[11]S. C. Freden and R. S. White, J. Geophys. Res. 67, 25 (1962).
[12]W. W. Macy, R. S. White, R. C. Filz, and E. Holeman, J. Geophys. Res. 67, 25 (1962).
[13]T. A. Farley, A. D. Tomassian, and M. Walt, Phys. Rev. Lett. 25, 47 (1970).

Cosmic-Ray Proton and Helium Spectra above 50 GeV

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Differential energy spectra of cosmic-ray protons and He nuclei have been measured for the first time by an ionization spectrometer flown at balloon altitudes. The energy range extended from 50 to >1000 GeV. The observed differential intensities can be represented with power-law spectra with a slope of -2.75 ± 0.03 for protons and of -2.77 ± 0.05 for He nuclei. The proton-to-He ratio is 26 ± 3 at 40 GeV/nucleon and is constant within errors up to 400 GeV/nucleon.

In recent years there has been much interest generated by the measurement of the cosmic-ray proton spectrum by Akimov, Grigorov, and coworkers[1-4] using an ionization spectrometer flown on the "proton" series of satellites. These workers have found a steepening of the integral proton spectrum at an energy of about 1000 GeV. Below 1000 GeV the exponent of the integral spectrum is -1.6, and above 2000 GeV the exponent is -2.3. In addition, they have found a discontinuity in the "all-particle" spectrum[5] (the integral burst spectrum of all cosmic rays incident on their calorimeter), which they claim may be related to the steepening of the proton spectrum. It is difficult to unambiguously decide whether the spectral break is due to astrophysical processes, to changes in the properties of nuclear interactions at these energies, or to instrumental effects. In order to decide which of these effects is responsible, it is important to check these results by measuring differential spectra directly. A program of experiments to determine the charge composition of galactic cosmic rays in the energy range 10^{10} to 10^{14} eV was initiated at Goddard Space Flight Center. In this Letter, we report measurements on the differential spectra of proton and helium nuclei up to 2000 GeV. The results on heavier nuclei ($Z>2$) will be reported later. These measurements were carried out at balloon altitudes during November 1970 using an ionization spectrometer. (This instrument has historically been called a calorimeter; however, the more descriptive term ionization spectrometer has been used here.)

Details of the experimental operation will be published elsewhere. An outline of the detector is shown in Fig. 1. The spectrometer has a depth of 4 proton interaction mean free paths (mfp) and is made up of seven iron modules, each 0.5 mfp thick, topped by twelve tungsten modules, each 1 radiation length thick. Each iron module has three symmetrically placed scintillators, and this frequent sampling is essential for reducing the extreme fluctuations associated with the nuclear cascade process. The detectors on the top,

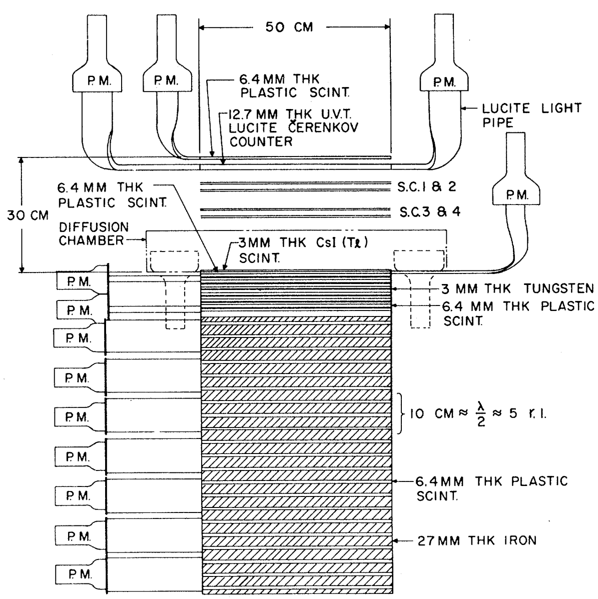

FIG. 1. Balloon-borne ionization spectrometer. SC_1, SC_2, SC_3, and SC_4 are wire-grid spark chambers with core readout to determine the trajectory of the incident particle and to help in rejecting background events. The detector system was designed so that a very wide range of charges and energies could be measured with a single instrument.

three scintillators and a Cherenkov detector, serve to identify the charge of the incident particles. The four-deck wire-grid spark chamber just below the Cherenkov detector defines the particle trajectory and is useful in rejecting background events. The 23 pulse heights, each with dynamic range of 10^4, are telemetered to the ground along with the spark coordinates. Each event is represented by over 10^3 bits.

