Mon. Not. R. Astron. Soc. **396,** 2065–2073 (2009) doi:10.1111/j.1365-2966.2008.14298.x

On the escape of particles from cosmic ray modified shocks

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Accepted 2008 November 25. Received 2008 October 22; in original form 2008 July 26

ABSTRACT

Stationary solutions to the problem of particle acceleration at shock waves in the non-linear regime, when the dynamical reaction of the accelerated particles on the shock cannot be neglected, are known to show a prominent energy flux escaping from the shock towards upstream infinity. On physical grounds, the escape of particles from the upstream region of a shock has to be expected in all those situations in which the maximum momentum of accelerated particles, $p_{\rm max}$, decreases with time, as is the case for the Sedov–Taylor phase of expansion of a shell supernova remnant, when both the shock velocity and the cosmic ray induced magnetization decrease. In this situation, at each time t, particles with momenta larger than $p_{\rm max}(t)$ leave the system from upstream, carrying away a large fraction of the energy if the shock is strongly modified by the presence of cosmic rays. This phenomenon is of crucial importance for explaining the cosmic ray spectrum detected at the Earth. In this paper, we discuss how this escape flux appears in the different approaches to non-linear diffusive shock acceleration, and especially in the quasi-stationary semi-analytical kinetic ones. We apply our calculations to the Sedov–Taylor phase of a typical supernova remnant, including in a self-consistent way particle acceleration, magnetic field amplification and the dynamical reaction on the shock structure of both particles and fields. Within this framework, we calculate the temporal evolution of the maximum energy reached by the accelerated particles and of the escape flux towards upstream infinity. The latter quantity is directly related to the cosmic ray spectrum detected at the Earth.

Key words: acceleration of particles – shock waves.

1 INTRODUCTION

Supernova remnants (SNRs) have long been suspected to be the main sources of galactic cosmic rays with energies up to the so-called 'knee' ($E \sim 3 \times 10^{15}$ eV). The existence of a flux of energetic particles escaping the accelerator from the upstream region is essential if this paradigm is indeed realized in Nature. The commonly accepted view of a typical SNR evolution is as follows. During the initial stage after the supernova explosion, the ejecta are in free-expansion with a high velocity $\sim 10^4$ km s^{-1}. Non-linear acceleration is expected to be at work: efficient particle acceleration is associated with shock modification and magnetic field amplification in a complex chain of causality. During this stage, the maximum momentum achieved by the accelerated particles increases with time up to $p_{\rm max} \sim 10^{16}$ eV c^{-1} (see e.g. Blasi, Amato & Caprioli 2007 and references therein). When the mass of the swept-up material becomes comparable with the mass of the ejecta, the Sedov–Taylor phase begins: the shock slows down and magnetic field damping is expected to become more effective (Ptuskin & Zirakashvili 2005). As a consequence, the maximum momentum of particles that can return to the shock is expected to decrease. At each time t, particles with momentum exceeding the current $p_{\rm max}(t)$ do not make it back to the shock and leave the system from upstream: this is the so-called 'escape flux'.

The escape flux plays an essential role in the formation of the cosmic ray spectrum detected at the Earth. If there was no escape from upstream during the Sedov–Taylor phase, all particles accelerated in a SNR would be advected downstream and undergo severe adiabatic energy losses before being injected into the ISM: in this case, SNRs could not be responsible for accelerating cosmic rays with energies around the knee (Caprioli et al., in preparation; and Ptuskin & Zirakashvili 2005).

While being of crucial importance during the Sedov–Taylor phase, it is clear that escape cannot occur during the free expansion phase of a SNR: the only particles that can leave the system from upstream are the ones at the highest energies, but, since $p_{\rm max}$ is now increasing with time, no appreciable energy flux can escape the accelerator during this phase.

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Kinetic approaches to non-linear particle acceleration (Malkov 1997; Malkov & Drury 2001; Blasi 2002, 2004; Amato & Blasi 2005) allow us to calculate the spectrum and the spatial distribution (including the absolute normalization) of the particles accelerated at the shock, even in the case when the diffusion coefficient is the result of magnetic field amplification by streaming instability induced by the accelerated particles themselves (Amato & Blasi 2006). These approaches, as well as others that have appeared in the literature (for instance Berezhko & Ellison 1999), are based on the assumption that the acceleration process (and the shock modification) may be assumed to reach some sort of stationarity. In all these cases, the calculations show that the shock is strongly modified by the presence of cosmic rays, and that the spectra are concave, with a slope at momenta close to the maximum momentum $p_{\rm max}$ which is flatter than p^{-4}. All these calculations, independently of the techniques used to solve the equations, predict an escape flux of particles (and energy) towards upstream infinity: the shock becomes radiative, which is one of the very reasons why the shock modification becomes effective (namely the total compression factor becomes larger than 4).

It is worth recalling that the assumption of stationarity was widely used also in the framework of two-fluid models (Drury & Völk 1981a,b), and its relevance in the case of modified shocks, leading to particle spectra harder than p^{-4} at high energies, was discussed by Malkov & Völk (1996) in the context of the so-called renormalized two-fluid models. It should be stressed, indeed, that the escape flux can arise both in the linear and non-linear regime of particle acceleration. The important difference between the two regimes, however, is that within the former the escaping particles carry away a negligible energy flux, while for modified shocks the bulk of the energy in the form of cosmic rays is stored in the highest energy particles and leaves the system with them.

The main weak point of the assumption of quasi-stationarity is that it leads to artificial escape fluxes in situations, such as the free expansion phase of a SNR, where escape would be unphysical (at least in the idealized cases of plane and perfectly spherical shocks). In this case, the appearance of an escape flux signals for the need of a fully time-dependent approach. In other words, what appears as an escape flux is the energy which is channelled into particles with ever increasing momentum. During the Sedov–Taylor phase, as emphasized earlier, escape is in fact possible if the magnetic field in the shock region decreases. In this case, the leakage of particles towards upstream allows the system to relax to a quasi-stationary situation.

A number of time-dependent studies of non-linear shock dynamics have been presented in the literature (e.g. Falle & Giddings 1987), in particular for SNRs (e.g. Berezhko & Völk 1997; Kang & Jones 2006 and references therein). All these calculations aim at solving the transport equations and the equation of fluid dynamics numerically. Most of these papers have however concentrated on the spectrum of accelerated particles rather than on the escape flux. In fact, in many of these papers, the magnetic field is introduced by hand and is not a function of time, thereby not leading to a decrease with time of the maximum momentum of accelerated particles. In other cases, the boundary conditions are imposed at upstream infinity, therefore the escape flux, even if it is present, would appear in the form of concentration of high-energy particles at large distances from the shock. In these cases, a recipe is needed to label these particles as *escaping* the system (see for instance Lee, Kamae & Ellison 2008).

