Energy-Dependent Propagation of High Energy Electrons

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Abstract

A correct treatment of the formation of the spectra of cosmic ray (CR) electrons in the interstellar medium (ISM) requires a separate consideration of the contributions of a nearby and of distant sources. To implement this approach, the problem of energy-dependent diffusive propagation of CRs from a single nonstationary source is considered, and the analytical solution to the diffusion equation is found. The results are applied to the interpretation of the energy spectrum and charge composition of CR electrons.

1. Introduction

It is commonly believed that the energy spectrum of CRs is well described in the framework of conventional (leaky-box or diffusion) CR propagation models which assume a uniform and continuous (both in space and time) distribution of sources in the Galaxy. Whereas for CR protons this assumption may be considered as a reasonable working hypothesis, the validity of this approach is limited for high energy electrons the spectrum of which extends up to $E \approx 2$ TeV \cite{1}. Indeed, from comparison of the radiative (synchrotron and Compton) cooling time of electrons ($\tau_c \approx 3 \cdot 10^5 (E/1 \text{ TeV})^{-1}$ yr) with the diffusive propagation time it follows that the distance to sources of the observed TeV electrons cannot exceed a few hundreds parsecs \cite{2,3}. In Fig.1 the total spectrum of CR electrons (solid curve), calculated assuming a homogeneous and continuous distribution of the sources in the galactic disk, is decomposed to show the contributions from sources located at different distances. Only about 10\% of the flux of 2 TeV electrons is contributed by sources located beyond 500 pc. Therefore, the assumption of a continuous distribution of sources would have to be valid down to scales of a few hundreds parsecs. Otherwise, a correct interpretation of the observed CR electron fluxes requires a separate treatment of the two different components of the primary electrons: (1) the electrons from the nearby local source(s) (the $L$-component), and (2) the electrons from sources at large distances, typically $r \geq 1$ kpc, which may be treated still in the traditional framework of a continuous distribution of CR sources in the Galaxy (the $G$-component). Apart from these directly accelerated electrons, there is some contribution to the total flux from secondary electrons (the $S$-component) produced in interactions of galactic CRs with interstellar gas at any point of the ISM. Note that only for these electrons the hypothesis of a continuous distribution of sources is justified for all distances.

2. Energy-dependent diffusion from a single source

Recently we found a simple Green's function solution to the general nonstationary energy-dependent diffusion equation for an arbitrary injection spectrum and an arbitrary energy losses of electrons \cite{4}. In the particular case when only synchrotron and Compton energy losses, $\frac{dE}{dt}_c = -bE^2$, are important (i.e. for $t \leq 10^7$ yr after the
injection), and the primary spectrum is a power-law, \( Q(E) \propto E^{-\alpha_0} \), the density of electrons at time \( t \) after their injection from a source at a distance \( r \) is equal to

\[
f_e(r, t, E) = \frac{Q(E)}{\pi^2/2} \left( 1 - b t E \right)^{\alpha_0 - 2} \left( \frac{r}{r_{\text{diff}}} \right)^3 e^{-\left( r/r_{\text{diff}} \right)^2},
\]

where \( E \leq E_{\text{max}} = 1/b t \). At energies \( E \leq 0.5 E_{\text{max}} \) the diffusion radius \( r_{\text{diff}} \approx 2 \sqrt{D(E) t} \), and Eq. (1) becomes similar to the expression obtained earlier [5] in the particular case \( D(E) = \text{const} \).

