IS THE RESIDENCE TIME OF COSMIC RAYS IN THE GALAXY ENERGY-DEPENDENT?

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Recent observational data indicate that (1) the spectra of primary cosmic ray nuclei in the energy range 3-50 GeV/nucleon become progressively flatter with increasing Z, and (2) the ratios of the fluxes of secondary nuclei to that of their parents are decreasing functions of energy in the same range.

We present here a model to explain these observations. We assume that a. all cosmic ray nuclei are accelerated to identical spectra, $\sim E^{-2.6}$;

b. subsequent to the acceleration process the cosmic rays diffuse away from the sources, in an energy dependent fashion, and
c. further transport in the interstellar medium and leakage from the Galaxy are energy-independent.

Abundances and spectral shapes calculated on this basis are in agreement with the observations, if the effective grammage during diffusion in the neighborhood of the sources varies as $\sim 1.7 \exp(-E/7.85$ Gev/n) g cm^{-2}, with a subsequent traversal of ~ 1.6 g cm^{-2} of interstellar matter during their residence time in the Galaxy. The present model is also consistent with the early observations at higher energies using giant emulsion stacks, leads to no serious conflict with limits on anisotropy at high energies and does not aggravate the power requirement from the cosmic ray sources. Finally, we point out a test to distinguish between the various models.

1. Introduction. Recent observational data on the composition of cosmic ray nuclei in the energy range of ~3-70 GeV/nucleon (Juliusson et al., 1972; Smith et al., 1973; Ormes and Balasubrahmanyan, 1973; and Webber et al., 1973) have given rise to much speculation that the spectra of different cosmic ray nuclei are different at the sources and that the mean residence time of cosmic rays in the galaxy is energy dependent. In this paper we present a simple, elegant solution to the cosmic ray spallation and transport equation and show that

1) One can explain all the observational data on the spectra and the relative abundances of secondaries and primaries by assuming that the sources accelerate all nuclei to identical speeds, that cosmic rays subsequently leak from the sources in an energy dependent fashion, and that the subsequent leakage from the galactic volume is energy independent.

2) The derived source abundances are closely correlated with the so-called universal abundances, lending support to our earlier suggestions (Cowsik, 1971; Cowsik and Price, 1971; Price et al., 1971) that the bulk composition near the cosmic ray sources need not be different from the universal abundances.

2. Cosmic Ray Propagation and Spallation. Let cosmic rays be generated in a volume v_α, where they will also suffer fragmentation and leakage loss. If we assume that the boundaries of this volume have a high reflectivity for the

cosmic rays, we can characterize the leakage from this volume by a mean leakage time τ_α. Then the equation governing the density of nuclei in v_α is

$$\frac{dN_i}{dt} = -\frac{N_i}{t_\alpha} - N_i b_i \beta c n_H + \sum_{j>i} \sigma_{ij} \beta c n_H N_j + S_i, \tag{2.1}$$

Here N_i = density of nuclei of the i^{th} kind

$\frac{1}{\tau_\alpha}$ = probability of leakage from the volume per unit time

b_i = total cross-section for break up

βc = velocity of the nuclei $\approx$ c

n_H = number density of target atoms in the volume

σ_{ij} = partial cross-section for break up of nuclei of the j^{th} kind into the i^{th} kind, and

S_i = average source strength per unit volume.

Consider a diagonal matrix D_{ii} with elements $(\frac{1}{\tau} + \beta c n_H b_i)$ and a triangular

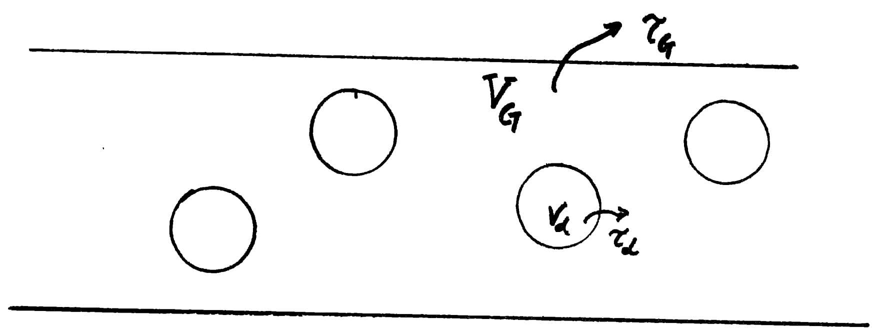

Fig. 1.

