Effect of random nature of cosmic ray sources – Supernova remnants – on cosmic ray intensity fluctuations, anisotropy, and electron energy spectrum

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Abstract

The random nature of sources (the supernova remnants) leads to the fluctuations of cosmic ray intensity in space and time. We calculate the expected fluctuations in a flat-halo diffusion model for particles with energies from 0.1 to 10^3 TeV. The data on energy spectra and anisotropy of very high energy protons, nuclei and electrons, and the astronomical data on supernova remnants, the potential sources of cosmic rays, are used to constrain the value of the cosmic-ray diffusion coefficient and its dependence on energy.
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1. Introduction

The SN explosions that are thought to give rise to the galactic cosmic rays are essentially statistical events, discrete in space and time. This poses the question as to whether the fluctuations of cosmic ray density and anisotropy are significant (Jones, 1969). The problem can be approached by an analytical calculation of the average values and their fluctuations in the frameworks of "statistical mechanics of supernovae" (Jones, 1969; Lee, 1979; Berezinskii et al., 1990), by a corresponding numerical simulations (Pohl and Esposito, 1998; Strong and Moskalenko, 2001), and by a calculation of the cosmic ray distribution based on the astronomical information about the actual characteristics of local SNRs (Dorman et al., 1985; Nishimura et al., 1997; Kobayashi et al., 2004). Below we study the effects of cosmic ray fluctuations produced by random SN

bursts in the diffusion model of energetic particle transport in the Galaxy with flat cosmic-ray halo.

2. Statistical fluctuations of Galactic cosmic rays

2.1. Model of cosmic ray transport

We consider a simple flat-halo Galaxy model of cosmic ray transport. The cosmic ray density $N(E, \mathbf{r}, t)$ obeys the diffusion equation

$$\frac{\partial N}{\partial t} - \nabla D \nabla N + \frac{\partial}{\partial E}\left(\left(\frac{dE}{dt}\right)_{\text{loss}} N\right) = q\,\delta(z). \tag{1}$$

where $D(E)$ is the scalar diffusion coefficient that does not depend on position; the term $(dE/dt)_{\text{loss}} < 0$ describes the synchrotron and inverse Compton energy losses for very high energy electrons moving in the interstellar medium; $q(E, x, y, t)\,\delta(z)$ is the source term that represents the cosmic ray production by supernova bursts in a thin disk at $z = 0$. The coordinate z is perpendicular to the galactic

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plane. There are cosmic-ray halo boundaries at $|z| = H (H = 4 \text{ kpc})$ and $r = R$ ($R = 20 \text{ kpc}$) where cosmic rays freely exit from the Galaxy. We consider the proton–nucleon and the electron components of very high energy cosmic rays $E > 0.1$ TeV because it turns out that the fluctuation effects are not significant at smaller energies. Also, the data on cosmic ray anisotropy are probably not affected by modulation in the heliosphere at $E > 1$ TeV.

At such high energies, one can ignore the ionization energy losses, the nuclear interactions with interstellar gas, and the possible reacceleration on interstellar turbulence. The cosmic ray diffusion coefficient in this model was determined at energies 10^{-4}–10^{-1} TeV/nucleon (Jones et al., 2001), where the data on secondary nuclei are available. (The primary nuclei are accelerated in the SNRs whereas the secondary nuclei are produced in the process of spallation of primary nuclei in the interstellar gas.) There are two basic versions of the diffusion model that explain, each in different terms, the observed peaks in the ratios of fluxes of secondary and primary nuclei at an energy of about 10^{-3} TeV/nucleon. The two models differ in the values of the diffusion coefficients. We assume that the diffusion does not change its character up to the knee at 4×10^{3} TeV. Thus, based on results of Jones et al. (2001), we accept for very high energy particles with a charge Z the values $D = 1.55 \times (E/Z)^{a} H_5 \text{ kpc}^2/\text{Myr}$, $a = 0.3$ in the model with distributed reacceleration, and $D = 2.76 \times (E/Z)^{a} H_5 \text{ kpc}^2/\text{Myr}$ $a = 0.54$ in the plain diffusion model, where E is in TeV and $H = 5 H_5$ kpc. It is worth noting that the process of cosmic-ray reacceleration in the interstellar medium may be important at energies less than about 50 GeV/n and it should be taken into account in Eq. (1) but the corresponding dependence of diffusion coefficient on energy extends to high energies.

