sigma_{\rm ML} = 40e^{-1/2R}$ respectively. In addition, L has a total spallation cross section of $\sigma_L = 100e^{-1/2R}$ mbar and L' undergoes β decay with a half life $\tau_{\rm D} = 2$ Myr.

We can use the computed ratios of L, L' to M to compute apparent mean grammages $\lambda(R)$ and ages $\tau_\beta(R)$ for our models, defined as follows:

$$\lambda(R) \equiv \frac{Y_L m_p}{Y_M \sigma_{ML} - \sigma_L Y_L}; \tag{38}$$

$$\tau_\beta(R) \equiv \left[\frac{Y_M \sigma_{ML'} \lambda(R)}{Y_{L'} m_p} - 1\right](1 + u^2)^{1/2} \tau_D . \tag{39}$$

$\lambda(R)$ and $\tau_\beta(R)$ are, respectively, the grammages and escape times that would be inferred on the basis of the simplest energy-independent leaky box model. The cross sections and decay time are representative of true light and medium nuclei. We stress that (τ_β, λ) are the apparent ages and grammage. It is far more complicated in our model to calculate true mean values for these quantities, and we have not yet done so.

vii) *Heavy Nuclides*

Heavy nuclei are best exemplified by a heavy nucleus H $(A = 56, Z = 28)$, similar in properties to Fe which has a large ionization loss and spallation cross section $\sigma_H = 700e^{-2/R}$ mbar. We shall assume that H is injected with the same spectrum as the protons.

b) *Solution of the General Equation*

i) *Time-Dependent Equation*

Collecting together all of the above terms and adding them to equation (24), the equation for the time evolution of species Y_i is

$$\begin{aligned}\frac{\partial Y_i}{\partial t} = \frac{1}{\tau_{\rm sh}} \int dx' \phi(x - x') Y_i(x') \\ - \frac{Y_i}{\tau_{\rm sh}} - \frac{Y_i}{\tau_e}(R) - R\frac{\partial}{\partial x}\left[\frac{Y_i}{R^2}\frac{dR}{dt}\right] \\ + 5.7 \times 10^{-4} \rho_{-24} \beta \left[-\sigma_i Y_i + \sum_j \sigma_{ji} Y_i\right] \\ - 0.94\left(\frac{A\beta Y_i}{ZR\tau_D}\right) + V_1 R^{-\delta} .\end{aligned} \tag{40}$$

ii) *Energy Budget*

It is clearly of interest to understand the overall energy balance in this model. In addition, this provides a check on the accuracy and self-consistency of the numerical computations. Most of the power is associated with the protons, and for simplicity we ignore the other components. Taking a kinetic energy density moment of equation (40) we find that in the steady state, cosmic rays of rigidity R in the range $R_{\rm min} = \exp(x_{\rm min}) \leq R \leq R_{\rm max} = \exp(x_{\rm max})$ gain energy at rates W_a, W_r from the initial acceleration and redistribution, respectively, and lose an equivalent amount in heating the interstellar medium through ionization loss (W_i) and escape (W_e) where

$$W_a = 4\pi\left(\frac{\lambda_D}{\bar\rho}\right) V_1 \int_{x_{\rm min}}^{x_{\rm max}} dx R^{-\delta} g(R) , \tag{41a}$$

$$W_r = \frac{4\pi}{\tau_{\rm sh}}\left(\frac{\lambda_D}{\bar\rho}\right) \int_{x_{\rm min}}^{x_{\rm max}} dx \times \left[\int dx' \phi(x - x') Y(x') - Y(x)\right] g(R) , \tag{41b}$$

$$\begin{aligned}W_i = -4\pi\left(\frac{\lambda_D}{\bar\rho}\right) \int_{x_{\rm min}}^{x_{\rm max}} dx Y(x)(R^2 + m_p{}^2)^{-1/2} \frac{dR}{dt} \\ - 4\pi\left[Y(x_{\rm min}) g(R_{\rm min}) \frac{\dot R(R_{\rm min})}{R_{\rm min}} \right. \\ \left. - Y(x_{\rm max}) g(R_{\rm max}) \frac{\dot R(R_{\rm max})}{R_{\rm max}}\right] ,\end{aligned} \tag{41c}$$