matrix T_{ij} with elements $\sigma_{ij}\beta c n_H$ for j>i and zero for j≤i. Defining $M_{ij}=T_{ij}-D_{ij}$ we can write

$$\frac{dN_i}{dt} = S_i + M_{ij} N_j \tag{2.2}$$

where N_i and S_i now are column vectors and summation over repeated indices is implied throughout this paper unless otherwise stated. This equation can be solved for an arbitrary S_i by making a similarity transformation of the equation 2.2 so that M_{ij} is diagonalized. The solution of the cosmic ray propagation equations under conditions of steady state is almost trivial when viewed in the notation of equation 2.2. Assuming steady state, $\frac{dN_i}{dt} = 0$, so that $$S_i = -M_{ij}N_j \qquad (2.3)$$

with an inverse $$N_j = -(M)^{-1}_{ji}\ S_i \qquad (2.4)$$

Now consider a set of volumes v_α embedded in a much larger volume V_G as shown in Fig. 1. Let the volume V_G be characterized by a leakage time τ_G. It is clear that the cosmic rays leaking into V_G from v_α will be the source for cosmic rays in V_G. This source strength is

$$q_i = \frac{1}{V_G}\frac{v_\alpha}{\tau_\alpha} N_j \approx \frac{\nu}{V_G}\frac{<v>_\alpha}{<\tau>_\alpha}\ N_j \qquad (2.5)$$

Here q_i = source strength in the volume V_G

ν = number of volumes of the kind discussed above embedded in V_G and the brackets around the variables indicates their averages over the ensemble.

Now let a matrix m_{ij} describe the fragmentation and leakage in volume V_G. This matrix is similar to the matrix M_{ij} but uses parameters relevant to volume V_G. As before, we have for the steady state density of nuclei in V_G

$$n_j = -(m)^{-1}_{ji}q_i \equiv -(m)^{-1}_{ji}\ \frac{\nu}{V_G}\frac{<v>_\alpha}{<\tau>_\alpha} N_i = (m)^{-1}_{ji}\ \frac{\nu}{V_G}\frac{<v>_\alpha}{<\tau>_\alpha}(M)^{-1}_{ik}\ S_k \qquad (2.6)$$

and conversely

$$S_j = \frac{V_G<\tau>_\alpha}{\nu<v>_\alpha}\ M_{jk}m_{k\ell}n_\ell \qquad (2.7)$$

The equations 2.3, 2.4, 2.6 and 2.7 show many interesting features. First of all, they are simple algebraic equations and the problem of solving either for the equilibrium cosmic ray composition or the source composition becomes extremely simple and straightforward. One does not have to go through the painful and onerous program of numerically integrating the equations, in a manner which has been customary in the field of cosmic ray physics. Concentrating now on equations 2.6 and 2.7 we see that the matrices M and m degererate into diagonal forms when the fragmentation can be neglected. The diagonal terms are proportional to $\frac{1}{<\tau>_\alpha}$ and $\frac{1}{\tau_G}$ respectively. Therefore in conditions under which nuclear fragmentation can be neglected the equilibrium densities n_j in the volume V_G are independent of $<\tau>_\alpha$.

3. The Model. We make the customary assumption that the cosmic ray sources are distributed quasi-uniformly in the Galactic volume. It is natural to expect that there might be an envelope of shocked gas around the sources as is seen in the case of supernova remnants and pulsars. In our model we identify with v_α the volumes defined by these envelopes. The volume V_G is identified with the Galactic disc.

The escape probability ($\sim\frac{1}{<\tau>_\alpha}$) from the source envelope is assumed to be dependent upon the particle energy. This is reasonable in analogy with the behavior of cosmic rays in the solar modulation region. If the length scales of magnetic inhomogeneities are comparable with the gyroradii of the GeV cosmic rays, the scattering mean free path and consequently diffusive escape will become dependent on particle energy. On the other hand, in the interstellar space, the length scales for the magnetic inhomogeneities are $\sim 10^{18}$ cm and become comparable to the gyroradii of cosmic rays of $\sim 10^{15}$ eV. Indeed we notice in the air shower data evidence indicating strongly energy dependent leakage at these high energies. Therefore in our present discussion, which is confined to much smaller energies, it is reasonable to assume that the leakage from the Galaxy characterized by τ_G is independent of particle energy.