The identification of the escape flux is all but a mathematical detail: as discussed by Ptuskin & Zirakashvili (2005), the escape flux is crucial for establishing a connection between SNRs and the origin of cosmic rays, in that it allows the escaping particles to avoid the adiabatic losses related to the expansion of the supernova shell, and therefore become cosmic rays at the knee. In the absence of escape towards upstream infinity at the beginning of the Sedov–Taylor phase, SNRs cannot be the sources of cosmic rays up to the knee.

The contribution of the escape flux to the galactic cosmic ray spectrum was first estimated by Ptuskin & Zirakashvili (2005). These authors included the magnetic field damping and the consequent decrease of $p_{\rm max}$ during the Sedov–Taylor expansion, but in their study the shock structure and particle spectrum are fixed rather than self-consistently calculated.

In this paper, we present the first attempt at carrying out a calculation of the escape flux within the framework of a kinetic approach to non-linear shock acceleration, self-consistently including particle acceleration, magnetic field amplification and the dynamical reaction of both on the shock. We focus our attention on shocks in shell-type SNRs, discussing the physical meaning of the escape flux during the different phases of their evolution. In particular, the flux of energetic particles leaving the remnant during the Sedov–Taylor phase is explicitely computed and its phenomenological implications for the origin of cosmic rays are discussed.

The paper is organized as follows. In Section 2, we discuss the implications of the assumption of stationarity of the acceleration process. In Section 3, we discuss the escape flux based on the most general version of the conservation equations. In Section 4, we discuss how the escape flux is connected to the existence of a maximum momentum in the distribution of accelerated particles. In Section 5, we apply our calculations to the different stages of evolution of a SNR. We conclude in Section 6.

2 THE ASSUMPTION OF STATIONARITY

The standard solution of the stationary transport equation

$$u(x)\frac{\partial f(x,p)}{\partial x} = \frac{\partial}{\partial x}\left[D(p)\frac{\partial f(x,p)}{\partial x}\right] + \frac{p}{3}\frac{{\rm d}u}{{\rm d}x}\frac{\partial f}{\partial p} + Q \qquad (1)$$

leads, in the test-particle regime, to the well-known power-law spectrum of accelerated particles $f(p) \propto p^{-\alpha}$, with $\alpha = 3r/(r-1)$ where r is the compression factor at the shock.

The power law extends to infinitely large momenta. Since for ordinary non-relativistic gaseous shocks $r < 4$ (namely $\alpha > 4$), the total energy in the form of accelerated particles remains finite. This solution is found by imposing as boundary condition at upstream infinity ($x = -\infty$) that $f(-\infty) = 0$ and $\partial f(-\infty)/\partial x = 0$.

If the boundary condition $f(x = x_0) = 0$ is used, instead, at some finite distance $x_0 < 0$ upstream, the solution of the transport equation is easily calculated to be

$$f(x,p) = \frac{f_0(p)}{1-\exp\left[\frac{u_1 x_0}{D(p)}\right]}\left\{\exp\left[\frac{u_1 x}{D(p)}\right] - \exp\left[\frac{u_1 x_0}{D(p)}\right]\right\}, \qquad (2)$$

where

$$f_0(p) = K\exp\left\{-\frac{3u_1}{u_1-u_2}\int_{p_{\rm inj}}^{p}\frac{{\rm d}p'}{p'}\frac{1}{1-\exp\left[\frac{x_0 u_1}{D(p')}\right]}\right\}. \qquad (3)$$

In the case of Bohm diffusion, $D(p) = D_0(p/m_{\rm p}c)$ and one obtains

$$f_0(p) = K\exp\left\{-\frac{3u_1}{u_1-u_2}\int_{p_{\rm inj}}^{p}\frac{{\rm d}p'}{p'}\frac{1}{1-\exp\left(-\frac{p_*}{p'}\right)}\right\}, \qquad (4)$$

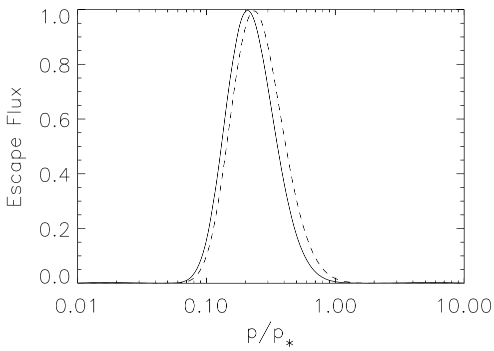

Figure 1. We plot the escape flux $\phi(x_0, p)$ as a function of momentum. The curves refer to two different values of the shock compression ratio: $r = 4$ (solid line) and $r = 7$ (dashed line). The computation is carried out in the test-particle regime. The x-axis is in units of the reference momentum $p_* = r/(r-1)\, p_{\max}$, while units along the y-axis are arbitrary.

where $p_* = |x_0|\ u_1 m_p c/D_0$. Now one can show that for $p \ll p_*$, $f_0(p) \propto (p/p_*)^{-3r/(r-1)}$, with $r = u_1/u_2$, the standard result. However, for $p \gg p_*$, $f_0(p) \propto \exp[-\frac{3r}{r-1}\frac{p}{p_*}]$. The quantity $p_{\max} = p_*(r-1)/3r$ plays the role of maximum momentum of the accelerated particles.

This simple example shows how a maximum momentum can be obtained in a stationary approach only by imposing the boundary condition at a finite boundary. Physically this corresponds to particles' escape, as shown by the fact that the flux of particles at $x = x_0$ is

$$\phi(x_0, p) = u_1 f(x_0, p) - D(p)\frac{\partial f(x_0)}{\partial x} = -\frac{u_1 f_0(p)}{1 - \exp\left[\frac{u_1 x_0}{D(p)}\right]} \times \exp\left[\frac{u_1 x_0}{D(p)}\right] < 0. \quad (5)$$

The fact that $\phi(x_0, p) < 0$ shows that the flux of particles is directed towards upstream infinity. Moreover, the escape flux as a function of momentum, $\phi(x_0, p)$, is negligible for all p with the exception of a narrow region around $p_{\max}$: only particles with momentum close to $p_{\max}$ can escape the system towards upstream infinity. The escape flux as a function of momentum is plotted in Fig. 1, for two values of the compression factor, $r = 4$ (solid line) and $r = 7$ (dashed line). The normalizations are arbitrary, since the calculations are carried out in the context of test particle theory. The latter value of r cannot be realized at purely gaseous shocks, but we have adopted this value to mimic the effect of shock modification, which leads to total compression factors larger than 4.

