Very Heavy Nuclei in the Primary Cosmic Radiation. II. Interpretation of the Energy Spectra*

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An attempt has been made to understand the form of the observed energy spectrum of the highly charged ($20\leq Z\leq 30$) nuclei in the primary cosmic radiation. It is shown that the behavior of this spectrum relative to that of the helium nuclei also present in the cosmic radiation can be explained by assuming that the highly charged nuclei have two components. The first of these components is dominant at high energies and has a spectrum at the source that can be represented as a simple power law with a slope somewhat flatter than that of the source spectrum of the helium nuclei. The nuclei in this component may be regarded as having traversed several g/cm² of matter between the times of acceleration and of detection. The second component is of importance only at relatively low energies (a few hundred MeV/nucleon), has a spectrum that falls steeply with increasing energy, and cannot simply be represented as a power law. The possible consequences of such a model are briefly considered.

INTRODUCTION

IN a previous paper,[1] henceforth referred to as Paper I, we have presented our experimental observations on the energy spectrum of the very heavy nuclei, $20\leq Z\leq 30$, found in the primary cosmic radiation. In this paper we discuss our interpretation of these data in terms of the propagation of these nuclei through the interstellar medium and the possible spectra at the "source" of cosmic rays.

The energetic heavy nuclei that we observe near the earth are the survivors of a complex series of physical processes that are poorly understood and not well defined. We can identify some of the principal stages that we believe characterize these processes by tracing these nuclei backwards in time from the region of observation. Closest to us, and thus one of the most well studied of these processes, is the interaction of the nuclei with the solar environment. This interaction leads to a solar modulation of the intensity of the nuclei, decreasing the intensity detected near the earth compared to that outside the solar system. The magnitude of this solar modulation is dependent both on the energy of the nuclei and on the level of solar activity, and the precise physical mechanism that is involved is not yet fully understood. In particular, even at times of low solar activity such as existed during the observations reported in Paper I, there is considerable uncertainty regarding the amount and form of the residual modulation. However, there are good theoretical reasons for believing that nuclei having similar charge-to-mass ratios will be equally affected by the solar modulation process. The modulation appears to be some function of the velocity and rigidity (momentum per unit charge) of the particles and consequently for nuclei of a similar A/Z, such as the heavy nuclei considered here, will be the same for all the nuclei. An experimental comparison of the modulation of helium nuclei and of all nuclei with $Z\geq 3$ made by us a few years ago[2] is consistent with this assumption, as are the observations on $Z\geq 6$ nuclei made by many workers.[3] Since in this study we are interested in the heavy nuclei as indicators of the astrophysical aspects of the cosmic radiation rather than with the local environment, the solar modulation is merely a complication to our analysis and we will take the appropriate measures to eliminate its influence as far as possible. In particular, where possible, we will consider the ratio of the helium nuclei to the very heavy (VH) nuclei at each energy E, $\Gamma_{\alpha VH}(E)$. If the modulation function for i-type particles $\Psi_i(E)$ is given by $\Psi_i(E) = j_1^\infty(E_i)/j_i^{\text{earth}}(E_2)$ where $j_i^\infty(E_1)$ and $j_i^{\text{earth}}(E_2)$ are, respectively, the differential intensities of the i-type nuclei outside and inside the modulation region *and if* $E_1=E_2=E$, then

$$\Gamma_{\alpha VH}(E)=\frac{j_\alpha^{\text{earth}}(E)}{j_{VH}^{\text{earth}}(E)}=\frac{j_\alpha^\infty(E)\Psi_{VH}(E)}{j_{VH}^\infty(E)\Psi_\alpha(E)};$$

thus if $\Psi_{VH}(E)=\Psi_\alpha(E)$, then $\Gamma_{\alpha VH}(E)=j_\alpha^\infty(E)/j_{VH}^\infty(E)$ and the effect of modulation has been eliminated. In fact for VH nuclei $A/Z\simeq 2.1$, while for helium nuclei $A/Z\simeq 1.95$, due to the approximately 10% abundance of He^3, and so $\Psi_{VH}(E)$ is not exactly equal to $\Psi_\alpha(E)$ but the effect is unlikely to be large and can be neglected at this level of analysis.

Note that the effects of interstellar propagation cannot be treated in a similar manner because clearly in this case $E_1\neq E_2$ nor are the E_1's and E_2's the same for helium and VH nuclei. Finally, if solar modulation were due to an energy loss effect, so that $E_1>E_2$, which was, however, similar for nuclei with similar A/Z ratios, then $\Gamma_{\alpha VH}^{\text{earth}}(E_2)=\Gamma_{\alpha VH}^\infty(E_1)$, and the effect of modulation would be to shift the observed ratios to a higher energy outside the solar cavity.

After modulation the next effect that will have been experienced by the nuclei is that produced by the com-

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[1] P. S. Freier and C. J. Waddington, preceding paper, Phys. Rev. **175**, 1641 (1968).

[2] P. S. Freier and C. J. Waddington, Phys. Rev. **135**, B724 (1964).

[3] W. R. Webber, in *Handbuch der Physik* (Springer-Verlag, Berlin, 1967), Vol. XLVI, p. 1.

bination of physical processes that we include under the heading of interstellar propagation. It is with these processes and their effects on assumed source characteristics that we will be primarily concerned. During the propagation of the nuclei through the interstellar and/or possibly the intergalactic medium, they will be subject to the effects of the matter and magnetic fields traversed. The matter will produce the customary effects of energy loss due to ionization processes and charge degradation due to nuclear interactions. The magnitude of these effects will depend both on the nature and the amount of the matter traversed, which in turn will depend in some manner on the configuration and strength of the magnetic fields experienced by the nuclei during their traversal of the medium. In addition, the magnetic field may conceivably affect the energies of the individual nuclei, producing either acceleration or deceleration, by, for example, a Fermi-type process. The nuclei that we are considering here predominantly have energies in the range $10^{9\pm1}$ eV/nucleon and consequently in magnetic fields of the sort typical of the interstellar medium, $10^{-(5\pm1)}$ G, have gyroradii of $10^{11.5\pm2.0}$ cm, or of the order of a microparsec. Their motion is thus entirely dominated by the geometrical form of the magnetic field lines over a very small scale length. From observations[4] of the depolarization of 21-cm radio sources it can be assumed that the microstructure of the galactic field has a scale size less than 1 parsec, but it would seem improbable that it should be as small as these gyroradii. Hence, in order to describe the motion of the nuclei it is necessary to make some assumptions as to the general configuration of the galactic field. In addition, it appears, in general, as we shall see later, that the amount of matter traversed by the nuclei is much greater than that which would be encountered during a single rectilinear traversal of the interstellar material in the galaxy, which is at most 0.1 g/cm^2. Hence it is possible that these nuclei will have moved through many different regions of the galaxy and, unless the field has a relatively efficient trapping configuration, have a good chance of being lost from the galaxy to intergalactic space, so that the factor controlling the duration of interstellar propagation may be the escape probability. It has been customary to assume that the cosmic radiation is mainly confined to the galaxy simply because of the high energy density that is observed locally, but with the observation of the quasi-stellar sources as regions of almost unlimited energy even this assumption seems to have little justification, and the possibility that the radiation is mainly extragalactic cannot be ignored. On the other hand, it can be argued that the very small gyroradii of these nuclei implies that they are unable to traverse distances of even a galactic scale without passing through considerable amounts of matter. In this case, these particles would have to be regarded as being of a rather local origin, produced, for example, within our own galactic arm.

