

Cosmic ray lepton puzzle in the light of cosmological N -body simulations

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The PAMELA, ATIC, and Fermi collaborations have recently reported an excess in the cosmic ray positron and electron fluxes. These lepton anomalies might be related to cold dark matter particles annihilating within a nearby dark matter clump. We outline regions of the parameter space for both the dark matter subhalo and particle model, where data from the different experiments are reproduced. We then confront this interpretation of the data with the results of the cosmological N -body simulation Via Lactea II. Having a sizable clump ($V_{\max} = 9 \text{ km s}^{-1}$) at a distance of only 1.2 kpc could explain the PAMELA excess, but such a configuration has a probability of only 0.37%. Reproducing also the ATIC bump would require a very large, nearby subhalo, which is extremely unlikely ($p \approx 3 \times 10^{-5}$). It is even less probable for the smaller Fermi bump to be caused by the presence of such an object. In either case, we predict Fermi will detect the gamma-ray emission from the subhalo. We conclude that under canonical assumptions, the cosmic ray lepton anomalies are unlikely to originate from a nearby cold dark matter subhalo.

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Recent measurements [1–3] of cosmic ray positrons and electrons (hereafter dubbed leptons) have stirred a lot of interest, since they may be the first indirect hint of the presence of particle dark matter (DM) in the halo of the Milky Way (MW) [4–7]. In the standard cosmic ray picture, positrons are secondary species produced by the spallation of cosmic ray protons and helium nuclei on the interstellar medium [8]. But secondary positrons quite clearly fail to reproduce the PAMELA [1], ATIC [2], or Fermi [3] measurements. Note that H.E.S.S. also measured cosmic ray leptons spectrum [9,10], but with higher threshold, making these measurement less relevant for dark matter interpretations. While astrophysical primary positron sources exist (the most obvious class of candidates being pulsars) and can account for the recent measured anomalies [11–13], a more exotic origin is possible. In particular, the observed excess could be sourced by the annihilations of the massive and weakly interacting particles (WIMPs) proposed to explain the astronomical DM. Estimates based on standard assumptions on the annihilation cross section and on the DM halo fail, however, to reproduce the measured cosmic ray lepton anomalies. Enforcing a standard thermal DM production in the early universe sets the pair-annihilation rate a couple of orders of

magnitude below what is needed to explain the data. This issue is usually tackled by assuming *boost factors* which can arise from different arguments.

A first possibility lies in the existence of a particle physics boost factor which increases the annihilation rate as a result of a mechanism evading the relic density constraint (e.g. velocity-dependent annihilation cross sections, nonthermal primordial production). Alternatively, an effective astrophysical boost factor could stem from the *clumpiness* of the Galactic DM halo. Indeed, the annihilation rate being proportional to the squared DM density ρ^2 , the “clumpier” the halo, the higher the signal in a $\sim \langle \rho^2 \rangle / \langle \rho \rangle^2$ proportion. Although the boost factor is actually sensitive to the energy [14–16], most analyses of the cosmic ray lepton anomalies have so far treated clumpiness either assuming an energy-independent enhancement of the signal or modifying its spectral shape regardless of the overall normalization. Moreover, the various contributions to the positron flux at the Earth have not been derived within the same cosmic ray propagation model and are in general inconsistent with each other.

In this letter, we reassess [17–19] the possibility that a single nearby DM clump contributes substantially to the lepton anomalies. Particular attention is paid to cosmic ray

propagation. We adjust the clump distance D and luminosity L in order to reproduce the PAMELA, ATIC, and Fermi data. The probabilities of these clump configurations are calculated based on the cosmological N -body simulation Via Lactea II (VL-II) [20], which allows us to quantify for the first time how unlikely a sufficiently bright nearby CDM subhalo is. We eventually comment about the interplay between the DM particle properties and the clump parameters. As an illustration, we point out extreme configurations where these mutual effects lead to subtle, but relevant modulations of the DM signal.