On the ground each detector output is normalized in units of minimum-ionizing cosmic-ray muons. When a proton intereacts in the instrument producing pion secondaries, a nuclear-electromagnetic cascade is initiated. If neutral pions have been produced, the number of particles increases rapidly to a broad maximum and then decreases with an exponential dependence on depth. The longitudinal development of the cascade showers from two-proton and two-helium events from the balloon flight are shown in Fig. 2. The frequent measurements of the cascade made by this instrument allow the first interaction point of the incoming particle to be estimated to the nearest module ($\frac{1}{2}\lambda$). This and the knowledge of the path length of the shower through the experiment remove the major source of fluctuations in shower development. The remaining fluctuations in nuclear disintegration energy and the fraction of absorbed energy limit the accuracy of energy estimation to ± 25%. This spectrometer incorporates design improvements which allow it to measure differential spectra for the first time in this energy range.

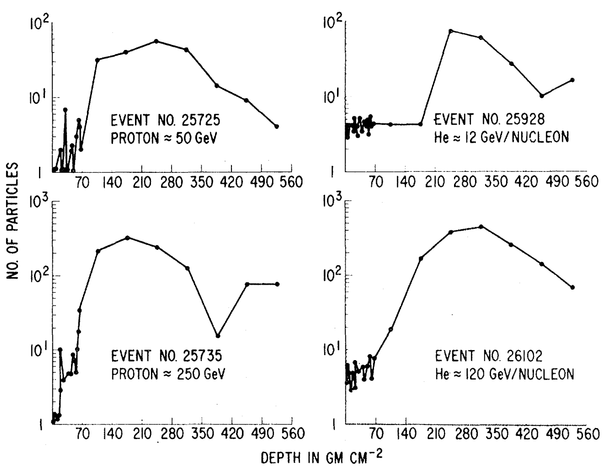

FIG. 2. Examples to show the development of nucleonic cascades in the spectrometer. The showers reach a flat maximum and generally decay away. Though individual shower curves fluctuate a great deal, most of this is because of the location of the first interaction. The area enclosed by the shower curves gives a good measure of the total energy of the incident particle.

The spectrometer was calibrated with protons up to 20 GeV energy at the alternating gradient synchrotron (AGS) at Brookhaven National Laboratory. At AGS energies the total spectrometer output formed by summing the seven iron modules was found to be directly proportional to the incident energy. The measured outputs implied that (60 ± 10)% of the incident energy was available for sampling in the form of electron-photon cascades.

The currently available accelerator energies are lower than those of the cosmic rays of interest, and so it is planned to calibrate a similar instrument in the 300-GeV beam at the National Accelerator Laboratory. At higher energies, mountaintop experiments (by Jones *et al.*[6] and Murzin[7]) with very deep spectrometers have shown that the proportionality between incident energy and light output holds up to energies greater than 2000 GeV. This has been confirmed by Monte Carlo calculations which have been normalized to the observed distributions at lower energies.[8,9]

With a spectrometer only 4 mfp deep, it is necessary to make a correction for the energy which escapes out the bottom of the instrument. This correction is energy dependent because the shower attenuation length increases with primary energy. Based upon the results of the mountaintop experiments and Monte Carlo calculations, it is estimated that the fraction of energy escaping increases from 3% at 40 GeV, instrument threshold, to 36% at 2000 GeV, with an approximately logarithmic dependence on energy.

In this Letter, data are included from 16.6 h of the balloon flight at an altitude of 6 g cm^{-2}. The sensitive lifetime was (42 ± 2)% and the geometrical factor 358 cm^2 sr. The observed fluxes have been corrected to the top of the atmosphere (15%), for particles which interact beyond module 3 in the spectrometer (30%), and for spark-chamber inefficiencies [(20 ± 5)%].

