Very Heavy Nuclei in the Primary Cosmic Radiation. I. Observations on the Energy Spectrum*

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The intensities and energy spectrum of the medium- and very heavy, *MH* and *VH*, nuclei, $16 \leq Z \leq 30$, present in the primary cosmic radiation have been studied using three stacks of nuclear emulsions exposed on high-altitude balloons flown over Fort Churchill (Canada), Texas, and India. Integral intensities of $Z \geq 20$ nuclei above energies of 0.225, 1.58, and 7.1 BeV per nucleon were, respectively, 1.50 ± 0.06, 0.403 ± 0.023, and 0.090 ± 0.006 (*VH* nuclei)/m^2 sr sec. The energy spectrum of these nuclei was measured in detail between 225 MeV per nucleon and 1 BeV per nucleon, and was found to exhibit a maximum in the 300- to 400-MeV per nucleon range, where the differential intensity was of the order of 1.7×10^{-3} nuclei/m^2 sr sec (MeV per nucleon). Integral intensities of $16 \leq Z \leq 19$ nuclei above energies of 1.58 and 7.1 BeV per nucleon were, respectively, 0.086 ± 0.010 and 0.020 ± 0.003 nuclei/m^2 sr sec; differential intensities were also measured at lower energies.

INTRODUCTION

In this and the following papers we present our observations and interpretation of the results obtained from a study of the heaviest nuclei, those with $16 \leq Z \leq 30$, customarily found in the primary cosmic radiation. In this, the first paper, henceforth referred to as Paper I, we will give our observations on the energy spectra and intensities of these nuclei. Paper II describes our interpretation of these results in terms of the nature of the cosmic-ray source and the effects of propagation through interstellar matter. In a future paper we will present our measurements on the detailed charge spectrum of these nuclei. A study of these nuclei is of particular interest because of the large ionization energy losses and short mean free paths against nucleon interaction that characterize their passage through matter. Preliminary results of this experimental program were reported previously.[1-3]

EXPERIMENTAL PROCEDURE

The data discussed here have all been obtained from an analysis of three large stacks of photographic nuclear emulsions flown on high-altitude balloons. Two of these stacks, those flown in Canada and India, were exposed in an emulsion camera having horizontal shutters made of glass-backed emulsions mounted above them. These shutters were swung away from the top surface of the stacks when they reached ceiling altitude. As a consequence, it was possible to make a distinction between nuclei which were recorded during the ascent phase of the balloon flight and those recorded at ceiling. The third stack, that flown in Texas, was only rotated through 90° on reaching ceiling, which reduced but did not eliminate the correction for ascent. The characteristics and exposure details of each stack are given in Table I(a).

In this table we have listed the mean value of the vertical cutoff rigidity, averaged over the trajectory of the flight. However, since in this experiment particles were detected which had zenith angles of as great as 60° we have also calculated the mean effective cutoff rigidity, taking into account the response of the detector at each zenith angle and assuming an energy spectrum similar to that obtained in our later analysis. This calculation was done most accurately for the India exposure since there we could use the results obtained by Daniel and Stephens[4] on the zenith and azimuthal dependence of the cutoff rigidity in this locality. The result of this calculation was to raise the effective cutoff from 16.7 to 17.3 BV, which was such a small percentage change that we have neglected it in comparison to the statistical uncertainties. Similar detailed geomagnetic calculations do not appear to be available for Texas, but it seems unlikely that the effect can be much more serious than in India. It should be emphasized that unlike most detectors, although we do observe particles with zenith angles out to 60°, the majority of the particles have much smaller zenith angles and hence the east-west effect is greatly reduced.

It may be noticed in Table I that these stacks were not exposed simultaneously, but that instead the exposures cover a period of nearly two years from mid 1964 to mid 1966. As a result any comparison of the results obtained from one flight with those from other flights should take into account the possible role of intensity variations introduced by solar modulation effects. We have estimated the magnitude of such variations by considering the sea-level neutron monitor counting rates recorded during the duration of each flight. Somewhat fortuitously, it turns out that the exposure made at a time that shows the greatest

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[1] P. S. Freier and C. J. Waddington, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965* (the Institute of Physics and the Physical Society, London, 1965), Vol. 1, p. 390.

[2] P. S. Freier and C. J. Waddington, Can. J. Phys. 46, S578 (1968).

[3] O. Mathiesen, C. E. Long, P. S. Freier, and C. J. Waddington, Can. J. Phys. 46, S583 (1968).

[4] R. R. Daniel and S. A. Stephens, Proc. Indian Acad. Sci. 63, 275 (1966).

TABLE I. Details of each stack and flight.

