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THE NATURE OF THE COSMIC-RAY ELECTRON SPECTRUM, AND SUPERNOVA REMNANT CONTRIBUTIONS

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ABSTRACT

We examine the observed cosmic-ray (CR) electron spectrum and positron fraction $e^+/(e^- + e^+)$ spectrum above 1 GeV, and find that a deconvolution of the total spectrum into three components is necessary because of the increase of $e^+/(e^- + e^+)$ above 5 GeV: (1) Secondary electrons $e^{\pm}$ from the interaction of the CR protons with the interstellar gas provide the total e^+ for energies less than 3 GeV, but for energies above 3 GeV these electrons cannot account for the observed positron flux; (2) Electrons (e^-) generally thought to derive from supernova remnants (SNRs), probably via shock acceleration, dominate the total spectrum for $E \leq 10$ GeV but definitely decline relative to total at higher energies; (3) Another $(e^- + e^+)$ source dominates the total spectrum at $E \geq 40$ GeV. The derived spectrum of (2) is consistent in its energy cutoff (though gradual) with that deduced from the observed synchrotron emission of some old SNRs and follows naturally from shock acceleration with synchrotron and inverse Compton scattering losses taken into account. As for (3), nearby pulsars may be important contributors.

Subject headings: cosmic rays: general — nebulae: supernova remnants — pulsars — radiation mechanisms

I. INTRODUCTION

The observed CR electron and nuclei spectra are different, partly because of their different energy losses during the acceleration and propagation phases. These losses affect the spectra if the radiation time is comparable to the acceleration time or the escape time from the Galaxy. For electrons, these losses increase with energy ($\propto E^2$) making both acceleration and escape of very high energy (e.g., 10^4 GeV) electrons very difficult if not impossible.

One potential source of both electron and nuclear CRs is supernova remnants (shell-like SNRs) in which shock acceleration is expected to yield a power-law spectrum similar to that observed at Earth (e.g., Blandford and Eichler 1987). However, it is difficult to reconcile between electrons and nuclei if we try to apply the same constraints on both. For example, if both are assumed to escape from the Galaxy with the same escape time derived for the nuclei, one is unable to obtain the correct spectral index for the electrons at high energies from the same source spectrum (Ormes and Protheroe 1983). Or, if we assume the same propagation model (e.g., simple or nested leaky-box models), the escape time is different from that of the nuclei, both in magnitude at low energies and in energy dependence at high energies (Ismail *et al.* 1987). All these problems as we will see later may have been the consequence of assuming a single source spectrum for the electrons.

Recently, there have been some new measurements of the positron content in the electron flux which indicate a relative increase above 5 GeV (Golden *et al.* 1987; Müller and Tang 1987). This prompted some interpretations such as an energy cutoff in the primary electron spectrum (Tang and Müller 1987) or a dominance of a flatter pulsar source above ~20 GeV (Harding and Ramaty 1987).

In this paper we analyze the electron spectrum in light of another astrophysical observation, the presence of spectral breaks in the radio spectra of some old SNRs. This may indicate that the steepening of the radio emission spectra is just a consequence of the steepening of the electron spectrum in the SNRs themselves. If we assume that shock acceleration is taking place, then such an energy cutoff in the electron spectrum results from synchrotron losses and inverse Compton scattering (e.g., Webb, Drury, and Biermann 1984). Furthermore, the required flatter positron spectrum above ~10 GeV may indicate that the Crab-like SNRs, and their associated pulsars, dominate the electron spectrum with equal amounts of positive and negative electrons.

In the next section, we use both the measured total electron flux, and the $e^+/(e^- + e^+)$ ratio fit to decompose the electron flux into three components. Next, in § III, we discuss the origin of the primary component by evaluating a cutoff energy in the shell-like SNR shocks, and a possible importance of the Crab-like SNRs in the very high energy part of the electron spectrum.

II. COSMIC-RAY ELECTRON SPECTRA

The CR electrons observed at Earth have spectra which are generally represented by power laws (i.e., of the form $E^{-\gamma}$), where the spectrum index γ varies from less than 1.5 for $E \leq 1$ GeV to more than 3 for $E \geq$ about 10 GeV (Webber 1983; Tang 1984). At very high energies (>100 GeV) the spectral index is uncertain, but may reach 3.3 (Webber 1983) or 3.7 (Müller and Tang 1985). However, a spectral index less than 3 is not ruled out by the experiments (Tang 1984). The change of γ with energy is generally attributed to the dominance of different energy losses to which the electrons are subjected (Ramaty 1974). At high energies ($E > 10$ GeV), the main energy losses are synchrotron radiation and inverse Compton scattering.