The lepton density $\psi = dn/dE$ is related to its source q through the stationary cosmic ray diffusion equation described in [8]. Lepton propagation throughout the diffusive halo is dominated by space diffusion and energy losses via synchrotron emission and inverse Compton scattering on the CMB and stellar light. The diffusion equation is solved in the framework of the two-zone diffusion model detailed in [21] and yields the flux $\phi = c\psi/4\pi$. The propagation parameters best-fit the B/C ratio and correspond to model med of [22]. A value of 7×10^{15} sec is taken for the energy loss time scale.

The positron flux at the Earth $\phi_{e^+} = \phi_{e^+}^{\text{sec}} + \phi_{e^+}^s + \phi_{e^+}^c$ results here from three contributions. The astrophysical background $\phi_{e^+}^{\text{sec}}$ is provided by the secondary species produced by primary cosmic rays impinging on the interstellar material and is computed as in [8]. The smooth DM halo contributes the source term

$$q_{\text{DM}}^s(\mathbf{x}, E) = \frac{1}{2} \langle \sigma v \rangle \left\{ \frac{\rho_s(\mathbf{x})}{m_\chi} \right\}^2 f(E), \quad (1)$$

where $f(E)$ is the energy spectrum of the positrons created in the annihilation process. The Galactic DM halo density

ρ_s is borrowed from the results of the Via Lactea II simulation, with a spherical profile featuring an inner (outer) logarithmic slope of $-1.24(-3)$ and a 28.1 kpc scale parameter. The local density ρ_\odot is equal to 0.37 GeV cm^{-3} . The galactocentric distance of the Earth is $r_\odot = 8.5 \text{ kpc}$. Finally, the contribution of a nearby clump located at \mathbf{x}_c can be expressed as

$$q_{\text{DM}}^c(\mathbf{x}, E) = \frac{1}{2} \langle \sigma v \rangle \frac{L}{m_\chi^2} \delta^3(\mathbf{x} - \mathbf{x}_c) f(E), \quad (2)$$

where $L = \int_{\text{clump}} \rho_c^2(\mathbf{x}) d^3\mathbf{x}$ is defined as the subhalo luminosity. Furthermore, as clumps are treated here as point-like objects [23], the source terms q_{DM}^s and q_{DM}^c add up directly to yield the total DM lepton signal.

The annihilation cross section of the DM particles under scrutiny is set equal to $\langle \sigma v \rangle = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. This canonical value matches a thermal production of DM in the early universe. We also consider 100 GeV and 1 TeV WIMPs as benchmark cases. Finally, inspired by [4,7], we concentrate here on leptophilic species and considered either a pure e^+e^- annihilation final state (positronic line) or an equal production of charged leptons $e^\pm + \mu^\pm + \tau^\pm$. Notice that a case for both scenarios can be made from the model-building perspective. We also tried to use models featuring pure $b\bar{b}$ or W^+W^- annihilation final state, but we disregard them since the required clump configurations are extremely unlikely ($p < 10^{-6}$).

The particle physics framework being set, we perform fits to the PAMELA, ATIC, and Fermi data which include a smooth DM component plus a contribution from a DM subhalo whose luminosity L and distance D are free parameters. For PAMELA, we compute the positron fraction $\phi_{e^+}/\phi_{e^+} + \phi_{e^-}$ where we use for ϕ_{e^-} the observed cos-