(a) Characteristics and exposure details of each stack

Location and date	Launch point	Impact point	Cutoff rigidity (BV)	Launch time GMT	Cut-down time GMT	Ceiling time (sec)	Residual matter (g/cm^2)
Churchill, 22 July 1964	94° 04′ W, 58° 45′ N	108° 00′ W, 58° 22′ N	Air cutOff	0722	2130	3.98×10^4	2.3 ± 0.3
Texas, 1 July 1966	96° W, 32° N	112° W, 33° N	4.91	1100	0100, 2 July	3.75×10^4	2.8 ± 0.2
India, 27 March 1965	78.5° E, 17.6° N	79° E, 21° N	16.7	2326, 26 March	1046	3.22×10^4	4.4 ± 0.3

(b) Neutron monitor counting rates for each flight

	Sulphur Mountain	Deep River	Mt. Washington	Mean % change
Churchill	208958 (−3.86%)	6207 (−3.38%)	2370 (−2.4%)	−3.24%
Texas	207155 (−4.69%)	6154 (−4.20%)	2354 (−3.13%)	−4.01%
India	217340 (0%)	6424 (0%)	2430 (0%)	0%

deviation in counting rate from the other two was that made over India, where, because of the high cutoff energy, the effects of solar modulation should be least pronounced. Table I(b) shows the percentage changes in the neutron monitor counting rates of the other two flights with respect to that made in India. These values have been obtained by averaging the counting rates reported by the neutron monitors at Sulphur Mountain, Deep River, and Mount Washington. It can be seen that the rates for the Canada and Texas exposures only differ by about 0.8%. Since the regression curve between the neutron monitor counting rates and the intensities of helium nuclei,[5] which presumably have a similar modulation to that experienced by the heavy nuclei due to their similar Z/A ratios, shows that at the cutoff energy typical of Texas the percentage change in the primary intensity is only about 1.2 times that in the neutron monitor, this 0.8% change will only correspond to about a 1% change in the intensity of the heavy nuclei. This change is sufficiently small compared to our statistical uncertainties that we may reasonably neglect it.

After conventional processing of the exposed emulsions they were scanned to detect the tracks produced by suitable heavy nuclei. These scans were line scans made at various depths below the top edge of the emulsions with various criteria. Table II summarizes these various scans and criteria employed during the course of this analysis. It should be particularly noticed that by scanning the Canadian stack for all nuclei of charge $Z\geq 18$, relativistic or not, we also efficiently detected considerable numbers of low-energy lower-charge nuclei. These nuclei can be used to obtain differential intensities for these nuclei over energy ranges that are a function of the charge being considered. However, we can not obtain integral intensities for these nuclei, whereas we can do so in Texas and India, which were scanned down to a lower charge limit.

MEASUREMENT OF ENERGY SPECTRUM

Because of the limited number of nuclei detected and the problems of individual charge resolution, we did not attempt to determine the energy spectra of the individual elements, but instead principally confined our attention to those nuclei heavier than potassium, $Z\geq 20$, although we also obtained data on the somewhat lighter nuclei. These nuclei are conventionally designated as the VH nuclei of the primary cosmic radiation, and we shall use the same designation. A convenient feature of this grouping is that the fraction of these nuclei that have charges near the lower boundary of the group is relatively small and hence the measurements were not particularly sensitive to misidentifications of charge between potassium and calcium nuclei. This is particularly important since although the measurements that will be described in a future paper

TABLE II. Scanning conditions.

Place	No. $Z\geq 20$	$\theta_{\max}$	Scan area (cm^2)	Location of scan line below T.E. (cm)	Dip angle l/plate (mm)	$A\Omega$ (cm^2 sr)	Min relativistic charge scanned for	Ascent correction	Scanning eff. for $Z\geq 15$
Churchill	883	35°	297.4	0.20	2.0	184.1	18	−2.1%	95.2%
India	277	60°	716.8	0.20	1.0	1207.8	15	−6.3%	98.1%
Texas	315	60°	148.2	1.02	1.0	252.9	15	−4%−16%	98.7%

[5] P. S. Freier and C. J. Waddington, Space Sci. Rev. 4, 313 (1965).

show that it is possible to resolve, or nearly resolve, individual charges in this charge region, the techniques involved are laborious and thus far have only been applied to nuclei having relativistic velocities. For the lower-energy nuclei observed in the Canadian stack our charge measurements were not capable of charge resolution, and the possibility of misidentification does exist.

The charges of all the nuclei were determined by δ-ray counting. In addition, all nuclei producing tracks of suitable geometry in the Texas and India exposures have been measured with a photometric densitometer and the charge spectra have been determined. The charges of the nuclei in the Churchill stack have yet to be measured by the densitometer, so that all of the charge determinations of these nuclei depend on δ-ray counting.