The positron spectrum has been measured between 10 MeV and 30 GeV (Agrinier *et al.* 1969; Fanselow *et al.* 1969; Daugherty, Hartman, and Schmidt 1975; Buffington, Orth, and Smoot 1975; Golden *et al.* 1987; Müller and Tang 1987). Because of solar modulation, the results are usually presented in the form of a ratio of the positron to the total electron fluxes [i.e., $e^+/(e^+ + e^-)$]. The main features of this ratio, although with large uncertainties, are (1) it is large (~30%–50%) at low energies (~0.1 GeV), indicating a large flux of positrons; (2) it decreases unambiguously with energy between 0.1 GeV and

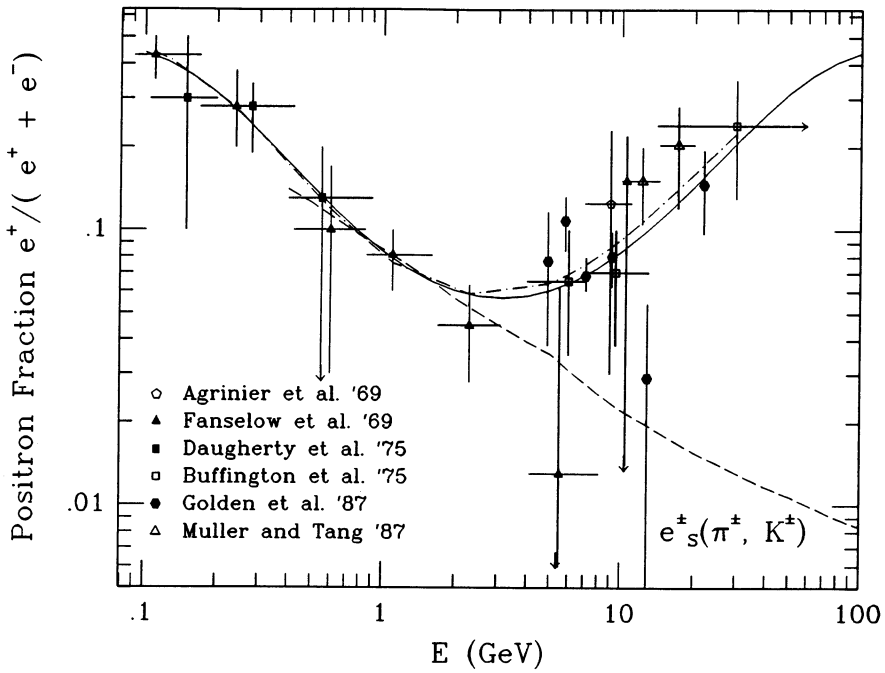

FIG. 1.—Positron ratio best fitting (*solid curve*) and the predicted ratio (*dashed line*) from secondary production above 0.4 GeV for an energy-independent escape time of 2×10^7 yr, and a proton pathlength $x_0 = 6$ g cm^{-2}. The analytical expression of the fitted curve is given by eq. (7), and the reduced χ^2 value is 1. The dot-dashed curve represents the best fit for the data without the two lowest ratios (<0.02) of Fanselow *et al.* (1969) and Golden *et al.* (1987). It is clear that these lowest ratios do not affect the fit trend and can be considered as outliers in the fit.

about 4 GeV, reaching a value of about 5%; and (3) beyond 4 GeV the data are not very precise; nonetheless, the measurements quoting better precisions (see Fig. 1) indicate an increase with energy.

a) Decomposition of the Electron Spectrum

We decompose the observed CR electron flux into SNR and non–SNR parts. The total electron flux is used according to the fit of Protheroe (1982). The positron–to–total electron fluxes ratio and the total electron spectrum can be written, respectively, as follows:

$$R = \frac{J_{\rm pos}}{J_{\rm tot}} \; ; \quad \text{with } J_{\rm tot} = J_{\rm pos} + J_{\rm neg} + J_{\rm SNR} \,, \tag{1}$$

where $J_{\rm pos}$ is the positron flux; $J_{\rm neg}$ is the electron flux of the same origin as the positron; and $J_{\rm SNR}$ is the electron flux from SNRs (we assume that SNRs accelerate only negative electrons).[1] We will assume also that all the non-SNR sources contribute equally to the electron and positron fluxes (e.g., secondary sources such as mesons or pulsars), that is $J_{\rm pos} \approx J_{\rm neg} \equiv J_{\rm NSNR}/2$ for all energies. Therefore, $J_{\rm SNR}$ and $J_{\rm NSNR}$ can be written

$$J_{\rm NSNR}(E) = 2R(E)J_{\rm tot}(E) \,, \tag{2}$$

$$J_{\rm SNR}(E) = [1 - 2R(E)]J_{\rm tot}(E) \,. \tag{3}$$

We fit the positron ratio by analytical functions of energy, and then we decompose the total flux into $J_{\rm SNR}$ and $J_{\rm NSNR}$.

