COSMIC RAY ELECTRONS: A DISCUSSION OF RECENT OBSERVATIONS

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Our recent measurement of the spectrum of cosmic ray electrons has provided statistically significant evidence for a spectral shape that is much steeper than that of protons. The electron spectrum does not fit well to a single power law, and the abundance of electrons relative to that of protons decreases from 4% at 10 GeV to 0.1% at 300 GeV. This result is consistent with a galactic escape lifetime for electrons exceeding 10^7 years. We shall discuss our data in light of current models for the propagation of cosmic rays in the galaxy, and conclusions will be drawn concerning the consistency of various models with the observations.

1. Introduction. Significant information can be obtained from an accurate measurement of the high energy cosmic ray electron spectrum because of the interaction of electrons with the magnetic and photon fields in the galaxy. Electrons lose energy by synchrotron radiation and inverse Compton collisions (in the Thomson limit) at a rate: \[ \frac{dE}{dt} = -\frac{E^2}{\tau_{\text{r}}} \] where \( E \) is the electron energy, \( \tau_{\text{r}} \approx 10^{-3} \left[ W_{\text{ph}} + \frac{1}{2} (H_{\perp}^2/8\pi) \right] \) (Gev sec)^{-1}, \( H_{\perp} \) is the mean perpendicular magnetic field component (in gauss), and \( W_{\text{ph}} \) is the ambient photon energy density (in eV/cm³). Such energy losses define a radiative lifetime \( \tau_{\text{r}} = (\kappa E)^{-1} \) which can be compared with the time scale of other loss processes, such as leakage from the galaxy, which are expected to affect all cosmic ray species.

In this paper, we will interpret our new measurement of the electron spectrum above 10 GeV (Hartmann et al. 1977) in the context of models for the storage and propagation of cosmic rays in the galaxy. The most important feature of our data that must be considered is a spectral index \( \alpha \geq 3.0 \) above 10 GeV (Figures 1-3). Such a spectral index is considerably steeper than that of the primary nuclear component (\( \alpha \approx 2.7 \)). This feature strongly suggests the influence of radiative energy losses on the shape of the electron spectrum and we shall discuss its implications in some detail.

2. Comparison of Electron Measurements. We wish to first compare and contrast our measurement of the high energy electron spectrum with the results of other experimenters. A multidecade logarithmic plot tends to mask the differences between the various results, so in Figure 1 we have plotted all measurements of the differential energy spectrum of electrons divided by a reference energy spectrum of \( E^{-3.0} \). Large discrepancies in absolute flux outside the quoted error bars are immediately apparent, and are indicative of systematic errors in at least some of the experiments. Besides these differences, significant discrepancies also exist in the quoted spectral indices which range from \( \alpha = 2.7 \) to \( \alpha = 3.4 \). Figure 1 indicates how difficult it is for most experiments to make a definitive statement about the spectral index above 40 GeV due to the size of the experimental errors. We believe that the combination of good statistics, good background rejection, and

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extensive high energy calibration make our measurement very reliable. Comparing our results to those of other experiments (Figure 1), we find ourselves in good agreement with the results of Meegan and Earl (1975). Our data are also qualitatively similar to those of Silverberg (1976), although the differences are outside the quoted errors. We also note that our measurements below 40 GeV are consistent with the results of Fulks (1973), but disagree with the low energy data of Müller and Meyer (1973).* 

Although our data are consistent with at least one other experiment, they differ from previous results in one significant feature. Other experiments have found their data to be consistent with a single power law spectrum. We find that a single power law is not a good fit to our data. Further, our data suggest a gradual steepening in spectral index from $\propto \propto 3.0$ at 15 GeV to $\propto \propto 3.4$ above 40 GeV.

3. Interpretation of Results. We now wish to discuss various models for the propagation of cosmic rays in the galaxy. It should be stressed that any interpretation of the high energy electron results is extremely model dependent. One is immediately confronted with the problem of an abundance of free parameters in the models, many of which are only approximately known. The electron data themselves do not specify a unique model. Rather, given a

* As noted by Müller and Meyer (1973), their data below $\sim$30 GeV may be too low due to an erroneous dead-time correction. It appears now that this was the case, and these previous low energy results should be considered withdrawn.
model, the electron data can be used to put constraints on the parameters of the model. We therefore wish to concentrate on the simplest models with the fewest free parameters. We first discuss the homogenous model and later the disk-halo models.