The energy-dependent diffusive propagation from a single source results in a strong modification of the primary spectrum of electrons (and CRs) during times much less than their cooling times. The modification of the primary spectrum is defined mainly by a factor \( s(E, t) = r/r_{\text{diff}} \). Assuming a burst-like source (when injection occurred during \( \Delta t_{\text{inj}} \ll t \)) at distance \( r \), for a given energy \( E(< E_{\text{max}}) \) the maximum flux \( J^{\text{max}}(\propto r^{-3}) \) is reached at times \( t \approx t_{\text{max}}(E) = r^2/6D(E) \), corresponding to \( s \approx 1 \). At \( s \gg 1 \), when \( t \ll t_{\text{max}}(E) \), the electrons have not yet reached the observer \( (J_{\text{e}} = (c/4\pi) f_e \propto \exp(-s)) \), while at \( t \gg t_{\text{max}}(E) \) the flux decreases as \( r_{\text{diff}}^{-3} \propto (D(E) \times t)^{-3/2} \) due to spherical expansion of the region occupied by the particles. Therefore, at high energies for which the time of maximum flux has already passed \( (s \ll 1) \), the electron flux acquires the power-law form with exponent \( \alpha = \alpha_0 + (3/2) \delta \), where \( D(E) \propto E^\delta \). Obviously, for \( D(E) = \text{const} \) the electrons just repeat the injection spectrum with a time-dependent flux \( \propto s^3 \exp(-s^2) \). Note that for a continuous source, when \( \Delta t_{\text{inj}} \approx t \), the exponent is different, namely \( \alpha = \alpha_0 + \delta \) [4].

3. Two-component approach

The spectrum of electrons produced \( t = 10^5 \text{ yr} \) ago by a single burst-like source at distance \( r = 100 \text{ pc} \) is shown in Fig. 2. A differential power-law injection spectrum with an index \( \alpha_0 = 2.2 \) was assumed. The diffusion coefficient was approximated as \( D(E) = D_0 (1 + E/E_0)^{\delta} \) which contains both the power-law behavior of \( D \) for \( E \gg E_0 \), and its tendency to energy-independence below \( E_0 \) (typically \( \sim 3 \text{ GeV} \)). \( D_0 \) is chosen from the condition that \( D(10 \text{ GeV}) = 5 \cdot 10^{27} \text{ cm}^2/\text{s} \) (see e.g. [6]). The typical values \( \alpha_0 = 2.2 \), and \( \delta = 0.6 \) have been used in calculations (for both the L- and G-components) which give a power-law index of the spectrum of high energy electrons formed during their propagation from the nearby source, \( \alpha = 3.1 \). This fits well the observed spectrum above 100 GeV. However, at lower energies the single source cannot contribute much to the observed flux; \( 10^5 \text{ yr} \) is not enough for low energy electrons to effectively reach us. At energies \( E < 100 \text{ GeV} \), the G-component (dashed curve in Fig. 2) contributes more effectively.

It is seen from Fig. 2 that the two-component ("G+L") approach to primary electrons explains fairly well the spectrum observed from 1 GeV to 2 TeV. Due to strong radiative losses of electrons, this model, in contrast to leaky-box type models, predicts a sharp cutoff of the spectrum at TeV energies. The cutoff energy \( E_{\text{max}} \) depends on the age the nearby source. Actually, it is quite possible that there are \( > 1 \) nearby sources contributing to the high energy electron flux. Since it is unlikely that the electron fluxes from different sources would have the same cutoff energy (as well as intensities, which depend on their distances \( r_j \)), one might expect fluctuations at the high energy end of the electron spectrum [2]. Thus the search for possible irregularities in the electron spectrum at multi-hundred GeV energies in future experiments...
would answer the question: is there one or are there many sources nearby?

The spectrum of the G-component has been calculated integrating the contributions of sources over the galactic disk starting from \( r_0 \geq 1 \) kpc. In Fig.3 the ratio \( \zeta(E) = t_{\text{dir}}(E)/t_e(E) \) of the effective propagation time of electrons from 1 kpc to the total (including also ionization and bremsstrahlung) energy loss time is shown. The minimum of \( \zeta(E) \) is reached for \( E \sim 10 \) GeV. Therefore these electrons reach us from large distances most effectively, whereas both at \( E \gg 10 \) GeV, and \( E \ll 10 \) GeV, the fluxes of electrons of the G-component are suppressed (by a factor \( \propto \exp(-\zeta) \)) due to significant energy losses during their propagation.