Many attempts have been made to deduce the amount of matter traversed by the cosmic-ray nuclei during their propagation from the source region.[5–8] These estimates have been predominantly based on the observed charge composition of the heavy nuclei and in particular on the relative abundances of those elements that should be rare or absent from the source region. Studies of the abundances of the "L nuclei"—lithium, beryllium and boron—as well as of the abundances of He^3 and H^2 have all concluded that "on average" the heavier nuclei presumed to produce these elements by spallation have traversed the equivalent of 2–6 g/cm^2 of matter. The problem of what form of distribution of path lengths goes to make up this average is not well understood. Present models range from a distribution that is a δ function, through a Gaussian,[9] to an exponential.[10] In any case, this average value represents the mean amount of matter traversed by the cosmic-ray nuclei from the moment that the nuclei acquired sufficient energy to make nuclear spallation reactions. It is thus conceivable that much of this matter was traversed by the nuclei when they had energies quite different from those at which they are observed. Consequently, it seems well worthwhile to attempt to obtain another measure of the amount of matter traversed which is based on a different physical concept. Such a measure should, in principle, be provided by studying the effects of ionization energy losses on the shape of the energy spectra of the nuclei of different charges. Since these energy losses are proportional to the square of the nuclear charge, a comparison of the spectra of two different charge groups could provide an indication of the amount of matter traversed by these nuclei. This measure might be quite different from that obtained from the charge distribution since it is indicative only of that matter traversed since the particles ceased to experience appreciable acceleration.

EXPERIMENTAL OBSERVATIONS

The distribution in velocity of cosmic-ray particles can be described in terms of several different "motion parameters." Of these parameters, those that appear to have the most physical significance are the total, kinetic plus rest mass, energy per nucleon, E; the kinetic energy per nucleon, T; the magnetic rigidity R. These parameters are analytically related, and a description

[4] J. M. Bologna, E. F. McClain, W. R. Rose, and R. M. Soanaker, Astrophys. J. **142**, 106 (1965).

[5] R. R. Daniel and N. Durgaprasad, Progr. Theoret. Phys. (Kyoto) **35**, 36 (1966).
[6] N. Durgaprasad, J. Geophys. Res. **72**, 965 (1967).
[7] S. Biswas, S. Ramaduri, and N. Sreenivasan, Can. J. Phys. **46**, S593 (1968).
[8] C. E. Fichtel and D. Reames, Phys. Rev. **149**, 995 (1966).
[9] V. K. Balasubrahmanyan, E. Bolt, and R. A. R. Palmeira, Phys. Rev. **140**, B1157 (1965).
[10] R. Cowsik, Yash Pal, S. N. Tandon, and R. P. Verma, Phys. Rev. **158**, 1238 (1967).

of a class of particles in terms of one automatically defines the description in terms of the other two. However, it is generally true that experimental uncertainties are sufficient to allow plausible representations of observed data in different parameters that are analytically inconsistent. In particular, it is convenient, and conventional, to attempt to represent experimental spectral data in the form of power laws. Thus if ψ represents a motion parameter, the integral intensity of i-type particles will often be given by

$$J_i(>\psi)=K_i(\psi)\psi^{-n_i(\psi)}, \qquad (1)$$

where $K_i(\psi)$ and $n_i(\psi)$ are constants over the range of ψ considered. Similarly, the differential intensity will be given by

$$j_i(\psi)=dJ_i(>\psi)/d\psi=C_i(\psi)\psi^{-\gamma_i(\psi)}, \qquad (2)$$

where

$$C_i(\psi)=-n_i(\psi)K_i(\psi) \text{ and } \gamma_i(\psi)=n_i(\psi)+1.$$

The representation of these spectra by power laws is based partly on the experimental observations that most components of the cosmic radiation can be described by such spectra over limited ranges, except in the region where solar modulation is most important, and partly on the theoretical consideration of acceleration and propagation models, which generally lead to power laws. In addition, at high energies, where $E\simeq T \simeq(A/Z)R$, the energy spectra do seem to be consistent with power laws over a wide range of motion parameter.

We have, as a consequence, plotted the integral and differential intensity data for VH and helium nuclei as functions of each of these three motion parameters, being careful to take the appropriate mean values in the manner described in Paper I.

In Figs. 1 and 2 we show the integral and differential spectra of both the VH nuclei and the helium nuclei, plotted as function of T, E, and R. These spectra for the helium nuclei represent a composite of experimental data obtained by several groups.[11] So far as possible only data obtained when solar modulation was essentially similar to that prevailing when the measurements of Paper I were made have been used.

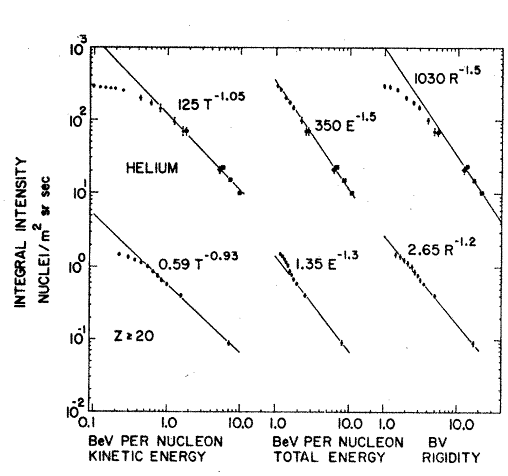

FIG. 1. Integral spectra of helium and $Z\geq 20$ nuclei plotted as functions of three different motion parameters. The power-law best fit to the data is given for each parameter.

