THE AGE DISTRIBUTION OF GALACTIC COSMIC RAYS

II. *Interacting, Decaying, and Secondary Particles*

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Abstract. The distribution of ages for secondary and interacting cosmic-ray nuclei is derived for a class of steady-state, bounded models of diffusion in the galaxy. Results are presented in detail for the model in which diffusion is in one dimension, sources are uniformly distributed throughout the scattering region, and particles are observed near the central plane of the galaxy. The leakage-lifetime approximation is shown to be accurate as long as the ratio of the cosmic-ray diffusive lifetime to the mean interaction or decay lifetime is less than about ten.

1. Introduction

Information on the average age of galactic cosmic rays can be obtained from the abundances of the light elements (Li, Be, B) in the cosmic rays (Meyer *et al.*, 1974) and from the abundances of radioactive nuclei in cosmic rays (Webber *et al.*, 1973). Models used to obtain the average age from these observations have usually approximated the diffusion process by a 'leaky box' model (Cowsik *et al.*, 1967; Gloeckler and Jokipii, 1969; Webber *et al.*, 1973) in which a term n_i/τ_0 is introduced into the transport equation for particle species i. The resulting age distribution is an exponential with characteristic time τ_0. In a previous paper (Owens, 1975; subsequently referred to as Paper I), the age distribution for non-interacting primary cosmic rays was obtained through the use of the Green's function of the full time-dependent diffusion equation in several models of cosmic-ray production and confinement. The conclusion was that the leaky-box model gives a good approximation to the age distribution for ages larger than the average age, although the shape of the distribution for small ages is dependent on the details of the model. The purpose of this paper is to extend the analysis to the case in which the particles are secondaries, decaying particles, or particles which suffer spallation losses.

2. The Diffusion Models

As in Paper I, consider homogeneous diffusion models with free escape of particles at the outer boundaries of the galaxy. Consider first the *disk* models, in which the thickness of the confinement region (2L) is so much smaller than the galactic radius that a one-dimensional diffusive model is appropriate. The density of particles of species i will be determined by the differential equation

$$\frac{\partial n_i}{\partial t} = K\frac{\partial^2 n_i}{\partial z^2} - \frac{n_i}{\tau_{ir}} - wn_{\mathrm{H}}\sigma_i n_i + \sum_{j\neq i}\sigma_{ji}wn_{\mathrm{H}}n_j + S_i, \tag{1}$$

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where the successive terms on the right-hand side of the equation represent diffusion, radioactive decay, spallation loss, spallation production and production by cosmic-ray sources, where K is the diffusion coefficient; w, the velocity; τ_{ir}, the radioactive exponential decay time; $n_H \sim 1/\text{cm}^3$, the density of hydrogen in the interstellar medium; σ_i, the cross-section for spallation loss; σ_{ji}, the cross-section for production of species i from species j by spallation; and S_i, the rate of production (particles/sec) of particles of species i. Energy loss can be neglected for nuclei with energies $\gtrsim 1$ GeV/nucleon if the cosmic-ray lifetime is $\sim 10^6 - 10^7$ years (Ginzburg and Syrovatskii, 1964). For simplicity, assume that K and n_H are constant throughout the confinement volume and define

$$\alpha_i = \frac{1}{\tau_{ir}} + wn_H\sigma_i, \tag{2}$$

which is the rate of loss of particles of species i due to radioactive decay and interaction in the interstellar medium. Then Equation (1) can be written as

$$\left\{\frac{\partial}{\partial t} - K\frac{\partial^2}{\partial z^2} + \alpha_i\right\}n_i = S_i + \sum_{j\neq i} \sigma_{ji} w n_H n_j. \tag{3}$$

Equation (3) is an inhomogeneous partial differential equation for the species i, with the terms on the right corresponding to the sources.