The escape phenomenon is basically irrelevant in the test-particle regime because of the negligible fraction of energy carried by particles with $p \sim p_{\max}$, but it becomes extremely important in the calculation of the shock modification induced by accelerated particles. For strongly modified shocks, the slope of the spectrum at high energies is flatter than p^{-4} and the fraction of energy that leaves the system towards upstream infinity may dominate the energy budget. This is the escape flux which appears in all approaches to cosmic ray modified shocks.

In the context of kinetic calculations of the shock modification in the stationary regime, the escape flux appears however not as a consequence of imposing a boundary condition at a finite distance upstream, but rather as an apparent violation of the equation of energy conservation (Berezhko & Ellison 1999), that requires the introduction of an escape term at upstream infinity. In the next section, we discuss this effect, which reveals the true nature of the escape flux, as related to the form of the conservation equations and the assumption of stationarity.

3 CONSERVATION EQUATIONS AND ESCAPE FLUX

In this section, we rederive the conservation equations for cosmic ray modified shocks in their general form, in order to emphasize the mathematical origin of the escape flux.

The time-dependent conservation equations in the presence of accelerated particles at a shock can be written in the following form:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x} \quad (6)$$

$$\frac{\partial(\rho u)}{\partial t} = -\frac{\partial}{\partial x}\left[\rho u^2 + P_g + P_c + P_W\right] \quad (7)$$

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho u^2 + \frac{P_g}{\gamma_g - 1}\right] = -\frac{\partial}{\partial x}\left[\frac{1}{2}\rho u^3 + \frac{\gamma_g P_g u}{\gamma_g - 1}\right] - u\frac{\partial}{\partial x}\left[P_c + P_W\right] + \Gamma E_W. \quad (8)$$

Here P_g, P_c and P_W are, respectively, the gas pressure, the cosmic ray pressure and the pressure in the form of waves. E_W is the energy density in the form of waves and Γ is the rate at which the background plasma is heated due to the damping of waves on to the plasma. The rate of change of the gas temperature is related to ΓE_W through

$$\frac{\partial P_g}{\partial t} + u\frac{\partial P_g}{\partial x} + \gamma_g P_g \frac{du}{dx} = (\gamma_g - 1)\Gamma E_W. \quad (9)$$

The cosmic ray pressure can be calculated from the transport equation:

$$\frac{\partial f(t,x,p)}{\partial t} + \tilde{u}(x)\frac{\partial f(t,x,p)}{\partial x} = \frac{\partial}{\partial x}\left[D(x,p)\frac{\partial f(t,x,p)}{\partial x}\right] + \frac{p}{3}\frac{\partial f(t,x,p)}{\partial p}\frac{d\tilde{u}(x)}{dx}, \quad (10)$$

where we put $\tilde{u}(x) = u(x) - v_W(x)$ and $v_W(x)$ is the wave velocity. For our purposes, here, we are neglecting the injection term.

Multiplying this equation by the kinetic energy $T(p) = m_p c^2 (\gamma - 1)$, where γ is the Lorentz factor of a particle with momentum p, and integrating the transport equation in momentum, one has

$$\frac{\partial E_c}{\partial t} + \frac{\partial(\tilde{u} E_c)}{\partial x} = \frac{\partial}{\partial x}\left[\bar{D}\frac{\partial E_c}{\partial x}\right] - P_c\frac{d\tilde{u}}{dx}, \quad (11)$$

where

$$E_c = \int_0^\infty dp\, 4\pi p^2\, T(p) f(p)$$

and

$$P_c = \int_0^\infty dp\, \frac{4\pi}{3} p^3 v(p) f(p) \quad (12)$$

are the energy density and pressure in the form of accelerated particles. Moreover, we introduced the mean diffusion coefficient:

$$\bar{D}(x) = \frac{\int_0^\infty 4\pi p^2 T(p) D(p)\frac{\partial f}{\partial x}}{\int_0^\infty 4\pi p^2 T(p)\frac{\partial f}{\partial x}}. \quad (13)$$

The only assumption that we made here is that $f(p) \to 0$ for $p \to \infty$. We will comment later (Section 4) on what would happen if the spectrum were truncated at some fixed $p_{\max}$, instead.

Introducing the adiabatic index for cosmic rays γ_c as $E_c = P_c/(\gamma_c - 1)$, we can rewrite equation (11) as

$$\frac{\partial E_c}{\partial t} + \frac{\partial}{\partial x}\left[\frac{\gamma_c \tilde{u} P_c}{\gamma_c - 1}\right] = \frac{\partial}{\partial x}\left[\bar{D}\frac{\partial E_c}{\partial x}\right] + \tilde{u}\frac{dP_c}{dx}, \quad (14)$$

and use it to derive $u\partial P_c/\partial x$. In this way equation (8) becomes

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho u^2 + \frac{P_g}{\gamma_g - 1} + E_c\right] = -\frac{\partial}{\partial x}\left[\frac{1}{2}\rho u^3 + \frac{\gamma_g P_g u}{\gamma_g - 1} + \frac{\gamma_c P_c \tilde{u}}{\gamma_c - 1}\right] + \frac{\partial}{\partial x}\left[\bar{D}(x)\frac{\partial E_c}{\partial x}\right] - v_W\frac{\partial P_c}{\partial x} - u\frac{\partial P_W}{\partial x} + \Gamma E_W. \quad (15)$$

At this point, we can make use of the equation describing the evolution of the waves:

$$\frac{\partial E_W}{\partial t} + \frac{\partial F_W}{\partial x} = u\frac{\partial P_W}{\partial x} + \sigma E_W - \Gamma E_W, \quad (16)$$

where σ is the growth rate of waves, integrated over wavenumber and F_W is the flux of energy in the shape of waves. These quantities can be calculated once it is known how particles with given momentum p interact with waves with wavenumber k and how the turbulence energy is shared between the kinetic and the properly magnetic contributions. Substituting into equation (15), we get

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho u^2 + \frac{P_g}{\gamma_g - 1} + E_c + E_W\right] = -\frac{\partial}{\partial x}\left[\frac{1}{2}\rho u^3 + \frac{\gamma_g P_g u}{\gamma_g - 1} + \frac{\gamma_c P_c \tilde{u}}{\gamma_c - 1} + F_W\right] + \frac{\partial}{\partial x}\left[\bar{D}(x)\frac{\partial E_c}{\partial x}\right] - v_W\frac{\partial P_c}{\partial x} + \sigma E_W. \quad (17)$$

