Thus $\sin\alpha$, and therefore the current, will have a sign depending on n mod 2. An odd number of rotations will reverse the current relative to the current in the unrotated situation.

In order to have a large current, the many-electron state must be symmetric in the coordinate α, which means symmetric in the direction of current flow and therefore antisymmetric in the other coordinates.

In a future paper we shall discuss the relevance of these considerations to the fermion superselection rule.[4–6]

[4] G. C. Wick, A. S. Wightman, and E. P. Wigner, Phys. Rev. **88**, 101 (1952).
[5] R. F. Streater and A. S. Wightman, *TCP, Spin and Statistics and All That* (W. A. Benjamin, Inc., New York, 1964).
[6] Y. Aharonov and L. Susskind (to be published).

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Steady State of Cosmic-Ray Nuclei—Their Spectral Shape and Path Length at Low Energies

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The steady state of cosmic-ray nuclei in interstellar space is discussed. It is shown that for a steady-state situation (or for any mode of propagation in which the allowed path lengths between the source and observer have a wide distribution), the generally used matter-slab approximation for the interstellar matter traversed by cosmic rays leads to erroneous conclusions. The steady-state energy spectra of heavy nuclei are found to have negative slopes down to energies $\sim$50 MeV/N, if the injection spectra are like a rigidity power law; this offers an explanation for the apparently surprising observation of flat spectra for heavy nuclei down to energies $\sim$50 MeV/N. Further it is found that the L/M ratio cannot keep on increasing at low energies but must decrease continuously below a few hundred MeV/N, even for energy-independent fragmentation cross sections; this also is in accord with recent experimental results.

INTRODUCTION

TRAVERSAL through interstellar space modifies both the spectral shape and the chemical composition of cosmic rays. It is well recognized that an understanding of these modification processes combined with the experimentally observed properties of cosmic rays near the solar system can provide useful information about their propagation history. One of the gross parameters characterizing this propagation is the amount of matter traversed by cosmic rays at the time of their arrival. This is usually determined by measuring the relative abundances of nuclei such as Li, Be, B (L nuclei), which have a cosmic-ray abundance much larger than their universal abundance and hence should be products of the fragmentation suffered by heavier nuclei, mainly C, N, O (M nuclei). The relative abundance L/M should be a measure of the number of collisions suffered by M nuclei and hence of the amount of matter traversed by cosmic rays. The basic advantage of such a procedure is that the L/M ratio is affected very little by solar modulation.

The analysis procedure adopted by most of the authors (see for example Appa Rao and Kaplon[1] and Balasubrahmanyan *et al.*[2]) is briefly as follows: It is assumed that the amount of matter traversed by particles of a given rigidity is unique with zero or a small spread;[2] in other words, a slab of matter (hydrogen) of definite thickness is assumed to exist between the source of cosmic rays and the observer. Then, starting with a reasonable source spectrum, a diffusion equation is set up in which the various fragmentation processes and energy loss due to ionization are properly taken into account. In this way the changes in the composition and the spectral shape are calculated and a comparison with the experimental values of L/M (or He^3/He^4) ratio[3] gives the appropriate value of the thickness of the matter slab, which is then called the amount of matter traversed by cosmic rays.

Here we would like to emphasize that such an approximation is not valid and leads to an erroneous interpretation of the experimental data at low energies. Lack of any anisotropies in the observed flux of cosmic rays has led to the conclusion that after production their directions of motion are randomized by the irregular magnetic fields existing in the galaxy. Thus for particles of any given rigidity there must exist a host of allowed trajectories, of widely varying lengths, connecting the point of origin and the point of observation; hence, the distribution in the amount of matter (in

[1] M. V. K. Appa Rao and M. F. Kaplon, Nuovo Cimento **27**, 700 (1963).
[2] V. K. Balasubrahmanyan, E. Boldt, and R. A. R. Palmeira, Phys. Rev. **140**, B1157 (1965).