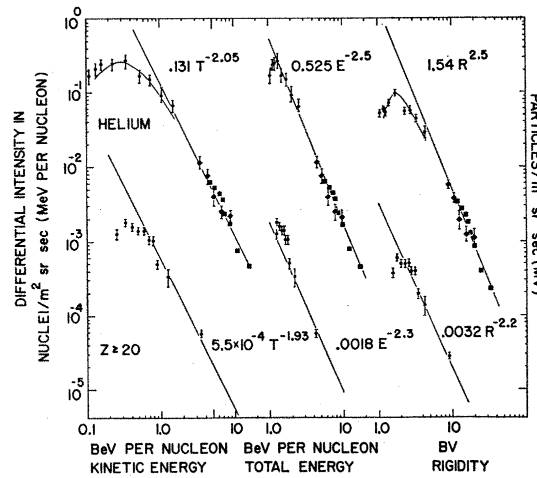

FIG. 2. Differential spectra corresponding to integral spectra in Fig. 1.

An examination of these figures shows that in general power-law representations are feasible for each of the motion parameters above values corresponding to $T\simeq 1$ BeV/nucleon. Below this the typical "bending over" of the spectrum usually attributed to solar modulation is generally observed except for the behavior of the integral E spectrum which shows a marked *upward* break. This behavior is, as far as we know, quite unique and we shall return to a consideration of its significance later.

From these data we can calculate the experimental values of $\Gamma_{\alpha VH}(T)$ and these are shown in Fig. 3.[12] It is the shape of this curve that we must try to explain in terms of interstellar propagation and assumed source characteristics, since we assume that these measured ratios do represent those prevailing outside the solar influence. Note that this curve is not as simple as might be expected and that the influence of the greater ionization energy losses of the VH nuclei at low energies is not very marked.

ANALYSIS

We start with the simplest possible model of propagation, the "slab" model in which it is assumed the path length distribution of the nuclei is a δ function, so that all the nuclei have traversed a unique x g/cm^2 of matter. We shall also assume that this matter is all neutral

[11] P. S. Freier and C. J. Waddington, J. Geophys. Res. **73**, 4361 (1968).

[12] In constructing this figure the differential ratio at the equator cutoff energy has been obtained from the integral ratio by mutiplying by a factor of n_α/n_{VH}, with $n_\alpha=1.5$ and the value shown later to provide the best fit to the high-energy data $n_{VH}=1.3$. Letting $n_\alpha=n_{VH}$ would further *increase* the difference between the ratios at intermediate and high energies.

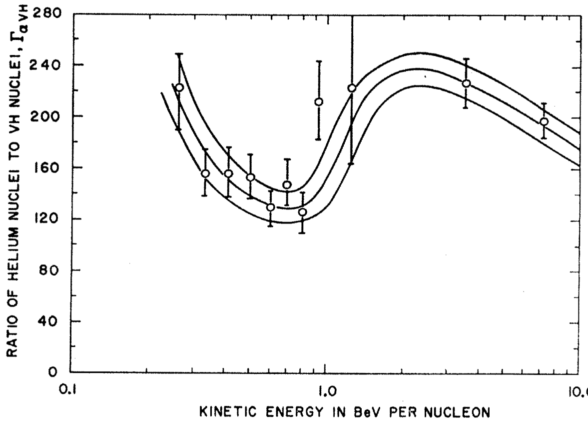

FIG. 3. The experimental values of the ratio of helium to VH nuclei plotted as a function of kinetic energy. The curves drawn represent our visual estimate of the most reasonable fit to the data, together with an indication of the probable uncertainty in this estimate.

hydrogen. The differential intensity after traversing x g/cm^2, $j_i(\psi,x)$, is then given in terms of the intensity at the source, $j_i(\psi,0)$, for particle components that are not seriously influenced by the fragmentation of heavier nuclei feeding additional particles into the group, which is true for both the VH and helium nuclei, by

$$j_i(\psi,x)=j_i(\psi',0)\frac{dT'/dx}{dT/dx}\exp[-x/\Lambda_i(\bar{\psi})], \qquad (3)$$

where ψ' is the value the motion parameter must have at the source in order for it to have the value ψ after traversing x g/cm^2, dT'/dx and dT/dx are the energy loss rates appropriate to ψ' and ψ, and Λ_i is the absorption mean free path at the mean value of the motion parameter $\bar{\psi}$. The use of a value of Λ_i at an arithmetic mean value of ψ is only an approximation, but appears amply justified in view of the other uncertainties involved in this calculation. To solve this equation for $j_i(\psi,x)$, it is then necessary to assume some form for the source spectra, $j_i(\psi',0)$, and to be able to calculate $\Lambda_i(\bar{\psi})$. We have assumed power laws for $j_i(\psi',0)$. This assumption corresponds not only to assuming that the acceleration mechanism produces particles with a power-law spectrum, which is a plausible assumption, but also implies that the particles traverse essentially no matter during this acceleration phase of their life, which is perhaps less plausible. However, if the nuclei do traverse appreciable amounts of matter during the acceleration phase, the effects on the charge distribution would presumably be much more pronounced than those on the energy spectrum, which to first order would be unaltered. Since we wish to calculate intensity ratios which can be arbitrarily normalized, the values of C_i, Eq. (2), are immaterial. However, those of γ_i are not, and it is necessary to study the effects of varying these parameters. Since $\Lambda_i(\psi)$ is essentially independent of ψ at high energies, where also $dT'/dx=dT/dx$, $j_i(\psi',0)$ should have the same shape as $j_i(\psi,x)$ when ψ has a value equivalent to $T>3$ BeV/nucleon. Now at very high energies where all ψ are equivalent it seems well established that $\gamma_i\simeq 2.5$ and it is thus plausible to start by assuming that $\gamma_i(\psi,0)$ has this value. For the E spectra this is in reasonable accord with the observed values, but for the T and R spectra it becomes necessary to assume that the slopes have been significantly affected by solar modulation even in the range $3\leq T\leq 10$ BeV/nucleon. An alternative possiblity is to assume that the source spectra closely resemble those actually observed in this energy range, i.e., that residual solar modulation is unimportant, and to put $\gamma_i(R,0)=2.1$ and $\gamma_i(T,0)=1.9$. In this case, it is necessary also to assume that these values increase quite markedly as T and R increase, so that the source spectra are only power laws over a limited range.

In order to discuss the effects of matter traversal on the cosmic-ray nuclei it is necessary to know the interaction mean free paths λ_i and fragmentation probabilities P_{ii} of the particular nuclide in the particular medium being considered, since the absorption mean free paths are given by $\Lambda_i==\lambda_i/(1-P_{ii})$. Further, since in this case, unlike that of the extrapolation through the residual atmosphere, we are dealing with traversal of amounts of matter that are as much, or more, than a mean free path, the conclusions are strongly dependent on the values chosen for these parameters. It is thus necessary to discuss the selection of the relevant parameters in detail.