At low energies, this effect allows one to explain the measured fluxes below a few GeV and, perhaps, to understand the discrepancy (see Fig.2) between these fluxes and the ones derived from the radio data which give information on the CR electron density integrated along the line of sight. Then, it allows also to explain the high ratio \( C_+ = e^+/(e^- + e^+) \) observed below 1 GeV (see Fig.4), if one assumes that the G-component consists (only) of accelerated negatrons \( e^- \), and takes into account the S-component which consists of equal amounts of secondary \( e^+ \) and \( e^- \). Since the secondaries are produced by CRs at any point in the ISM (formally even infinitely close to us), at low energies the S-component flux is less affected by the energy losses than the G-component flux. This results in a ratio \( C_+(E) \) significantly increasing with decreasing \( E \) even below 1 GeV, which is not the case if at low energies the electron flux derived from radio data is used for \( (e^- + e^-) \).

At high energies, the suppression of the G-component flux results in the gradual increase of the fraction of the L-component electrons in the total flux with increasing \( E \) (see Fig.2). Therefore, at energies \( E \geq 10 \) GeV the ratio \( C_+(E) \) essentially depends on the charge composition of the L-component. Thus it contains important information about the origin of the local source(s).

4. The origin of the L-component

To produce the flux of the L-component electrons as shown in Fig.2, only \( W_e \approx 10^{48}\rho_{100}^2 \) erg of energy injected in electrons above 100 MeV is required (\( \rho_{100} = \rho/100 \) pc). This can be easily explained by acceleration of electrons in one (or few) nearby SNRs, e.g. one or all of the Loop SNRs, as suggested by J. Nishimura. Assuming that the local source is a "typical" galactic accelerator of negatrons like a SNR, one should expect a monotonically decreasing behavior of \( C_+(E) \) with energy (see dashed curve in Fig.4) due to essentially different energy spectra of the "G-L" \( (J_{G-L} \propto E^{-\gamma}) \) and the S-components \( (J_S \propto E^{-\gamma}) \).

In fact, we may expect also secondary \( (\pi^+ \text{ decay}) \) positrons from SNRs due to interactions of accelerated protons with the shocked ambient gas. The expected contribution of these positrons above 10 GeV to \( C_+ \) is \( C_+^{(\pi)} \approx 3 \cdot 10^{-5} n_0 t_5 W_{50}/r_{100}^2 \) [3], where \( W_{50} = W_p/10^{50} \text{ erg} \), \( t_5 = t/10^5 \) yr, and \( n_0 = n/1 \text{ cm}^{-3} \) are the total energy accelerated in protons, their confinement time in the SNR, and the number density of the ambient gas, respectively. Since the measured CR fluxes limit the value \( (W_{50}/r_{100}^2) \leq 3 \) (see Fig.2), the contribution of positrons from a nearby SNR can exceed the flux of the S-component positrons at 10 GeV only for \( n_0 t_5 \gtrsim 3 \). In Fig.3 the contribution of the secondary \( \pi^+ \text{- decay} \) positrons from the SNR corresponds to \( n_0 t_5 = 2 \). It dominates at \( E \gtrsim 10 \) GeV due to the hard SNR source spectrum.

Actually, the existing data indicate \( \geq 10\% \) positron content at \( E \gtrsim 10 \) GeV (e.g.
which can hardly be explained by secondary positrons from a nearby SNR, requiring \( n_0 t_0 > 10 \). Thus, in the case of confirmation of these data by new measurements we have to suggest another source(s) of positrons nearby, e.g. the Geminga pulsar [3].

References


Fig. 1 The electron fluxes from galactic sources located at \( r \geq r_0 \). The compilation of the measured fluxes is from [1]. The dashed region corresponds to the electron fluxes deduced from the radio data.

Fig. 2 Energy spectra of electrons of \( L_\gamma \), \( G_\gamma \), and \( S \)-components shown by solid, dashed and dash-dotted curves, respectively. The total electron spectrum is shown by the heavy solid line. The distance to the local source is \( r = 100 \) pc. The spectrum of CR protons from the local source calculated for \( W_p = 3 \cdot 10^{30} \) erg (dotted curve), and the measured CR fluxes are also presented.

Fig. 3 The ratio of the diffusion time to the cooling time of electrons.

Fig. 4 The positron content due to different components.