[3] Unlike the L/M ratio, the He^3/He^4 ratio is considerably affected by solar modulation, owing to the large difference in the A/Z ratios of He^3 and He^4.

g/cm^2) traversed would be quite broad and the slab approximation cannot, in general, be valid.[4]

All the allowed trajectories are realized only if the ionization and interaction losses are unimportant; therefore we call these trajectories "vacuum trajecjectories." The distribution of vacuum trajectories probably depends only on the particle rigidity, the average path length increasing with decreasing rigidity. In practice the very long vacuum trajectories are suppressed because of ionization and interaction losses. This effect becomes progressively more important with decreasing energy as the ionization loss increases, and the mean of the distribution shifts progressively to smaller values. This effect is illustrated schematically in Fig. 1. Thus with decreasing energy the experiment will sample predominantly those nuclei which have come after traversing smaller and smaller amounts of matter. For a steady-state situation the effective amount of matter traversed, X, is given by

$$\frac{1}{X}=\frac{1}{\Lambda_l}+\frac{1}{\Lambda_{\text{int}}}+\frac{1}{\Lambda_{\text{ion}}} \equiv\frac{1}{\Lambda}+\frac{1}{\Lambda_{\text{ion}}}, \tag{1}$$

where Λ_l and Λ_{int} are the attenuation lengths against leakage and interaction and Λ_{ion} is an equivalent attenuation length for the energy loss by ionization. For different nuclei Λ_l may be same at a given rigidity, but Λ_{int} and Λ_{ion} will depend on charge and mass of the nucleus; hence the effective slab thickness corresponding to different experimentally measured parameters will be different. For example the effective slab thickness derived from the L/M ratio, at a specific energy per nucleon, cannot be used to derive the spectral shapes of nuclei and the He^3/He^4 ratio. At relativistic energies Λ_{ion} and Λ_{int} are much longer than Λ_l, and X will be same for all nuclei. However, below the energy where the various attenuation lengths become comparable, X will depend not only on energy but also on the charge and mass of the nucleus concerned, and hence the slab concept will no longer be useful.[5]

In this paper we investigate the steady state of cosmic rays in interstellar space in a class of models

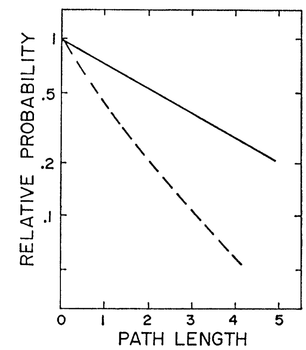

FIG. 1. The effect of ionization and interaction losses during propagation on the path-length distribution is illustrated *schematically*. The solid curve gives the distribution of vacuum trajectories, while the dashed curve gives the modified distribution in the presence of matter. The abscissa is in arbitrary units.

which have the following features:

(a) The sources of particles are constant in time.

(b) The length distribution of vacuum trajectories linking the sources to the point of observation has an exponential form. (This will be realized, for example, if the rate of leakage is independent of time, and the space distribution of sources is uniform, or in any case, if the diffusion time of particles is much smaller than the leakage life.)

(c) There is no acceleration during propagation.

Taking into account the ionization energy loss and interactions during propagation, we calculate the low-energy spectra of medium and heavy nuclei and the ratio of light to medium nuclei as a function of energy. It is shown that very general considerations of propagation discussed above lead to a steady-state spectrum of M and H (heavy) nuclei outside the solar system which, for an assumed rigidity-power-law injection spectrum, has a negative slope down to energies as low as 50 MeV/N. As can already be inferred from the qualitative discussion given above, at low energies, below about 300 MeV/N, the L/M ratio decreases with decreasing energy; further if the leakage lifetime of cosmic rays is taken to vary inversely as rigidity at low energies, then the calculation reproduces the observed peak in the L/M ratio at a few hundred MeV/N, even if the fragmentation parameters are taken to be energy-independent. [It has to be emphasized that although the results presented here are for a steady state of cosmic rays, the general features of the results, such as the fall in L/M ratio at very low energies and the negative slopes of spectra down to very low energies, would follow for any mode of propagation in which a broad range of path lengths (in g/cm^2) is allowed.]