i) *Positron Ratio Fitting*

We used the nonlinear least-square Marquardt method (Press *et al.* 1986) to obtain the best fit with the minimum $\chi^2 = 1$. One of the representations studied is (Boulares 1988)

$$R = \frac{1}{2 + 10^{-G(E)}} \,, \tag{4}$$

$$G(E) = a_0 + a_1 \log(E) + a_2[\log(E)]^2 + a_3[\log(E)]^3 \,. \tag{5}$$

This representation has the property that R can be anywhere between 0 and 0.5, which is what it should be if we assume that for each e^+ there is a corresponding e^-. With E in GeV, the best fit which corresponds to a reduced χ^2 value of 1 has the following parameters:

$$a_0 = -1.00 \pm 0.10 \,, \quad a_1 = -0.78 \pm 0.17 \,, \quad a_2 = 0.78 \pm 0.2 \,, \tag{6}$$

and a_3 is negligible. The ratio R is then given by

$$R = \frac{1}{2 + 10.0E^{0.78}10^{-0.78[\log E]^2}} \,. \tag{7}$$

This expression is plotted in Figure 1, along with the observed data. The fit is excellent, and the two lowest data points (5.5 GeV, 0.02) of Fanselow *et al.* (1969); and (12.9 GeV, 0.03) of Golden *et al.* (1987) are found to be outliers in the fit with little effect on it, as shown also in Figure 1 by the dot-dashed curve. Therefore, according to this fit, the increase of the ratio $e^+/(e^+ + e^-)$ with energy is very plausible, and the e^+ secondary production in the ISM cannot account for this increase as shown by the dashed curve in Figure 1. The secondary flux equilibrium spectrum $J_{\rm es}$ (e.g., Ormes and Protheroe 1983; Ismail *et al.* 1987) is computed for an escape time of the form $2 \times 10^7(E/5.5 \text{ GeV})^{-0.7}$ yr, an interstellar pathlength $\sim$6 g cm^{-2}, and an interstellar magnetic field of 5 μG. The dashed curve in Figure 1 represents the ratio of $J_{\rm es}$ to the total electron flux.

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[1] There is no known mechanism that can produce positrons at SNRs' shocks.

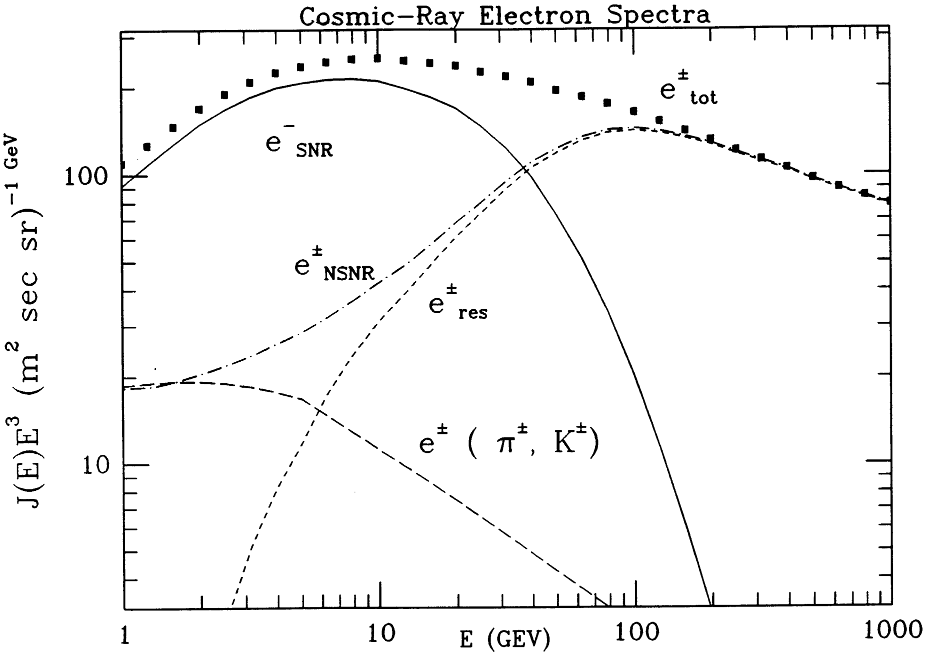

FIG. 2.—Electron spectra: the square dotted curve is the total electron flux (multiplied by E^3) as given by Protheroe (1982). The curve marked $e^-_{\rm SNR}$ is the spectrum from shell-like SNRs, $e^{\pm}$ is the total flux which is not from shell-like SNRs, and $e^{\pm}_{\rm res}$ is the new component required to explain both the total flux and the positron ratio.

ii) *Decomposition of the Electron Spectrum*

The fluxes $J_{\rm NSNR}$ and $J_{\rm SNR}$ are computed and plotted in Figure 2. As we can see the SNR electron spectrum does have a sharp cutoff at about 20 GeV, while the other NSNR spectrum is flatter at high energies than at low energies. (Note that the plot is of E^3J so that the rising curve of $J_{\rm NSNR}$ below 100 GeV corresponds still to falling J.) We expect $J_{\rm SNR}$ to continue to decline exponentially with energy as shown by Webb *et al.* (1984), while $J_{\rm NSNR}$ bends over at about 100 GeV. The latter is nevertheless the main source of electrons and positrons at high energies.