$$W_e = 4\pi \int_{x_{\rm min}}^{x_{\rm max}} dx Y(x) \tau_e{}^{-1}(R) , \tag{41d}$$

where

$$g(R) = [(R^2 + m_p{}^2)^{1/2} - m_p]/R . \tag{42}$$

In this form, the quantities W are expressed as powers per unit area of galactic disk. Energy balance implies that

$$W_a + W_r = W_i + W_e . \tag{43}$$

The ambient pressure and energy density are given respectively by

$$P_{\rm cr} = \frac{4\pi}{3} \int_{x_{\rm min}}^{x_{\rm max}} dx YR(R^2 + m_p{}^2)^{-1/2} ; \tag{44a}$$

$$e_{\rm cr} = 4\pi \int_{x_{\rm min}}^{x_{\rm max}} dx Y g(R) . \tag{44b}$$

iii) *Numerical Method*

We seek a stationary solution to the coupled equations (40) for each relevant nucleon species. This we obtain by equating the right-hand side to zero, writing the integro-differential equation as a matrix equation, and inverting the matrix. The solution for $Y_i(x)$ can then be expressed in the form

$$Y_i = \mathfrak{M}^{-1} Z_i \tag{45}$$

where $\mathfrak{M}^{-1}$ is the inverted matrix, and for primary particles, the vector $\boldsymbol{Z}_i$ is proportional to the source term. For secondary particles, $\boldsymbol{Z}_i$ is related to the solution $\boldsymbol{Y}_i$ for the relevant parent nuclei. In the computations, $Y_i(x)$ was a 31 component vector evaluated at equally spaced values of x for R in the range $0.1 \text{ GV} \leq R \leq 100 \text{ GV}$. Below 0.1 GV, ionization losses dominate and the steady state distribution function is very small. Above 100 GV, escape or synchrotron loss dominates and the distribution function can be found by extrapolation.

V. STEADY-STATE COSMIC RAY SPECTRA

a) *Models*

We have computed solutions to equation (40) using different redistribution functions and varying the parameters associated with injection, escape, and loss. We illustrate our results in Figure 3 by displaying spectra using two extreme supernova models, an evaporative solution truncated at $M_c = 1.5$ (model I) and a Sedov solution truncated at $M_c = 10$ (model II).

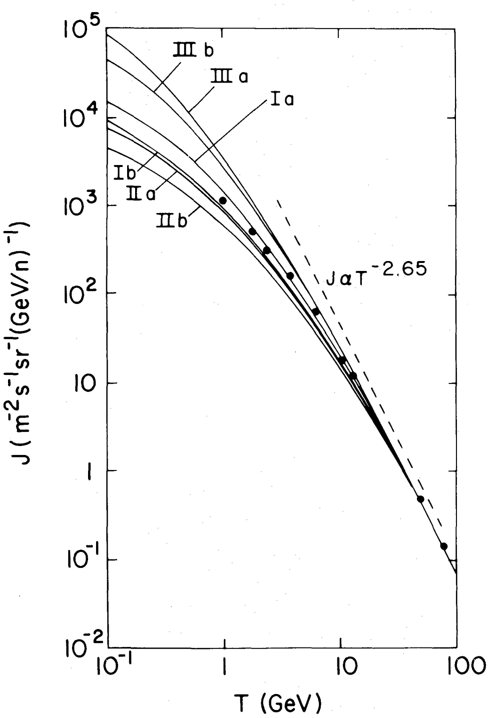

FIG. 3.—Calculated proton spectra for models incorporating (*a*) one- and (*b*) two-component escape laws. Model I utilizes an evaporative solution truncated at $M_c = 1.5$; model II, a Sedov solution truncated at $M_c = 10$; and model III, a conventional model with power-law injection and no redistribution. The data points above 1 GeV are taken from the compilation of Ormes and Freier (1978). Measured intensities below 1 GeV are subject to strong solar modulation and are not shown.