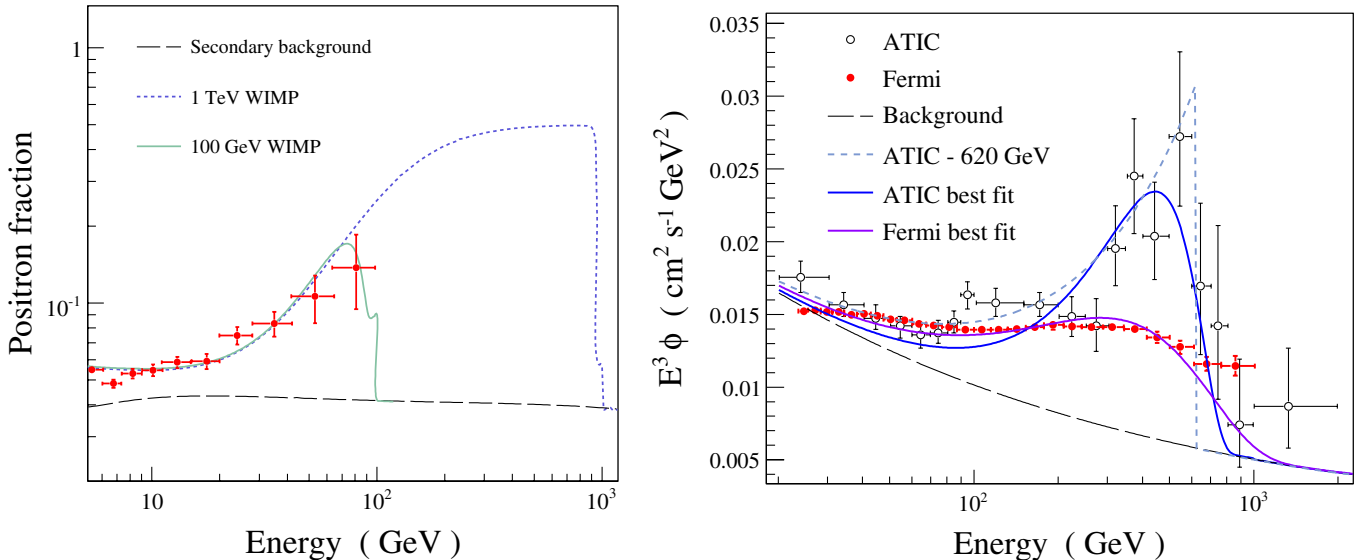


FIG. 1 (color online). Best fits to the PAMELA data in the case of a positronic line (see the e^+/e^- row of Table I) (left panel) and fits to the ATIC data (right panel).

mic ray electron flux measured by AMS [24] and HEAT [25] and parametrized by Casadei and Bindi [26]. As regards ATIC and Fermi, we derive the total lepton flux $\phi_{e^+} + \phi_{e^-}$, assuming that the electron background $\phi_{e^-}^{\text{back}}$ is given at high energy by the Casadei and Bindi fit and adding a DM contribution equal to $\phi_{e^+}^s + \phi_{e^+}^c$. Solar modulation is implemented using the force field approximation [27] with a Fisk potential of 300 MV. Figure 1 illustrates the fit results in the case of e^\pm direct production. For PAMELA, both 100 GeV and 1 TeV WIMPs can accommodate the excess. As far as ATIC is concerned, we reproduce the observed feature in the case of a 1 TeV WIMP if we assume a DM clump with luminosity $2.98 \times 10^9 M_\odot^2 \text{ pc}^{-3}$ lying at a distance of 1.52 kpc from the Earth. The ATIC excess was reported in [2] where it was interpreted as evidence for a 620 GeV Kaluza-Klein species. We confirm that result in the case of a positronic line. In that case, no satisfying adjustment can be found adding a nearby subhalo, WIMP annihilations take place only inside a smooth Galactic DM distribution and the required cross section $< \sim 10^{-24} \text{ cm}^3 \text{ s}^{-1}$, *i.e.* 2 orders of magnitude above our canonical value. As we shall see in the following, the fit to the Fermi data points towards an incredibly bright clump. Indeed, the feature seen by Fermi is not very peaked and quite spread and requires, because of propagation effect, both a very large mass for the DM particle and a quite far away clump. We therefore exclude the combination of a thermal relic density plus a bright clump as a solution to this puzzle and we do not mention this case when dealing with other messengers. All parameters found in the best-fit cases are displayed in Table I.

DM annihilations within these clumps not only produce charged leptons but also antiprotons and γ -rays. We first allow a small branching ratio $F_{\bar{p}}$ into antiprotons either through the $b\bar{b}$ channel when the WIMP is light or through the W^+W^- channel for a 1 TeV species. We compute the total antiproton flux $\phi_{\bar{p}} = \phi_{\bar{p}}^{\text{sc}} + \phi_{\bar{p}}^s + \phi_{\bar{p}}^c$ with the same cosmic ray and DM models as for positrons. Requiring that the resulting signal does not exceed the PAMELA antiproton data [28] by more than 1σ , we derive the upper limits on $F_{\bar{p}}$ featured in Table II. Antiprotons are not forbidden if they are produced together with leptons but their abundance in the annihilation debris is sufficiently suppressed. We also present in Table II conservative estimates for the detectability of the γ -ray emission from the lepton-fitting clumps of Table I. The γ -ray flux at the Earth is expressed in units of Fermi 5σ sensitivity over 1 yr of data taking for