As a first approximation to the solution of the coupled Equations (3), consider two types of cosmic rays, with the idealized titles of primary and secondary particles. Assume that the primary particles are produced by the cosmic ray sources but not by spallation processes, and that the secondary particles are produced by spallation with primary cosmic rays and not by the sources. Then the equation for primary cosmic rays, with the subscript p, is given by

$$\left\{\frac{\partial}{\partial t} - K\frac{\partial^2}{\partial x^2} + \alpha_p\right\}n_p = S. \tag{4}$$

This equation is similar to Equation (2) of Paper I, the only difference being the addition here of the term α to denote decay and spallation destruction. The equation for the secondaries is more complex, because it involves a sum over all species. If one assumes, however, that spallation due to secondaries is small compared to primaries, and that few of the primaries are destroyed, one has, approximately,

$$\left\{\frac{\partial}{\partial t} - K\frac{\partial^2}{\partial x^2} + \alpha_s\right\}n_s = \{\sigma_{ps} n_H w\} n_p^0, \tag{5}$$

where n_p^0 represents the primary density with $\alpha = 0$ and σ_{ps} is the effective cross-section for spallation (obtained by weighting the cross-sections by the source abundances). The approximations involved in going from Equation (3) to Equation (5) could be eliminated by solving Equation (4) for each primary nucleus and using the weighted sum in place of the simplified right-hand side of Equation (5), but in view of the observational uncertainties such a complex calculation does not presently appear appropriate. The validity of replacing $n_p(z)$ by $n_p^0(z)$ will be discussed below. The density of primaries n_p^0 is given in Equations (18) of Paper I.

3. Distribution of Ages

Given the differential Equations (4) and (5), the calculation of the age distributions proceeds as in Paper I: the Green's function of the differential operator is found and then integrated over the appropriate source function. The Green's function is identical in Equations (4) and (5), being the solution to the equation

$$\left\{\frac{\partial}{\partial t} - K\frac{\partial^2}{\partial z^2} + \alpha\right\} g(z, z_0, t, t_0) = \delta(t - t_0)\delta(z - z_0), \tag{6}$$

subject to the boundary conditions $n(z=L)=n(z=-L)=0$. First take the Fourier transform of Equation (6) with respect to t, choosing $t_0=0$ for simplicity. The result is

$$\left\{(-i\omega + \alpha) - K\frac{\partial^2}{\partial z^2}\right\}\tilde{g}(z, z_0, \omega) = \delta(z - z_0), \tag{7}$$

where

$$\tilde{g}(z, z_0, \omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t} g(z, z_0, t). \tag{8}$$

Then solve Equation (7) as a function of z by the eigenfunction method of Carslaw and Jaeger (1969) to obtain

$$\tilde{g}(z, z_0, \omega) = \frac{1}{L}\sum_{n=1}^{\infty} \frac{\sin(n\pi[z + L]/2L)\sin(n\pi[z_0 + L]/2L)}{(n^2\pi^2K/4L^2) + \alpha - i\omega}. \tag{9}$$

Finally, take the inverse Fourier transform and re-insert the t_0 to obtain the Green's function

$$g(z, z_0, t, t_0) = \frac{1}{L}\sum_{n=1}^{\infty} \exp\left[-\left(\frac{Kn^2\pi^2}{4L^2} + \alpha\right)(t - t_0)\right] \times \\ \times \sin(n\pi[z + L](2L)\sin(n\pi[z_0 + L]/2L). \tag{10}$$

Comparing Equation (10) with Equation (6) of Paper I, we see that the term involving α in Equation (6) above merely introduces a factor of $\exp[-\alpha(t-t_0)]$ in the Green's function. As in Paper I, we construct the age distribution in the steady state by inserting the appropriate position-dependent source function $S(z)$ and taking t_0 to zero, we obtain

$$f(\tau) = \int_{-L}^{L} \mathrm{d}z_0 g(z, z_0, \tau, 0)S(z_0). \tag{11}$$

It can easily be shown that a similar result is obtained in the three-dimensional ('halo') model discussed in Paper I, where again the term α merely introduces a term $\exp[-\alpha(t-t_0)]$ into the Green's function. For *primary* cosmic rays the solution to Equation (4) gives the same results as in Paper I for the age distributions except for the additional multiplicative term $\exp(-\alpha\tau)$. For the *secondary* models, the density of primary cosmic rays from Paper I's Equations (18) are used for the source function. For the disk model with sources filling the confinement region, for example, the primary density

$$n_p^0(z) = A[1-(z^2/L^2)]. \tag{12}$$

In what follows, for the sake of simplicity only this model (disk, uniform) will be considered, although it is straightforward to extend the analysis to other models from Paper I or to more complicated source functions $S(z_0)$. From Equations (10)–(12), we can derive the age distribution for *primary* cosmic rays