Spectra obtained using evaporative and Sedov supernova models and the same value of M_c were not distinguishable. In model I the cosmic rays were decompressed to the ambient pressure, and in model II to the ambient density. Changing the prescription for dealing with expansion made less than a 30% difference to the results. For comparison purposes, we also computed spectra with a conventional power-law source term without redistribution (model III). Model I (in contrast to models II and III) contains a substantial stochastic component. Models using one- and two-dimensional escape laws, which are substantially different, are displayed in Figure 3 and designated *a* and *b*.

The parameters were adjusted to give a rough fit to observed cosmic ray intensities as follows:

i) The exponent μ in the escape law, equation (32), was determined from the difference in slope between the high energy tail of the redistribution function and the observed high energy proton spectrum.

ii) The product τ_{sh}/τ_e which measures the importance of redistribution relative to initial injection was adjusted to fit the low energy proton spectrum.

iii) The product $\rho\tau_e$ was adjusted to give the observed apparent grammage $\lambda = 5.5 \text{ g cm}^{-2}$ at 1 GeV per nucleon.

iv) The escape time scale τ_e was adjusted to give a value of the apparent lifetime $\tau_\beta = 20$ Myr at 300 MeV per nucleon.

v) The mean electromagnetic energy density U was adjusted to give the observed high energy steepening of the electron spectrum.

vi) The spectra were normalized to the observations at 10 GeV per nucleon.

As discussed in § II, the only significant time scale in the problem is that provided by the ^{10}Be chronometer, which at present gives only a lower bound on τ_e. (In the absence of a reliable estimate of the field strength above the galactic disk, synchrotron losses probably do not as yet provide a good chronometer.) All of the spectral shapes (except that of L') are unaffected by the transformation $\tau_e \rightarrow K\tau_e$, $\rho \rightarrow K^{-1}\rho$, $\tau_{sh} \rightarrow \tau_{sh}$, $V_1 \rightarrow K^{-1}V_1$, $U \rightarrow K^{-1}U$. (Note that for a given value of the cooling radius r_c, the inferred surface density of supernovae in the galactic disk Σ is unchanged by this transformation.)

b) *Protons*

In Figure 3 we compare the proton spectra for these three models with the observed spectrum. In models I and II at low energy ($R \lesssim R_1$), the steady state spectrum is determined by balancing injection (roughly proportional to $R^{-\delta}$) with ionization loss ($\dot{R} \propto -R^{-2}$) and so Y is roughly proportional to $R^{3-\delta}$, or $J(T)$ is roughly proportional to $T^{(1-\delta)/2}$. We find that $R_1 \approx 0.3$ GV ($T_1 \approx 10$ eV) typically, and that the spectrum is somewhat shallower than $J \approx T^{(1-\delta)/2}$ on account of redistribution. At high energy $R > R_2$ we balance reacceleration of the protons with

$R_1 \lesssim R \lesssim R_2$ by escape ($\propto R^{-\mu}$) to give $J(T)$ is roughly proportional to $T^{-(2+\delta+\mu)}$. We find that $R_2 \approx 3$ GV ($T_2 \approx 3$ GeV), typically. What is happening is that nonrelativistic protons are injected into strong shock waves and a small fraction is accelerated in those shock waves up to a rigidity $R \gtrsim R_1$ such that subsequent acceleration (partly by weak shocks) will be more important than ionization loss. This can only occur up to R_2, where for an individual proton, escape starts to become more important than acceleration. For $R \gg R_2$, most protons have been directly accelerated from suprathermal or GeV energies. The typical age of a proton will then be the time required to accelerate from R_1 to R_2—i.e., $\sim(\tau_{sh}/k)\ln(R_2/R_1)$. As might be expected, model III, which includes no redistribution, leads to a steeper low-energy proton spectrum than models I and II. In principle, an accurate determination of the demodulated proton spectrum could distinguish between these two extremes.