First of all, it is necessary to make some assumption as to the composition of the medium being traversed by the particles. This assumption is necessarily model-dependent, since the composition of the medium may well be different for particles assumed to spend their lives in interstellar space than for particles trapped near the source region. Not only may the composition be different, but so may the degree of ionization, thus affecting the rate of energy loss due to ionization. The influence of different composition and different ionizations has been considered by several authors,[13–15] and can have quite a large influence of the results since, for example, the energy loss in ionized hydrogen may be some two to four times as great as that in neutral hydrogen. However, only a small part of the hydrogen in interstellar space is ionized and so, unless the particles are confined to particular regions, we can largely neglect this effect.

In the course of this analysis we shall require values for the fragmentation probabilities both for group to group and element to element; consequently, it is necessary to appeal to data other than the directly measured parameters. Many studies have been made of partial spallation cross sections using artificially ac-

[13] D. Reames and C. E. Fichtel, Phys. Rev. **162**, 7291 (1967).
[14] M. V. K. Appa Rao and S. Ramaduri, TIFR Report No. NE 67-2, 1967 (unpublished).
[15] N. Dugraprasad, NASA Report No. X-611-67-413, 1967 (unpublished).

celerated protons and generally detecting the production of radioactive isotopes. Large numbers of these cross sections are known for a wide range of target nuclei and incident energies, These data have been used by Rudstam[16] to obtain a five-parameter analytical expression for the formation cross section of a spallation product of atomic number Z and mass number A, $\sigma(Z,A)$, in terms of the mass number of the target nucleus, A_t, of the form

$$\sigma(Z,A) = \frac{\hat{\sigma} P d^{2/3} A_t^{-2e/3}}{1.79[\exp(PA_t)(1-2e/3PA_t)-1+\frac{2}{3}e+\frac{2}{3}e/PA_t]} \times \exp(PA-R|Z-SA+VA^2|^{3/2}),$$

where $\hat{\sigma}$ is given in terms of the interaction cross section σ_i by[17]

$$\hat{\sigma}=\sigma_i f_1(A_t) f_2(T)$$

and

$$f_1(A_t)=\exp(-g+hA_t),$$

$$f_2(T)=\exp(+k-lT) \quad \text{for } T<240 \text{ MeV}$$
$$=1 \quad \text{for } T\geq 240 \text{ MeV}.$$

Here d, e, g, h, k, and l are constants having the values of 11.8, 0.45, 0.25, 0.0074, 1.73, and 0.0071, respectively. The parameters P, R, S, and V are given by

$$P=20E^{-0.77} \quad \text{for } T<2100 \text{ MeV}$$
$$=0.056 \quad \text{for } T\geq 2100 \text{ MeV},$$
$$R=dA^{-e},$$
$$S=0.486,$$
$$V=3.8\times 10^{-4}.$$

This expression has been found to provide a reasonable representation of the experimental data, predicting

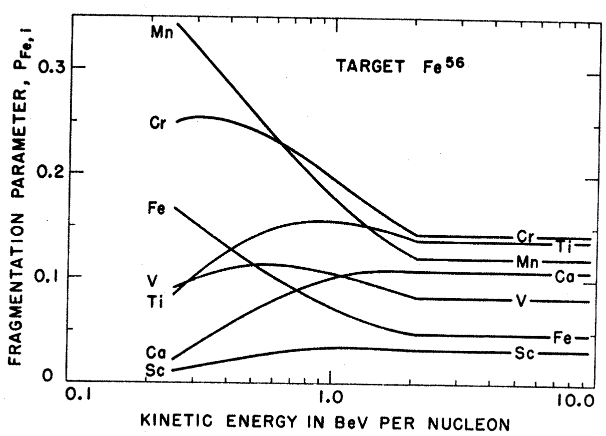

FIG. 4. The variation with energy of the fragmentation param eter for elements in the VH group produced from the passage o iron, Fe^{56}, nuclei through hydrogen. Calculated from the Rudstam equation (Ref. 16); see text.

[16] G. Rudstam, Z. Naturforsch. **21a**, 1027 (1966).

[17] Note that Durgaprasad (Ref. 6) apparently assumed that $\hat{\sigma}=\sigma_i$, which would imply an approximate 20% error for $A_t=56$, $T>240$ MeV.

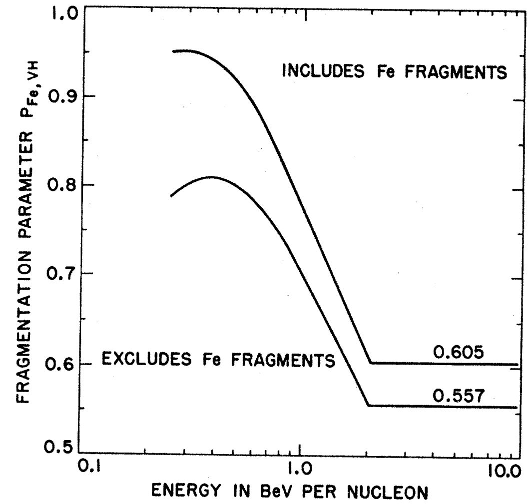

FIG. 5. The variation with energy of the fragmentation parameter for iron, Fe^{56}, target nuclei to produce fragments in the VH group. Curves are shown for the cases where lighter iron isotopes are included and excluded. See text for further details.

the individual cross sections to within a factor of 2.4. A study of the variation of the ratio of the experimentally observed cross section to that calculated, as a function of Z, shows that in the charge range of interest to us the only major divergences are for Sc, Ca, and S, all of which have $\sigma_{expt}/\sigma_{calc}\simeq 0.5$.

We have used this equation to calculate the fragmentation probabilities, for each of the elements between S and the assumed target as a function of energy, for target nuclei that are the most abundant isotope of each element between Cl and Fe. In order to make this calculation $\sigma(Z,A)$ has been determined for all the isotopes that have any appreciable yield and, in order to simulate the conditions in the interstellar propagation, all decays except K capture, having a lifetime of less than 10^9 years, have been allowed to go to the stable end product. Note that a selection of 10^6 years instead would make no difference, there being no isotopes in this charge range with half-lives between that of K^{40}, 1.3×10^9 yr, and Cl^{36}, 3.1×10^5 yr. The fragmentation parameters were then calculated from $\Sigma\sigma(Z,A)/\sigma_i$, summed over all the A values at a particular Z. Figure 4 shows an example of the calculated values for Fe^{56} target nuclei. Note that from the discussion given previously the values for Sc, Ca, and S should presumably be reduced by approximately one-half. From these values for individual elements we can also obtain the values for charge groups by suitable summing. Figure 5 shows the parameter $P_{Fe,VH}$, both after omitting and including the secondary Fe nuclei and reducing Sc and Ca by half. Since we believe[18] that the majority of the nuclei in the VH group were originally either iron or manganese, the above parameter is the suitable one to use in the first-order approximations to the interstellar propagation calculation.