$$f(\tau) = \left\{\frac{4S_0}{\pi}\right\} \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \left(\frac{1}{n}\right) \exp(-\alpha_s\tau) \exp(-Kn^2\pi^2\tau/4L^2) \sin[n\pi\{z + L\}/2L] \tag{13}$$

using $S(z_0)=S_0$, and the age distribution for *secondary* cosmic rays

$$f(\tau) = \left\{\frac{32}{\pi^3}\sigma_{ps}n_H wA\right\} \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \left(\frac{1}{n^3}\right) \exp(-\alpha_s\tau) \times \\ \times \exp(Kn^2\pi^2\tau/4L^2) \sin[n\pi\{z + L\}/2L] \tag{14}$$

using Equations (5) and (12) for $S(z_0)$.

The similarity between the formulae for the age distributions (13) and (14) is striking. Apart from the initial amplitude (in braces) and the different interaction rates α_p and α_s, the only difference is that the series for secondaries has a factor of n^{-3} instead of n^{-1} in the sum. This means that the secondary series is more rapidly convergent, and the secondary age distribution is even more closely represented by an exponential than the primary distribution. This result is shown in Figure 1, where the normalized age distributions for primaries and secondaries near the galactic central plane are given. The parameters on the figure and others below are

$$X = K\pi^2\tau/4L^2 = \tau/t_d \tag{15a}$$

and

$$R = 4L^2\alpha/K\pi^2 = t_d/t_{\text{int}}, \tag{15b}$$

where X is the ratio of the age to the typical diffusion time $(K\pi^2/4L^2)^{-1} \equiv t_d$ and R is the ratio of the typical diffusion time to the interaction or decay time $(\alpha)^{-1} \equiv t_{\text{int}}$. The results in Figure 1 are for positions near the center of the scattering region ($z=0$). For other positions the difference between the primary and secondary distributions is somewhat larger.

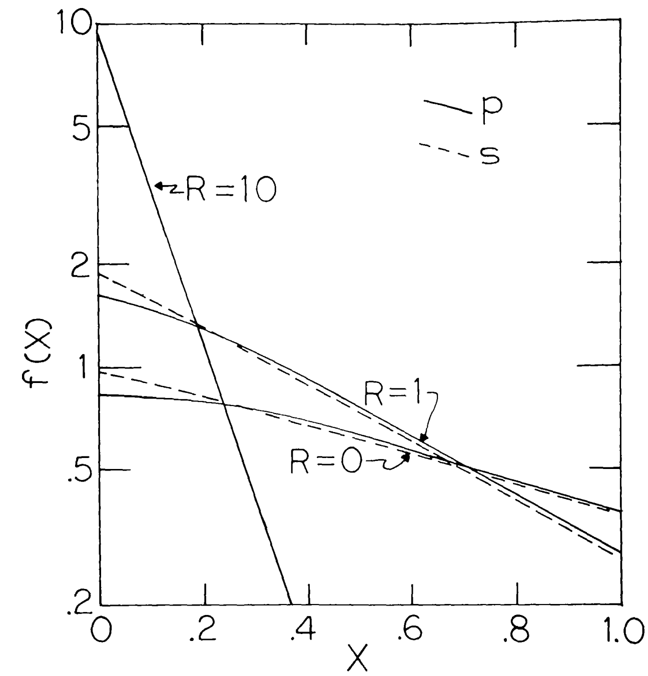

Fig. 1. The age distribution of cosmic rays. X is the age in terms of t_d, and $R = t_d/t_{int}$. The solid and dashed curves are for primary and secondary particles, respectively, with the two curves coinciding for $R = 10$. Curves are for the disk, uniform model at $z = 0$.