The choice of escape time also has a noticeable effect on the spectrum around $T \approx 1$ GeV. The two-component power time spectra are systematically flatter than the one-component spectra in which particles with $R \lesssim (\tau_{e0}/\tau_0)^{1/\mu}$ are relatively less likely to escape from the galaxy. For model IIIa the spectrum is almost a power law in rigidity and perhaps can be excluded on observational grounds because it has an associated energy density of $\sim 1.5 \times 10^{-12}$ ergs cm^{-3}, which may be too large to be retained in the interstellar medium (see Spitzer 1977).

c) *Electrons*

The corresponding electron spectra are plotted in Figure 4. At low energy these are steeper than the proton spectra because ionization loss is less important for relativistic electrons than nonrelativistic protons. From comparing models I and II with model III, we see that one consequence of redistribution is to flatten the low energy spectrum below ~ 1 GeV. This is just what is indicated by the radio data (e.g., Cummings, Stone, and Vogt 1973) which gives $J(T) \propto T^{-1.8} \times (T \lesssim 2$ GeV). In models with significant redistribution, ionization loss can be ignored for $T \gtrsim 20$ MeV because the effective acceleration time is shorter than the loss time. Furthermore, for $T \lesssim 1$ GeV, escape can be ignored, particularly for a one-component escape law. Therefore, in the energy range 20 MeV $\lesssim T \lesssim 1$ GeV, the electrons will be steadily accelerated by the peak in the redistribution function at a rate $\dot{T} \propto T$. If the current in energy space were rigorously conserved, then a flux $J(T) \propto T^{-1}$ would be established. End effects and losses prevent the spectrum from ever becoming as flat as this, but the tendency of the redistribution is nevertheless to effect some spectral flattening below ~ 1 GeV. This flattening is difficult to reproduce with a pure source function model (see Jokipii and Higdon 1979).

At high energy, synchrotron and inverse Compton losses ($\propto T^2$) are more important than escape ($\propto T^{3/2}$),

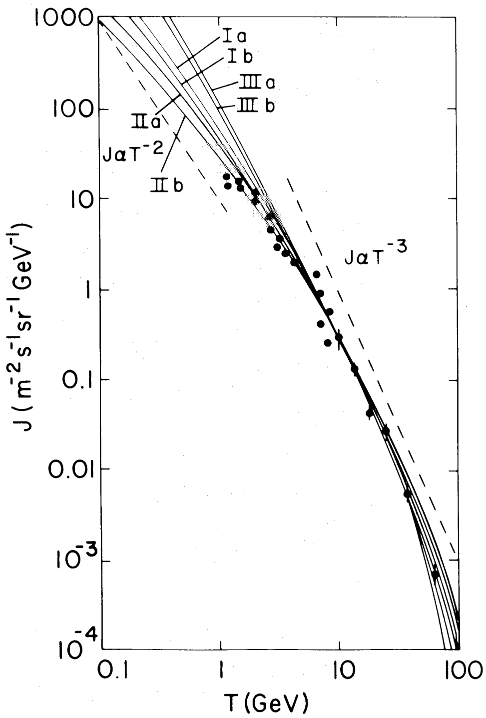

FIG. 4.—Calculated electron spectra for models I, II and III (see Fig. 3). The data points in the range 1–10 GeV are taken from the compilation of Ormes and Freier (1978) and above 10 GeV from Prince (1979). The spectra are normalized to agree with Prince's measurement at 10 GeV.

and the spectrum will steepen. The required electromagnetic energy densities range from $U \approx 0.8$ eV cm^{-3} (model I) to $U \approx 1.6$ eV cm^{-3} (model II). This is within the range of plausible values after averaging over the confinement volume. Note that when there is a substantial stochastic component to the redistribution function, a smaller value of U suffices to steepen the electron spectrum.