The usual interpretation of the positron content in the CR electron flux is that it is secondary in origin. It is produced by the decay of π-mesons in the ISM. For the simple leaky-box model, the equilibrium spectrum (e.g., Protheroe 1982) is plotted in Figure 2. Between 0.4 and 3 GeV, the secondary production may account for all the NSNR flux. Above 3 GeV the secondary production cannot account for the e^+ flux, even if the recent estimates (Webber 1987), which yield higher fluxes, are used. In fact the secondary production alone predicts the wrong slope above 3 GeV. The difference between $J_{\rm NSNR}$ and $J_{\rm es}$ is also shown as a residual spectrum. This spectrum looks more or less like a power law below ~50 GeV, with spectral index between 1 and 2, although other shapes are not excluded. At 100 GeV, the spectral index is 3 and steepens further with increasing energy.

III. ORIGIN OF PRIMARY ELECTRONS AND POSITRONS

There are two different kinds of cosmic-ray electrons in the observed spectrum (e.g., Webber 1983): primary ($e_p^{\pm}$), and secondary ($e_s^{\pm}$) electrons. The $e_p^{\pm}$ kind consists of electrons of positrons produced and accelerated in a source before propagation in the ISM. It has many possible contributors, and each source may be dominant in an energy range and negligible in others. At low energy (keV–MeV range), the source of observed electrons are solar flares (Toptygin 1985). The $e_s^{\pm}$ origin is mostly β-decaying nuclei.

At higher energies (>100 MeV) a Galactic origin must be invoked, and shock acceleration by supernova remnants is likely to be a significant source. A primary positron component from these remnants is unlikely because there is no known mechanism to create and accelerate them in the remnant's shock. In addition secondary production accounts more or less for the observed positron flux between 0.1 and 3 GeV.

At energies above ~20 GeV there is a deficit of primary electrons from SNR's, and other *nearby* sources must dominate. There are many suggested sources of electrons and positrons at high energy: Type I SN explosions (Colgate and Johnson 1960; Colgate 1983); pulsar magnetospheres (Gunn and Ostriker 1969; Arons 1983); dark matter annihilation (Rudaz and Stecker 1988). Most of the positrons are produced by pair production in these sources.

a) *Shock Acceleration in SNRs: Energy Cutoff*

i) *Radio Emission from Shell-like SNRs*

There are two types of SNRs: shell-like remnants with steep radio spectra ($0.3 < \alpha < 0.8$) and filled-center remnants (Crab-like or plerions) with flatter spectra. For Crab-like remnants, a pulsar source for both electrons and magnetic field is most likely (e.g., Reynolds 1988); while for shell-like remnants a localized enhancement near the shells is required.

In old remnants, the intensity of the emission can be accounted for by the large compression behind cooling shock fronts of both ambient interstellar magnetic field and CR electron density (van der Laan 1962). This is supported by the general spatial correlation of optical and radio structures seen in most old remnants (McCray, Stein, and Kafatos 1975;

Straka *et al.* 1986). However, there are some observed features which may be inconsistent with this interpretation.

First, the presence of thermal X-ray emission (i.e., temperatures $\geq 10^6$ K) makes radiative shocks (temperatures between 10^4 and 10^5 K) unlikely and suggests a rather adiabatic phase for most old SNRs. Blandford and Cowie (1982) used the fact that the ISM may be inhomogeneous (cloudy) (Cox and Smith 1974; McKee and Ostriker 1977) and allowed for some particle shock acceleration to modify the van der Laan theory. They found that if the old remnants sit in a relatively diffuse ISM ($n \sim 5$ cm^{-3}) and if the shock compression is very large (McKee and Cowie 1975), then only a modest particle injection and acceleration is needed at the shock. Unfortunately, the actual inhomogeneity is not well understood, and the relation of the optical emission to X-ray emission is not agreed upon (Hester and Cox 1986).

Second, the correlation between the radio and the optical emissions is not perfect. For instance, the radio emission is found to have two components in contrast to the optical emission (e.g., Fürst and Reich 1986 for S 147; Straka *et al.* 1986 for Cygnus Loop): one component along the optical filaments, and another diffuse component which may be associated with a warm more diffuse interstellar gas. It is suggested that the diffuse emission is due to shock acceleration in all the remnent shocks, while the emission along the optical filaments is mostly due to the large radiative shock compressions (e.g., Lawson *et al.* 1987). This is consistent with the observations of S147 where the spectrum of the diffuse emission is steeper than that associated with the optical filaments (Fürst and Reich 1986).