FORMULATION FOR THE PROPAGATION OF COSMIC-RAY NUCLEI IN A STEADY STATE

In a steady state the flux of particles at any energy is obtained by summing over the contributions from the production of particles of various appropriate energies at all times in the past. Production here includes direct injection and production through fragmentation of heavier nuclei. For a uniform matter density the time

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[4] The only physical situation where the slab assumption is applicable is when there exists a one-to-one correspondence between the time of observation and the time of production, and the density distribution is fairly uniform in the region of propagation.

[5] This approach has been discussed in some form by Kaplon and Skadron [M. F. Kaplon and G. Skadron, Department of Physics and Astronomy, the University of Rochester Report No. URPA-124, 1966 (unpublished)], by Kuzhevskii and Syrovatskii (Kuzhevskii and Syrovatskii, Zh. Eksperim. i Teor. Fiz. **49**, 1950 (1965) [English transl.: Soviet Phys.—JETP **22**, 1331 (1966)]), and by Davis [L. Davis, Jr., in *Proceedings of the Moscow Cosmic-Ray Conference, 1959* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. 3, p. 220]. However, to our knowledge, its consequences for the problems discussed in this paper have not been so far pointed out.

can be converted into the amount of matter travelled, and one can assume that the production spectrum $P(E)$ per g/cm² is independent of the time of production. Then the flux of nuclei of type j at any energy E can be written as

$$F_j(E)dE=dE\int_0^\infty P_j[E'(E,x)]\frac{dE'/dx}{dE/dx}\times\exp\left(-\int_0^x\frac{dy}{\Lambda[E'(E,y)]}\right)dx. \quad (2)$$

Here E' is the energy which the particle should have had in order to arrive with an energy E after having traversed x g/cm² of matter, $P_j(E)$ is the production spectrum per g/cm², $(dE'/dx)/(dE/dx)$ is the Jacobian which takes care of the expansion in the energy interval due to the shift of energy produced by ionization loss, and the exponential term gives the probability for survival of a particle of initial energy E' against leakage and interaction losses; the attenuation length $\Lambda(E')$ is given by

$$\frac{1}{\Lambda(E')}=\frac{1}{\Lambda_l(E')}+\frac{1}{\Lambda_{\rm int}(E')}. \quad (3)$$

Ionization energy loss favors smaller values of the amount of matter traversed through the factor $P_j[E'(E,x)](dE'/dx)/(dE/dx)$, which decreases with increasing x. At high energies, where ionization loss is not important, $E'(E,x)\sim E$, and Eq. (2) reduces to

$$F_j(E)dE=\Lambda(E)P_j(E)dE, \quad (4)$$

which is just the production per g/cm² multiplied by the attenuation length. It may be noticed that the path-length distribution for vacuum trajectories is an exponential with a mean value Λ_l.

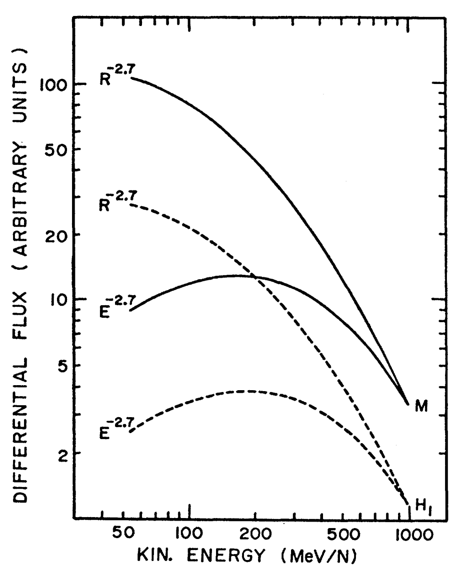

FIG. 2. Shape of the steady-state differential energy spectra, before solar modulation, of M ($Z=7$) and H_1 ($Z=12$) nuclei is shown for the two assumed injection spectra, namely, a power law in total energy or rigidity with differential exponent -2.7.