The sensitivity to the escape law is similar to that found for the proton spectrum.

d) *Medium and Light Nuclides*

The M spectra (not plotted) are similar to the proton spectra and consistent with the observations. However, the increased importance of ionization loss and the presence of a significant nuclear cross section leads to slightly larger values of R_1, R_2. Instead of plotting the L and L' spectra, we present graphs of the apparent grammage $\lambda(T)$ (Fig. 5), and lifetime $\tau_\beta(T)$ (Fig. 6). Neither of these functions is found to be especially sensitive to the choice of cross sections. With model Ia, $\lambda(T)$ rises proportional to $T^{0.3}$ for $T < 1$ GeV per nucleon and falls proportional to $T^{-0.25}$ above 3 GeV per neutrino. Model IIa is similar, but gives a smaller grammage at large T. By contrast, in the pure injection models (III), $\lambda(T)$ rises slower and falls more rapidly. This is directly attributable to the

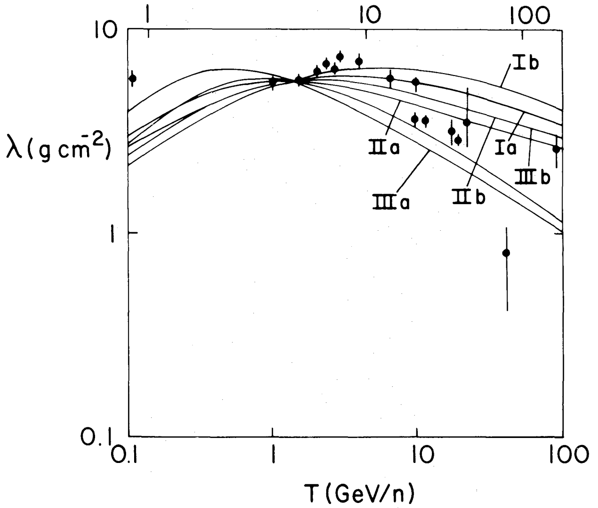

FIG. 5.—Apparent grammage $\lambda(T)$ (defined in eq. [38]) as a function of the kinetic energy per nucleon T for models I, II and III (see Fig. 3). The data points are the calculated escape grammages taken from the compilation of Ormes and Freier (1978), and the curves are normalized to a value of 5.5 g cm^{-2} at 1 GeV per nucleon. As discussed in the text, the apparent grammage below 1 GeV per nucleon is influenced by ionization loss and the velocity dependence in the escape grammage.

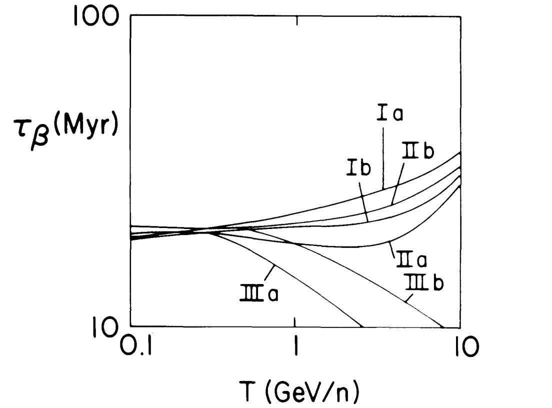

FIG. 6.—Apparent age τ_β computed from the ratio of L' to M (eq. [39]) as a function of kinetic energy per nucleon T for models I, II, and III. The curves are normalized to $\tau_\beta = 20$ Myr at 300 MeV per nucleon.

decreasing importance of redistribution as we go from model I to model III. The reacceleration of the secondary light nuclides leads to a flattening of their steep production spectrum and a consequent flattening of $\lambda(T)$. The observed 3–30 GeV slope of $\lambda(T)$ is probably steeper than that predicted by the redistributive models, although more accurate abundance ratios are needed to confirm this. The difference between calculation and observations below 1 GeV per nucleon is partly artificial because the formula in equation (38) does not take account of ionization losses which have to be included in low energy grammage estimates (e.g., Reames 1974). It is also partly a consequence of the choice of escape law. This can be seen from the pure source function models (III). In the one-component escape law case, the true escape grammage is roughly constant at subrelativistic speeds and falls $\propto T^{-0.5}$ at relativistic speeds. By contrast, $\lambda(T)$ exhibits an increase below 300 MeV per nucleon. In the two-component case, the grammage increases faster for $T \lesssim 1$ GeV per nucleon because escape is relatively easier at the lower energies. For $T \gtrsim 3$ GeV per nucleon, the grammage falls more slowly than in the one-component models. When the assumed two-component law is combined with redistribution as in model Ib, the results seem to be incompatible with the observations. In general, the variation of the grammage with energy is probably the most sensitive indicator of the importance of redistribution. A careful multi-species calculation incorporating accurate cross section should show several observable spectral signatures of redistribution.