The charges of fast nuclei from the India and Texas exposures were determined from counting 4-grain δ rays. The count was made for 500 μ in each of two adjacent plates and at two or three places along the track, as it entered the stack, as it left or made an interaction, and in between if the other two counts were more than 3 cm apart. The charge scale was calibrated from the oxygen peak and was checked with the changes in charges observed in interactions. Any particle having a charge measured to be in the range $14 \leq Z \leq 22$ which could not be measured by the densitometer was independently remeasured by the δ-ray method. These particles made up 25% of all the $Z \geq 16$ nuclei observed in Texas and 34% of those observed in India. Figure 1 shows the charge differences between the initial δ-ray determination and the densitometer determinations, $Z_\delta - Z_D$, for particles in the ranges $14 \leq Z_D \leq 18$ and $18 \leq Z_D \leq 22$ for both Texas and India. From these distributions and the numbers of particles assigned particular charges we can ascribe maximum errors on the intensities of the VH and MH nuclei. In Texas these considerations lead to maximum errors due to charge misidentification of 2.2% on J_{VH} and 7% on J_{MH}. In fact these errors must be even smaller since the remeasurement of the charges that could not be determined by the densitometer presumably leads to a more accurately determined value.

For particles in the Churchill stack the δ-ray determinations of charge were made with greater care than in Texas or India since we knew that densitometer charge values would not be immediately available. As a consequence, we paid considerable attention to the problems of maintaining consistent δ-ray counts by the continual recounting of standard tracks and the intercomparison of different observers. Since the tracks in Churchill were produced both by slow and fast particles, it was necessary to use both integral and differential determinations of δ rays and to compare these methods with each other.

Those particles which ended in the emulsions had

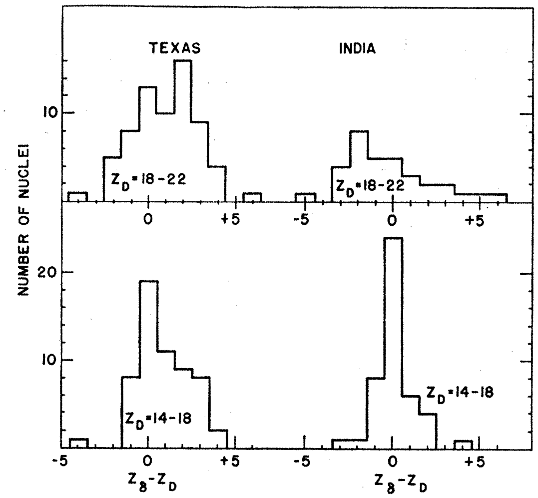

FIG. 1. The differences between charges determined from initial δ-ray counting and densitometer measurements for all particles in both Texas and India with charges measured by the densitometer as $14 \leq Z_D \leq 22$.

their charges determined from integral δ-ray counts using the procedure originally adopted by Lim and Fukui,[6] and employing a calibration based on setting the highest charge group of appreciable abundance equal to $Z = 26$ and by recognition of the abundant oxygen peak.

The differential δ-ray density as a function of residual range was empirically determined from a group of nuclei identified from the cumulative δ-ray count as $Z = 26$. Some iron nuclei with ranges of 15 cm stopped in the stack and were used for this calibration. The δ-ray densities for particles of the same velocity were assumed to vary as Z^2 in deriving the differential δ-ray curves for other charges.

The charge of ending particles was determined from both differential and cumulative δ-ray counts. If the two determinations differed by more than two charges, the particles were remeasured. The maximum error in the intensity of slow $Z \geq 20$ nuclei due to improper charge determination is consequently estimated to be 6%.

In the Canadian stack we have measured the energies of those individual nuclei that had energies of less than approximately 1 BeV per nucleon, while in the Texas and Indian stacks we have only measured the integral intensity of nuclei and assumed that they are all above the geomagnetic cutoff energy typical of the latitude. As a consequence we have obtained a rather detailed differential energy spectrum between the air cutoff energy and about 900 MeV/nucleon, but only an indication of the spectrum above this energy.

The energies of these low-energy nuclei were measured by the following methods:

[6] Y. K. Lin and K. Fukui, Nuovo Cimento 40, 102 (1965).