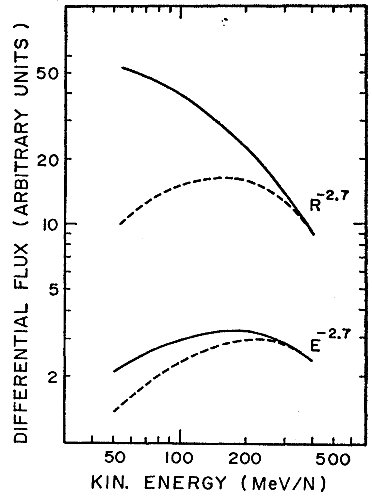

FIG. 3. Steady-state spectra of M nuclei (solid curves) are compared with those predicted by assuming a matter slab of 5 g/cm² thickness between the source and the observer (dashed curves) for the two assumed shapes of the injection spectrum. It is seen that unlike the predictions of slab approximation, the steady-state spectrum depends quite strongly on the shape of the injection spectrum down to energies ~50 MeV/N. Further, for a rigidity-power-law injection spectrum the steady-state spectrum has a negative slope down to ~50 MeV/N, thereby offering an explanation for the observed flat spectra, which imply a negative slope before solar modulation (see text).

Before any numerical results are obtained, the energy dependence of the leakage length Λ_l still remains to be defined. Experimental observations on L/M ratio as a function of energy clearly indicate an increase, with decreasing energy, in the effective amount of matter travelled by heavy nuclei of cosmic rays, down to an energy of ~400 MeV/N (see Biswas *et al.*[6]) This is consistent with the expectation that the interstellar magnetic fields will provide a better containment for low-rigidity particles. The exact dependence of leakage length on rigidity cannot be guessed; however, for the sake of definiteness we have assumed that in the lower energy range $\Lambda_l\propto 1/R$, and we normalize it to a value of 3 g/cm² at 3 GeV/N kinetic energy (~8 GV rigidity for M nuclei); this corresponds to a $1/R\beta$ dependence of leakage lifetime. Any weaker dependence of Λ_l will lead to flatter steady-state spectra at energies $\gtrsim$400 MeV/N, where leakage is important; on the other hand, spectra at lower energies are affected little by such a change because leakage is not important here and energy loss by ionization is the dominant effect. For the rate of ionization energy loss and the range-energy relation in hydrogen, the tables given by Barkas and Berger[7] have

[6] S. Biswas, S. Ramadurai, and N. Sreenivasan, Phys. Rev. **149**, 1037 (1966).

[7] W. H. Barkas and M. J. Berger, Nat'l. Aeron. Space Admin. Report No. NASA SP-3013, 1964.

been used. The numerical computation was done with the help of a CDC 3600 computer.

RESULTS

I. Low-Energy Spectra of Medium and Heavy Nuclei

Low-energy steady-state spectra of M $(Z=7)$ and H_1 $(Z=12)$ nuclei outside the solar system have been calculated assuming absorption lengths of 10 g/cm^2 (M nuclei) and 7 g/cm^2 (H_1 nuclei). Two alternative shapes for the injection spectrum were used, namely a power law in total energy or a power law in rigidity. The contribution from fragmentation of heavier nuclei to M and H_1 nuclei has not been considered. These spectra are shown in Fig. 2. *Notice that the shape of the expected spectrum at very low energies is quite sensitive to the injection spectrum.* Experimentally it is observed that the spectra of medium and heavy nuclei are flat going down from $\sim$500 to $\sim$50 MeV/N; if the effect of solar modulation is removed one expects that, outside the solar system, these spectra will have a negative slope down to the lowest energies.[8] Hence we can conclude, referring to Fig. 2, that the injection spectrum (per g/cm^2) in the low-energy range must be about as steep as a rigidity power law. A more definite statement is not possible because of uncertainty in the character and degree of solar modulation.

It is useful to compare the predictions of the steady-state model discussed here with those of the matter-slab approximation. For the above mentioned two shapes of the injection spectra, the expected spectra of M nuclei below $\sim$300 MeV/N for a slab thickness of 5 g/cm^2 are shown in Fig. 3 along with the steady-state spectra.[9] It is seen that the shape of the calculated spectrum for the slab approximation is very insensitive to the nature of the injection spectrum. In order to obtain a negative slope down to the lowest energies, the injection spectrum above $\sim$300 MeV/N (which is the energy at the source corresponding to $\sim$50 MeV/N at observation, for a slab of 5 g/cm^2) will have to be extremely steep. On the other hand, for the steady-state picture the observed spectrum is very sensitive to the input spectrum, which can be easily adjusted to fit the experimental data once the solar modulation parameters are better understood and defined.

