Cosmic-ray reacceleration in the interstellar medium

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After relativistic cosmic-ray particles of energy $E \gtrsim 1$–3 GeV/nucleon leave their compact sources they may undergo a weak, distributed reacceleration and spatial diffusion as they interact with interstellar MHD turbulence. When weak reacceleration is included, a galactic diffusion model for cosmic-ray propagation and nuclear fragmentation closely fits observational data for cosmic rays and the interstellar medium.

1. INTRODUCTION

Galactic cosmic rays of energy $E \gtrsim 1$-3 GeV/nucleon cannot be accelerated by some mechanism distributed throughout the interstellar medium and accompanied by nuclear fragmentation in the interstellar gas. In fact it has been shown[1-3] that cosmic-ray acceleration accompanying nuclear fragmentation would cause the density ratio s/p of secondary nuclei (fragments) and primary nuclei to increase with energy, contrary to the observational evidence.[4,5]

Nevertheless, relativistic particles accelerated in compact sources might well experience some further acceleration after they escape into the ISM. There are certain indications that such "reacceleration" could play a significant role for low-energy cosmic rays.[6,7] At energies $E \gtrsim 1$-3 GeV/nucleon even a comparatively weak reacceleration might distort the measured s/p ratio. This is the outcome of numerical calculations by Simon et al.,[8] who have investigated the possibility of weak reacceleration by shock waves in a simple homogeneous (leaky box) model for cosmic-ray propagation in the Galaxy (other work of this character was reported at the Moscow cosmic-ray conference[9,10]).

Actually it seems almost inevitable that cosmic rays will undergo a certain amount of acceleration in the ISM. If scattering of relativistic particles by magnetohydrodynamic turbulence is responsible for their spatial diffusion, then the scattering should also serve statistically to accelerate the particles. If one adopts a standard set of ISM parameters, one finds that the effective acceleration time for $E \sim 1$ GeV/nucleon particles will be only slightly longer than the time scale for accelerating cosmic rays in the Galaxy.[11,12] Such statistical acceleration becomes less efficient as the particle energy increases.

In this letter we offer an analytic solution to the problem of nuclear cosmic-ray fragmentation in a galactic diffusion model when allowance is made for particle reacceleration.

2. DIFFUSION MODEL WITH WEAK REACCELERATION

With suitable simplifying assumptions (for details see Chap. 3 of the book by Berezinskii et al.[13]) the problem of the diffusion and fragmentation of the nuclear cosmic-ray component in the Galaxy reduces to that of solving the equation of transfer for a single species of primary nuclei:

$$-D(p)\frac{\partial^2 f(z,p)}{\partial z^2} + n(z)v\sigma f(z,p) - \frac{1}{p^2}\frac{\partial}{\partial p}p^2\varkappa(p)\frac{\partial f(z,p)}{\partial p} = Q(z,p). \tag{1}$$

A steady-state, one-dimensional diffusion model is considered here; f(z, p) represents the distribution function of cosmic rays with respect to the absolute value p of the momentum, so that the nuclei of the species in question have a number density $N(>p) = \int fp^2dp$. The functions D(p), $\kappa(p)$ denote the diffusion coefficients in coordinate and momentum space, respectively; n(z) is the number density of interstellar gas particles, the function Q(z, p) describes the distribution of cosmic-ray sources, $v \sim c$ is the velocity of the nuclei, and σ is the total cross section for inelastic interactions of the cosmic-ray nuclei with interstellar gas nuclei (we regard σ as independent of energy).

We shall assume that cosmic-ray diffusion in the Galaxy is limited to the zone $|z| \le H$, where H is the diameter of the galactic halo, and that the cosmic-ray sources are distributed uniformly over the region $|z| \le h_s$ ($h_s \ll H$), so that Q(z, p) takes the form q(p). All the gas will be regarded as concentrated in a disk $|z| \le h_g$ ($h_g \ll H$) of constant density n_g. (The gas in the halo will have no appreciable effect on cosmic-ray fragmentation if its density $n_h \ll n_g h_g/H$.)