TABLE II. Upper rows: maximal values of the branching ratio $F_{\bar{p}}$ of WIMP annihilation into $b\bar{b}$ (100 GeV) or W^+W^- (1 TeV) pairs allowed by the PAMELA antiproton data [28]. Lower rows: the clump γ -ray flux above 0.1 GeV is expressed in units of $3 \times 10^{-9} \gamma \text{ cm}^{-2} \text{ s}^{-1}$ [29].

	PAMELA		ATIC
m_χ (GeV)	100	1 000	1 000
e^\pm/e^-	0.23 ($b\bar{b}$)	0.066 (W^+W^-)	0.13 (W^+W^-)
$e^\pm + \mu^\pm + \tau^\pm$	0.063 ($b\bar{b}$)	0.0074 (W^+W^-)	0.055 (W^+W^-)
e^+/e^-	0.95	12.7	3.9
$e^\pm + \mu^\pm + \tau^\pm$	28.6	370	12.9

high-latitude pointlike sources [29]. This is probably roughly close to the best the LAT instrument can do for a low-latitude source over its lifetime. We conclude that in all cases the LAT will detect the clumps. Notice that one of the leptophilic 1 TeV PAMELA cases has a very large flux of $1.11 \times 10^{-6} \gamma \text{ cm}^{-2} \text{ s}^{-1}$ and is already of the order of the EGRET point-source sensitivity of $2 \times 10^{-7} \gamma \text{ cm}^{-2} \text{ s}^{-1}$ [30], and should have been already detected and correspond to one of EGRET unidentified sources.

Figure 2 shows the probability of having the nearest DM clump of luminosity L within a distance D from the Sun. The abundance of nearby clumps and their properties are taken directly from the VL-II simulation [20]. The high mass and force resolution, combined with a physical time step criterion [31], allow VL-II to resolve subhalos even in the dense environment near the solar circle. The mean separation of subhalos with peak circular velocities $V_{\text{max}} > 5 \text{ km s}^{-1}$ is 9.6 kpc and their luminosities are

$$L = 7.91 \times 10^5 M_\odot^2 \text{ pc}^{-3} \left(\frac{V_{\text{max}}}{5 \text{ km s}^{-1}} \right)^3 \sqrt{\frac{c_V}{2 \times 10^6}}. \quad (3)$$

For comparison, the smooth VL-II main halo has a luminosity of $L = 3.4 \times 10^9 M_\odot^2 \text{ pc}^{-3}$, while the total luminosity is about 10 times higher [20]. At a given V_{max} we assume a log-normal distribution of luminosities with factor of 3 scatter, motivated by the substantial variance in the concentration c_V found in nearby subhalos [20]. The bold line gives the median distances calculated from a random sample of observer positions. The long-dashed, dashed and dotted lines stand for the 10th, 1st, and 0.1st percentiles, respectively. The points represent the locations of the best fits to the data in the $L - D$ plane while the surrounding contours display the 1σ excursions around these best-fit

TABLE I. Best-fit values of the ($D; L$) couple in units of (kpc; $M_\odot^2 \text{ pc}^{-3}$) for various DM particle masses and annihilation channels.

	PAMELA		ATIC	Fermi
m_χ (GeV)	100	1 000	1 000	2500
e^\pm/e^-	$1.22 - 1.07 \times 10^7$	$0.78 - 3.56 \times 10^9$	$1.52 - 2.98 \times 10^9$	$2.68 - 5.53 \times 10^{10}$
$e^\pm + \mu^\pm + \tau^\pm$	$0.44 - 2.51 \times 10^7$	$0.27 - 9.84 \times 10^9$	$XX - XX \times 10^9$	$2.81 - 2.17 \times 10^{11}$