Next consider the variation of the cosmic-ray omnidirectional intensity

$$n(z) = \int_0^\infty f(X)\,\mathrm{d}X, \tag{16a}$$

and the average cosmic-ray age

$$\langle\tau(z)\rangle = \langle X\rangle t_d = (1/n)t_d \int_0^\infty Xf(X)\,\mathrm{d}X. \tag{16b}$$

The variation of the intensity is shown on Figure 2, where $n(z)$ has been set equal to 10 at $z = 0$ for ease of comparison. The secondary particles are more concentrated toward the center because the sources (12) are more numerous near $z = 0$. Primary cosmic rays become less numerous near $z = 0$ for a larger interaction cross-section (proportional to R) because they are depleted in producing secondaries more rapidly near the center. Notice, however, that the difference between the primary curves for $R = 0$ and $R = 1$ is not very large. This means that primary species which propagate with 100% of the source abundance ($R = 0$) have a distribution in the galaxy nearly the same as those particles whose intensity is reduced to about 50% of the source abundance through interactions ($R = 1$), and the assumption that all primary species have the same distribution with z (assumed in deriving Equation 5) is fairly well satisfied. The variation

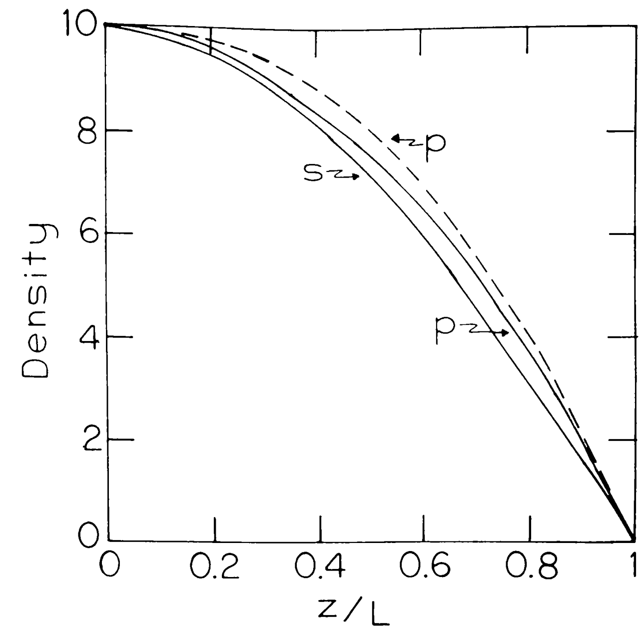

Fig. 2. The density of cosmic rays. The densities are normalized to 10 at $z=0$. The solid curves are for $R=0$ and the dashed curve for $R=1$, where p and s represent primary and secondary particles. The curve for $R=1$ secondaries coincides with the curve for $R=0$ secondaries to within 1%.

of $\langle\tau\rangle$ with z is small in the uniform disk model being discussed here, both for primaries and for secondaries. For primaries, the variation is typically 20% for small R and about a factor of 2 for R larger than 100, while for secondaries the variation of $\langle\tau\rangle$ with z is only a few percent for all R.

As the discussion above indicates, the leakage-lifetime approximation, with an exponential age distribution and the same average age for primary and secondary particles, is a fairly good approximation to the actual diffusion models as long as R=(diffusion time)/(interaction time) is not too large. This conclusion is somewhat similar to that of Jones (1970), who considered the effects of energy-change terms in the diffusion equation (which have been neglected here) and found that the leakage-lifetime approximation is valid if the energy change is small enough.

4. Particles Detected Near Earth

The Earth appears to be near the galactic central plane ($z/L \lesssim 0.1$), so for simplicity consider the cosmic-ray density and average age at $z=0$. From Equations (13)–(16), one has for the density

$$n_p = (32/\pi^3)A \sum_{n=1}^{\infty}{}^{*} (1/n)(R + n^2)^{-1}, \tag{17a}$$

$$n_s = (32/\pi^3)A[\sigma_{ps} n_{\mathrm{H}} w t_d] \sum_{n=1}^{\infty}{}^{*} (1/n)^3 (R + n^2)^{-1}. \tag{17b}$$

where $A=L^2S_0/2K$ and the symbol $\sum^*$ represents the sum over odd integers with alternating signs: i.e.,

$$\sum_{n=1}^{9}{}^{*}\, n = 1 - 3 + 5 - 7 + 9. \tag{18}$$

The subscript p represents primary and s secondary particles. Similarly, for the average age it is easy to show that

$$\langle\tau\rangle_p = t_d \sum_{n=1}^{\infty}{}^{*} (1/n)(R + n^2)^{-2} \Big/ \sum_{n=1}^{\infty}{}^{*} (1/n)(R + n^2)^{-1}, \tag{19a}$$

$$\langle\tau\rangle_s = t_d \sum_{n=1}^{\infty}{}^{*} (1/n)^3(R + n^2)^{-2} \Big/ \sum_{n=1}^{\infty}{}^{*} (1/n)^3(R + n^2)^{-1}. \tag{19b}$$

From the relations above, recall that $t_a=4L^2/K\pi^2$ and that $R=t_d/t_{\text{int}}$, where t_{int} is the decay time due to radioactive decay or spallation interactions.