Finally, the radio emission spectrum of many old remnants has spectral breaks at about 1 GHz. It is found that the van der Laan theory alone cannot explain these breaks (DeNoyer 1974) through a mere compression of Galactic electrons and fields.

In young remnants, the emission is very large ($\sim 10^5$ times the ISM background emission) and cannot be due merely to shock compression of the background radiation. In addition, the inferred magnetic field from the radio emission (Cowsik and Sarkar 1980) must be ≥ 80 μG for the concentration of the electrons not to produce too much γ-ray radiation. This means that at the early stages of a SNR, the magnetic activity is very powerful near the shell, which results in a very effective hydromagnetic turbulent acceleration. However, it declines rapidly after a few hundred years, when the SNR enters its adiabatic Sedov phase, leading the way to the more efficient first-order Fermi acceleration at the shock front itself. This process lasts for the subsequent lifetime of the remnant.

If cosmic rays are generated by SNRs as generally assumed by CR origin theories, especially shock acceleration models, we expect them to be plentiful near the remnant's shocks and to have a nonnegligible role in their dynamics (e.g., Völk *et al.* 1984; Boulares and Cox 1988). Therefore, shock acceleration must be consistent with the different radio emission properties of SNRs if the emission is mostly associated with the remnant's shock fronts.

The highest energy which can be acquired by the particles in this process depends on energy losses, age, and size of the remnants. *The high-energy CRs are generated by the large and old SNRs.* The electrons lose much more energy than the nuclei do at high energies. Therefore, their SNR spectrum could display a cutoff at an energy much smaller than that of the nuclei.

The shell-like SNRs do not radiate nonthermally in the X-ray energy domain, and all their X-ray emission is thermal and is consistent with atomic line emission (e.g., Holt 1983 for young SNRs; Canizares *et al.* 1983 for old SNRs). Infrared emission is due mostly to shock-heated dust (e.g., Dwek 1988). Some old SNRs have spectral breaks in the GHz-frequency range, while others must have spectral breaks either in the IR or optical similar to those of compact sources (Schlickeiser 1984) where continuous Fermi acceleration is dominant. Unfortunately, observations of the nonthermal breaks which occur at IR frequencies are hampered by the intense dust emission from the remnants themselves and all along the line of sight.

Among the 17 observed old SNRs whose fluxes have been measured at 3 or more frequencies with the highest frequency above 1 GHz, those which do show spectral breaks in their radio emission are the Cygnus Loop, S147, HB 9, G33.2−0.6, and G126.2+1.6, in addition to the Galactic loops (I and III) (Lawson *et al.* 1987). They all have radio spectral breaks at about 1 GHz. Furthermore, an electron of energy E and in a magnetic field $B_\perp$ radiates most at a characteristic frequency ν_c given by (Cummings 1973)

$$\nu_c = 16 B_\perp(\mu\mathrm{G}) E^2(\mathrm{GeV})\ \mathrm{MHz} \sim 1000\ \mathrm{MHz}\,, \tag{8}$$

then for an average magnetic field of about 15 μG, the energy corresponding to the break is about 2 GeV. This is consistent with the spectrum J_{SNR} found earlier. Therefore, if the SNRs are indeed a significant source of Galactic CRs, the spectral breaks seen in some old SNRs is an indication that the other remnants must have spectral breaks at higher frequencies.

There are many possible interpretations of the break both in SNRs and the Galactic emissions (Fürst and Reich 1986). For example, it has been suggested that the break can occur because of a break in the electron spectrum, because of synchrotron losses on the continuously injected spectrum, or because of compression of the ISM magnetic field and the Galactic cosmic-ray electrons by the remnants. We will see that a cutoff in the electron spectrum generated at the shock and affected by synchrotron losses (also inverse Compton scattering) at the shock can explain the different features of the radiation of the remnants. Furthermore, if we assume that old SNRs contribute to the Galactic cosmic-ray electrons with different energy cutoffs, then the observed spectrum at Earth will have a component (SNR electrons), as suggested earlier, which starts steepening at about 2 GeV and continues to be steeper more gradually than an exponential up to maximum energy cutoff above 100 GeV.

ii) *The Source Spectrum: Energy Cutoff*

The mechanism most favored in the acceleration of cosmic rays (nuclei, protons, and electrons), is the first-order Fermi process in interplanetary and interstellar shock waves (see Blandford and Eichler 1987 for review and references). The energy gain of a relativistic CR particle when it crosses a shock front once and the average residence time spent between each crossing upstream (1) and downstream (2) are given, respectively, by (Drury 1983):

$$\langle \Delta E\rangle = \frac{2}{3c} E(U_1 - U_2)\ ; \quad \langle \Delta t_1\rangle = \frac{4\kappa_1}{cU_1}\ ; \quad \langle \Delta t_2\rangle = \frac{4\kappa_2}{cU_2}\,. \tag{9}$$