(a) For those nuclei that come to rest, the energy was determined from the residual range of the nuclei in the nuclear emulsion. This method is by far the most accurate available in this sort of study, because of the precision with which ranges can be measured. The greatest uncertainty inherent to this method is that due to the uncertainty in the charge of the nucleus. Since the residual range R_n of a nucleus of charge Z and mass A is given in terms of the range of a proton, R_p, by $R_p=(Z^2/A)\ R_n$ and since the energy E_p of a proton of range R_p can be represented approximately over the energy range of interest by $E_p \simeq 0.25\ R_p^{0.58}$, the error in the energy estimate is given by

$$\frac{\delta E_p}{E_p}=0.58\frac{\delta R_p}{R_p}=0.58\left(\frac{2\delta Z}{Z}-\frac{\delta A}{A}\right). \qquad (1)$$

Thus, for example, an iron nucleus misidentified by two charges and consequently four mass units would have a most probable value for $\delta E_p/E_p$ of $\simeq 10\%$. These errors are less than those present in other forms of energy determination, and are much less than the size of the energy groups that are used in constructing an energy spectrum. The fraction of the total number of nuclei at any particular energy whose energy was determined from the range is illustrated in Fig. 2 as a function of the energy extrapolated to the top of the atmosphere, and can be seen to exceed 50% for $E<600$ MeV/nucleon.

(b) The energies of those nuclei that suffered nuclear interactions or left the emulsions before they were brought to rest were estimated either from a measurement of the change in ionization or the characteristics of the nuclear interactions and the related nuclear fragments.

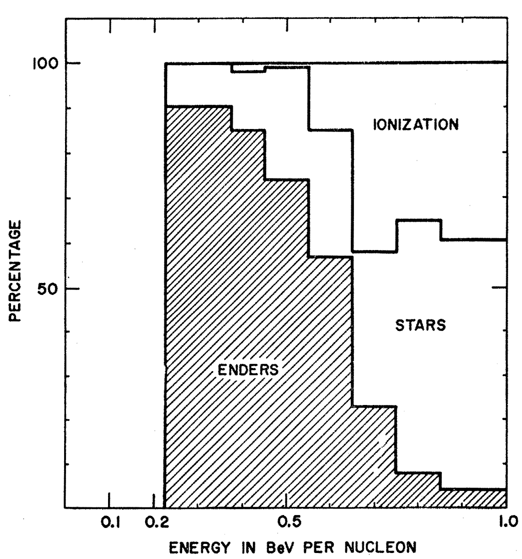

FIG. 2. The percentage of VH nuclei whose energy is determined by either range, characteristics of interaction, or change in ionization.

The fractions of nuclei whose energies were determined in this way are also shown in Fig. 2. Although the individual ionization measurements were made with statistical errors of less than 7% we only consider them to be reliable to within 10%. Such an error implies that the charges are determined to ± 1 unit of charge and consequently that energies of nuclei with $E<600$ MeV/nucleon are known to $\pm 10\%$ which is similar to case (a). This error in the energy increases with increasing energy so that at $E=800$ MeV/nucleon the error is (+140, −120) and at $E=1000$ it is (+400, −150) MeV/nucleon.

ATMOSPHERIC CORRECTIONS

The result of these energy measurements was to give the energy or a lower limit to the energy for each of the 883 VH nuclei, as well as for the 450 nuclei of lighter charge, detected in the Canadian stack. Similarly the scanning and measurement in the Texas and Indian stacks give the total numbers of nuclei in each stack having energies at the top of the atmosphere greater than the appropriate geomagnetic cutoff value. In every case these measurements had to be translated into differential or integral intensity values at the top of the atmosphere by making appropriate allowances for the geometry of the detectors, the energy losses between the region of measurement and the top of the atmosphere and the effects of nuclear interactions in the overlying matter.

Detector Geometry

The detector geometry in each flight was determined from the total area scanned, the maximum projected angle accepted, and the maximum dip angle, i.e., minimum length per plate, accepted. The resulting geometrical factor for each flight, expressed in m^2 sr, is shown in Table II. The error on the geometrical factor comes principally from the uncertainty in the mean thickness of the emulsions, and in the bowing of the emulsions, and is about 2%. The intensity of any group of nuclei can then be obtained simply by dividing the number of nuclei observed by the appropriate exposure factor.

Energy Degradation

The energy of each nucleus was corrected for the matter between the point of determination and the top of the atmosphere on an individual basis, using the standard range-energy curves. These corrections took account of the charge and inclination of each particle and, apart from the problem of defining exactly at what point on a track the energy had been determined, were as accurate as the energy estimates made on the residual ranges of stopping nuclei.