In solving Eq. (1) we invoke the condition that cosmic rays escape freely from the outer halo boundary into intergalactic space, where the cosmic-ray density is negligible; in other words we assume that $f(|z| = H) = 0$. At the surfaces $|z| = h_g$, h_s we require that both f and $\partial f/\partial z$ be continuous.

The last term on the left in Eq. (1) will be treated as a small perturbation (as mentioned in Sec. 1, the s/p ratio would otherwise be observed to increase with energy). We shall consider cosmic rays of energy $E \gtrsim 1$-3 GeV/nucleon.

To describe the diffusion output, nuclear fragmentation, and acceleration of cosmic rays let us introduce the effective reciprocal time scales ν_ℓ, ν_n, ν_a, respectively, as quantities measuring the relative contribution of the corresponding terms in Eq. (1). Expanding the distribution function in

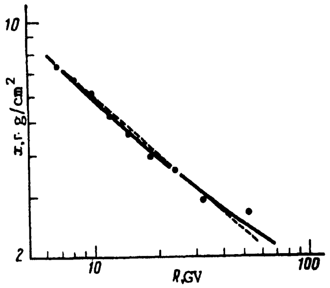

FIG. 1. The average thickness x of matter traversed by cosmic rays as a function of the particle rigidity R = pc/q, where q denotes the charge on the particles. Dots represent the points determined by Margolis [14] from the observed abundance of secondary nuclei. The dashed line indicates the best fit that can be achieved by the standard leaky-box model without reacceleration: $x(R) = 8.2(R/R_0)^{-0.56}$ g/cm^2, with R_0 = 5.5 GV. The solid curve demonstrates the improvement obtained for the model proposed in this letter: $x_{eff}(R) = 4.2(R/R_0)^{-0.33}[1 + (R/R_0)^{-0.66}]$ g/cm^2.

the small parameter $v_a/(v_\ell + v_n) \ll 1$,

$$f(z, p) = f_0(z, p) + f_1(z, p) + \dots, \tag{2}$$

where $f_k \sim O[v_a/(v_\ell + v_n)^k]$, we obtain from Eq. (1) the two lowest-order equations

$$-D\frac{\partial^2 f_0}{\partial z^2} + nv\sigma f_0 = Q, \tag{3}$$

$$-D\frac{\partial^2 f_1}{\partial z^2} + nv\sigma f_1 = \frac{1}{p^2}\frac{\partial}{\partial p}p^2\varkappa\frac{\partial f}{\partial p}. \tag{4}$$

Upon solving Eqs. (3), (4) for a height z = 0 (a point close to the sun) subject to

$$n_g v\sigma h_g^2 D^{-1} \ll 1, \tag{5}$$

and neglecting the high-order terms in h_s/H, h_g/H, we find that

$$f(z=0, p) \approx f_0 + f_1 \approx \left[1 + \frac{1}{3}(\nu_l + \nu_n)^{-1}\frac{1}{p^2}\frac{\partial}{\partial p}p^4\nu_a\frac{\partial}{\partial p}\right]\frac{\bar{Q}}{\nu_l + \nu_n}. \tag{6}$$

Here

$$\nu_l = DH^{-2},\ \nu_n = n_g h_g H^{-1} v\sigma,\ \nu_a = \varkappa p^{-2},\ \bar{Q} = qh_s H^{-1}. \tag{7}$$

The second term in brackets in Eq. (6) is assumed small, producing weak reacceleration.

If v_a = 0 there is no reacceleration, and Eq. (6) is equivalent to the corresponding formula for a leaky-box model in which the average thickness of matter traversed by cosmic-ray particles in the ISM is

$$x_l = n_g h_g H v D^{-1}. \tag{8}$$

Equation (5) then implies that $x_\ell\sigma \ll Hh_s^{-1}$. The equivalence between the diffusion and leaky-box models will break down, however, for rapidly decaying radioactive nuclei or major energy losses (Berezinskii et al.,[13] Chap. 3).