[18] O. Mathiesen, C. Long, P. S. Freier, and C. J. Waddington Can. J. Phys. **46**, S583 (1968).

Note that because of the increase in $P_{Fe,VH}$ as T decreases, the fraction of nuclei removed from the VH group by nuclear interactions is reduced as the energy decreases. This effect tends to compensate the increasing energy loss that occurs at lower energies and, consequently, the distortion of an initial energy spectrum by passage through matter is less than would otherwise be expected.

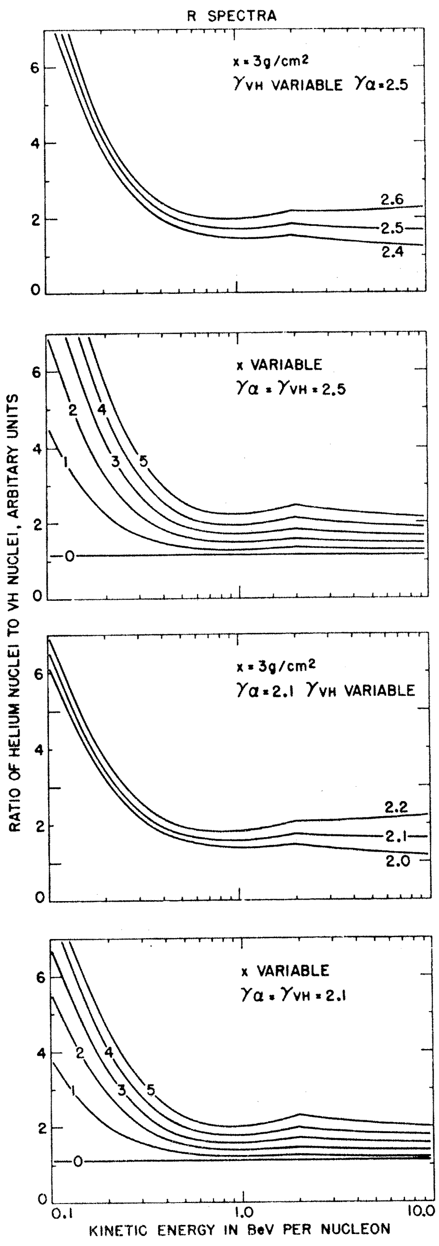

FIG. 6. Calculated values for the ratio of helium to VH nuclei as a function of kinetic energy, assuming power laws in rigidity R at the source. From top to bottom there are shown the effects (a) of varying γ_{VH} when $\gamma_\alpha=2.5$ and $x=3$ g/cm^2; (b) of varying x between 0 and 5 g/cm^2 when $\gamma_{VH}=\gamma_\alpha=2.5$; (c) of varying γ_{VH} when $\gamma_\alpha=2.1$ and $x=3$ g/cm^2; and (d) of varying x between 0 and 5 g/cm when $\gamma_{VH}=\gamma_\alpha=2.1$.

From the value of $P_{Fe,VH}$ and the experimentally observed interaction mean free path of protons in iron, equivalent to $\lambda_{Fe}=2.5$ g/cm^2 of hydrogen, $\Lambda_i(T)$ can then be calculated.

Having obtained an expression for Λ_i we can now use

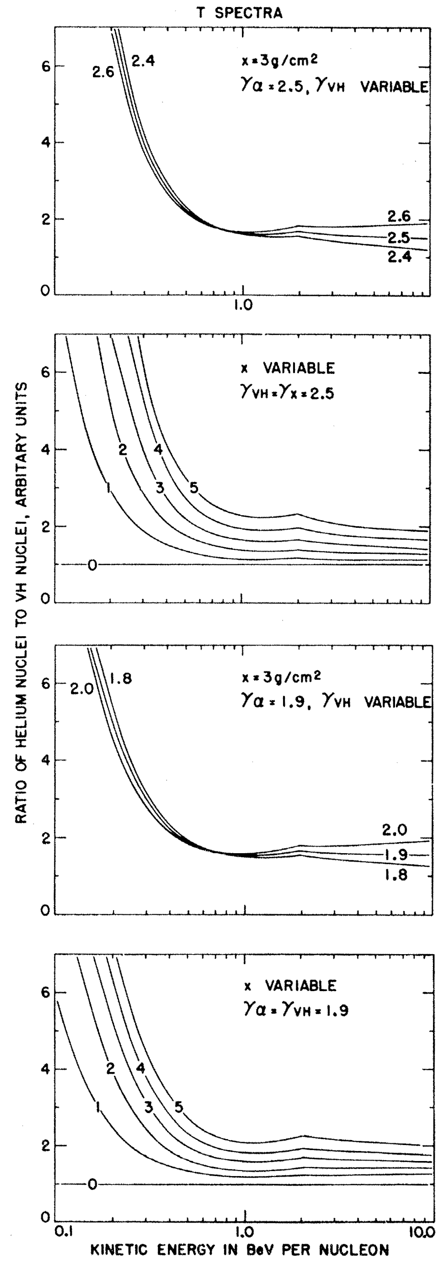

FIG. 7. Similar to Fig. 6 except assuming power laws in kinetic energy T at the source.

Eq. (3) to calculate $j_i(\psi,x)$ both for VH nuclei and for helium nuclei, and thus produce $\Gamma_{\alpha VH}(\psi,x)$ on the basis of various assumptions. Some of the calculated variations of this ratio with kinetic energy are shown in Figs. 6–8, where we have arbitrarily assumed that at the source $C_\alpha = C_{VH}$. For each ψ and for $\gamma_{VH}=2.5$ or the observed value, we show the situation when $\gamma_\alpha=\gamma_{VH}$ and x is varied from 1 to 5 g/cm^2 in 1-g/cm^2 steps and that for $x=3$ g/cm^2 and $\gamma_\alpha=\gamma_{VH}\pm0.1$ or zero.

In making these calculations we assumed that the absorption mean free path for helium nuclei, Λ_α, was 23.6 g/cm^2 in hydrogen and energy-independent. This value may be compared with that of 15 g/cm^2 deduced for the interaction mean free path from machine experiments,[19] and that of between 150 and ∞ proposed by Durgaprasad,[6] who suggests $P_{\alpha\alpha}$ varies between 0.9 and 1.0 with energy. Fortunately all these values are appreciably greater than the amounts of matter that we shall consider the particles to have traversed; consequently, the results are not strongly dependent on the exact value chosen.