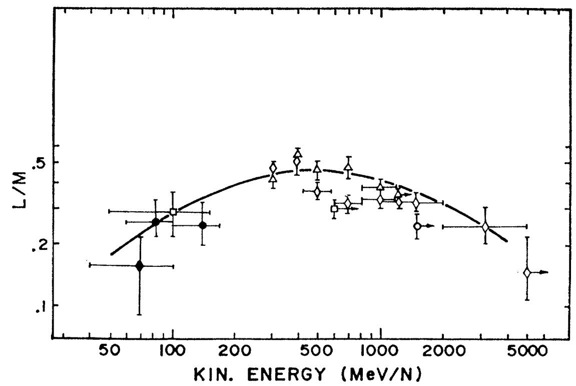

FIG. 5. The calculated L/M ratio as a function of energy is shown for a rigidity-power-law injection spectrum of M nuclei (see text). The calculated curve is normalized to experiment at $\sim$3 GeV/N. The rise in L/M ratio is due to better containment at lower energies, and the fall beyond the peak is due to the increasing importance of attenuation due to ionization energy loss. (The experimental points with arrows correspond to integral flux measurements; the effective average energy would be higher than the cutoff energy by a factor $\sim$2). Experimental points are from (i) V. K. Balasubrahmanyam, D. E. Hagge, G. H. Ludwig, and F. B. McDonald, in *Proceedings of the International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and The Physical Society, London, 1966), Vol. 1, p. 427—open square; (ii) W. R. Webber, *ibid.*, Vol. 1, p. 403—open triangle (1963 data), open diamond (1964 data); (iii) D. V. Reames and C. E. Fichtel, Phys. Rev. **149**, 991 (1966)—closed diamond; (iv) Comstock *et al.*, Ref. 8—closed circle; and (v) F. W. O'Dell, M. M. Shapiro and B. Stiller, J. Phys. Soc. Japan **17**, Suppl. AIII 23 (1962)—open circle.

Thus we find that some serious difficulties encountered while using the slab approximation for the traversal through interstellar matter disappear when one considers the proper steady state of cosmic-ray nuclei in the galaxy.

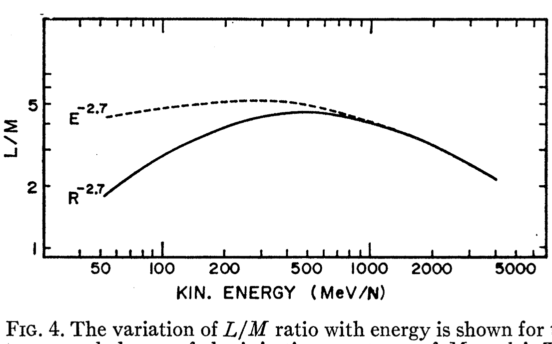

FIG. 4. The variation of L/M ratio with energy is shown for the two assumed shapes of the injection spectrum of M nuclei. The ordinate has an arbitrary normalization. Only the contribution of M nuclei to L nuclei has been considered (see text).

II. Energy Dependence of the L/M Ratio

The ratio L/M as a function of energy has been calculated for rigidity and for total-energy power-law injection spectra. Since the interest here is only in the energy dependence of this ratio, only the contribution to L nuclei arising from collisions of M nuclei has been taken into account. The cross section for the reaction $M+p\rightarrow L$ has been assumed to be energy-independent,[10]

[8] S. M. Comstock, C. Y. Fan and J. A. Simpson, Astrophys. J. **146**, 51 (1966).

[9] The slab thickness of 5 g/cm^2 has been selected in view of the fact that from an analysis of the experimental results Biswas *et al.* (Ref. 6) find that for the two energy ranges of 50–150 and 200–500 MeV/N the amount of matter traversed is $\sim$5 g/cm^2 and $\sim$9 g/cm^2, respectively.