It is also necessary to make an allowance for those events in which backscattering of particles from the main cascade occurs. It has been found from accelerator data at 17.5 GeV that this correction is less than 20% for events interacting in the first 0.5 mfp. However, little is known of the details of the backscattering process. This factor may be energy dependent and so distort the incident spectrum. The influence of backscattering has been checked by observing that the spectrum has the same shape for groups of events which interact in the first, second, and third modules. The loss of events due to backscattering of multiple particles to the top of the instrument should vary as a function of depth. If an energy dependence in the backscattering were important, it should be reflected in a depth dependence of the spectral exponent. The backscatter from interactions at 1.5 mfp depth is not able to reach the charge module as a result of the thickness of material and the small solid angle (0.60 sr) into which high-energy products would have to be backscattered.

The differential energy spectra obtained are shown in Fig. 3. The errors for the intensities are statistical only. A systematic uncertainty in the absolute intensity of ± 20% should be allowed. The solid lines represent differential spectral exponents. The proton and helium spectra are represented by

$$dN_p/dE = (8.6 \pm 0.8)\times 10^6 E^{-2.55 \pm 0.03} \text{ protons m}^{-2}\text{ sr}^{-1}\text{ sec}^{-1}\text{ GeV}^{-1},$$

$$dN_{\text{He}}/dE = (3.6 \pm 0.6)\times 10^5 E^{-2.77 \pm 0.05} \text{ He m}^{-2}\text{ sr}^{-1}\text{ sec}^{-1}\text{ (GeV/nucleon)}^{-1},$$

where E is in GeV. The spectra slopes are identical within the errors. The 1σ errors are obtained using Poisson statistics and a maximum-likelihood fitting method. When plotted on an energy per nucleon scale, the ratio of protons to helium nuclei is equal to 26 ± 3 at 60 GeV/nucleon and is constant within errors of up to 400 GeV/nucleon.

These spectra can be converted to integral form, and they are consistent with the results of Akimov *et al.*[1] and of Pinkau *et al.*[10] While these data do not extend as high in energy as the "proton" results, there is no statistically significant evidence for any abrupt change of slope in the region of 1000 GeV. On the other hand, our data are not inconsistent with the "proton" spectrum.

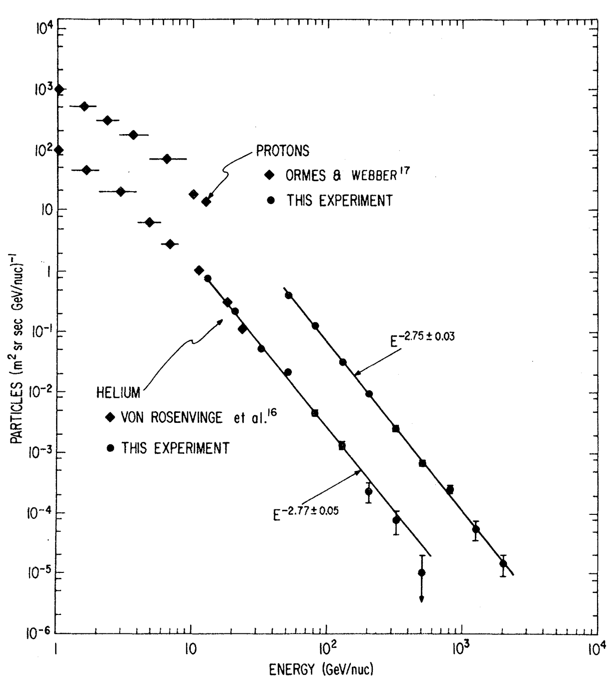

FIG. 3. Differential spectra of protons and helium nuclei. Both components are well represented by power laws and the data agree well with previously published results at lower energy.

It should be emphasized that the "proton" spectrometers were, of necessity, limited in the amount of detailed information obtainable with each event. They had neither trajectory-determining devices nor the ability to localize the first interaction point. These factors, coupled with the relatively shallow (2.5 mfp) depth of the spectrometers, made the estimation of primary energy extremely difficult and possible energy dependent. No correction for energy escaping out the back has been discussed in reference to their data. It is probably for these reasons that they report only integral spectra.

Using data from the same experiment, Akimov *et al.* have also reported[11] that the inelastic scattering cross section for cosmic-ray protons on carbon nuclei increases with energy. Yodh and co-workers,[12,13] analyzing the spectra of unaccompanied protons in the atmosphere, conclude that the inelastic cross section for the interaction of protons on air nuclei also increases with energy. If this is the case, the spectrometer effectively becomes deeper with increasing energy, and so particle energy will be overestimated. This will cause the spectrum to appear flattened. This effect tends to compensate for the increase in energy going out the back. By observing an energy-dependent depletion in the high-energy surviving cosmic-ray protons at mountain altitudes, Kamata *et al.*[14] and Kaneko *et al.*[15] have concluded that either the cosmic-ray proton spectrum has a break, or the cross section increases with energy. Their results are not consistent with both effects at once.