Quite clearly none of these calculated curves can come close to fitting that observed experimentally, and it must be necessary to use either a more complex model of the propagation than the simple slab approximation, or a more complex source spectrum. Unfortunately, there is no redundancy in the calculation of $\Gamma_{\alpha VH}$ and hence any model that introduces additional parameters cannot be explicitly solved. However, we can discuss the possible situation in a general way on the basis of the slab-model curves. One quite plausible explanation for the shape of the experimental $\Gamma_{\alpha VH}$ curve is that it is due to a dependence of the path length on the motion parameter. Now, to a first degree of approximation we can study the effects of such a path length dependence by considering that the effect on the value $\Gamma_{\alpha VH}$ is simply given by varying x as ψ varies in the slab-model approximation. Thus, on one of the figures that show the effects of a variable x, the true ratio can be considered by moving from one curve to another as ψ is varied. Such a variation of x with ψ is most likely to be a rigidity effect and so as an example let us consider a source spectrum that is a power law in R with $\gamma_\alpha=\gamma_{VH}=2.5$, Fig. 6. If, for the sake of argument, we assume that the broad plateau in $\Gamma_{\alpha VH}$ for $1.5\lesssim T\lesssim 4.0$ BeV/nucleon corresponds to the traversal of 4 g/cm^2, then in order to account for the rapid decrease between 1.5 and 0.7 BeV/nucleon, x would have to be reduced to 1 g/cm^2 or less. Such a small value of x, even if it remained constant at lower energies, would not be sufficient to explain the increase in $\Gamma_{\alpha VH}$ at still lower energies, making it necessary to invoke an increase in x at these energies. Furthermore, above 4.0 BeV the apparent decline in $\Gamma_{\alpha VH}$ also suggests that x must again decrease although this could be explained by having $\gamma_{VH}<\gamma_\alpha$. Similar conclusions would be drawn for power spectra in T or E. Such a model seems so artificial that one cannot place much reliance on it. An alternative possibility could be that the source spectra are not indeed power laws, but instead γ changes with energy. However, it can be seen

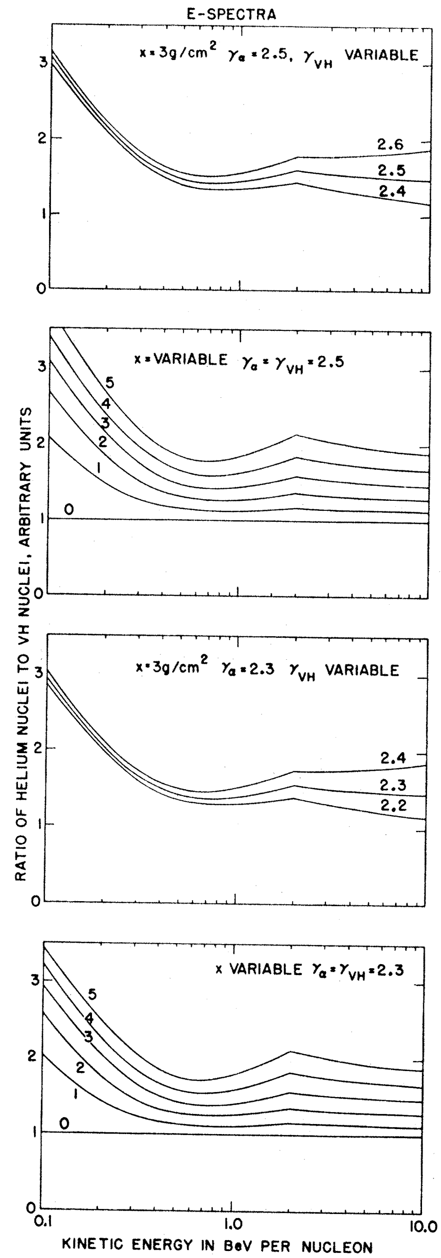

FIG. 8. Similar to Fig. 6 except assuming power laws in total energy E at the source. Note that the arbitrary scale of ratio values in this figure is half those in Figs. 6 and 7.

by comparing the effects of assuming different values of source γ's that $\Gamma_{\alpha VH}$ is not particularly sensitive to quite large changes in γ, so that such an effect can hardly explain the observations. Another possible manner in which one might attempt to explain the experimental curve of $\Gamma_{\alpha VH}$ is to consider the effect of a broad path length distribution at every energy, so that the nuclei observed would represent a sample that had traversed a wide range of x. The effect of such a distribution would be to produce VH nuclei that had on averages traversed less matter than the corresponding helium nuclei, since

[19] F. F. Chen, C. P. Leavitt, and A. M. Shapiro, Phys. Rev. **99**, 857 (1955).

the VH nuclei are more efficiently removed by nuclear interactions. Since the energy spectrum of the helium nuclei is not markedly affected by traversal through a few g/cm^2 of matter, except at the very lowest energies, the variation of the calculated ratio will approximately resemble that typical of a value of x appropriate to the mean of the distribution of the observed VH nuclei. Thus such a model can be treated in the same manner as that just discussed above in which the path length is a function of the energy only. For these reasons we do not believe that the observations can be explained by any reasonable propagation model based on relatively simple source characteristics. Instead we are forced towards the concept of a two-component model of the cosmic radiation, or at least for the VH nuclei.

It was remarked previously that the integral total energy spectrum of the VH nuclei as measured by us showed a remarkable and unique feature in that at low energies there was an apparent break in the spectrum in an upward direction. It is tempting to attribute this break to the appearance of additional particles having a steep energy spectrum that only contributes appreciably to the total distribution at low energies, Since the other components of the cosmic radiation, and specifically the helium nuclei, do not show such an effect, we have assumed that this second distribution is made up purely of VH nuclei, although we cannot exclude the possibility that lighter nuclei show a similar behavior. For example, it has been suggested[20,21] that these nuclei have a second component at low energies that has traversed little if any matter.