A. Homogeneous Model. The homogeneous or "leaky box" model is governed by the equation:

\[
\frac{N(E)}{\mathcal{T}(E)} + \frac{d}{dE} \left( -kE^2 N(E) \right) = Q(E)
\] (1)

where \(N(E)\) is the density of electrons of energy \(E\), \(\mathcal{T}(E)\) is the escape lifetime from the confinement volume, \(k\) is the energy loss coefficient defined in the introduction, and \(Q(E)\) is the source function for electrons. All quantities are assumed to be position independent. We also assume the escape lifetime \(\mathcal{T}(E)\) to be energy dependent in the form of a power law: \(\mathcal{T}(E) = \mathcal{T}_0 \left( \frac{E}{E_0} \right)^{-\delta} \) (\(E_0 = 1\) GeV). This assumption with \(\delta \approx 0.5\) would be in agreement with measurements of the nuclear cosmic ray composition (e.g. Caldwell 1977). We further assume that the injection spectrum of electrons follows a power law:

\[Q(E) = A E^{-\Gamma} \]

The solution to equation 1 is discussed by Ramaty and Silverberg (1974). Up to a normalization factor \(N_0\), \(N(E)\) is determined by 3 free parameters: \(\Gamma\), \(\delta\), and \(E_c = (k \mathcal{T}_0)^{-1}\). This can be seen from the explicit solution of equation 1:

\[N(E) = N_0 E^{-(\Gamma+1)} \int_1^\infty d\xi \xi^{\Gamma-\delta} \exp \left[ \frac{-(1-\xi^{\delta-1})}{(1-\delta)} \right] E^{\delta-1} \] (2)

where all energies are in GeV.

We now make the usual assumption that the electrons are injected with the same spectral index as the nuclei. With a leakage lifetime \(\tau = \mathcal{T}_0 \left( \frac{E}{E_0} \right)^{-\delta}\), the observed spectral index \(\Gamma'\) of the primary nuclei must be larger than the source spectral index by an amount \(\delta\), i.e. \(\Gamma' = \Gamma + \delta\).

We now take the primary nuclei above 5 GeV to have a spectral index \(\Gamma' = 2.7\) (Caldwell 1977). This reduces the number of free parameters to two, namely \(\mathcal{T}_0\) and \(\Gamma\) (see text).

![Figure 2](image-url)

Figure 2

Differential energy spectrum of electrons multiplied by \(E^{3.0}\). Curved lines are fits to the homogeneous model of cosmic ray propagation with energy independent leakage lifetime. (see text).
\[ E_c = (k \tau_o)^{-1}, \]
and \( \delta \). In Figures 2 and 3, we show the results of numerical fits of equation 2 for various values of the parameters \( E_c \) and \( \delta \). (All fits are normalized to a flux of 2.9 \( \times 10^{-1} \) at 10 GeV.) Figure 2 shows the case for which there is no leakage lifetime for electrons i.e. \( \delta = 0 \). As can be seen, the best fit occurs for \( E_c \approx 30-50 \) GeV. Taking a canonical value of \( k = 1 \times 10^{-16} \) GeV\(^{-1}\) sec\(^{-1}\), (corresponding to a combined energy density for magnetic and photon fields of 1 eV/cm\(^3\)), we arrive at an energy independent leakage lifetime of \( \tau_o = (7-10) \times 10^6 \) years. This lifetime is consistent with the age derived from measurements of the Be\(^{10} \) abundance (Garcia-Munoz et al. 1975 and 1977). Figure 3 shows numerical fits for an energy dependent leakage lifetime with \( \delta = 0.3 \). The best fits suggest values of \( E_c \approx 3-6 \) GeV and corresponding lifetimes at 1 GeV of \( (5-10) \times 10^7 \) years. No good fit can be found for values of \( \delta \) significantly larger than 0.3.

In summary, under the assumption that nuclei and electrons have the same injection spectrum, and in the context of the homogeneous model, the data tend to support a rather weak dependence of the leakage lifetime on energy, \( \delta \leq 0.3 \) and a rather long leakage lifetime at 1 GeV, \( \tau_o \approx 1 \times 10^7 \) years (with \( k = 1 \times 10^{-16} \) GeV\(^{-1}\) sec\(^{-1}\)). We note that a value of \( \delta = 0 \) for electrons implies that another mechanism besides an energy dependent leakage lifetime would have to be found to explain the energy dependent pathlength of the primary nuclei.

It should be stressed that the homogeneous model as described by equation 1 is a considerably simplified model of cosmic rays in the galaxy. This is due to the fact that diffusion, boundary conditions, and position dependence have been replaced by a leakage lifetime \( \tau_o(E) \) which is purely phenomenological. The parameter \( E_c = (k \tau_o)^{-1} \) measures the energy for which radiative loss from electrons becomes important in comparison to all other propagation and source effects. This is both the attraction and the drawback of the homogeneous model. On the one hand, the homogeneous model does not make excessive assumptions about the details of cosmic ray dynamics, but on the other hand, the information it yields is limited and the physical interpretation of the model is necessarily incomplete.

B. Disk-Halo Model. Various models for cosmic ray propagation have
been proposed that make explicit assumptions about the structure and dimensions of the galaxy, and about the diffusion or convection processes governing the propagation of cosmic rays. Usually, these models confine cosmic ray sources to a galactic disk, but allow the cosmic rays to escape into a confinement volume, the "halo", whose dimensions are larger than those of the disk. We wish to briefly discuss the simplest of such models: cosmic rays are generated uniformly within a disk of radius R and thickness 2L, and are allowed to diffuse isotropically without boundary constraints. This model is similar to that discussed by Jokipii and Meyer (1968). The diffusion equation for this model is:

\[ \nabla \cdot (D \nabla N) + \frac{d}{dE} (-kE^2 N) = Q(E,z) \]  

with

\[ Q(E,z) = \begin{cases} 
    A E^{-\Gamma}, & |z| \leq L \\
    0, & |z| > L 
\end{cases} \]

This yields a solution for \( z = 0 \) and \( R/L \to \infty \) of

\[ N(E) = N_0 E^{-(\Gamma+1)} \int_0^\infty dE' E'^{-\Gamma} \text{erf} \left[ \left( \frac{L^2 kE}{4D(1-\frac{1}{\Gamma})} \right)^{3/2} \right] \]

where \( N_0 \) is an overall normalization constant.