We can immediately see several consequences of the model described above. For protons and other low Z primary nuclei the effect of nuclear interaction during their transport can be neglected. Therefore their equilibrium abundances (n_j) in the Galactic volume are essentially independent of their leakage lifetimes ($<\tau>_\alpha$) from the source envelopes. In such cases the measured cosmic ray spectra truly reflect the spectra accelerated by the sources (S_j). But the spectra of the primary high Z nuclei would be modified because of fragmentation in the volume v_α. Should we assume that $<\tau>_\alpha$ is a decreasing function of particle energy then at high energies, the effect of fragmentation in the sources can be neglected even for high Z nuclei and their equilibrium spectra in the Galactic volume will approach the source function. At low energies the nuclei would be depleted by fragmentation and their equilibrium spectra would be flatter than their source spectra. Now consider the equilibrium abundances of nuclei like Li, Be, and B which arise in cosmic rays predominantly through the breakup of heavier nuclei. With $<\tau>_\alpha$ decreasing with energy, the fraction of heavy nuclei which fragment in the source envelope to generate these lighter secondaries will decrease with energy and therefore the secondary to primary ratio will decrease with energy. However, this ratio will be expected to reach a constant value at high energies because of spallation in the Galactic volume V_G, τ_G being energy independent. Another consequence of the constancy of τ_G is that this model would predict an anisotropy which does not increase with increasing energy so that it is easy to chose τ_G such that the experimental limits on the anisotropy (Speller et al., 1972) are left inviolate. We find that by chosing

$$\begin{aligned} &\beta c n_H(\text{Galaxy})\tau_G \approx 1.6 \text{ g cm}^{-2} \\ \text{and} \quad &\beta c n_H(\text{source})<\tau>_\alpha \approx 1.7 \exp(-E/7.85 \text{ GeV/n}) \text{g cm}^{-2} \end{aligned} \tag{3.1}$$

we can generate a very good fit to all experimental data. The procedure adopted in this fitting program is discussed in the next section.

4. <u>Comparison with Experiment</u>. We start by using the composition data of Webber et al. (1972) to generate the column vector n_j. We average the relative abundances of nuclei with very low flux levels, such as fluorine, phosphorous, etc. with the data quoted by Simpson (1971) to improve the statistical accuracy. Using equation (2.7) and the expressions for τ_G and $<\tau>_\alpha$ from equation (3.1), we derive S_j representing the abundances of the various nuclei at a source. These are shown in Fig. 2. We show the abundances as a ratio of the universal abundances (Cameron, 1967) only for those nuclei which have less than 15% contamination by secondaries at observation. One can notice the very striking correlation between the universal abundances and the composition of cosmic rays accelerated by the sources. Here we wish to reiterate our point of view that cosmic rays can arise from regions where the elemental abundances are akin to the universal abundances with a preferential acceleration of heavy nuclei as in the case of solar flares (Price et al., 1971, Crawford et al., 1972).

We wish to point out two other features in Fig. 2. The oxygen abundance seems to be about a factor of 2 lower than universal. This feature has been recognized for sometime, however we know of no sound explanation for it. The apparent low abundance of the sulphur nuclei is most probably explained as due to an erroneous estimation of its universal value. The same discrepancy has been noticed in solar flare accelerated nuclei (Sullivan et al., 1973).

The reason that we chose to plot only nuclei which were not contaminated by secondaries is because of the large errors involved in the estimation of their source abundances. This can be made clear with an example. Let us say one has measured ~50 deuterium nuclei to quote a flux of $(3.6\pm0.5)\times10^7$ on scale where silicon is 10^6. Our calculation gives us a flux of 3.5×10^7 as due to secondaries. Even if we assume that all the cross-sections and all the rest of the fluxes were exactly known, we get $(0.1\pm0.5)\times10^7$ as the direct contribution from the source. Therefore our prescription for obtaining the source abundances of nuclei dominated by secondaries is to interpolate their ratio to the universal abundance from Fig. 2 and to use the product of this ratio.