Nuclear Absorption

The overlying material has a threefold effect on the measured intensities of the nuclei in a particular charge group. Firstly, some nuclei will make interactions and be removed from consideration; secondly, some nuclei will make interactions which give rise to fragments of lighter charge which will, however, still be in the charge group being considered; and thirdly, some of the nuclei in heavier charge groups will produce fragments in the group. The correction to the intensity for this absorption-fragmentation process can be treated in terms of a one-dimensional diffusion process.[7] Then for VH nuclei, where there are essentially no still heavier nuclei, the intensity at a depth $x=0$, $J_i(0)$, is given by

$$J_{VH}(0)=J_{VH}(x)e^{+x/\Lambda_{VH}}, \tag{2}$$

where Λ_i is the absorption mean free path of i-type nuclei. For those nuclei in the next lighter group between sulphur and potassium, the MH nuclei, where in general $x \ll \Lambda_{VH}\Lambda_{MH}/(\Lambda_{MH}-\Lambda_{VH})$,

$$J_{MH}(0) \simeq J_{MH}(x)e^{x/\Lambda_{MH}}-J_{VH}(0)P_{VH,MH}x/\lambda_{VH} \tag{3}$$

if P_{ij}, is the fragmentation probability of an i-type nucleus producing a j-type fragment.

These equations show that in order to calculate primary intensities it is necessary to know the values of the mean free paths and fragmentation parameters applicable to the atmosphere. The fractional error in the intensity of VH nuclei at the top of the amtosphere is given by

$$\frac{\delta J_{VH}(0)}{J_{VH}(0)}=\frac{\delta J_{VH}(x)}{J_{VH}(x)}+\frac{\delta x}{\Lambda_{VH}}-\frac{x\delta\Lambda_{VH}}{\Lambda_{VH}^2},$$

while that for the MH nuclei is given by a similar but appreciably more complicated expression. In the conditions of the observations described here the first term on the right-hand side will typically be of the order of 0.1, representing the statistical fluctuation on about 100 nuclei, while the second and third terms will depend on x and Λ_{VH}. Thus in the Canadian stack, where $x=2.4\pm0.1$ g/cm^2, if $\Lambda_{VH}=15\pm5$ g/cm^2 then the second term is 0.006 and the third is 0.05, so that even a 30% uncertainty in Λ_{VH} is relatively unimportant. This is particularly fortunate since Λ_{VH} is not well known and its true value appears to be subject to some controversy. The same is true for Λ_{MH} and the appropriate P_{ji} values.

Interaction mean free paths and fragmentation parameters may be measured directly in media such as carbon or Teflon that in their nuclear makeup resemble air, or may be estimated from measurements made in other media. From these measurements or estimates it is possible to calculate the appropriate absorption mean free paths. Alternatively, attenuation mean free paths may be measured directly by measuring the variation in intensity of a particular group of nuclei as a function of depth in the atmosphere. However, except for the VH nuclei, it is clear from Eqs. (2) and (3) that these attenuation mean free paths are not the same as the absorption mean free paths defined above and may quite probably be themselves functions of the depth x.

The direct measurement of these parameters,[8,9] has not yet been undertaken with sufficient statistical accuracy, particularly for these rare VH and MH nuclei, to provide more than an indication of the true values. As a result it has been necessary to attempt to deduce the values in air from measurements made in media such as nuclear emulsion, e.g., those by Waddington.[10] By far the most extensive values for VH and MH nuclei in emulsion have been obtained as a by-product of the studies described here. These have been reported elsewhere,[11,12] and lead to a plausible parametric representation of the mean free paths as a function of the type of medium, thus giving predicted values for an air medium. The values of P_{ji} and λ_i that result from these studies are given in Table III. These values may be compared with those deduced from the attenuation measurements of Webber and Ormes[13] for VH nuclei. Unfortunately, these workers were unable to make similar measurements on the MH nuclei due to statistical limitations and, indeed, corrected their observations on the MH nuclei in some unspecified manner. However, for the VH nuclei the attenuation mean free path should be directly comparable with the absorption mean free path given in Table III. Webber and Ormes[13] find a value of 16 ± 2 g/cm^2 after making a correction for ionization losses which depends on their measured energy spectrum. This value is significantly lower than that in Table III. This difference is presumably related to the difference in the forms of the lower end of the energy spectra observed in these two experiments that will be discussed later, but is clearly not the cause for these different energy spectra. Essentially, in comparison with our spectrum that of Webber and Ormes predicts too few low-energy nuclei and consequently would lead to an underestimate of the correction necessary to the observed attenuation mean free path.

TABLE III. Parameters in air and emulsion.

			Emulsion	
	(g/cm^2)		(g/cm^2)	(cm of emulsion)
VH nuclei	$\lambda_{VH}=16.2$	$P_{VH,VH}=0.25$ $P_{VH,MH}=0.15$	$\Lambda_{VH}=21.6$	$\lambda=9.6\pm0.6$
MH nuclei	$\lambda_{MH}=19.1$	$P_{MH,MH}=0.15$	$\Lambda_{MH}=22.5$	$\lambda=9.8\pm0.6$

[7] M. F. Kaplon, J. H. Noon, and J. H. Racette, Phys. Rev. **96**, 1408 (1954).