Although Eq. (6) should hold true for any diffusion and acceleration processes, we shall assume that the spatial diffusion and the acceleration of cosmic rays result from identical processes of particle scattering in a turbulent medium. Suppose that weak MHD waves propagate preferentially along the magnetic field and have a spectral energy density w(k) $\propto k^{-4+\mu}$, where k is the wave number. Then cyclotron resonance scattering of the particles will give[12]

$$\nu_a/\nu_l = \frac{3}{4\mu(4-\mu^2)(4-\mu)}\left(\frac{v_a x_l}{v n_g h_g}\right)^2,\quad \nu_l^{-1} \propto \nu_a \propto p^{-\mu}, \tag{9}$$

where v_a is the Alfvén velocity. Recognizing that $v_a/(v_\ell + v_n) \ll 1$ and taking $\bar{Q}(p) \propto p^{-\gamma_s-2}$, $v\sigma \approx c\sigma$ = const, we obtain

$$f(0, p) \approx \nu_l^{-1}\bar{Q}(1 + \alpha\nu_a/\nu_l)\cdot[1 + \sigma x_l(1 + \alpha\nu_a/\nu_l)]^{-1}, \tag{10}$$

where

$$\alpha = \frac{1}{3}[(\gamma_s + 2)(\gamma_s + \mu - 1) + (2\gamma_s + 1)\mu\nu_l(\nu_l + \nu_n)^{-1} + 2\mu^2\nu_l^2(\nu_l + \nu_n)^{-2}]. \tag{11}$$

Equation (10) implies that in the weak-reacceleration case the problem of nuclear fragmentation in the diffusion model will again reduce to a simple leaky-box model with an effective thickness

$$x_{eff} \approx x_l(1 + \alpha\nu_a/\nu_l), \tag{12}$$

where the second term in parentheses is assumed to be small. Accordingly we can also evaluate the effective spectral index

$$\mu_{eff} = -\frac{d\ln x_{eff}}{d\ln p} \approx \mu\left(1 + 2\alpha\frac{\nu_a}{\nu_l} - \frac{\nu_a}{\nu_l}p\frac{d\alpha}{dp}\right). \tag{13}$$

It is the quantity x_{eff} which measures the contribution of diffusive escape and reacceleration to the overall balance of cosmic rays in the Galaxy, as well as the relative abundance of secondary nuclei.

The expressions (12), (13) bear out the numerical results[8] that indicate a stronger energy dependence for the s/p ratio in models with reacceleration of cosmic rays in the ISM than in the standard model without reacceleration.

To illustrate this important point let us suppose that x_ℓ = 5 g/cm^2 (in a pure hydrogen medium) for E_0 = 2 GeV/nucleon and μ = 0.3; then $x_\ell = 5(p/p_0)^{-0.3}$ g/cm^2. Using Eqs. (8), (11), (12), we find that

$$x_{eff} \approx 5\,(p/p_0)^{-0.3}\,(1 + 0.5\,(p/p_0)^{-0.6}). \tag{14}$$

if we set $v_a = 3\cdot10^6$ cm/sec, $n_g h_g = 3\cdot10^{20}$ cm^{-2}, γ_s = 2.4. Then Eq. (13) will give

$$\mu_{eff} \approx 0.3\,(1 + (p/p_0)^{-0.6}). \tag{15}$$

Thus x_{eff}(2 GeV/nucleon) ≈ 1.5 x_ℓ(2 GeV /nucleon) 8 g/cm^2; the effective index μ_{eff}(2 GeV/nucleon) ≈ 0.6, and μ_{eff}(≳20 GeV/nucleon) ≈ 0.3. Figure 1 demonstrates how well the model we have described reproduces the theoretical particle-rigidity dependence of the average thickness of matter traversed by cosmic rays, as derived from the observational data.