In Section 2, we assume statistical uniformity of SN bursts in the infinite galactic disk and do not take into account a large scale gradient of the galactic SNR distribution. It is assumed that the SN rate in the galactic disk is $\sigma_{sn} = 50 \text{ kpc}^{-2}/\text{Myr}$, and each SNR instantly injects $S(E) \sim E^{-\gamma}$ energetic particles into interstellar space where the source spectrum index is $\gamma = 2.4$ in the model with reacceleration and $\gamma = 2.16$ in the model with no reacceleration so that the observed proton spectrum $\sim E^{-2.7}$ is reproduced in both models. The method of calculations of the average values and dispersions of cosmic ray density and anisotropy was presented in (Lee, 1979; Berezinskii et al., 1990) where it was applied to the not realistic case of an unbounded three-dimensional distribution of sources.

2.2. The case of protons and nuclei

The energy loss term can be omitted for protons and nuclei. Using the technique described by Berezinskii et al. (1990) and taking into account that $H \ll R$, one can find the average cosmic ray density in the galactic disk $\langle N \rangle \approx S\sigma_{sn} H(2D)^{-1} \sim E^{-\gamma - a}$ and the amplitude of fluctuations

$$\frac{\delta N}{N} \equiv \frac{\langle (\delta N)^2 \rangle^{1/2}}{\langle N \rangle}$$

$$= \frac{D^{1/2}}{(2\pi\sigma_{sn})^{1/2} H^2}$$

$$\times \left(\sum_{n,m} -Ei\left(-\left(\left(n - \frac{1}{2}\right)^2 + \left(m - \frac{1}{2}\right)^2 \right) \frac{\pi^2 D\tau}{H^2} \right) \right)^{1/2}$$

$$\approx \frac{1}{2^{3/2}\pi\sigma_{sn}^{1/2} H\tau^{1/2}} \approx \frac{D^{1/4}}{2\pi^{3/4}\sigma_{sn}^{1/4} H} \propto (E/Z)^{a/4}$$

$$\text{at } \tau = (4\pi\sigma_{sn}D)^{1/2}, \quad \tau \ll H^2/D. \tag{2}$$

Here, the averaging is taken over an ensemble of source configurations at the fixed observer location in the Galaxy; $m, n = 1, 2, \ldots$; $Ei(x) = \int_{-\infty}^{x} dt\, t^{-1} e^t$ is the exponential integral; the cut-off parameter τ takes into account the absence of very young and nearby sources (Lee, 1979; Berezinskii et al., 1990; Lagutin and Nikulin, 1995), its typical value can be estimated as $\tau = (4\pi\sigma_{sn}D)^{-1/2}$ for the accepted distribution of cosmic ray sources in the galactic plane.

Eq. (2) allows one to estimate that for protons $\delta N/N = 0.018 \times H_5^{-1} E^{0.075}$ in the model with reacceleration, and $\delta N/N = 0.021 \times H_5^{-1} E^{0.14}$ in the plain diffusion model. Thus, the "typical" fluctuation of proton density is relatively small and come to only 2–6% of the average intensity.

The amplitude of the anisotropy in the diffusion approximation is calculated by the equation $\delta = -3D\nabla N(cN)^{-1}$. The average $\langle \delta_{fl} \rangle = 0$ for a statistically uniform distribution of sources whereas the fluctuation anisotropy at $\tau \ll H^2/D$ is

$$\delta_{fl} \approx \frac{3}{2^{5/2}\pi^{1/2} c\sigma_{sn}^{1/2} H\tau} = \frac{3D}{2^{3/2} cH} \propto (E/Z)^{a} \text{ at } \tau = (4\pi\sigma_{sn}D)^{1/2}. \tag{3}$$

The amplitudes of fluctuation anisotropy expected at $H = 4$ kpc in the two models under consideration are shown in Fig. 1. The complicated composition of cosmic rays including protons and nuclei up to Iron was taken into account in these calculations.