[8] M. W. Friedlander, K. A. Neelakantan, S. Tokunga, G. R. Stevenson, and C. J. Waddington, Phil. Mag. **8**, 1691 (1963).

[9] R. R. Daniel and N. Dugaprasad, Nuovo Cimento **23**, Suppl. 82, (1962).

[10] C. J. Waddington, Progr. Nucl. Phys. **8**, 1 (1960).

[11] T. F. Cleghorn P. S. Freier, and C. J. Waddington, Can. J. Phys. **46**, S572 (1968).

[12] T. F. Cleghorn, University of Minnesota Technical Report No. CR-104, 1967 (unpublished).

[13] W. R. Webber and J. F. Ormes, J. Geophys. Rev. **72**, 5957 (1967).

TABLE IV. Integral and differential intensities of VH and MH nuclei.

VH nuclei				MH nuclei		
$J_{VH}(>E)$ (parts/m^2 sr sec)	E (BeV/ Nucleon)	$j_{VH}(E)$ [parts/m^2 sr sec (MeV/nucleon)]	$\bar{E}$ (BeV/ Nucleon)	$J_{MH}(>E)$ (parts/m^2 sr sec)	$j_{MH}(E)$ (parts/m^2 sr sec MeV/nucleon)	$\bar{E}$ (BeV/ nucleon)
1.50 ± 0.060	0.225	$(1.27\pm0.19)\times10^{-3}$	0.262			
		$(1.77\pm0.20)\times10^{-3}$	0.337		$(0.64\pm0.08)\times10^{-3}$	0.280
		$(1.59\pm0.19)\times10^{-3}$	0.412		$(0.40\pm0.06)\times10^{-3}$	0.425
		$(1.39\pm0.15)\times10^{-3}$	0.50			
		$(1.38\pm0.15)\times10^{-3}$	0.60			
		$(1.03\pm0.13)\times10^{-3}$	0.70			
		$(1.00\pm0.13)\times10^{-3}$	0.80			
		$(0.49\pm0.07)\times10^{-3}$	0.925			
0.594 ± 0.03	1.00					
0.405 ± 0.023	1.58	$(5.7\ \pm0.5)\times10^{-5}$	3.45	0.086 ± 0.010	$(1.8\ \pm0.33)\times10^{-5}$	3.45
0.090 ± 0.006	7.1			0.020 ± 0.003		

If we use the values in Table III it is relatively straightforward to calculate the true intensities of the VH and MH nuclei at the top of the atmosphere from those observed in the emulsions. There remains, however, the problem of those nuclei that were actually fragments when detected. These will be assigned incorrect energies at the top of the atmosphere and at low energies the effect on an individual nucleus can be quite large, since in an extreme case a fragment may be detected when the primary nucleus would fail even to reach the scan line. The number of such nuclei is given by the ratio of the number of fragments to the number of nuclei observed $\simeq xP_{ii}/\lambda_i$ so long as $x\ll\lambda_i$. Thus for the Churchill stack, where $x=2.4$ g/cm^2, $P_{ii}=0.25$, and $\lambda_i=16.2$ g/cm^2, the percentage of nuclei observed that are fragments is 3.7%. Since only a fraction of these will have their energies seriously affected, it is reasonable to neglect the error involved.

Using these intensities, we have plotted the differential energy spectrum as a function of kinetic energy, and the values are tabulated in Table IV. In doing this it is necessary to be careful as to how one plots those differential values measured over wide energy intervals. In particular, our data on VH nuclei contain one differential value obtained by comparison of the integral values at Texas and India. This value was plotted at a mean value of the kinetic energy T, given by

$$\bar{T}=\left(\frac{(T_2-T_1)n(T)}{T_1^{-n(T)}-T_2^{-n(T)}}\right)^{\{1/[1+n(T)]\}}, \qquad (4)$$

where $n(T)$ is the exponent of the power-law representation of the integral spectrum and $T_1<T_2$ and are the cutoff values at Texas and India, respectively. Clearly, in order to determine $\bar{T}$ it is necessary first to know $n(T)$ and to assume that over the range T_1 to T_2 the spectrum is a power law. Note that this value is somewhat different, and higher, than that obtained from the conventional representation that

$$\bar{T}=\int_{T_1}^{T_2}Tj(T)dT\bigg/\int_{T_1}^{T_2}j(T)dT, \qquad (5)$$

where $j(T)$ is the differential intensity given by $j(T)=nKT^{-(n+1)}$. This difference appears to come from the manner in which such an experimental value of $j(T)$ is calculated:

$$j(T)=[J(>T_1)-J(>T_2)]/(T_2-T_1).$$

Then since $j(T)$ is given,

$$n(T)K(T)\bar{T}^{-[1+n(T)]}=K(T)(T_1^{-n(T)}-T_2^{-n(T)})/(T_2-T_1),$$

where $\bar{T}$ is the effective mean energy, and hence Eq. (4). Note that Eq. (4) is equally valid for rigidity or total-energy representations when expressed in the appropriate parameters.