Because of the limited exposure factor available on balloon platforms, it is planned to have an experiment similar to the one described above (5 mfp thick) on the first of the High Energy Astronomical Observatories satellites. This experiment will measure the spectra of all cosmic rays up to 10^{14} eV. The spectrometer to be flown will be deeper, and the improved statistics from the planned two-year exposure will help to resolve the discrepancy at 1000 GeV.

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[1]V. V. Akimov *et al.*, Acta Phys., Suppl. No. 1, 29, 517 (1970).

[2]N. L. Grigorov *et al.*, Can. J. Phys. 46, S687 (1968).

[3]N. L. Grigorov *et al.*, Acta Phys., Suppl. No. 1, 29, 518 (1970).

[4]N. L. Grigorov *et al.*, in Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, Tasmania, Australia, August 1971 (unpublished), Vol. 5, p. 1746.

[5]N. L. Grigorov *et al.*, in Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, Tasmania, Australia, August 1971 (unpublished), Vol. 5, p. 1752.

[6]L. W. Jones *et al.*, in *Proceedings of the Sixth Inter-American Seminar on Cosmic Rays, La Paz, Bolivia, 1970* (Universidad Mayor de San Andrēs, La Paz, 1970).

[7]V. S. Murzin, in *Progress in Elementary Particles and Cosmic Ray Physics*, edited by J. G. Wilson and S. A. Wouthuysen (North-Holland, Amsterdam, 1971), Vol. 9.

[8]W. V. Jones, Phys. Rev. 187, 1868 (1969).

[9]W. V. Jones, in *Proceedings of the Twelfth International Conference on Cosmic Rays, 1969. Abstracts* (Central Research Institute for Physics, Budapest, 1969), Vol. 2, paper No. 27/1.

[10]K. Pinkau *et al.*, Acta Phys., Suppl. No. 3, 29, 291 (1970).

[11]V. V. Akimov *et al.*, Acta Phys., Suppl. No. 3, 29, 211 (1970).

[12]G. B. Yodh, J. R. Wayland, and Y. Pal, Fields Quanta 1, 233 (1971).

[13]G. B. Yodh, J. Trefil, and Y. Pal, to be published.

[14]K. Kamata *et al.*, Acta Phys., Suppl. No. 3, 29, 49 (1970).

[15]Kaneko *et al.*, in Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, Tasmania, Australia, August 1971 (unpublished), Vol. 3, p. 1235.

[16]T. T. von Rosenvinge, W. R. Webber, and J. F. Ormes, Astrophys. Space Sci. 5, 342 (1969).

[17]J. F. Ormes and W. R. Webber, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, England, 1965*, edited by A. C. Stickland (The Institute of Physics and The Physical Society, London, England, 1966), Vol. I.

Generalization of the Concept of Invariance of Differential Equations. Results of Applications to Some Schrödinger Equations

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We have found that differential equations can be form invariant under a larger class of infinitesimal transformations than those considered by Lie and Ovsjannikov. We give a generalization of the concept of point transformation. It is necessary for the systematic determination of the generators of continuous invariance groups of, e.g., the partial differential equations of physics. Applying it to Schrödinger's equation, time-dependent constants of the motion are found systematically, as illustrated here for the hydrogen atom.

Lie[1] based his group theoretical treatment of systems S of differential equations (linear and/or nonlinear) of order π,

$$F^r(x,u,\partial_j u,\partial_j\partial_k u,\cdots)=0;\quad r=1,2,\cdots,R, \tag{1}$$

upon the concept of an infinitesimal point transformation of an N-dimensional Euclidean manifold

$$(x,u)\to(\bar{x},\bar{u}) \tag{2}$$

with coordinates of two types,[1]

$$x=(x^1,x^2,\cdots,x^n) \tag{3a}$$

and

$$u=[u^1(x),u^2(x),\cdots,u^m(x)]. \tag{3b}$$

The functions u^j may be taken to be the original unknown functions appearing in S, or they may be new functions that have arisen by reduction of S to an "equivalent" set S' of quasilinear first-order equations through the device of introducing