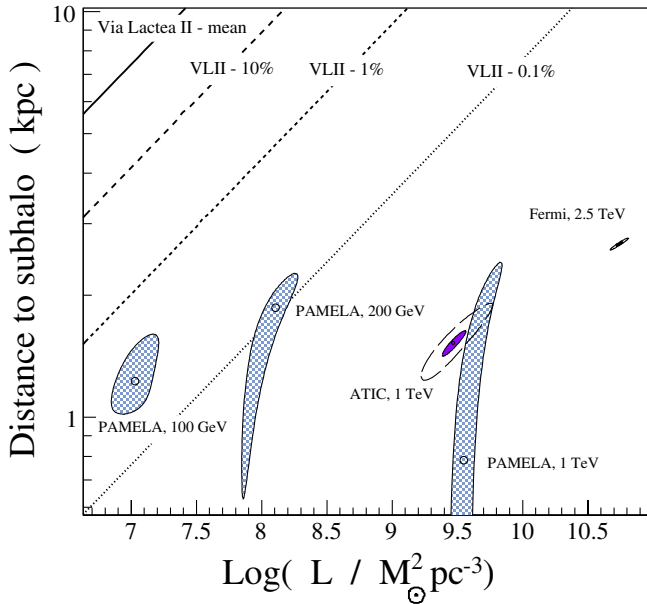


FIG. 2 (color online). Best-fit results in a clump luminosity-distance plane for different configurations, together with probabilities inferred from Via Lactea-II results.

values (as well as 3σ for ATIC and Fermi). We find that clumps fitting the PAMELA, ATIC, and Fermi data are far from the natural values indicated by VL-II. The most probable configuration is the PAMELA fit with a 100 GeV WIMP, which is inside the Via Lactea 3σ contours. That configuration is found in 0.37% of all realizations. However, this scenario cannot accommodate ATIC data because m_χ is too small. Increasing the mass of the DM particle requires even brighter and less likely clumps at $D \approx 1$ kpc. As illustrated by the different PAMELA fit contours of Fig. 2, the parameter degeneracy also increases as m_χ gets higher. Basically, the PAMELA measurements do not constrain the spectral shape of the signal above 100 GeV and leave more lever-arm to the fits when WIMPs are heavy. For TeV WIMPs, there are clump properties which reproduce both the ATIC and PAMELA excess (see Fig. 2), and such a source would be well within the reach of Fermi (Table II). However, in the standard CDM halo it is very unlikely to exist ($p \approx 3 \times 10^{-5}$). A DM spike or a higher cross section would be required to get the needed luminosity from a smaller, more probable nearby subhalo. As shown in Table II, the corresponding subhalo is within reach of Fermi. Concerning the best fit to the Fermi data, it points at the need for a clump that should be brighter than the whole Milky Way. We can safely associate a zero probability to this configuration. Note finally that the Via Lactea II contours extrapolate at lower values of the distance and clump luminosity. The corresponding clumps are not of particular interest for this particular study as for mean Via Lactea clumps, the natural luminosity decreases faster than what we gain from placing the clump closer.

Our investigation of the lepton anomalies in the presence of a nearby DM clump has finally led us to discover a subtle interplay between the injection spectrum, the position of the substructure and the signal at the Earth. In particular, the commonly used criterion of a sharp cutoff as a smoking-gun signature of DM models producing leptons (e.g. in universal extra dimension Kaluza-Klein models [32]) is misleading if most of the DM signal stems from a highly clumpy halo. At TeV energies, leptons detected at the Earth are produced within a distance of ~ 1 kpc and a clump becomes less visible if it lies outside that region. The opposite case is also possible since one can obtain very peaked lepton spectra even with a soft injection spectrum, as long as the nearest clump is close enough. In general, as far as spectral shapes are concerned, it is important to bear in mind the spectral distortions induced by cosmic ray propagation.