COMPARISON WITH OTHER RESULTS

A number of other workers have measured the intensities of VH and MH nuclei at various times during the last solar cycle. With the exception of the results reported by Webber and Ormes,[13] none of these determinations is of even remotely comparable statistical weight and we therefore start by comparing our results with those of these later authors. In a series of seven balloon flights, Webber and Ormes[13] used a Cherenkov-scintillation detector to study the detailed charge spectrum of all the nuclei with $Z\geq3$ with unprecedented statistical accuracy. In the course of this study they observed some 240 VH nuclei in flights where the air cutoff determined the energy limit they could observe and another 46 at higher energies. These flights were at mean altitudes of approximately 5 g/cm^2, but most of the low-energy data were obtained from two flights at an average of approximately 2.5 g/cm^2. As a consequence, the influence of slightly differing atmospheric corrections should be relatively neglible in a comparison with our results. These data were adjusted to a mean level of solar modulation which, as measured by the Mt. Washington neutron monitor, was 3% higher than that prevailing at the time of our Canadian flight. If we temporarily disregard this difference and directly

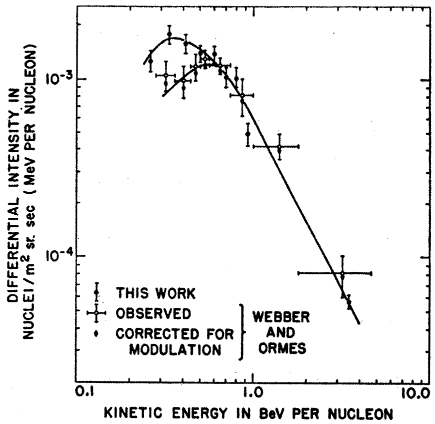

FIG. 3. Differential intensities of VH nuclei measured in this experiment and by Webber and Ormes (Ref. 13).

compare our measured spectrum for VH nuclei with that obtained by Webber and Ormes[13] we see the spectra shown in Fig. 3.[14] The agreement for $T \geq 500$ MeV/nucleon is very good, but there is a serious divergence between the spectra at lower energies, with our values rising above those of Webber and Ormes. This is in the opposite direction to what would be expected as a consequence of solar modulation and is far too large to be statistical. This difference is particularly serious since this low-energy region is just that where the influence of interstellar ionization energy losses would be most noticeable. For this reason, it is necessary to consider the other available results in an attempt to distinguish between these divergent results. In order to do this, it becomes essential to consider the effects of modulation.

Regression curves of the intensities of helium nuclei as a function of modulation as measured by neutron monitors have been presented by several workers, e.g., Freier and Waddington,[5] and Webber.[15] We have assumed that the VH nuclei, with their closely similar A/Z ratios, will be modulated in a similar manner to that of the helium nuclei. Figure 4 shows the percentage modulation of the primary intensity as a function of the Mt. Washington neutron monitor counting rate at various energies, normalized to zero modulation when the counting rate is 2400. The effects of the resulting correction to the data of Webber and Ormes[13] in comparison to ours are shown in Fig. 3 and clearly it merely enhances the discrepancy already noted.

Other observations of VH nuclei have been made by several groups, although these have been of poor

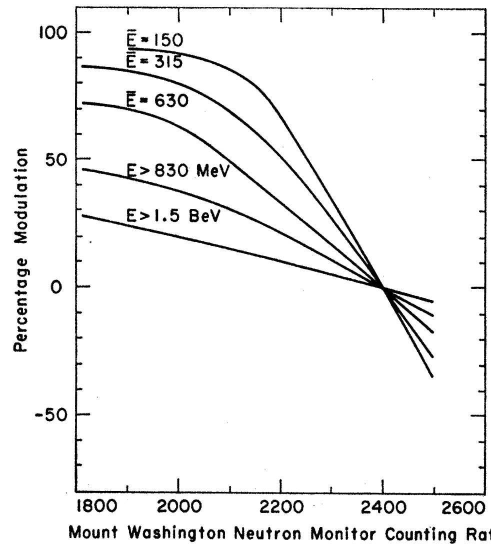

FIG. 4. Modulation of helium nuclei of various energies as a function of Mt. Washington neutron monitor counting rate.

statistical weight.[16–21] These results have, in some cases, been obtained by using rather different corrections for atmospheric absorption than those used here; however, because of the large statistical errors and uncertainties of the modulation corrections, it was not thought worthwhile to attempt to adjust them for this relatively minor effect.