In the case of Alfvén waves resonant with the Larmor radius of the accelerated particles, one has $v_W = v_A = B/(4\pi\rho)^{1/2}$ and (Skilling 1975)

$$\sigma E_W = v_A\frac{\partial P_c}{\partial x}, \quad (18)$$

so that the energy conservation equation reads

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho u^2 + \frac{P_g}{\gamma_g - 1} + E_c + E_W\right] = -\frac{\partial}{\partial x}\left[\frac{1}{2}\rho u^3 + \frac{\gamma_g P_g u}{\gamma_g - 1} + \frac{\gamma_c P_c \tilde{u}}{\gamma_c - 1} + F_W\right] + \frac{\partial}{\partial x}\left[\bar{D}(x)\frac{\partial E_c}{\partial x}\right]. \quad (19)$$

In the general case of waves other than resonant Alfvén waves, equation (18) does not hold and even the equality $v_W = v_A$ may be questionable: as a consequence one cannot use the standard equation (19), while equation (17) is still correct. However, in order to be able to solve the problem, the detailed form of the cosmic ray transport equation, as well as an expression analogous to equation (18), relating the growth of the wave energy to the cosmic ray dynamics, is still needed. It is worth stressing, in fact, that the magnetic turbulence has often been proposed to not show up in the form of standard resonant Alfvén waves, but rather as non-resonant purely growing modes Bell (2004), or as generic 'magnetic structures' in the phenomenological model of Bell & Lucek (2001). This also causes the connection between F_W and P_W to be generally different from the standard $F_W \approx 3uP_W$, thus the contribution of these waves to the energy conservation equation does not necessarily lead to equation (19), which was nevertheless used by Vladimirov, Ellison & Bykov (2006) in the implementation of the scenario suggested by Bell & Lucek (2001).

In the following, we limit ourselves to the case of Alfvén waves, which interact resonantly with particles, since in this case the calculations are all well defined.

Note that in the stationary regime, equation (14), integrated around the subshock leads to

$$\frac{\gamma_c}{\gamma_c - 1}\tilde{u}P_c - \bar{D}\frac{dE_c}{dx} = \text{constant}, \quad (20)$$

because of the continuity of the cosmic ray distribution function. On the other hand, equation (19) (again in the stationary case), once integrated around the shock, leads to conclude that

$$\frac{1}{2}\rho u^3 + \frac{\gamma_g P_g u}{\gamma_g - 1} + F_W = \text{constant}. \quad (21)$$

In other words, at the subshock the energy fluxes of the gaseous and cosmic ray components are conserved separately. This is what is usually meant when we refer to the subshock as an ordinary gas shock. In the following, we use the stationary version of equation (19),

$$\frac{\partial}{\partial x}\left[\frac{1}{2}\rho u^3 + \frac{\gamma_g P_g u}{\gamma_g - 1} + \frac{\gamma_c P_c \tilde{u}}{\gamma_c - 1} + F_W - \bar{D}(x)\frac{\partial E_c}{\partial x}\right] = 0. \quad (22)$$

4 ESCAPE FLUX AND THE NEED FOR A p_{max}

In non-linear theories of particle acceleration, the need for a maximum momentum is dictated by the fact that the spectrum at large momenta becomes harder than p^{-4}, so that in the absence of a high p cut-off the energy content of the accelerated particle distribution would diverge. Before this happens the dynamical reaction of the accelerated particles would inhibit further acceleration. In most approaches to non-linear calculations (Malkov 1997; Malkov, Diamond & Völk 2000; Blasi 2002, 2004; Berezhko & Ellison 1999), the maximum momentum is a given parameter, taken together with the assumption of stationarity of the acceleration process. The transport equation is then solved between the shock and upstream infinity. Both in the downstream region and at upstream infinity, one has $D\partial f/\partial x = 0$. Moreover, at upstream infinity there are no accelerated particles ($P_c = 0$) so that equation (22) becomes

$$\frac{1}{2}\rho_2 u_2^3 + \frac{\gamma_g P_{g,2} u_2}{\gamma_g - 1} + \frac{\gamma_c P_c u_2}{\gamma_c - 1} + F_W = \frac{1}{2}\rho_0 u_0^3 + \frac{\gamma_g P_{g,0} u_0}{\gamma_g - 1}. \quad (23)$$

None of the calculations of particle acceleration at modified shocks carried out so far satisfies equation (23) unless it is *completed* with an escape flux F_{esc} such that

$$\frac{1}{2}\rho_2 u_2^3 + \frac{\gamma_g P_{g,2} u_2}{\gamma_g - 1} + \frac{\gamma_c P_c u_2}{\gamma_c - 1} + F_W = \frac{1}{2}\rho_0 u_0^3 + \frac{\gamma_g P_{g,0} u_0}{\gamma_g - 1} - F_{esc}. \quad (24)$$

Unfortunately, as showed above, this apparently harmless step is inconsistent with $f(p, x)$ being a solution of the time-independent transport equation. In fact this is not surprising since, as we stressed above, the solution of such equation cannot be characterized by a finite p_{max} when the boundary condition of vanishing $f(p,x)$ and $\partial f/\partial x$ is imposed at upstream infinity.

On the other hand, as also discussed by Malkov & Völk (1996) in the context of the so-called renormalized two-fluid models, the requirement that there is a maximum momentum leads to the appearance of an additional term on the right-hand side of equation (11), which is

$$+\frac{1}{3}\frac{d\tilde{u}}{dx}\left[4\pi p^3 T(p) f(p, x)\right]_{p=0}^{p=p_{max}}. \quad (25)$$

This reflects in an additional term in equation (24):

$$\int_{-\infty}^{0^+} \mathrm{d}x \frac{1}{3}\frac{\mathrm{d}u}{\mathrm{d}x} 4\pi p_{\max}^3 T(p_{\max}) f(p_{\max}, x) = \int_{-\infty}^{0^-} \mathrm{d}x \frac{1}{3}\frac{\mathrm{d}u}{\mathrm{d}x} 4\pi p_{\max}^3 T(p_{\max}) f(p_{\max}, x) + (u_2 - u_1) T(p_{\max}) \times \frac{4\pi}{3} p_{\max}^3 f_0(p_{\max}). \tag{26}$$

Since $\mathrm{d}u/\mathrm{d}x < 0$ in the precursor, this term is negative and numerically coincides with the escape flux $F_{\rm esc}$.

As an alternative to obtaining the escape flux in this way, one could derive it by assuming that there is no $p_{\max}$ imposed by hand, and that a maximum momentum results from imposing the boundary condition at a finite distance upstream, rather than at upstream infinity (e.g. Vladimirov et al. 2006).