Qualitatively, this model leads to the following interpretation. An electron of energy \( E \) has a lifetime \( \tau_R = (kE)^{-1} \), against radiative losses. During this time it propagates an average distance \( \langle r \rangle \) which is proportional to the square root of diffusion coefficient and lifetime:

\[ \langle r \rangle = \left( 2D \tau_R \right)^{1/2} = \left( 2D/kE \right)^{1/2}. \]

For large energies, \( \langle r \rangle \) is smaller than the thickness of the disk, i.e. \( \langle r \rangle < L \), and a fully steepened spectrum will be observed: \( N(E) \propto E^{-(\Gamma+1)} \). For sufficiently small energies, \( \langle r \rangle \) exceeds the dimension of the source disk and thus defines the dimensions of a halo whose volume increases with the inverse square root of the electron energy. As a consequence, the spectrum observed within the disk will be proportional to \( E^{-(\Gamma+1)} / \sqrt{VE} \), i.e. \( N(E) \propto E^{-(\Gamma+1/2)} \). The halo dimension and the source dimension will be approximately equal at a critical energy \( E^* = 2D/kL^2 \) and the observed spectrum must steepen around this energy by half a power law unit (i.e. from \( E^{-(\Gamma+1/2)} \) to \( E^{-(\Gamma+1)} \)).

We now investigate whether it is this transition that is seen in our results as a gradual steepening from an \( E^{-3.0} \) to an \( E^{-3.5} \) spectrum. A reasonable fit of equation 4 to our data is obtained for a source spectral index of \( \Gamma = 2.5 \) and \( E^* \propto 25-75 \text{ GeV} \). With \( k = 1 \times 10^{-16} \text{ (GeV sec)}^{-1} \) and \( L = 200 \text{ pc} \), we find a diffusion coefficient \( D \propto 1 \times 10^{27} \text{ cm}^2 \text{ sec}^{-1} \). The value of the diffusion coefficient thus derived is smaller than the commonly accepted value of \( D \propto (10^{28} \text{ to } 10^{29}) \text{ cm}^2 \text{ sec}^{-1} \). This indicates that the model must be treated with caution. If this model is indeed a proper approximation, i.e. if the steepening of the spectrum is due to leakage of electrons into the halo, then this leakage process must be rather slow. This would imply a long residence time of cosmic rays in the galactic disk, exceeding \( 10^7 \text{ years} \). Also, the size of the electron halo could not significantly exceed the size of the galactic disk for electron energies above \( 10 \text{ GeV} \), and the sources of the observed high energy electrons could not be very distant.

We note that recent Be\(^{10}\) results (Garcia-Munoz et al. 1975 and 1977)
indicate that the cosmic rays traverse a medium of low average density (0.2 atom/cm$^3$). From this result and the 5 g/cm$^2$ pathlength derived from nuclear abundances, it has been inferred that cosmic rays must either propagate predominantly through regions of low interstellar density, or that the cosmic rays spend most of their lives in a sizeable halo and only a small fraction, approximately $10^6$ years, in the disk. The latter possibility would not agree with the conclusions of the simple diffusion model discussed here. The recent model of Owens and Jokipii (1977) in which equation 3 is augmented by a convective term may provide an alternative. We find that our data fit well to the numerical results of Owens and Jokipii if convective propagation dominates over diffusion.

4. Conclusions. Clearly, on the basis of our data alone, no decision can be made as to the proper model to describe the propagation of cosmic rays in the galaxy. We hope however, that the foregoing discussion at least illustrates the kind of implications that can be drawn from the high energy electron data.

The steepness of our measured spectrum strongly suggests a lifetime of cosmic rays that is at least as large as $\tau \approx 1 \times 10^7$ years, independent of the propagation model. However, the existence of a cosmic ray halo cannot be decided on the basis of our data due to the fact that radiative losses probably severely restrict the size of an electron halo above 10 GeV. Additional information concerning this question can be obtained from the Be$^{10}$ data. However, it may well be that because of their limited lifetime, neither Be$^{10}$ nor the high energy electrons indicate the full size of the cosmic ray halo.

Finally, we wish to point out that our conclusions always depend strongly on the assumed shape of the source spectrum of electrons. While there seems to be no direct way to determine the source spectrum of all electrons, the source spectrum of positrons, generated in p-p collisions, is known (e.g. Ramaty 1974). A measurement of the positron spectrum at high energies (up to 200 GeV) would therefore be an important step to sharpen up our conclusions. We hope that such a measurement becomes available in the future.

References
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