If the particle loses energy continuously as $\dot{E}_{\mathrm{loss}}(E)$, then the critical energy corresponding to a balance between loss and gain is given by

$$\Delta E_{\text{gain}} = \dot{E}_{\text{loss 1}}(E)\Delta t_1 + \dot{E}_{\text{loss 2}}(E)\Delta t_2 \,. \tag{10}$$

The diffusion coefficient parallel to the magnetic field for unidirectional Alfvén waves is of the form (e.g., Drury 1983; Blandford and Eichler 1987)

$$\kappa = \frac{c}{3}\,\lambda = \frac{c}{3}\,r_g\,\frac{B^2}{8\pi I(k)k} \equiv \kappa_0\left(\frac{E}{E_0}\right)^{2-\beta} , \tag{11}$$

where λ and r_g are the particle mean free path and gyroradius, respectively, and $I(k)$ is the Alfvén wave spectrum ($\propto k^{-\beta}$, where $\beta = 3/2$ for a Kraichnan spectrum, and $\beta = 5/3$ for a Kolmogorov spectrum) generated near the shock. The resonance condition from the streaming instability implies that $k \sim 1/r_g$. κ_0 corresponds to the diffusion of the bulk of CRs near shocks (for $E_0 \sim 1$ GeV). The energy balance yields

$$E_c^{\beta-1} = \frac{3}{E_0^{2-\beta}(U_1 - U_2)}\left[\frac{\kappa_{01}\,\dot{E}_{\text{loss 1}}(E_c)}{U_1} + \frac{\kappa_{02}\,\dot{E}_{\text{loss 2}}(E_c)}{U_2}\right] , \tag{12}$$

where E_c is a cutoff energy.

In the case of the protons, $\dot{E}_{\text{loss}}$ (E) is due mainly to CR collisions with ISM gas [$\dot{E}_{\text{loss}}(E) = \sigma_1 cnE$; $\sigma_1 = 10^{-26}$ cm^2]. The energy cutoff is given by

$$E_c^{2-\beta} = U_1^2\left(1 - \frac{1}{r}\right)\bigg/ 3c\sigma_1\,\kappa_{01}\,n_1\left(1 + \frac{\kappa_{02}}{\kappa_{01}}\,r^2\right) , \tag{13}$$

where r is the compression ratio ($U_1 = rU_2$).

For a compression ratio $r = 4$, a density of $n = 0.5$ cm^{-3}, an upstream velocity of $U_1 = 300$ km s^{-1}, and a diffusion coefficient $\kappa_{01} = \kappa_{02} = 10^{25}$ cm^2 s^{-1}, a cutoff in the proton spectrum is expected at an energy of about 3×10^7 GeV for a Kraichnan wave spectrum and at an energy of 10^{11} GeV for a Kolmogorov spectrum. It is interesting to note that in the Kraichnan case, the energy cutoff occurs exactly at the knee of the observed proton spectrum.

This energy cutoff is larger than most other estimates (e.g., Lagage and Cesarsky [1983], Hillas [1984], Völk [1987]) which are of the order of 10^5 GeV. In these estimates, the crucial criteria are the size and the lifetime of the single remnants. However, these criteria may be irrelevant if the diffusion mean free path of very high energy particles (e.g., $E \sim 10^6$ GeV) is of the order of the average distance between interstellar shocks (Boulares 1988) in which case, the acceleration is done by multiple shocks simultaneously, and the above result would have a significant impact on the interpretation of the proton spectrum.[2]

In the case of electrons, the energy loss is due to synchrotron radiation and Compton scattering: $\dot{E}_{\text{loss}}(E) = 2c\sigma_T(B_\perp^2/8\pi + 2w_p/3)(E/mc^2)^2$. Here, $B_\perp$ is the perpendicular component of the magnetic field to the shock front, and w_p is the photon energy density. The magnetic field $B_\perp$ will be compressed by r downstream (i.e., $B_{2\perp} = rB_{1\perp}$). With the diffusion coefficient (we assume $\kappa_{01} = \kappa_{02}$) in units of 10^{25} cm^2 s^{-1}, $E_0 = 1$ GeV, U_1 in 100 km s^{-1}, B in μG, and w_p in eV cm^{-3}, the cutoff energy is (in GeV)

$$E_c^{3-\beta} = 2.2 \times 10^4\left[\frac{U_1^2}{\kappa_{01}}\right]\frac{r-1}{r(r+1)} \times \frac{1}{[3.75 \times 10^{-2}B_{1\perp}^2(r^2 - r + 1) + w_p]} \,. \tag{14}$$

iii) *Relevant Supernova Remnant Parameters*

In principle this energy cutoff can be found in both young and old SNRs. However, the old remnants contribute most to the Galactic CR flux, because first, they are more numerous, second, the young ones may not have enough time to contribute significantly to the measured flux, and third, the large wave generation near shock fronts in young remnants makes the escape of the accelerated particles difficult. Furthermore, some estimates (Dickel 1974) indicate that the old SNRs may be enough to account for the total observed synchrotron radiation of the Galaxy. We concentrate here only on old SNRs, some of which do have a spectral break in their radio emission.