The apparent lifetime $\tau_\beta(T)$ shown in Figure 6 can also show a signature of redistribution, because it tends to increase with T in models where redistribution is important and to decrease when there is only a pure source function. However, this will only be apparent above 3 GeV where it is at present impossible to obtain a reliable measurement of the relevant ^{10}Be intensity. Fitting τ_β to an assumed value of 20 Myr gives estimates of the mean density $\bar{\rho}$ comparable to conventional estimates (see Table 2).

e) *Heavy Nuclides*

In Figure 7, we compare the H spectra with the observed spectrum of Fe. Again the results are not

TABLE 2

SHOCK ACCELERATION MODELS

Model	δ	μ	τ_{e0}	τ_0 (Myr)	τ_{sh}	ρ_{-24} (10^{-24} g cm^{-3})	U (eV cm^{-3})	Σ (kpc^{-2} Myr^{-1})	W_a (10^{51} ergs)	W_r (kpc^{-2} Myr^{-1})	P_{cr} (10^{-12} dyne cm^{-2})	e_{cr} (10^{-12} ergs cm^{-2})
Ia	0.2	0.4	30	...	9	0.35	0.65	8	1.1	1.9	0.3	0.7
Ib	0.2	0.5	80	50	11	0.28	0.5	9	1.0	1.3	0.2	0.5
IIa	0	0.65	37	...	20	0.37	1.6	350	0.9	2.7	0.2	0.5
IIb	0	0.75	89	58	22	0.31	1.4	370	0.8	2.0	0.2	0.4
IIIa	0.2	0.45	23	...	...	0.49	0.8	...	4.7	...	0.7	1.5
IIIb	0.2	0.55	63	42	...	0.37	0.7	...	4.1	...	0.5	1.1

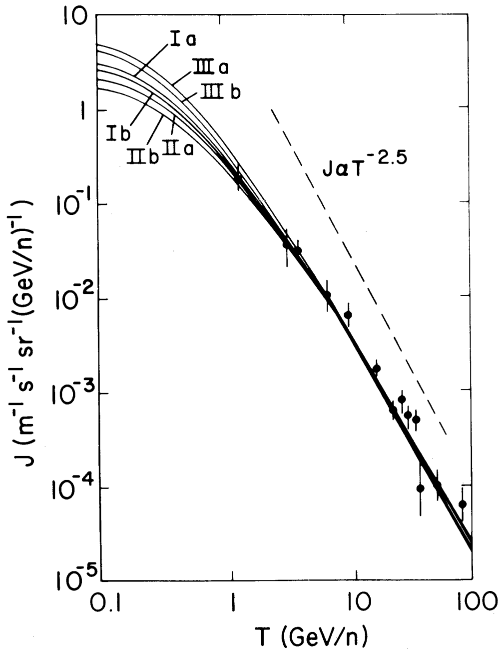

FIG. 7.—Calculated H spectrum for models I, II, and III. The data points are intensities of Fe cosmic rays taken from the compilation of Ormes and Freier (1978).

especially sensitive to the acceleration model. The large ionization and spallation losses give a spectrum $J(T) \propto T^{-2.4}$ in the 3–30 GeV per neutrino range. The slope approaches that of the protons at higher energies.

f) *Energetics*

We are now in a position to compute overall power requirements of the models that reproduce the observed cosmic-ray data and to compare the results of these numerical calculations with the estimates in § II. The principal derived quantities associated with the six models are displayed in Table 2.