The density and average age for primary cosmic rays are shown in Figure 3 at $z=0$ as a function of R, and Figure 4 gives the ratio of secondary to primary particles for the density and the age. As a crude approximation, for $R\lesssim 10$, both n and $\langle\tau\rangle$ vary approximately as $1/(R+1)$, as can be seen from the first term of the sum in the series of (17) and (19): i.e., one has, approximately,

$$1/\langle\tau\rangle \sim (R + 1)/t_d = 1/t_{\text{int}} + 1/t_d. \tag{20}$$

The approximate equality of the left and right elements of Equation (20) indicates that the leakage lifetime approximation is valid for the diffusive model under discussion, since the leaky-box model gives the interaction and diffusive lifetimes combining

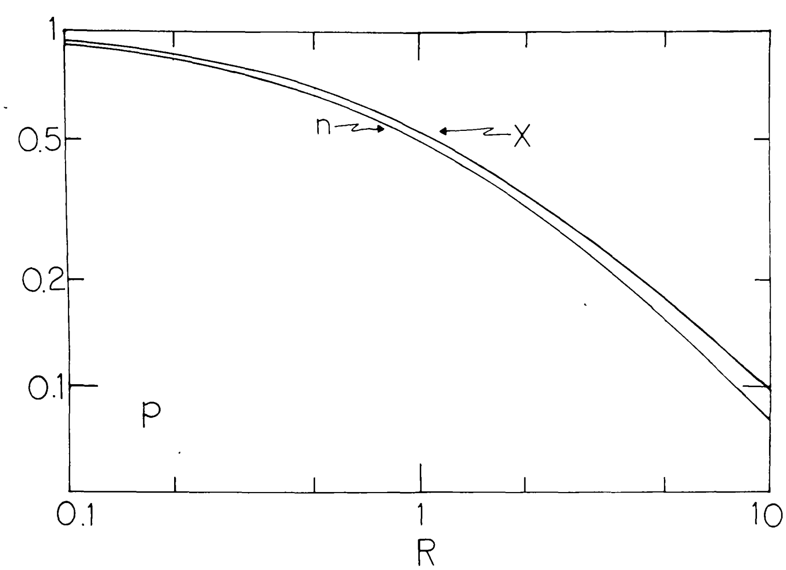

Fig. 3. The density and average age of cosmic rays. The density (n) is normalized to 1 at $R=0$, and the average age (X) is given in terms of t_d. The curves are the results for primary particles in the disk, uniform model at $z=0$.

as in Equation (20). As shown from Figure 4, the differences between secondaries and primaries amount to less than 20% in both the density and the average age.

For particles detected near Earth, one determines the ratio $R = t_d/t_{\text{int}}$ as follows. The ratio R is given by

$$R = t_d/t_{\text{int}} = (1/\langle X_0 \rangle)\tau_0/t_{\text{int}} = (1/\langle X_0 \rangle)\lambda_0/\lambda_{\text{int}}, \tag{21}$$

where $\langle X_0 \rangle$ represents the expected value of the cosmic-ray age divided by t_d for non-interacting particles (i.e., $R = 0$), τ_0 is the average age for non-interacting particles, λ_0 is the average pathlength (in g/cm^2) of non-interacting particles, and λ_{int} is the interaction mean-free path (in g/cm^2). For the particles ($\sim$1–5 GV rigidity) involved in spallation production, observations indicate that $\lambda_0 \sim 5$ g/cm^2. Adopting the mean-free pathlengths given by Webber (1967), one can construct Table I. (Use is made of the result $\langle X_0 \rangle = 1.03$ and 1.01 for primary and secondary particles, respectively.) Recalling from above that the leaky-box model gives a good approximation for $R \lesssim 10$, we see that the leaky-box model gives a good representation of a one-dimensional cosmic-ray diffusion process for all the species considered. The difference in the amount of matter traversed and the average ages between primaries and secondaries (see Figure 4) is less than 5%.