2.3. The case of very high energy electrons

The energy loss term in Eq. (1) is of the form $(dE/dt)_{loss} = -bE^2$ where $b \approx 4.3$ $(\text{Myr TeV})^{-1}$ for electrons with energies 0.1–10 TeV, see Kobayashi et al. (2004). The account of this term leads to rather cumbersome analytical solutions. The approximate solution can be easily found at high enough energies $E > 0.1$ TeV where the electrons loose their energy before reaching the halo boundaries and the system can be considered as unbounded (the formal condition is $(1 - a)bEH^2/D(E) \gg 1$). The average density of cosmic ray electrons and its fluctuations are then given by the following equations:
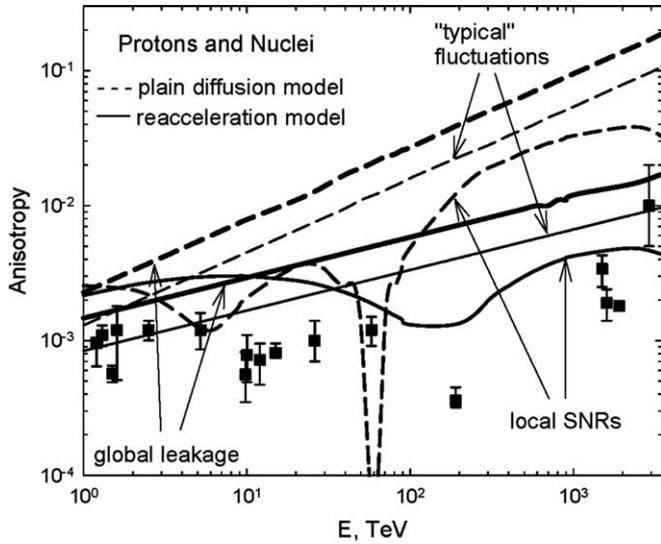
Fig. 1. The anisotropy of cosmic ray protons and nuclei in the reacceleration (solid curves) and the plain diffusion (dashed curves) models. Separately shown effects of the global leakage from the Galaxy, the "typical" fluctuations due to the random nature of SNRs, and the contribution from local SNRs. The data on cosmic ray anisotropy are taken from the paper Ambrosio et al. (2003).

$$\langle N \rangle \approx \frac{2\sigma_{sn}S(E)}{\sqrt{\pi(1-a)bED(E)}} \propto E^{-\gamma-\frac{1+a}{2}},$$

$$\frac{\delta N}{N} \approx \sqrt{\frac{(1-a)bE}{8\pi\sigma_{sn}D(E)\tau}} = \frac{((1-a)bE)^{1/2}}{2(\pi\sigma_{sn}D(E))^{1/4}} \propto E^{\frac{2-a}{4}}. \quad (4)$$

Eq. (4) allows one to estimate that $\delta N/N \approx 0.22E^{0.425}$ in the reacceleration model, and $\delta N/N \approx 0.15E^{0.365}$ in the plain diffusion model. The level of strong fluctuations $\delta N/N = 1/3$ is reached at energies about 3 and 9 TeV for the reacceleration model and the plain diffusion model, respectively.

The physical meaning of the divergence of expressions Eqs. (2)–(4) at $\tau \to 0$ lies in the dominance of nearby young SNs in the shaping of cosmic ray fluctuations at a given location in the galactic disk, see Berezinskii et al. (1990) for discussion. The formally calculated dispersion of fluctuations is infinite and the expressions presented above give only some characteristic amplitudes of the fluctuations (Lagutin and Nikulin, 1995). The knowledge of the properties of local recent SNs is needed to make accurate estimates of cosmic ray fluctuations.

3. Effect of individual supernova remnants

3.1. Cosmic ray anisotropy

The list of the local SNRs with determined parameters is probably representative for objects with distances from the Earth $r < 1$ kpc and the ages (the light-arrival times) $t < 0.05$ Myr. The following SNRs which belong to this group are included in our calculations: SN 185 ($r = 0.95$ kpc; $t = 1.8 \times 10^{-3}$ Myr), RX J1713.7-3946

(1; 2×10^{-3}), S 147 (0.8; 4.6×10^{-3}), G114.3 + 0.3 (0.7; 7.7×10^{-3}), Cygnus Loop (0.77; 2×10^{-2}), G65.3 + 5.7 (0.8; 2×10^{-2}), Vela (0.3; 1.1×10^{-2}), HB21 (0.8; 2.3×10^{-2}).

We do not take into account the very young close remnant RX J0852.0-4622/"Vela Junior" ($r = 0.2$ kpc; $t = 0.7 \times 10^{-3}$ Myr) recently discovered in ROSAT data (Aschenbach, 1998; Iyudin et al., 1998). With parameters indicated above, the inclusion of this SNR would give the anisotropy that is more than two orders of magnitude larger than the observed one. It well may be however that the accelerated high energy particles are still confined inside the envelope of this very young SNR (the confinement of accelerated cosmic rays in SNR envelopes was recently analyzed by Ptuskin and Zirakashvili, 2004). Also, the distance to "Vela Junior" is not well determined and it may be as large as 1.5 kpc (Slane et al., 2001) that would make this source unimportant in our consideration. The results of calculations of the "typical" statistical anisotropy of cosmic rays in the galactic disk based on Eq. (3) is very different from the calculations where the actual distribution of local SNRs is used, see Fig. 1. (It is assumed in both cases that the source term $S(E)$ is the same for all SNRs and that the background intensity of cosmic rays is maintained by the whole variety of galactic SNRs.) The discrepancy between these two approaches is apparently less at high energies. The main contribution to the anisotropy goes from Vela at $E < 150$ TeV in the reacceleration model, and $E < 6$ TeV in the plain diffusion model. The source S 147 dominates at higher energies up to the knee in the reacceleration model, and up to about 40 TeV in the plain diffusion model where SN 185 and RX J1713.7-3946 lead at higher energies till the knee.