As an illustration, we show in Fig. 3 how the signal from DM can exhibit a double-bump feature if, in addition to the contribution from a smooth DM halo, two nearby clumps are taken into account (solid light-colored line). In this example, the subhalos lie at a distance of 0.9 and 4.3 kpc from the Earth and their luminosities are of order 10^8 and $10^{10} M_\odot^2 \text{pc}^{-3}$ respectively. Indeed, a specificity of lepton propagation is that regardless of the overall normalization, a feature observed at energy E can always be produced by a source at distance D with an injection energy E_S as long as $D^2 \propto E^{-0.3} - E_S^{-0.3}$ [33]. However, without additional enhancements, having such bright nearby clumps is practically ruled out and we use them here for pedagogical purpose. CDM subhalos with $L > 10^8 M_\odot^2 \text{pc}^{-3}$ have $V_{\text{max}} > 25 \text{ km s}^{-1}$ and host relatively bright dwarf galaxies [34]. If such dwarfs existed nearby, they should have been observed. As regards the light-colored dashed curve of the

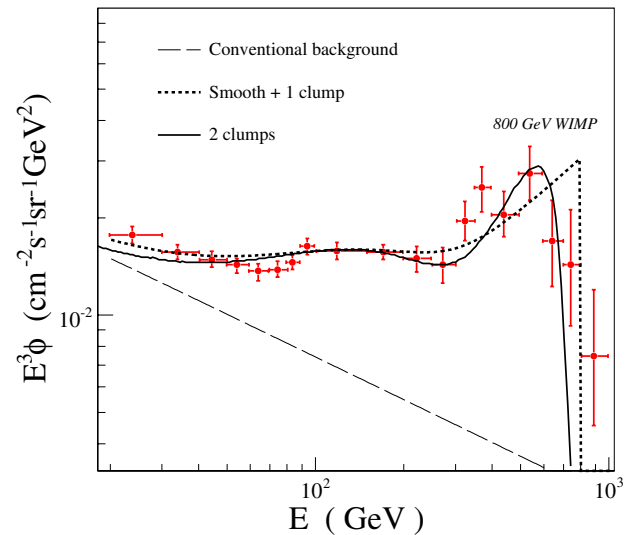


FIG. 3 (color online). Electron-positron spectra resulting from one (dotted) or two (solid) nearby subhalos. In the latter case, a 620 GeV DM species with thermal annihilation cross section is considered.

right panel of Fig. 3, the sharp edge at 800 GeV is associated to a strong local DM annihilation and is produced by the VL-II smooth halo whereas the bump at ~ 100 GeV comes from a single nearby clump located at 3.2 kpc. The cross section has been increased up to a value of $10^{-23} \text{ cm}^3 \text{ s}^{-1}$. This case seems somewhat more probable than the 2-clumps configuration. These examples illustrate how tricky boost factors are. Shifting upwards the DM cosmic ray fluxes turns out to be wrong especially in the light of the subtle effects introduced by propagation.

This paper is focused on the astrophysical boost associated to a nearby DM clump. We have restrained ourselves from using annihilation cross sections in excess of the canonical value of $3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. As a final remark, we point out that all the fitted spectra would have been obtained with significantly less luminous subhalos should the annihilation cross section be directly enhanced. If the Sommerfeld effect [35,36] is at play, the signal from small clumps is enhanced with respect to the contribution from larger substructures [37] since the velocity dispersion of DM particles decreases with the mass of the host subhalo. The blue contours of Fig. 2 are shifted towards smaller

values of L and get nearer to the mean predictions of the VL-II simulation, with a much larger probability of occurrence (by simply assuming $L \rightarrow L \times c/V_{\text{max}}$, the probability of ATIC best-fit case increases from $p \approx 3 \times 10^{-5}$ to 14%!). Because of the lack of recent data, we had to do some assumptions on the electron spectrum and propagation parameters. Therefore, new data may rule out some of these assumptions and hence lead to different conclusions. Future detailed measurements of the lepton spectrum from Fermi, and (on a much longer time-scale) of the positron fraction from e.g. PEBS [38] will provide us with much greater information on the nature and origin of the lepton anomalies, and perhaps shed light on galactic dark matter.