In this second case, $D\partial f/\partial x$ does not vanish at the upstream boundary and an escape flux appears in a natural way, namely

$$\phi_{\rm esc} = u_0 f(x_0,\ p) - D\left[\frac{\partial f}{\partial x}\right]_{x=x_0} = -D\left[\frac{\partial f}{\partial x}\right]_{x=x_0} < 0, \tag{27}$$

and the energy escape flux $F_{\rm esc}$ is related to $\phi_{\rm esc}$ through

$$F_{\rm esc} = \int_{p_{\rm inj}}^{p_{\max}} 4\pi\ p^2\ \mathrm{d}p\ \phi_{\rm esc}(p)\ T(p). \tag{28}$$

In other words, no artificial escape flux needs to be introduced if a boundary condition is imposed at a finite distance upstream, or if, as an alternative, the fully time-dependent solution of the problem can be found.

In next section, we explore the consequences of the existence of an escape flux for the origin of cosmic rays in SNRs.

5 PHYSICAL MEANING OF THE ESCAPE FLUX FOR SNRs

The acceleration process in SNRs is expected to work in qualitatively different ways during the free expansion and the Sedov–Taylor phases. Here, we restrict our attention to the propagation in a spatially uniform interstellar medium. During the free expansion phase, the velocity of the shell remains constant and the maximum momentum grows in time in a way that depends on the growth of the turbulent magnetic field in the upstream region. During this phase, particles cannot escape. Nevertheless, the standard approaches to the calculation of the shock modification would lead to predict an escape flux, a symptom of the need to carry out fully time-dependent calculations to treat this expansion regime. The lack of particles' escape implies an increase in the maximum momentum of the accelerated particles. This trend ends at the beginning of the Sedov–Taylor phase, when the inertia of the swept up material slows down the expanding shell. Physically, this is the reason why we expect that the highest energies for particles accelerated in SNRs are reached at the beginning of the Sedov–Taylor phase.

During this phase, the shock velocity decreases and the magnetic field amplification upstream, as due to streaming instability, becomes less efficient. The generation of magnetic turbulence via streaming instabilities may proceed through either resonant (Bell 1978a,b) or non-resonant (Bell 2004) coupling between particles and waves and the two channels are likely to dominate at different times in the history of the SNR (Pelletier, Lemoine & Marcowith 2006; Amato & Blasi 2009), as discussed below.

This general picture leads to a maximum momentum that decreases with time and to particles' escape towards upstream infinity: particles of momentum $p_{\max}(t_1)$ do not make it back to the shock at a time $t_2 > t_1$. In other words, during the time interval between t_1 and t_2, particles with momentum between $p_{\max}(t_1)$ and $p_{\max}(t_2)$ escape from the system. This happens at any time, and a net flux of particles (and energy) towards upstream infinity is realized. At any given time t, the spectrum of particles that escape is highly peaked around $p_{\max}(t)$ (see Fig. 1 for the test-particle case). The spectrum of accelerated particles that is confined in the accelerator and advected towards downstream is cut-off at a gradually lower maximum momentum, and this should reflect in the spectrum of secondary radiation, especially gamma-rays. The particles trapped downstream will also eventually escape the system, but at the time this happens they will have been affected by adiabatic losses due to the expansion of the shell. Therefore this part of the escaping flux, which will reflect the history of the remnant, is suspected to play a particularly important role only at the lowest energies in the cosmic ray spectrum at the Earth.

The flux of high-energy cosmic rays, close to the knee region, as we see below, is mainly generated during the early Sedov–Taylor phase and is made of particles escaping the accelerators from upstream. The actual flux of diffuse cosmic rays observed at the Earth results from the integration over time of all the instantaneous spectra of escaping particles, each peaked at $p_{\max}(t)$, and from the superposition of the supernova explosions that could contribute. This integration is affected by the accelerator properties, by the dynamics of the expanding shell and by the damping processes that may affect the way the magnetic field is amplified by streaming instabilities at any given time (Ptuskin & Zirakashvili 2005).

There is also another implication of the line of thought illustrated above: the spectrum of particles that escape, as integrated over time during the Sedov–Taylor phase, does not need to be identical to the spectrum of particles advected towards downstream. However, the latter are the particles which are responsible for the production of secondary radiation (radio, X-rays, gamma rays): the concave spectra predicted by the non-linear theory of particle acceleration and to some extent required to explain observations, might not be reflected in a concavity of the spectrum of escaping particles.

An estimate of the scalings of the relevant quantities during the Sedov–Taylor phase can be found as follows. The radius and velocity of the expanding shell can be written as

$$R_{\rm sh}(t) = 2.7 \times 10^{19}\ \mathrm{cm} \left(\frac{E_{51}}{n_0}\right)^{1/5} t_{\rm kyr}^{2/5}, \tag{29}$$

$$V_{\rm sh}(t) = 4.7 \times 10^{8}\ \mathrm{cm\,s^{-1}} \left(\frac{E_{51}}{n_0}\right)^{1/5} t_{\rm kyr}^{-3/5}, \tag{30}$$

where E_{51} is the kinetic energy of the shell in the free expansion phase in units of 10^{51} erg and n_0 is the number density of the plasma upstream in units of cm^{-3}. Here, we assumed the standard Sedov–Taylor time-scaling of $R_{\rm sh}$ and $V_{\rm sh}$, but the reader should bear in mind that the adiabatic solution may be affected by the fact that in this phase the shock is radiating energy in the form of cosmic rays. The maximum energy is estimated by requiring that the diffusion length upstream equals some fraction (say 10 per cent) of $R_{\rm sh}(t)$. If the diffusion coefficient is assumed to be Bohm-like and the magnetic field close to the shock is $\delta B(t)$, one obtains:

$$E_{\max}(t) = 3.8 \times 10^{4}\ \delta B_{\mu G}(t) \left(\frac{E_{51}}{n_0}\right)^{2/5} t_{\rm kyr}^{-1/5}\ \mathrm{GeV}. \tag{31}$$

The magnetic field in the shock vicinity is amplified by streaming instability, induced by the accelerated particles both resonantly