For the evaporative supernova model we find that a comparatively low supernova rate $\Sigma \approx 8\ \mathrm{kpc}^{-2}\ \mathrm{Myr}^{-1}$ suffices to replenish the galactic cosmic rays. For comparison, the estimated local pulsar birth rate is $50\ \mathrm{kpc}^{-2}\ \mathrm{Myr}^{-1}$ (Gunn and Ostriker 1970; Hansen 1979), comparable with the death rate of massive stars heavier than $4\ M_\odot$ (Ostriker, Richstone, and Thuan 1974) and the supernova rate estimated from observations of galaxies similar to our own (e.g., Tammann 1974). Thus roughly only one in five supernovae needs to expand out to a radius of 140 pc if shock acceleration is efficient out at this distance. For comparison, the required supernova rate for model II is $\sim 360\ \mathrm{kpc}^{-2}\ \mathrm{Myr}^{-1}$, about seven times larger than the above estimates. In fact, it can be concluded on these grounds that if redistribution is significant, then weak shocks have to be important and the redistribution function has an appreciable "stochastic" component.

If the SNRs cease to expand or at least cease to be efficient accelerators beyond ~ 50 pc, then most particles are only accelerated once and the redistribution acts only on freshly injected particles. It behaves like a simple source function that is a power law in rigidity as in model III. The energy required per supernova is sensitive to the proton spectrum or equivalently to the escape law. For model IIIb and an assumed value of $\Sigma \approx 50\ \mathrm{kpc}^{-2}\ \mathrm{Myr}^{-1}$, $E_{\mathrm{CR}} \approx 8 \times 10^{49}$ ergs. The cosmic rays must be injected and accelerated comparatively late in the evolution of the remnant in order to avoid severe expansion losses. This is because if the cosmic-ray energy in young SNRs were to exceed $\sim 3 \times 10^{50}$ ergs, then this would violate existing upper limits on the γ ray emissivity of nearby remnants (e.g., Montmerle 1979; Pinkau 1979).

Even if redistribution is important as in model I, the initial acceleration must account for $\sim$20–40% of the cosmic-ray energy. The results of the injection can have some observable spectral consequences as well as control the abundance ratios of different species. In particular, a large fraction of the high energy M and H nucleons are directly injected. By contrast, the power supplied in the injected particles at $R = R_0$ need not be so large. For example, if the volume injection rate is uniform and $R_0 = 1$ MV ($T_0 = 0.53$ keV) we calculate the injected particle power to be $\sim 10^{50}$ ergs kpc^{-2} Myr^{-1} for model I and $\sim 3 \times 10^{49}$ ergs kpc^{-2} Myr^{-1} for model III.

The ambient cosmic-ray pressure and energy density depends on the proton spectrum around ~ 1 GeV which is as yet unmeasured. In the models, the spectrum is sensitive to both the redistribution function and the escape law. From Table 2 we see that $E_{\mathrm{CR}} \approx 2.3 P_{\mathrm{CR}} \approx 0.5\text{–}1.0 \times 10^{-12}$ ergs cm^{-3}. In general, models involving significant reacceleration predict a smaller intensity of ~ 1 GeV protons than models without reacceleration.

g) *High Energy Galactic Cosmic Rays*

As emphasized in Blandford and Ostriker (1978), Bell (1978*a*) and Blandford (1979) the formalism used to describe the Fermi acceleration mechanism is invalid at high energy. If the scattering Alfvén waves are self-excited, then a simple estimate based on the quasi-linear theory growth rate yields a maximum energy for efficient acceleration of ~ 300 GeV. If, as advocated by Jokipii (1977), there exists an inertial range spectrum of interstellar turbulence between $\sim 10^{12}$ cm^{-1} and 30 pc^{-1}, then scattering may be efficient at higher energy. For a particle to be able to double its energy at energy T, the diffusion scale height ahead of the shock must be less than the radius of the shock. Equivalently, we require that the relative intensity in the hydromagnetic turbulence satisfy

$$\left(\frac{\delta B}{B}\right)^2 > 0.01\left(\frac{T}{1\ \mathrm{TeV}}\right)\left(\frac{v}{100\ \mathrm{km\ s^{-1}}}\right)^{-1}\left(\frac{r}{100\ \mathrm{pc}}\right)^{-1} .$$

For a Kolmogorov spectrum, $(\delta B/B)^2 \propto T^{2/3}$, and thus the maximum energy is a sensitive function of the turbulence level and SNR model. However, it is unlikely to exceed 3 TeV, and may be much smaller.