The results of more comprehensive calculations of cosmic ray anisotropy are presented in Fig. 1. The radial
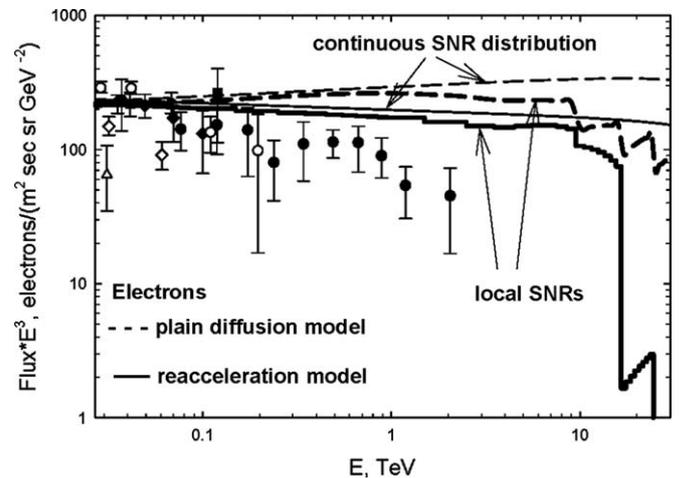

Fig. 2. The spectrum of high energy electrons calculated in the reacceleration (solid thick curve) and the plain diffusion (dashed thick curve) models with the effect of local SNRs included. The corresponding thing curves describe the case of continuous in space source distribution. The data on cosmic ray electron spectrum are taken from the paper Kobayashi et al. (2004).

dependence of the background SNR distribution (according to Case and Bhattacharya, 1996), the yield of local young SNRs listed above, the finite thickness of the galactic disk and the vertical displacement of the Sun position from the galactic plane, and the complicated elemental composition of cosmic rays were taken into account in these calculations.

3.2. Spectrum of very high energy electrons

The general procedure of calculation principally follows the approach used by Dorman et al. (1985) and Kobayashi et al. (2004). Fig. 2 shows the spectrum of very high energy electrons at the Earth calculated with the account of contribution from local galactic SNRs which have $r \leqslant 1$ kpc, $t \leqslant 5 \times 10^{-2}$ Myr plus the continuous distribution of more distant and more old SNRs.

4. Discussion and conclusion

The discrete nature of cosmic ray sources – the SNRs – is critical for the interpretation of data on cosmic ray anisotropy and on the spectrum of very high energy electrons. The diffusion model with reacceleration ($D \sim E^{0.3}$) is reasonably compatible with the data on cosmic ray anisotropy. The discrepancy between the calculated and measured anisotropy is roughly within the factor 3. The plain diffusion model ($D \sim E^{0.54}$) predicts too large an anisotropy at $E > 100$ TeV. The effect of global leakage of the cosmic rays from the Galaxy probably dominates at $E > 30$ TeV.

The Vela SNR mainly determines the observed cosmic ray anisotropy at 1–10 TeV and the flux of very high energy electrons. It is predicted that heavy energy losses cause the considerable steepening of the electron spectrum at $E > 10$ TeV even if the acceleration works up to higher energies.

The reliable determination of distance and age of the SNR G266.2-1.2 ("Vela Junior") is quite important for studies of cosmic ray acceleration in SNRs and the propagation of cosmic rays in the interstellar medium. This source was not included in the calculations shown in Figs. 1 and 2.

It is worth emphasizing the limited accuracy of our model. The assumption of isotropic and spatially uniform diffusion is the most questionable. The strong random magnetic field on the largest scale makes the diffusion close to isotropic on distances of the order of a few hundred parsecs but not at a short distance where the observed anisotropy is formed. Another critical point is the assumption of instant point sources of the same CR power. In reality the accelerated CRs can be confined inside the SNR for a time from 300 yr for the particles at the knee 4×10^{15} eV to 10^5 yr for GeV particles. Also, the kinetic energy of SN ejecta that is the energy reservoir for cosmic ray production has a considerable dispersion that should result in the dispersion of $S(E)$. We plan to improve the calculations in a forthcoming work.

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