TABLE I

Interaction Paths and the Parameter $R = t_d/t_{\text{int}}$

Species	λ_{int} (g/cm^2)	p or s	R
Fe	2.4	p	2.0
C, N, O	6.7	p	0.72
Li, Be, B	8.7	s	0.57
He	17	p	0.29
deuteron	32	s	0.15

As an application of the results given above, consider the problem of determining the average age T of galactic cosmic rays from the observed abundance of Be^{10}, an isotope with an exponential decay time $t_{\text{rad}} = 2.2$ million years. The average age T is conventionally obtained from the leakage lifetime (t_l) and the interaction time t_{int} by

$$1/T = 1/t_l + 1/t_{\text{int}}. \tag{22}$$

Then if the ratio of observed Be^{10} to the amount of Be^{10} to be expected if the species were radioactively stable is f, the age T is given by

$$T = [(1/f) - 1]\,\gamma t_{\text{rad}} \tag{23}$$

(e.g., Webber *et al.*, 1973, Hagen *et al.*, 1975), where γ is the relativistic Lorentz

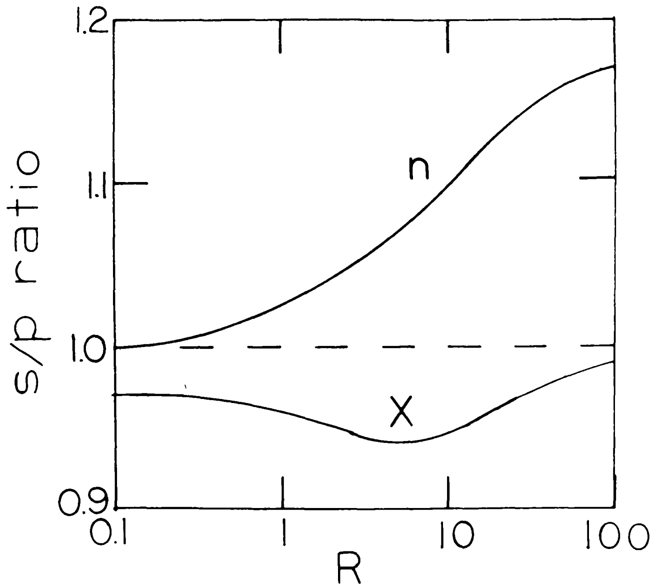

Fig. 4. The ratio of density and average age for secondary cosmic rays relative to primary cosmic rays. The secondary to primary ratio is given for density (n) and average age (X), as a function of R, for the disk, uniform model at $z=0$. The densities are normalized so that the ratio of secondaries to primaries is one at $R=0$, and the factor $\sigma_{ps}n_H w t_d$ in Equation (17b) is suppressed.

factor. In terms of the diffusion model discussed here, one finds the value of R (denoted by $R^{\#}$) such that

$$n(R^{\#})/n(R') = f \tag{24}$$

from Equation (17b) or Figures 3 and 4, where $R'=0.57$ is the value of R which a non-decaying species would have. Then from the definitions of R and X one can easily show that the average age $\langle\tau\rangle$ due to diffusion and interaction is

$$\langle\tau\rangle = \langle X'\rangle t_d = \langle X'\rangle(R^{\#} - R')\gamma t_{\text{rad}}, \tag{25}$$

where $\langle X'\rangle$ represents the expected value of X corresponding to $R=R'$. The results of the leakage-lifetime model (Equation 23) and the diffusion model (Equation 25) are shown in Table II for various values of f. Observed values of f range from the order of 0.1 (Garcia-Munoz *et al.*, 1975) to about 0.8 (Hagen *et al.*, 1975), so it is clear from the table that little error is involved in approximating the diffusion by the leakage-lifetime model.

Adopting the typical value $f\sim 0.4$ from the Be^{10} data above, and taking $\gamma\sim 1$, we obtain

$$t_d = 4L^2/K\pi^2 \approx 3.3 \text{ million years}. \tag{26}$$

For $L\sim 150$ pc as the half-thickness of the galactic confinement region, the diffusion coefficient $K\sim 8\times 10^{26}$ cm^2/sec.