The introduction of a two-distribution model for the VH nuclei makes it impossible to attempt to make a unique identification of the nature of the indivdiual spectra. However, we can study the consequences of making certain plausible initial assumptions in an effort to understand the restrictions that can be placed on such a model. For this reason we have considered what would be the consequences of assuming a main, or high-energy, distribution, component 1, which had the form of a power law in ψ at the source and propagated through x g/cm^2 of matter, together with a helium spectra that was only a power law in ψ. On this basis we will determine the form of the second VH distribution, component 2, necessary to explain the data. Due to the presence of solar modulation we will treat the data throughout in terms of the helium to VH ratio, Γ. The observed ratio $\Gamma(\text{obs})$ is related to the differential intensities at that particular energy by

$$\Gamma(\text{obs}) = j_\alpha/[j_{VH}(1)+j_{VH}(2)],$$

where $j_{VH}(1)$ and $j_{VH}(2)$ are the intensities of components 1 and 2, respectively.

Now $j_\alpha/j_{VH}(1)$ represents the calculated ratio $\Gamma(\text{calc})$ found for various parameters in the previous section. Thus the ratio for the second component $\Gamma(2)=j_\alpha/j_{VH}(2)$ is given by

$$\Gamma(2) = \frac{\Gamma(\text{calc})\Gamma(\text{obs})}{\Gamma(\text{calc})-\Gamma(\text{obs})}. \qquad (4)$$

In Fig. 9 we show a plot of $\Gamma(\text{obs})$ together with $\Gamma(\text{calc})$ normalized to $\Gamma(\text{obs})$ at 7 BeV/nucleon, for E, T, and R source spectra with $x=3$ g/cm^2, $\gamma_\alpha=2.5$, and $\gamma_{VH}=2.3$, Examination of the effects of varying x show that these curves are not very sensitive to changes in x, while varying γ_{VH} produces a considerable effect. In particular, making $\gamma_{VH}=\gamma_\alpha$ prevents any sort of reasonable fit to $\Gamma(\text{obs})$ in the 2–7 BeV/nucleon energy range. We have therefore calculated $\Gamma(2)$ from Eq. (4) using the above values of γ_α, γ_{VH}, and x.

The corresponding values of $j_{VH}(1)$, $j_{VH}(2)$ and of $j_{VH}(1)+j_{VH}(2)$, the total differential intensity, which are the intensities outside the solar system, are shown in Fig. 10. These values have all been normalized to the experimentally observed value at 7 BeV/nucleon, where solar modulation is presumably unimportant; hence they may be directly compared with the experimental spectrum. The difference between this experimental spectrum and the total spectra then presumably represents the effects of solar modulation. The resulting modulation functions $\Psi_{VH}(\psi)$ are given in Fig. 11, which shows that while there is essentially no residual modulation present if the component-1 source spectrum R an E spectrum, there is appreciable modulation for an is spectrum and very large amounts of modulation for a T spectrum. The nonsmoothness of the curves for $\Psi_{VH}(\psi)$ presumably represents a residual inconsistency in the model or the data since *a priori* one would expect these functions to show a smooth variation.

The spectra of component 2 all show a bending off at low energies from the apparent slope at higher energies.

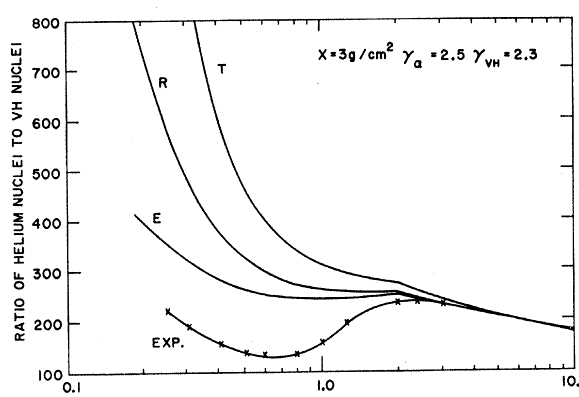

FIG. 9. The variation with energy of the observed ratio of helium to VH nuclei compared with the calculated variation for source E, R, and T spectra, assuming $x=3$ g/cm^2, $\gamma_\alpha=2.5$, and $\gamma_{VH}=2.3$. Al these curves are normalized at 7.0 BeV/nucle on to the experimental value there.

[20] G. C. Comstock, C. Y. Fan, and J. A. Simpson, Astrophys. J. 146, 51 (1966).
[21] S. Biswas, S. Ramaduri, and N. Sreenivasan, Phys. Rev. 149, 1037 (1966).

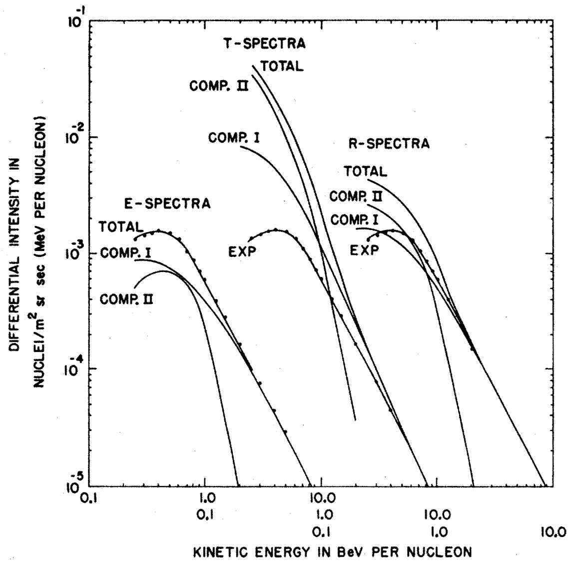

FIG. 10. The differential intensities of VH nuclei plotted on overlapping energy scales under the assumptions of E, T, or R spectra. In each case the experimental spectrum is shown by data —although for the E-spectrum case no curve is shown through those data because it would confuse the illustration of another spectrum. Also shown are the deduced spectra, outside the solar system, of component 1, component 2, and the sum of these two components, the total spectrum. The basis for the derivation of these spectra is given in the text.

These deviations are much too great to be explained in terms of the passage of a source power law through any reasonable amount of matter and we would prefer to attribute them to the source spectra themselves. It is possibly significant to note that the component-2 spectrum derived from a T source spectrum can be well represented over the entire range as an exponential spectrum in rigidity of the form $j_{VH}(2)\ \exp(-R/R_0)$, where $R_0\simeq600$ MV. This spectrum is very similar to that often observed for solar produced particles,[22] although with appreciably smaller values of R_0.

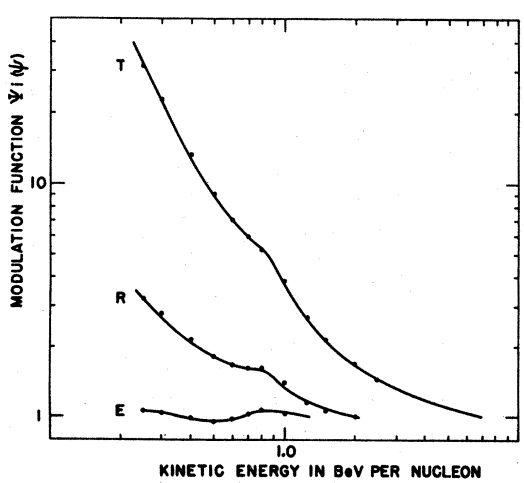

FIG. 11. The modulation functions deduced from the spectra of Fig. 10 as a function of kinetic energy for the case of E, T, or R spectra.