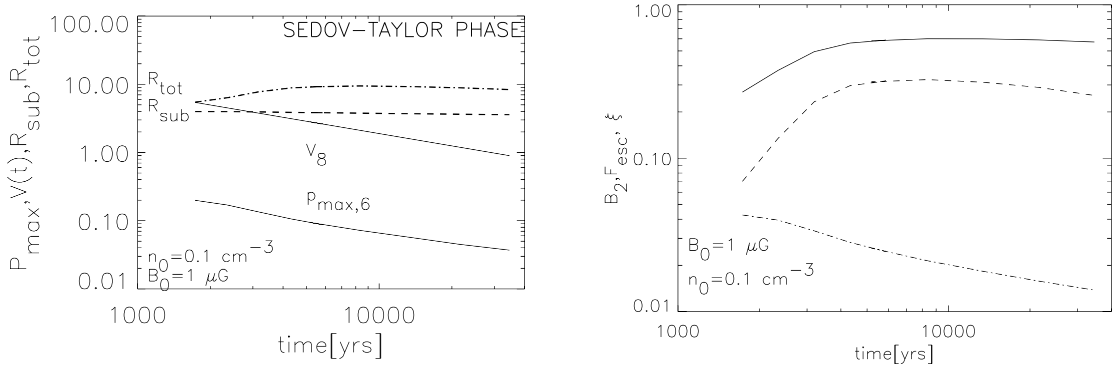

Figure 2. Left-hand panel: time-dependence of the maximum momentum of accelerated particles (solid curve labelled as $P_{\rm max,6}$) in units of $10^6 m_{\rm p}c$, of the shock velocity in units of $10^8\,{\rm cm\,s^{-1}}$ (solid curve labelled as V_8), of the compression factor at the subshock $R_{\rm sub}$ (dashed curve) and of the total compression factor $R_{\rm tot}$ (dot–dashed curve), during the Sedov–Taylor phase. Right-hand panel: time-dependence of the magnetic field strength downstream of the subshock in units of $10^3\ \mu G$ (dot–dashed curve), of the escape flux normalized to $\rho_0 V_{\rm sh}^3/2$ (dashed curve) and of the cosmic ray pressure at the shock location normalized to $\rho_0 V_{\rm sh}^2$. The magnetic field strength, B_0, and the number density of the background plasma, n_0, at upstream infinity are taken to be $B_0 = 1\ \mu G$ and $n_0 = 0.1\,{\rm cm^{-3}}$.

and non-resonantly. Let us introduce the acceleration efficiency as a function of time: $\xi_{\rm c}(t) = P_{\rm c}(t)/[\rho_0 V_{\rm sh}(t)^2]$. In terms of $\xi_{\rm c}$, the strength of the resonantly amplified magnetic field at the saturation level can be estimated as: $\delta B^2 = 8\pi\rho_0 V^2 \xi_{\rm c}/M_{\rm A}$ ($M_{\rm A}$ is the Alfvén Mach number), which leads to

$$\delta B(t) = 65\, n_0^{1/4}\, B_{0,\mu G}^{1/2} \left(\frac{E_{51}}{n_0}\right)^{1/10} t_{\rm kyr}^{-3/10}\, \xi_{\rm c}(t)^{1/2}\ \mu G. \qquad (32)$$

In a similar way, the strength of the field in the case of non-resonant amplification can be estimated from $\delta B^2 = 2\pi\rho_0[V_{\rm sh}(t)^3/c]\ \xi_{\rm c}(t)$ and leads to

$$\delta B(t) = 198\, n_0^{1/2} \left(\frac{E_{51}}{n_0}\right)^{3/10} t_{\rm kyr}^{-9/10}\, \xi_{\rm c}(t)^{1/2}\ \mu G. \qquad (33)$$

In general the two channels of magnetic field amplification work together but the non-resonant channel dominates at earlier times and leads to stronger magnetic field amplification.

The maximum momentum in the two cases is as follows:

$$E_{\rm max}(t) = 2.5\times 10^6 \left(\frac{E_{51}}{n_0}\right)^{1/2} n_0^{1/4} B_{0,\mu G}^{1/2}\, \xi_{\rm c}(t)^{1/2}\, t_{\rm kyr}^{-1/2}\ {\rm GeV}, \quad (34)$$

in the resonant case, and

$$E_{\rm max}(t) = 7.3\times 10^6 \left(\frac{E_{51}}{n_0}\right)^{7/10} n_0^{1/2}\, \xi_{\rm c}(t)^{1/2}\, t_{\rm kyr}^{-11/10}\ {\rm GeV} \qquad (35)$$

in the non-resonant regime.

In the naive assumption that the acceleration efficiency is constant in time, we see that $E_{\rm max}(t)$ scales with time as $t^{-11/10}$ at earlier times and as $t^{-1/2}$ at later times, when resonant scattering dominates. In actuality the scalings will be more complex because of the non-linear effects (especially the formation of a precursor upstream) induced by accelerated particles, which also lead to a time-dependence of $\xi_{\rm c}(t)$.

As discussed in the previous sections, it is not clear how to describe the non-resonant waves in the context of the conservation equations. A calculation of the dynamical effect of these modes on the shock is therefore not reliable at the present time. For this reason, here we confine ourselves to the investigation of the effects of resonant waves, for which there is no ambiguity. It is however worth keeping in mind that the introduction of the non-resonant waves is likely to result in significantly higher maximum energies at the early stages of the Sedov–Taylor phase.

Our complete calculations, including the non-linear dynamical reaction of the accelerated particles, the resonant amplification of magnetic field and the dynamical reaction of the field itself have been carried out as discussed by Caprioli et al. (2009). The results are illustrated in Figs 2–4. In the left-hand panels, we plot the maximum momentum ($p_{\rm max}$), the shock velocity (V) and the two compression factors ($R_{\rm sub}$ and $R_{\rm tot}$) as functions of time. The right-hand panels show the acceleration efficiency and the escape flux normalized to $\rho_0 V_{\rm sh}^2$ and $(1/2)\rho_0 V_{\rm sh}^3$, respectively, and the strength of the downstream magnetic field in units of $10^3\,\mu G$. The maximum momentum and the shock speed are in units of $10^6 m_{\rm p}c$ and $10^8\,{\rm cm\,s^{-1}}$, respectively. The three figures refer to the following sets of parameters: $n_0 = 0.1{\rm cm^{-3}}$, $B_0 = 1\ \mu G$ (Fig. 2), $n_0 = 0.1\,{\rm cm^{-3}}, B_0 = 5\mu G$ (Fig. 3) and $n_0 = 0.03\,{\rm cm^{-3}}, B_0 = 1\mu G$ (Fig. 4).

The maximum momentum is determined at each time by requiring that the diffusion length in the upstream section equals $0.1\ R_{\rm sh}(t)$. The quantities $p_{\rm max}(t), R_{\rm sub}(t)$ and $R_{\rm tot}(t)$ are all outputs of the non-linear calculations at the time t. The first point in time in all figures corresponds to the beginning of the Sedov phase. The time at which the Sedov–Taylor expansion begins was determined assuming $E_{51} = 1$ and that the mass of the ejecta is $M_{\rm ej} = 5\,{\rm M_\odot}$. The other relevant parameters are the temperature of the ISM in which the SNR is expanding, for which we assumed $T_0 = 10^4\,{\rm K}$, and the momentum threshold for particles to be injected into the accelerator, which was chosen to be $p_{\rm inj} = \psi_{\rm inj}\sqrt{2k_B T_2 m_{\rm p}}$, with $\psi_{\rm inj} = 3.8$ and T_2 the temperature downstream of the shock (see Caprioli et al. 2009 for details).