The validity of the approximations involved in obtaining Equation (5) from Equation (3) can be discussed in light of the results obtained above. As shown in Table I,

TABLE II

Age of Cosmic Rays from Be^{10} Measurements

f	$\langle\tau\rangle/(\gamma t_{rad})$	$T/(\gamma t_{rad})$
0.02	46	49
0.05	18	19
0.10	8.6	9.0
0.20	3.9	4.0
0.40	1.5	1.5
0.60	0.67	0.67
0.80	0.25	0.25
0.90	0.11	0.11

the parameter R_p is 2 or less for the primary particles of interest. Figures 1 and 2 show that $n_p(z)$ for these values of R_p is within a few percent of $n_p^0(z)$ for $R=0$. Thus the use of Equation (5), with n_p^0 as the source function, is reasonable. The effect of a finite R_p upon $n_p(z)$ is clear; it makes the distribution function more uniform in z. In the limit $R_p \to \infty$, the actual $n_p(z)$ will be the same shape as the source function $S_0(z)$, which has been taken as a constant. Since $n_p(z)$ is the source function of secondaries (as in Equation 3), for finite R_p the source function will vary between $n_p^0(z)$ (for $R_p=0$) and $S_0=$constant (for $R_p \to \infty$). Thus the distribution of secondaries will vary between what above was labelled 'secondaries' (with $R_p=0$) and what above was labelled 'primaries' (with $R_p \to \infty$). The approximations made in obtaining Equation (5) thus are valid, and if anything the results discussed slightly overstate the differences between primaries and secondaries. The differences between secondaries and primaries for $R>1$ in Figure 4 are thus overstated in the figure if $R_p \sim R_s \gg 1$. Finally, notice that probably the only isotope of current interest with R significantly larger than 1 is Be^{10}, which is a secondary. Since Be^{10} is formed mainly from carbon and oxygen (with $R_p \sim 0.74$ as in Table I), Equation (5) will be valid even though R_s may be much larger than 1.

5. Conclusions

For the one-dimensional diffusion model discussed above, with sources uniformly located throughout the scattering region and with the point of observation at the central plane (the disk, uniform model at $z=0$), the actual distribution of ages of both primary and secondary cosmic-ray particles is nearly exponential. The leakage-lifetime approximation is an accurate representation of the diffusion as long as $R=$ $=$(diffusion time)/(interaction time)$\lesssim 10$. Primary and secondary particles have very similar distributions of ages and average ages. It should be emphasized that these conclusions are sensitive to the type of diffusion model specified and to the location of the observer within the scattering region; for other models, the formalism derived in Sections 2 and 3 above gives the method for solving the problem.

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References

Carslaw, H. S. and Jaeger, J. C.: 1969, *Conduction of Heat in Solids*, Clarendon Press, Oxford.
Cowsik, R., Pal, Y., Tandon, S. N., and Verma, R. P.: 1967, *Phys. Rev.* **158**, 1238.
Garcia-Munoz, M., Mason, G. M., and Simpson, J. A.: 1975, *14th Int. Cosmic Ray Conf.*, Munich, **1**, 331.
Ginzburg, V. L. and Syrovatskii, S. I.: 1964, *The Origin of Cosmic Rays*, Pergamon Press, New York.
Gloeckler, G. and Jokipii, J. R.: 1969, *Phys. Rev. Lett.* **22**, 1448.
Hagen, F. A., Fisher, A. J., Ormes, J. F., and Arens, J. F.: 1975, *14th Int. Cosmic Ray Conf.*, Munich, **1**, 361.
Jones, F. C.: 1970, *Phys. Rev. D* **2**, 2787.
Meyer, R., Ramaty, R., and Webber, W. R.: 1974, *Physics Today* 23, October.
Owens, A. J.: 1975, *Astrophys. Space Sci.* **40**, 357 (Paper I).
Webber, W. R.: 1967, 'The Spectrum and Charge Composition of the Primary Cosmic Radiation', in K. Sitte (ed.), *Handbuch der Physik* **46/2**, 181, Springer-Verlag, New York.
Webber, W. R., Lezniak, J. A., Kish, J., and Damle, S. V.: 1973, *Astrophys. Space Sci.* **24**, 17.