Having derived the source composition S_i, we assume that all the cosmic ray nuclei are accelerated to identical spectra $\sim S_i E^{-2.6}$ and calculate the abundance near the earth $N_i(E)$ and compare it with observational data in Figs. 3, 4 and 5. It is seen that the abundances of all the nculei are well reproduced by our calculations. We wish to particularly emphasize the abundances of Na, Al, and P have been reproduced with their abundance at the source relative to S_i and Fe no more than that implied by universal abundances. Cartwright (1971) and Simpson (1971) were unable to generate the abundances of these elements without assuming a substantially higher abundance of these elements in the source than that which was implied by universal abundances. We do not know exactly the cause of this discrepancy. It might have arisen because of neglect of tertiaries in the propagation calculations. In steady state the importance of any nucleus as a source of secondaries depends merely on its equilibrium abundance rather than on its abundance at the source. In any case, the procedure described in Sec. 2, all the multiple interactions have been taken into account in a straightforward manner and without any complication. We wish to emphasize that the procedure adopted by us involves a one step algebraic process to either propagate from source abundance to the observed cosmic ray abundances or derive the source composition from observation. It is therefore free from errors that might accumulate through stepwise solution of the propagation equation.

The comparison of the calculated ratios $\frac{\text{Li+Be+B}}{\text{C+O}}$ with the measured ratios as a function of energy in Fig. 4 shows adequate agreement. The calculated energy spectra of the various nuclei agree well with observation, as seen from Fig. 5. The Fe spectrum at relatively low energies has become flatter than the primary accelerated spectrum ($\gamma=2.6$) because of spallation in source regions. But at high energies where the accelerated cosmic rays escape readily from the sources, the spectrum of all nuclei reaches to their accelerated value with $\gamma=2.6$. This behavior of the Fe spectrum is supported by the fact that the work on the huge emulsion stacks by the Japenese, Bristol and the Tata groups have shown no substantial changes in composition up to total energies of 10 TeV.

An explicit way to test the constancy of τ_G at energies ~10 GeV is that the positron spectrum in the region of ~10 GeV will have to have the same spectral slope as protons at ~100 GeV. At such high energies protons escape readily from the sources so that most of the positrons would be generated in the interstellar space. In the models suggested by other authors in which τ_G is assumed to be energy dependent the energy dependence of the positron flux would be similar to the flux of other secondaries such as Li, Be and B.