Above this maximum energy, the character of the acceleration will change. Provided that

$$\left(\frac{\delta B}{B}\right)^2 > 3 \times 10^{-6}\left(\frac{T}{1\ \mathrm{TeV}}\right)\left(\frac{r}{100\ \mathrm{pc}}\right),$$

cosmic rays can still be scattered more than once on either side of a shock front before they escape from its vicinity. They will therefore still be subject to first order acceleration, although they will not on average double their energy at an individual shock front. Their greater mobility does, however, enable them to interact with more shock fronts per unit of time. The work done by the shock on the cosmic rays crossing it is independent of the future or past history of those particles, although the difference between the energy gain and energy loss in escaping from the high density region behind the shock will change. We expect that the net acceleration rate would be less efficient in this intermediate energy regime although it would still have the Fermi character. The observation (still controversial) of a break in the proton spectrum near $T \approx 100$ TeV is highly relevant in the context. Only at the very highest energies, $T \gtrsim 10^{18}$ eV where the Larmor radii exceed 300 pc can we be sure that we have to invoke a separate cosmic-ray acceleration mechanism. We note here the possibility that similar processes acting in intergalactic space may accelerate those particles. The energy injected into the intergalactic medium by explosive events in galaxies (see Schwarz, Ostriker, and Yahil 1975) is ample for the purpose.

IV. CONCLUSIONS

In this paper, we have outlined a quantitative model for the acceleration of galactic cosmic rays based on the first order Fermi process acting at a shock front. The mechanism we investigated produces, in a more or less automatic fashion, spectra of the correct shape; we also find it capable of maintaining the local cosmic-ray energy density in the presence of escape and collisional loss if the typical (not mean) interstellar density is low ($n < 0.1\ \mathrm{cm}^{-3}$). However, it can operate in two distinct ways that may be distinguished on the basis of cosmic ray observations. Suprathermal particles directly injected behind the shock can be accelerated up to 1 GeV energy and above. If the supernova remnant is still able to accelerate particles after it has expanded out to a radius of ~ 100 pc, then reacceleration of 1 GeV particles is important. For a given escape law, reacceleration leads to flatter $\lesssim 1$ GeV proton and electron spectra, and larger estimates of $\lambda(T)$ above ~ 10 GeV per nucleon. The age estimate derived from the ^{10}Be abundance should increase with energy if reacceleration is important. Gamma ray observations of nearby SNRs and the measurement of curvature in the proton spectrum around 100 TeV may shed further light on the relevance of the acceleration scheme that we have discussed. The high energy cosmic rays may be produced by the same process acting in intergalactic space where galactic scale explosions ($\sim 10^{61}$ ergs) replace the supernova explosions ($\sim 10^{51}$ ergs) as the driving mechanism.

In general, tests of the proposed reacceleration mechanism should focus on the fact that spallation and acceleration are noncommuting processes. An example best explains the point. Suppose there were a process producing two isotopic species (A, B), the cross sections of which varied inversely with energy so that the spallation product ratio ([A]/[B]) varied strongly with the energy of the collision, say with A only produced at low energies; then a significant detection of A at high energies (large [A]/[B]) would indicate acceleration subsequent to spallation. The result would be more significant yet if A carried a clock indicating that the acceleration did not take place in a short time (the "source") but over a long time in interstellar space. Currently planned experiments capable of detecting isotopic ratios of fairly heavy species seem, in principle, capable of performing this type of test which asks if processes in interstellar space, of any kind, are important in accelerating galactic cosmic rays.

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R. D. BLANDFORD: Theoretical Astrophysics, California Institute of Technology, 130–33, Pasadena, CA 91125
JEREMIAH P. OSTRIKER: Princeton University Observatory, Peyton Hall, Princeton, NJ 08540