[10] Direct measurements on the cross section for the reaction $p+C\rightarrow Li$ have become available recently [R. Bernas, M. Epherre, E. Gradsztajn, R. Klapisch and F. Yiou, Phys. Letters **15**, 147 (1965)]. This cross section stays more or less constant for proton energies ranging from $\sim$50 to $\sim$500 MeV. However, a reasonable variation of fragmentation cross sections with energy would not affect the *basic* conclusions of this paper.

and the absorption length for L nuclei has been taken as 15 g/cm^2. The calculated energy dependence of L/M ratio is shown in Fig. 4. One finds (Fig. 4) that the L/M ratio above $\sim$400 MeV/N is the same for the two assumed injection spectra for M nuclei. However, as shown above, the injection spectrum should be nearer to a rigidity power law than to a total-energy power law at energies $\lesssim$400 MeV/N. Therefore we present, in Fig. 5, the energy dependence of the L/M ratio for a rigidity-power-law injection, along with the more accurate of the measured values at each energy. The curve is normalized to the experimental points at $\sim$3 GeV/N. It is seen that the calculated curve gives a remarkably good representation of the experimental data. We do not need to invoke different source regions for different energy ranges as suggested by Biswas *et al.*,[6] nor do we have to invoke an energy dependence, similar to that of the L/M ratio, for the fragmentation parameters, as done by Fichtel and Reames.[11] The observed variation of the effective amount of matter with energy is a natural consequence of the energy dependence of various losses in a steady-state situation; with decreasing energy the L/M ratio increases, down to $\sim$400 MeV/N, because of the increasing leakage life, whereas the fall below this energy is due to the increasing importance of ionization energy loss, so that particles traversing small amounts of matter reach the observer preferentially.

Two other values of the normalization for the leakage length, namely 2 and 4 g/cm^2 at 3 GeV/N, were also used for these calculations. It was found that this does not materially affect the spectral shapes and the L/M ratio at energies below $\sim$300 MeV/N, where ionization loss is the most important loss and leakage loss is unimporant. At high energies, where ionization and interaction losses are unimportant, this normalization is reflected linearly in the L/M ratio.

DISCUSSION

Recently Comstock *et al.*[8] have reported their extensive and beautiful set of measurements on the nuclear abundances and low-energy spectra of galactic cosmic rays. They obtain the apparently surprising result that the demodulated spectra of medium and heavy nuclei must have negative slopes down to the lowest energy ($\sim$50 MeV/N) and that these slopes must be more negative than for the He spectrum. As we have discussed in this paper, a proper consideration of cosmic-ray propagation does lead to negative spectral slopes even for heavy nuclei. However, our discussion up to now does not give slopes which increase with charge number. In other words, the relative intensities of various primary nuclei at low energies do not agree with experiment.[8] One way of achieving agreement has been suggested by Ramadurai,[12] who has investigated the effect of ionization loss on nuclei undergoing Fermi acceleration after being injected at very low energy. He has shown that particles tend to accumulate in the neighborhood of energies at which the rate of ionization loss is nearly equal to the rate of energy gain; thus for a given acceleration rate the resulting equilibrium spectra become steeper with increasing charge. If the rate of acceleration is kept low, the resulting spectral shapes are consistent with those observed. However, this attempt to ascribe the observed spectra to Fermi acceleration leads to serious difficulties on several counts. For example, very heavy nuclei have to traverse a path length corresponding to several interaction mean paths before reaching energies of several hundred GeV/N; this will reduce their intensity to unobservable values.

So far we have not been able to find a reasonable explanation for the difference between the helium and heavy nuclei spectra at low energies, *if we start with similar injection spectra for all the components;* maybe the source spectra are intrinsically different in this energy region.

The second apparently surprising result found by Comstock *et al.*,[8] that the L/M ratio does not go on increasing with decreasing energy is, as shown above, exactly according to the expectation for a steady state of cosmic rays.

ACKNOWLEDGMENTS

We wish to thank Prof. R. R. Daniel and Dr. M. V. K. Appa Rao for their comments and Shri S. Ramadurai for a discussion of his results prior to publication.

Note added in proof. Recently we have noticed that the calculated energy dependence of H_1/M is in agreement with experiment.

[11] C. E. Fichtel and D. V. Reames, Phys. Rev. **149**, 995 (1966).

[12] S. Ramadurai, Proc. Indian Acad. Sci. **65**, 219 (1967).