Some general comments are in order: one may note that the total compression factors obtained in our calculations are always lower than $\sim$10. This is uniquely due to the dynamical reaction of the amplified magnetic field. As shown by Caprioli et al. (2008), the effect of the amplified field on the plasma compressibility is relevant whenever the magnetic pressure becomes comparable with the thermal pressure of the background plasma upstream. The consequent decrease in the compression ratios allows us to be consistent with the values that have been inferred from observations of a few SNRs (Warren et al. 2005).

The highest momentum of accelerated particles, as expected, is reached at the beginning of the Sedov–Taylor phase and is of order $\sim 10^6$ GeV (about the knee). This should be considered as a lower

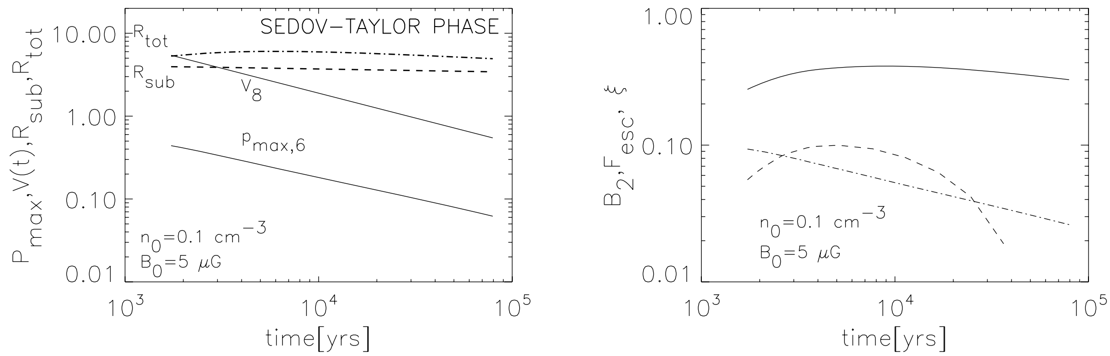

Figure 3. Same as Fig. 2 but for $B_0 = 5\ \mu G$ and $n_0 = 0.1\,\mathrm{cm}^{-3}$.

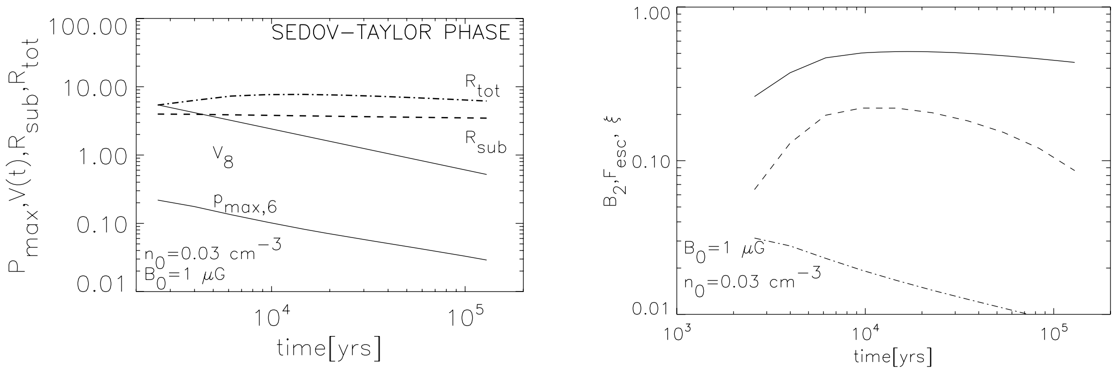

Figure 4. Same as Figs. 2 and 3 but for $B_0 = 1\ \mu G$ and $n_0 = 0.03\,\mathrm{cm}^{-3}$.

limit to the maximum energy reached at that time, since we have decided not to include the non-resonant channel of magnetic field amplification, which is very efficient when the shock velocity is large. During the following expansion, the time-dependence of $p_{\max}$ is reasonably well approximated by $p_{\max} \propto t^{-1/2}$, in agreement with equation (34), since $\xi_c(t)$ is roughly constant (see solid curve on the right-hand panels of Figs 2–4 and discussion below).

A crucial ingredient in calculating the maximum energy at a given time is the strength of the magnetic field. The magnetic field intensity in the downstream plasma is plotted in the right-hand panels (dot–dashed line) for the three cases considered here. The typical values are between $\sim$ a few - $10\mu G$ at late times and $\sim$30–100 μG at the beginning of the Sedov phase. After the first few thousand years, the dependence on time is not far from $\delta B_2 \propto t^{-3/10}$, as would result from equation (32), using the additional information on the approximate constancy of $\xi_c(t)$ and $R_{\rm sub}$ (dashed line in the left-hand panels of Figs 2–4). Despite these resemblances, the situation to which the plots refer is more complicated than that described by equation (32), where a number of effects have been neglected, first among these the presence of a precursor which evolves with time ($R_{\rm tot}$ is changing as can be seen from the dot–dashed curve on the left-hand panel of Figs 2–4) and the time varying adiabatic compression it entails.

The right-hand panels also show the acceleration efficiency (solid line) and normalized escape flux (dashed lines). One should note that even when the acceleration efficiency is very high, of order $\sim$50–60 per cent, the escape energy flux never exceeds $\sim$30 per cent. As discussed above, this latter quantity should be the one that is more directly related to the cosmic ray energetics in the Galaxy, at least at the highest energies, while the former is more relevant for the generation of secondary radiation due to cosmic ray interactions in the acceleration region.

The acceleration efficiency and the normalized escape flux initially increase with time during the Sedov–Taylor expansion phase. This behaviour is related to an analogous trend of the shock modification, as can be clearly seen from the time-dependence of $R_{\rm tot}$ (dash–dotted curve in the left-hand panel of Figs 2–4). In fact, at the beginning of the Sedov phase, the amplified magnetic field is at a maximum and its dynamical reaction on the shock is so strong that the acceleration efficiency is reduced. As soon as the magnetic field strength starts decreasing, the shock modification increases, and ξ_c and $F_{\rm esc}$ with it. Note, however, that this does not mean that the actual cosmic ray pressure and escape flux increase, because ξ_c and $F_{\rm esc}$ are normalized to $\rho_0 V_{\rm sh}^2(t)$ and $\rho_0 V_{\rm sh}^3(t)/2$, respectively, and both decrease with time rather quickly.

At later times, both ξ_c and $F_{\rm esc}$ start decreasing, with the latter showing a more rapid decline than the former. This is due to the fact that the shock is slowing down and progressively becoming unmodified: the maximum momentum is decreasing and the spectrum of accelerated particles is steepening. Recalling again that the plots show normalized quantities, one gathers that the decline with time of cosmic ray pressure and escaping energy flux is quite dramatic in this phase.