The relative importance of the two components can be seen from an examination of the spectra in Fig. 10. If these components have traversed different amounts of matter, or indeed if they represent a very different form of acceleration mechanism, then they may well show pronounced composition differences. The net effect of such differences will depend on the relative importances; therefore it may be possible to distinguish the source-spectrum model that should be used from an examination of the charge spectrum at low energies. Indeed the previous proposals for two-component models of the lighter nuclei have been based on an analysis of the charge distributions at low energies and require the second component to have traversed little if any matter.

SUMMARY

In the preceding sections we have given what we believe to be plausible reasons for concluding that at least the VH nuclei of the primary cosmic radiation are made up to two main components, one of which dominates at high energy and the other of which is only of importance at low energies. We must emphasize at this point that we do not claim to have *proved* the existence of these two components. Our entire analysis rests on the assumption that the physical processes responsible for accelerating energetic particles tend to produce spectra for both VH and helium nuclei that are representable over the range of energies $10^{9\pm1}$ BeV/nucleon by power spectra. If instead the production spectrum of the VH nuclei has some complex form widely different from a power spectrum and from that of the helium nuclei, then it would be unnecessary to invoke a two-component model. It is also possible, although we believe it is less likely, that some involved and apparently artificial-propagation model could explain the observed results. Finally, there is the never to be neglected possibility that a statistical fluctuation in our experimental results is the cause of our needing to invoke this model. While it is difficult to put a confidence limit on results of this nature, we would remark that to reduce our data to a form capable of explanation by a simple model would require several standard deviation fluctuations of several of the observed values.

If we accept the deduction that there are indeed two components, it is interesting to speculate at least briefly about their nature and origin. First of all it is clear that these two components are not those recently proposed by Burbidge *et al.*,[23] since neither of them can have traversed some 20 g/cm² of matter. Component 2 cannot have done so, since if such an appreciable number of VH nuclei were left after traversing some eight mean free paths of matter, the fragmentation products, with

[22] P. S. Freier and W. R. Webber, J. Geophys. Res. **68**, 1605 (1963).
[23] G. R. Burbidge, W. A. Fowler, and F. Hoyle, Nature **216**, 22 (1967).

their typical steep energy spectrum, would dominate the lower charge groups at low energies. Similarly component 1 cannot have traversed this amount of matter else the charge spectrum at high energies would be essentially the spallation spectrum of *VH* nuclei. Furthermore, the distribution of elements within the *VH* group would be sharply concentrated towards the low end, i.e. towards calcium. It is possibly more tempting to note that in the only example of particle acceleration in a stellar object that we have been able to observe directly, namely, solar particle emission, the resultant spectrum of these particles has been exponential in rigidity. The observation that with the correct assumptions component 2 can be similarly represented suggests that these nuclei might indeed be the product of general stellar activity throughout the galaxy. The fact that these particles appear to be mostly *VH* nuclei—which is the reverse of the case for solar particles—would then suggest that the principal contribution to this particle population came not from main-sequence stars but from special classes of sources such as magnetic variables, novas, and even supernovas, all of which can be rich in iron-type nuclei. It has been suggested[24] that acceleration mechanisms in such objects might have the property of accelerating only *VH* nuclei. Since these particles have small gyroradii it is quite possible that they are of rather local origin, originating only in the nearby regions of our galactic arm. Energetically such an assumption seems tenable since the energy density in these particles is small. Most of the energy is, of course, carried by the component-1 particles, and these then must be attributed to an entirely different production and propagation history. Whether they are predominantly of galactic or extragalactic origin would seem quite indeterminable, although it is tempting to try and explain the behavior of the very-high-energy ($\sim 10^{18}$eV) end of the spectrum as a transition from a galactic to an extragalactic population, which would imply a galactic origin for the component-1 nuclei.

[24] A. A. Korchak and S. I. Syrovatskii, Dokl. Akad. Nauk SSSR **5**, 792 (1958) [English transl.: Soviet Phys.—Doklady **3**, 983 (1959)].

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Equivalence Principle and Motion of a Gyroscope

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An incorrect application of the equivalence principle to the motion of a gyroscope in a uniform gravitational field is corrected, and a particular formulation of the equivalence principle is discussed. In applying the principle of equivalence, the entire physical situation, including the observer's reference frame, should be subjected to an equivalent acceleration in the absence of the field, and not just the system under observation. It is shown how the equivalence principle and the Thomas precession can be used to obtain the precessional angular velocity of a moving gyroscope in a uniform gravitational field. The result is confirmed by means of general relativity.

THE purpose of this paper is to correct an application of the equivalence principle to the motion of a gyroscope in a uniform gravitational field, and to discuss a particular formulation of the equivalence principle. Schiff has considered an apparent paradox which arises in the motion of a gyroscope.[1] If one considers a gyroscope moving quasistatically in the gravitational field external to a nonrotating, spherically symmetric mass, then one might at first expect that, when the distance of the gyroscope from the central body and the mass of the central body are simultaneously increased while keeping the value of the gravitational acceleration g in the vicinity of the gyroscope constant, the precessional angular velocity of the spin axis of the gyroscope will approach the value it has in a uniform gravitational field of gravitational acceleration g. Schiff pointed out that the expected limit is not approached, and resolved this apparent paradox by showing that if the precession in the Schwarzschild field is to be significant, then the angle of divergence between the gravitational field lines from the central body can not be ignored.

The precessional angular velocity of the spin axis of a gyroscope moving quasistatically in the gravitational field of the central mass is given, to lowest order, by[1]

$$\mathbf{\Omega}_A=(2g/c^2)(\hat{r}\times\mathbf{v}), \tag{1}$$

where $\hat{r}=\mathbf{r}/r$, $g=Gm/r^2$, $\mathbf{r}$ is the position vector of the gyroscope with respect to the center of the attracting body of mass m, G is the Newtonian constant, $\mathbf{v}$ is the velocity $d\mathbf{r}/dt$, and c is the speed of light. A gyroscope in quasistatic motion is one moving at nonrelativistic velocity and supported against the gravitational field

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[1] L. I. Schiff, Nuovo Cimento **17**, 124 (1960).