The decrease in the μ_{eff} value with energy, a feature typical of the reacceleration model, seems to be exhibited by data on the relative abundance of secondary nuclei.[15,16]

3. CONCLUDING REMARKS

When provision is made for weak reacceleration of relativistic particles in space, the conventional power-law behavior of x(p) is altered and good agreement is obtained with measurements of the contribution of secondary nuclei to the cosmic-ray composition. Such reacceleration occurs in a natural way as cosmic-ray particles diffuse through the turbulent ISM. Its influence can also explain the weakness of the observed energy dependence of the cosmic-ray anisotropy for energies from $5\cdot10^{11}$ to 10^{14} eV: a relation $D \propto p^{\mu}$ is required with[17] $\mu \approx 0.3$, compared with the stronger $x_{eff}(p)$ dependence for $E \sim 1$-30 GeV/nucleon. For a standard set of ISM parameters we have concluded (Sec. 2) that $v_a/v_\ell \approx 0.2$ for $E = 2$ GeV/nucleon, and this value will provide the observed energy dependence of the secondary-nucleon abundance if $v_\ell \propto v_a^{-1} \propto p^{-\mu}$ with $\mu \approx 0.3$. It is worth noting that this μ-value corresponds to a Kolmogorov spectrum for the random galactic magnetic field.

If as E drops below 2 GeV/nucleon the v_a/v_ℓ ratio increases by a power law with a constant spectral index 2μ, then our present approach, based on perturbation expansions in the small parameter $v_a/v_\ell \ll 1$, will become inapplicable to low-energy cosmic rays. One would then expect the spectra of cosmic-ray nuclei, protons, and electrons to flatten out significantly for rigidities below about 0.4 GV, due to strong reacceleration of those particles. The limiting form of the differential cosmic-ray spectrum would then be $N(p) \propto vp^{-1+\mu}$ (neglecting ionization and other possible energy losses), and the secondary/primary density ratio ought to diminish toward lower energies. We intend to examine this case more fully in a separate paper.

As for the weak-reacceleration case, we emphasize that the expressions (10) and (12) are valid for any model of cosmic-ray reacceleration in the Galaxy, with arbitrary functions $v_a(p)$, $v_\ell(p)$, but suitable changes should be made in the explicit formula (11) for the coefficient α.

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A solitary vortex sustained by the nonuniform rotation of a galaxy disk

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Based on the shallow-fluid model a numerical solution is obtained for a solitary vortex superimposed on the differentially rotating disk of a galaxy. The vortex parameters are determined by the differential rotation curve. Typical vortices are azimuthally elongated anticyclones.

In certain disk-type galaxies the kinematics of the flat subsystem exhibits localized departures from circular motion that are unrelated either to the spiral arms, to bar structure, or to close companion galaxies. Zasov and Kyazumov[1] cite one example of such a localized disturbance: in NGC 157 it takes the form of an oval region about 4 kpc in diameter drifting as an intact body, compared with the disk radius of ≈12 kpc. Other such cases have been encountered as well.[2,3]

We have suggested in a previous letter[4] that formations of this kind might represent vortices that can occur in the gas–dust component of a disk galaxy, much like the eddies encountered in giant-planet atmospheres[5] or in shallow rotating fluids.[6,7] We investigated perturbations[4] that alter the thickness of a rigidly rotating galaxy disk; the solution we obtained was a solitary circular vortex drifting at low velocity relative to the disk.

In planetary atmospheres, however, the vortices are superimposed on a background of nonuniform (zonal) flows which appear to maintain the eddies. A vortex may persist within a flow of fairly smooth profile that is stable in the linear approximation. For example, the northern Jovian hemisphere lacks a vortex analogous to the Great Red Spot, even