6 CONCLUSIONS

The escape of accelerated particles from the shock region towards upstream appears in basically all approaches to non-linear particle acceleration at shock fronts. In kinetic approaches that solve the relevant equations assuming quasi-stationarity at any given time, the escape flux appears either as a corrective term in the equation of energy conservation between downstream and upstream infinity or as a result of imposing as a boundary condition that the distribution function of accelerated particles vanishes at a finite distance x_0 upstream of the shock. In this second case, the escape flux is described by the non-vanishing spatial derivative of the distribution function at x_0. Such a quasi-stationary description of the acceleration process is meaningful only whenever the shock velocity decreases in time and the magnetization at the shock, as due to streaming of cosmic rays, also decreases in time. These two conditions lead to escape of the particles with momentum close to the highest achievable at any given time. This would be the case during the Sedov–Taylor phase of the expansion of a supernova shock. These approaches are therefore unfit to describe the phase of free expansion, where the maximum momentum increases with time and no escape towards upstream takes place.

Fully time-dependent, numerical calculations apply to arbitrary situations and should be able to describe the increase of momentum during the free expansion as well as the escape of particles during the Sedov phase. However, in most cases, these calculations do not include the time-dependence of the magnetic field at the shock. This leads to an increase of the maximum momentum even during the Sedov phase. Probably for this reason, there is no detailed discussion in the literature of the escape flux from a SNR in the context of time dependent calculations. The appearance of the escape flux in numerical time dependent approaches would depend on the boundary conditions adopted in the specific case. If the distribution function is assumed to vanish at upstream infinity, the escaping particles should appear as a concentration of particles with large momenta [close to $p_{\max}(t)$] at large distances from the shock: since the acceleration box extends to infinity, there is no real escape. On the other hand, if the boundary condition $f(x_0) = 0$ is imposed at a finite distance from the shock, the escape flux (as a function of time) should appear automatically as a consequence of a non-vanishing $D\partial f(x_0)/\partial x$. It would be interesting to have confirmation of these expectations by running time dependent calculations with decreasing magnetization.

The existence of the escape flux is not simply a mathematical nuisance: as discussed by Ptuskin & Zirakashvili (2005), it is the very reason why SNRs can potentially be the sources of cosmic rays up to the knee. If the particles we observe as cosmic rays were the ones advected towards downstream of the shock, the adiabatic losses suffered during the expansion of the shell would drive these particles towards lower energies. It is therefore of the highest importance to go beyond the test particle calculation of Ptuskin & Zirakashvili (2005) and achieve an understanding of the escape flux in the context of the fully non-linear theory of diffusive shock acceleration.

In this paper we made a first step in this direction, by calculating the temporal evolution of the escape flux during the Sedov–Taylor phase of the expansion of a supernova in a medium with constant density. Our calculation included a recipe to describe the cosmic ray induced magnetic field, the dynamical reaction of the accelerated particles and the field itself. In addition to this, we also discussed in some detail the limitations of the conservation equations as they are usually written, in that they do not allow to describe the magnetic field perturbations when they are not Alfvén waves.

We showed that the escape flux may involve between few and 10–30 per cent of the shock ram pressure, while the particle acceleration efficiency at the same time reaches 50–60 per cent. This means that the energetics inferred from observations of secondary radiation from a remnant, for instance in the form of gamma rays, may not be the relevant one for the origin of cosmic rays, since it is not related in a trivial way to the energy flux escaping the accelerator.

The maximum energy up to which particles may get accelerated is reached at the beginning of the Sedov phase and is of order 10^{15} eV if only resonant amplification of the field is included. $E_{\max}$ might be somewhat larger for some SNRs that at the very beginning of the Sedov–Taylor phase may experience the effect of non-resonant streaming instability (Bell 2004). This mechanism provides extremely efficient field amplification as long as the shock velocity is high, and hence is expected to play a very important role in the early Sedov–Taylor phase (and possibly during the free expansion phase). At present, non-resonant modes cannot be formally accounted for in the conservation equations, since a detailed description of the energy transfer between particles and waves is not available yet. We carried out all calculations in the simpler case of Alfvén waves interacting with particles in a resonant way.

The amplification of the magnetic field, by either resonant or non-resonant streaming instability, has profound implications on the escape flux of particles towards upstream of a shock, and therefore on the spectrum of cosmic rays we observe at the Earth. The most obvious consequence of the magnetic field amplification is that of allowing for higher values of the maximum energy of accelerated particles, as shown by our equations (34) and (35). However large magnetic fields exert a dynamical reaction on the plasma leading to a reduction of the compression in the precursor. This happens whenever the magnetic pressure exceeds the pressure of the background gas (Caprioli et al. 2008). As a result, the concavity of the spectrum of accelerated particles (Caprioli et al. 2009) is reduced and at the same time the escape flux at $p \sim p_{\max}$ decreases. It follows that larger field implies larger $p_{\max}$ but not necessarily larger escape flux, as shown in Figs 2–4 (see the behaviour of the curves at early times).

The escape of accelerated particles from a cosmic ray modified shock has profound implications for the origin of cosmic rays, which will be discussed in detail in a forthcoming paper. Here, we want to emphasize some general points:

(1) the escape from upstream is the natural solution to the well-known problem of explaining how the highest energy particles (say, close to the knee energy) could escape the system without suffering substantial adiabatic energy losses;

(2) the magnetic field amplification is expected to switch from mainly non-resonant to mainly resonant at the beginning of the Sedov–Taylor phase. It can be easily understood that this may lead to peculiar changes in the spectrum of cosmic rays detected at the Earth, reflecting this transition;

(3) the flux of escaping particles, once integrated in time during the SNR evolution may be very different from the concave instantaneous spectrum which can potentially be observed in a SNR, for instance by looking at its multifrequency emission. This point is certainly relevant for the purpose of addressing the commonly raised point of how a concave spectrum of accelerated particles can reflect in an almost perfect power law over many orders of magnitude;

(4) there is a further complication of all the picture, due to the acceleration of nuclei at energies that may be expected to scale as the charge of the nucleus (in the case of Bohm diffusion). Any calculation of the flux of single chemical species observed at the Earth must take these complex effects into account.

ACKNOWLEDGMENTS

We are grateful to G. Cassam-Chenai and S. Gabici for reading an intermediate version of the present manuscript. We are also grateful to the anonymous referee for his/her precious comments. This work was partially supported by PRIN-2006, by ASI through contract ASI-INAF I/088/06/0 and (for PB) by the US DOE and by NASA grant NAG5-10842. Fermilab is operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States DOE.

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