1. The magnetic field interior to the shock wave of an old SNR is generally tangential and is enhanced not only by compression but also by the wave activity generated by the acceleration process. This activity is very efficient in producing a parallel magnetic field for diffusive acceleration to take place in a quasi-perpendicular ($\Theta > \pi/4$) magnetic field (Drury 1987; Jokipii 1987). Therefore, even upstream of the shock, we expect a field larger than the ambient magnetic field of about 4–6 μG. In fact, there has been some work done on evaluating the magnetic field of supernova remnant rims from their rotation measures, and is found to be around 15 μG (Milne 1987; Kim, Kronberg, and Landecker 1988; Raymond *et al.* 1988). An enhanced magnetic field of about 8 μG upstream of the shock is then reasonable. The Alfvén speed $V_A[\equiv B/(4\pi\rho)^{1/2}]$ in this case is about 20 km s^{-1} for a gas density of 0.5 cm^{-3}.

2. The shock acceleration process is no doubt a time-dependent mechanism; however, high cosmic-ray energies are reached only at relatively late times of SNRs. Therefore, the velocities of interest here are not very large ($\sim$100–400 km s^{-1}). The effect of the velocity on the cutoff energy goes as $U_1^{2/(3-\beta)}$. The sound speed is about 5 km s^{-1} for typical conditions near interstellar shocks ($P_g = 5 \times 10^{-13}$ dyn cm^{-2}; $n = 0.5$ cm^{-3}). This corresponds to a sonic Mach number of about 40 for $U = 300$ km s^{-1}, and an Alfvén Mach number of 15.

3. The cosmic-ray spectrum generated by SNRs has a spectral index of $2 + \epsilon$, where ϵ is given for arbitrary Alfvén Mach numbers ($M_A \equiv \bar{U}s/V_A$, U_s is the shock velocity) by (Bell 1978)

$$\epsilon = \frac{4 - r + 3r/M_A}{r - 1 - 2r/M_A} , \tag{15}$$

where the compression ratio r is derived from the Rankine-Hugoniot relations. Here, we assumed that the wave scattering centers move against the flow with the Alfvén speed on both sides of the shock. We notice that both the sonic and Alfvén Mach numbers are large and r is indeed very close to 4 and $\epsilon < 0.2$. We take $r = 4$.

4. The diffusion coefficient is inversely proportional to the wave amplitude $I(k)$ (Wentzel 1974), as seen earlier, but this amplitude increases as we approach the shock from upstream and may saturate. On the other hand, the higher energy particles sample a larger area (Eichler 1979) than the bulk does.

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[2] The acceleration of cosmic rays by more than one shock must be small at low energies (Simon *et al.* 1986) to be consistent with the secondary to primary ratios energy dependence; however, the simultaneous acceleration of very high energy particles by multiple shock systems is not constrained by the measurements.

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They map greater diffusion coefficients. Therefore, the overall diffusion coefficient is larger for higher energy particles than that of lower ones (see, e.g., Tan *et al.* 1987 for an analogy with interplanetary shocks). Furthermore, a typical estimate for the diffusion coefficient near a shock wave for the bulk of cosmic rays (~ 1 GeV) is about 10^{25} cm^2 s^{-1} (Völk 1986; see also Boulares and Cox 1988 for the Cygnus Loop).

5. Photon energy density does not contribute much to electron energy losses near SNR shocks because of the dominance of the synchrotron radiation. Nonetheless, it is equal to ~ 1.5 eV cm^{-3} (Ginzburg and Ptuskin 1985; Webber 1987) in the inner disk of the Galaxy, and the photon density from the remnants themselves is negligible.

As a consequence, for an old SNR with $U_1 = 300$ km s^{-1}, $r = 4$, $B_1 = 8$ μG, $\bar{w}_p = 1.5$ eV cm^{-3}, and $\kappa_{01} = 10^{25}$ cm^2 s^{-1}, the cutoff energy values from eq. [14] are 9, 30, 95, and 166 GeV for β equal to 0, 1, 3/2, and 5/3, respectively. We note here that Tang and Müller (1987) found a similar value (20 GeV), but they did so by using parameters for very young supernova remnants ($U_1 = 10^3$ km s^{-1}; $B_1 = 400$ μG), like Cas A, and neglected Compton scattering contribution. The cutoff value just found must be considered as a upper limit, because the loss effects become important well below this energy. The primary spectrum $J_{\rm SNR}$ found earlier is consistent with an energy cutoff between 10 and 90 GeV. Therefore, a wave spectral index $\beta \leq 3/2$ is sufficient to explain the cutoff.