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- [1] O. Adriani *et al.* (PAMELA), arXiv:0810.4995.
 - [2] J. Chang *et al.* (ATIC), Nature (London) **456**, 362 (2008).
 - [3] A. A. Abdo *et al.* (Fermi LAT), Phys. Rev. Lett. **102**, 181101 (2009).
 - [4] M. Cirelli, M. Kadastik, M. Raidal, and A. Strumia, Nucl. Phys. **B813**, 1 (2009).
 - [5] L. Bergstrom, T. Bringmann, and J. Edsjo, Phys. Rev. D **78**, 103520 (2008).
 - [6] I. Cholis, L. Goodenough, D. Hooper, M. Simet, and N. Weiner, arXiv:0809.1683.
 - [7] F. Donato, D. Maurin, P. Brun, T. Delahaye, and P. Salati, Phys. Rev. Lett. **102**, 071301 (2009).
 - [8] T. Delahaye *et al.*, arXiv:0809.5268.
 - [9] F. Aharonian *et al.* (H.E.S.S.), Phys. Rev. Lett. **101**, 261104 (2008).
 - [10] C. F. Aharonian (H.E.S.S.), arXiv:0905.0105.
 - [11] D. Hooper, P. Blasi, and P. D. Serpico, J. Cosmol. Astropart. Phys. 01 (2009) 025.
 - [12] H. Yuksel, M. D. Kistler, and T. Stanev, Phys. Rev. Lett. **103**, 051101 (2009).
 - [13] S. Profumo, arXiv:0812.4457.
 - [14] J. Lavalle, J. Pochon, P. Salati, and R. Taillet, Astron. Astrophys. **462**, 827 (2007).
 - [15] P. Brun, G. Bertone, J. Lavalle, P. Salati, and R. Taillet, Phys. Rev. D **76**, 083506 (2007).
 - [16] J. Lavalle, Q. Yuan, D. Maurin, and X.-J. Bi, Astron. Astrophys. **479**, 427 (2008).
 - [17] D. Hooper, A. Stebbins, and K. M. Zurek, Phys. Rev. D **79**, 103513 (2009).
 - [18] T. Bringmann, J. Lavalle, and P. Salati, arXiv:0902.3665.
 - [19] D. T. Cumberbatch and J. Silk, Mon. Not. R. Astron. Soc. **374**, 455 (2007).
 - [20] J. Diemand, M. Kuhlen, P. Madau, M. Zemp, B. Moore, D. Potter, and J. Stadel, Nature (London) **454**, 735 (2008).
 - [21] D. Maurin, F. Donato, R. Taillet, and P. Salati, Astrophys. J. **555**, 585 (2001).
 - [22] F. Donato, N. Fornengo, D. Maurin, P. Salati, and R. Taillet, Phys. Rev. D **69**, 063501 (2004).
 - [23] The scale radii of the clumps are always much smaller than the typical lepton diffusion length.
 - [24] J. Alcaraz *et al.* (AMS), Phys. Lett. B **484**, 10 (2000).
 - [25] J. J. Beatty *et al.*, Phys. Rev. Lett. **93**, 241102 (2004).
 - [26] D. Casadei and V. Bindi, Astrophys. J. **612**, 262 (2004).
 - [27] J. S. Perko, Astron. Astrophys. **184**, 119 (1987).
 - [28] O. Adriani *et al.*, Phys. Rev. Lett. **102**, 051101 (2009).
 - [29] F. W. B. Atwood (LAT), Astrophys. J. **697**, 1071 (2009).
 - [30] R. C. Hartman *et al.*, Astrophys. J. Suppl. Ser. **123**, 79 (1999).
 - [31] M. Zemp, J. Stadel, B. Moore, and C. M. Carollo, Mon. Not. R. Astron. Soc. **376**, 273 (2007).
 - [32] D. Hooper and S. Profumo, Phys. Rep. **453**, 29 (2007).
 - [33] The 0.3 exponent depends on the cosmic ray propagation model.
 - [34] M. Kuhlen, J. Diemand, and P. Madau, Astrophys. J. **671**, 1135 (2007).
 - [35] J. Hisano, S. Matsumoto, and M. M. Nojiri, Phys. Rev. Lett. **92**, 031303 (2004).
 - [36] J. D. March-Russell and S. M. West, arXiv:0812.0559.
 - [37] M. Lattanzi and J. I. Silk, Phys. Rev. D **79**, 083523 (2009).
 - [38] H. Gast, R. Greim, T. Kirn, G. R. Yearwood, and S. Schael, arXiv:0903.5404.