These data are plotted in Fig. 5 together with the

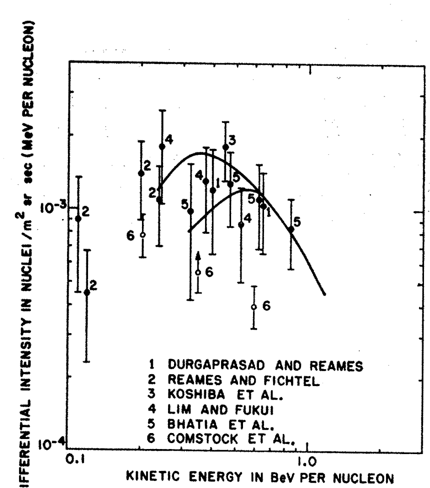

FIG. 5. Differential energy spectrum of VH nuclei measured by different observers. The two curves are from these data and Webber and Ormes (Ref. 13) and are from Fig. 3.

[14] The data of Webber and Ormes (Ref. 13) are plotted as given by them. Note that the high-energy differential points are placed at the center of the energy intervals and should, in fact, be at a mean energy that is somewhat less. See, e.g., Eq. (5).

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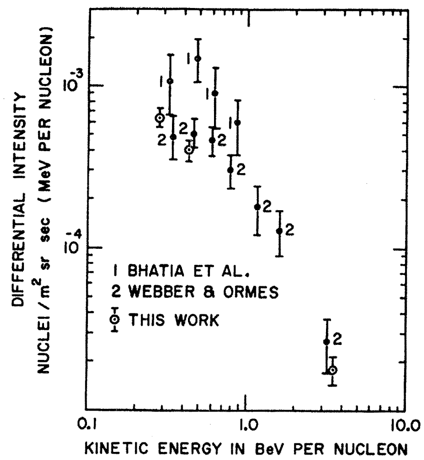

FIG. 6. Differential energy spectrum of *MH* nuclei measured by different observers. The results obtained in this work are shown by large open circles.

smooth curves that represent the best fits to our data and those of Webber and Ormes.[13] It can be seen that these other data do not clearly distinguish between these two curves and that this is true whether or not we include the data of Bhatia *et al.*[20] which, for reasons to be discussed below, we also suspect to be in error. However, the rocket observations marked (2) on this figure,[16] which are applicable to lower energies than those obtainable by balloon observations, do seem to be more consistent with the trend of our results than with that of Webber and Ormes.[13]

Since we are unable to make a clear resolution between these divergent spectra on the basis of the data on the *VH* nuclei alone, we now look at the available data on the *MH* nuclei. These nuclei are particularly rare in the primary cosmic radiation and hence we have only our own data, those of Webber and Ormes,[13] and those of Bhatia *et al.*[20] The resulting spectra, corrected for modulation, are shown in Fig. 6, from which it is clear that there is a serious discrepancy between the values reported by Bhatia *et al.*[20] and the others. The difference is made still more striking if it is noted that Webber and Ormes actually define *MH* nuclei as having $15 \leq Z \leq 19$, whereas Bhatia *et al.* and we define them as $16 \leq Z \leq 19$. As a result, one would expect the Webber-Ormes values to lie somewhat above those of Bhatia *et al.* and ourselves. Instead Bhatia *et al.* report values on an average a factor of 2 greater than the others. In view of the rather good agreement between our results and those of Webber and Ormes it seems probable that the results of Bhatia *et al.* are too high because they have included nuclei of the *LH*[22] or *VH* groups. The agreement between our values and those of Webber and Ormes, while reasonable, would also be consistent with Webber and Ormes having failed to correctly identify some of the lowest-energy nuclei.

This last observation provides, we believe, a clue to a reasonable explanation of the differences between our results. Nuclei of low energy and high charge, and thus consequently of high ionization power, are those which are most reliably detected and studied in a nuclear emulsion detector, but may well be those most difficult to study in a wide range electronic detector of the type used by Webber and Ormes. It therefore seems reasonable to us to postulate that these authors' results have been affected by some loss, or misidentification, of events of the most heavy ionization. Clearly, such a postulate can not be verified without additional measurements made preferably by an independent technique.[23] However, for the purposes of the interpretation of the observed energy spectrum given in Paper II we have used our measured spectrum shown in Fig. 3.

[22] *LH*-nuclei have $10 \leq Z \leq 14$.

[23] Dr. Webber tells us that with measurements made by a newly modified detector, he finds higher intensities at these low energies, even though solar modulation has increased.