Webb, Drury, and Biermann (1984) computed some electron spectra subjected to energy cutoff E_c and found that the spectrum must break at that energy. They found also that the spectrum drops like $E^{1/2} \exp(-E/E_c)$ to zeroth order for a monoenergetic injection spectrum. The primary spectrum $J_{\rm SNR}$ is found less steep than that. This is not surprising not only because $J_{\rm SNR}$ contains propagation effects but must be considered as a convolution with the distribution of SNRs with different energy cutoffs in their electron spectra. If we combine the above calculations and the recent results of Golden *et al.* (1987) and Müller and Tang (1987), we deduce that SNR electrons are no longer dominant beyond a few tens of GeV, and moreover, secondary electrons and positrons from π's alone cannot account for both the total electron flux and the $e^+/(e^- + e^+)$ ratio. A new component of electrons and positrons must dominate at high energies.

b) *Pulsar Origin of Very High Energy Electrons*

The most probable sources of $e^\pm$ at higher energies ($E > 10$ GeV) are nearby young pulsars (e.g., Vela) (Taylor and Stinebring 1986; Backer 1988). Although, a self-consistent quantitative description of pulsar electrodynamics does not yet exist, energy considerations and observed emission properties (e.g., radio, γ-rays) suggest this possibility. In fact, if the recent $e^+/(e^+ + e^-)$ measurements are reliable, this will definitely require a pulsar source, because no other nearby conventional astrophysical sources (within 100–500 pc) can generate both e^- and e^+ at high energies (of course, dark matter annihilation may be important if it exists).

The synchrotron radiation in the intense magnetic fields ($\sim 10^2$–10^3 μG; Reynolds 1988) of Crab-like SNRs has power-law spectra with spectral indices—α between 0 and -0.3 at radio wavelengths, and -1.1 for X-ray wavelengths. This corresponds to an electron spectral index—γ (with $\gamma \equiv 2\alpha + 1$) between -1 and -1.6 at about 1 GeV (using eq. [8] with $\nu \sim 1$ GHz and $B \sim 10^2$ μG) and decreasing to -3.2 at about 10^4 GeV (for X-ray emission at ~ 1 keV). With propagation effects taken into account, the observed spectrum should have a spectral index approaching -4.2 at 10^4 GeV. This synchrotron radiation spectral information is consistent with the residual spectrum of Figure 2, which has spectral indices of ~ -1.5 at few GeV, -2.0 at 10 GeV, and -3.0 at 100 GeV. It would be interesting to measure the continuous synchrotron spectrum of such objects from the radio to the X-ray; unfortunately, it is not available yet.

Recently, Harding and Ramaty (1987) estimated the positron flux and spectrum assuming that they are produced through magnetic pair creation ($\gamma \rightarrow e^\pm$) in the cascades near polar caps which may be the source of the observed γ-rays. Assuming that all Galactic pulsars emit γ-rays similarly to the Crab's pulsar, they derived a Galactic positron production rate given by

$$Q_+(E) = 8.6 \times 10^{39} b_{30} f_+ B_{12}^{-0.7} \left\{\frac{t_{\rm max}}{10^4\ {\rm yr}}\right\}^{0.15} E^{-2.2}\ {\rm s}^{-1}\,{\rm GeV}^{-1}\,,$$

where b_{30} is the Galactic pulsar birthrate in units of 30 yr; f_+ is the ratio of escaping positrons to γ-rays produced by pulsar; B_{12} is the pulsar magnetic field in units of 10^{12} G; and $t_{\rm max}$ is the time the pulsars emit γ-rays. The equilibrium spectrum is derived using a leaky-box propagation model with an energy dependent escape time (see Harding and Ramaty 1987). It is found that the spectrum is of the form $E^{-2.2}$ at low energies (<20 GeV) and is flatter than the spectrum from secondary production above 10 GeV. This is a good indication that the electron spectrum decomposition in the present paper may be true.

The expected flux of $e^\pm$ from pulsars is still very uncertain, although, the estimates of Harding and Ramaty (1987) predict a somewhat smaller flux than what is needed to account for the measured total electron flux. For example, at 100 GeV, the residual positron flux (see Fig. 2) is about 5×10^{-5} positrons/(m^2 s st GeV), while the predicted flux from pulsars is about 10^{-5} positrons/(m^2 s st GeV). Evidently, more work is needed to understand the magnetohydrodynamics of pulsars and to assess their contribution to the observed positron flux.

IV. CONCLUSION

As an attempt to understand the electron spectrum, we suggest that it is a composite of at least *three different contributions: a shell-like SNR* source providing the bulk of the primary electrons in the intermediate energies; a secondary source due mainly to *meson production* in the ISM from cosmic-ray nuclei (also supposed to originate from SNRs) interacting with the gas; and finally a source, probably *Crab-like SNRs* (and their pulsars), with flatter spectrum, which was negligible at low energies but dominates the total spectrum at high energies because of cutoffs in the spectra of the shell-like SNRs.

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