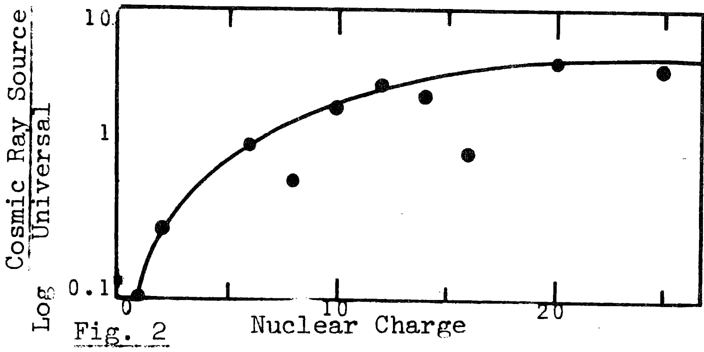

Fig. 2

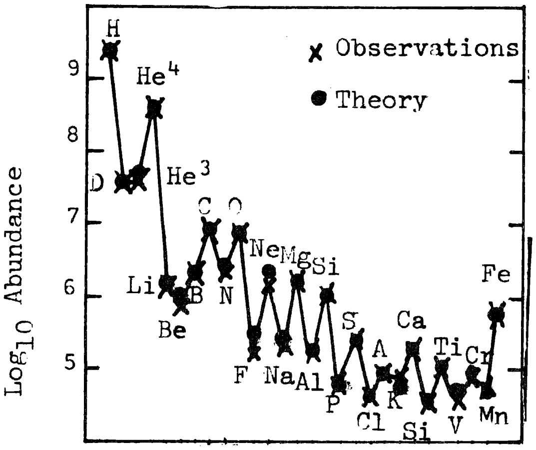

Fig. 3

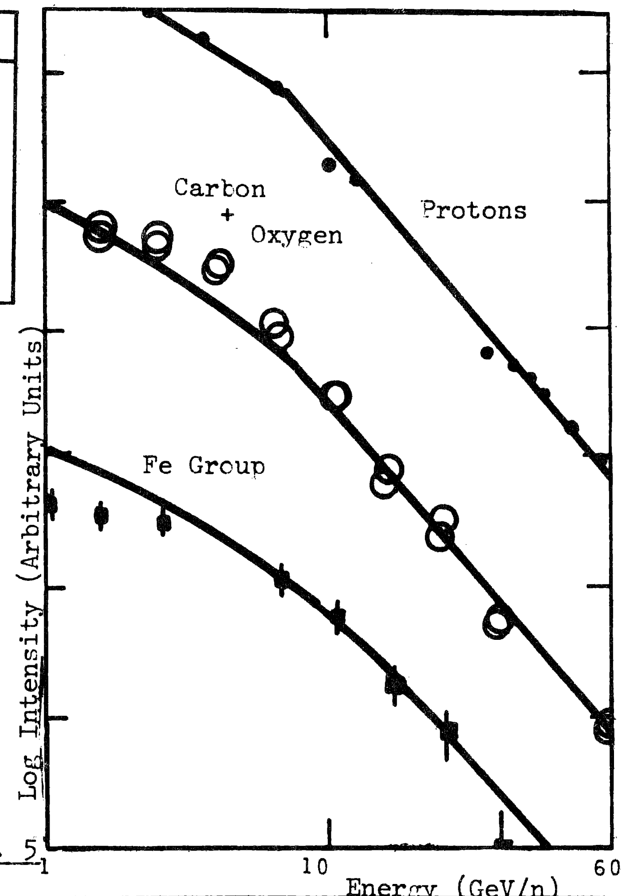

Fig. 5

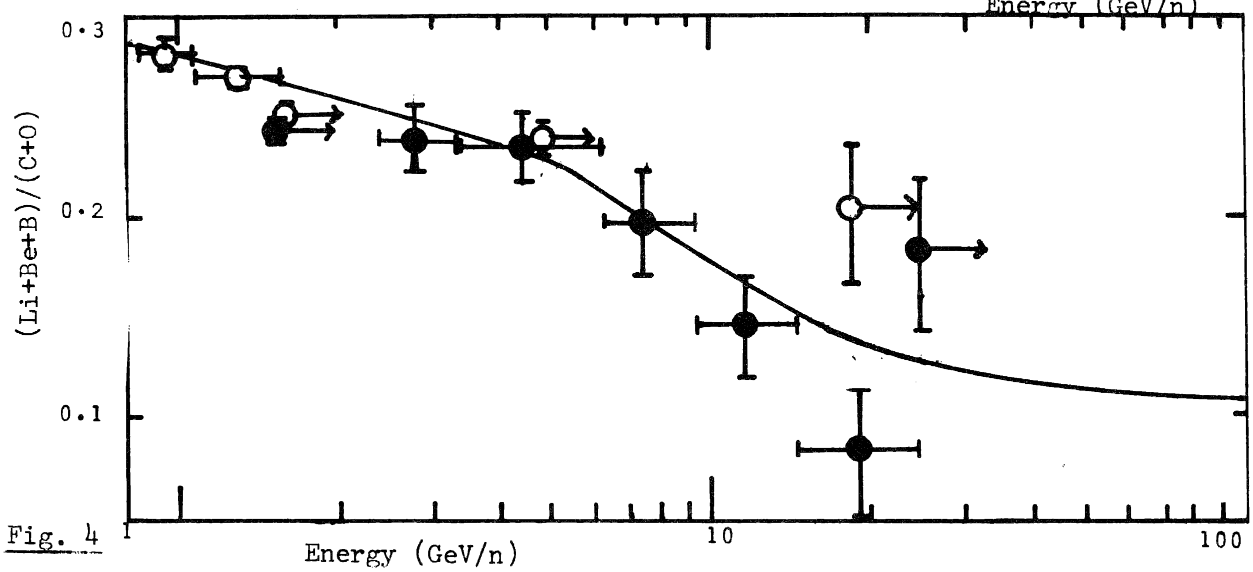

Fig. 4

5. Summary. We have presented an elegant and simple solution to the cosmic ray propagation equation and shown that 1) the cosmic ray abundances soon after acceleration are closely related to the universal abundances. 2) We propose a model in which the leakage from the source level is energy dependent governed by the equation $\beta c n_H(\text{source})\langle\tau\rangle_\alpha = 1.7\exp(-E/7.85\text{GeV/n})$g cm^{-2}, while on the other hand the leakage from the galactic volume is energy independent with $\beta c n_H(\text{galaxy})\tau_G \approx 1.6$ g cm^{-2}. With this model all the observational data is adequately explained but with fewer parameters than most others. Also in our model $\tau_G \approx 2$ million years with $n_H(\text{Galaxy}) \approx 0.5$, leaving the anisotropy limits inviolate.