COSMIC RAY PROPAGATION IN A CLOSED GALAXY

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Abstract. A simple model of cosmic ray propagation is proposed from which the major experimental results can be derived: The model reproduces the observed nuclear abundances and accounts for the observed changes of nuclear composition with energy, the high degree of isotropy of cosmic ray flux at all energies, and the high degree of its constancy throughout the history of the Solar System. It is consistent with the observed size distribution of extensive airshowers, the intensity and energy distribution of the electron component, and the diffuse emission of γ-rays and radio waves.

The model is characterized by the two basic assumptions: (1) that cosmic rays have been injected at an unchanging rate by sources located in the galactic spiral arms and (2) that a large-scale magnetic field retains all particles in our galaxy, where they interact with interstellar gas, so that all complex nuclei are finally fragmented and their energy dissipated in meson production and electro-magnetic interactions.

1. Introduction

Many years ago the question was examined whether the nuclear composition of cosmic radiation could be understood in terms of the injection of particles into a volume where they remain until they are fragmented and their energy is depleted by nuclear collisions in the interstellar gas (cf. Bradt and Peters, 1950). The answer was obtained from the abundance ratio between neighboring elements. For a given flux of carbon or oxygen one should observe, in an equilibrium state, a comparable flux of their principal fragmentation products, the elements Li, Be, B. In the cosmic radiation such fragmentation products are considerably rarer than required by equilibrium conditions and this made it necessary to abandon the equilibrium hypothesis. In order to explain the deviation from equilibrium one may assume that the particles escape before they have reached equilibrium with their collision products or one may assume that, in our neighborhood at least, a young component is present which has not yet reached equilibrium.

Generally the first hypothesis is preferred because of its apparent simplicity, and because one avoids assigning to the Solar System a preferred position with respect to the sources of cosmic ray particles. In the vicinity of a source one might expect considerable directional anisotropy in the cosmic ray flux and major intensity fluctuations during the age of the Solar System. Neither has been observed and the experimentally determined upper limits have decreased over the years.

The hypothesis that particles leak out from the galactic disk has been widely accepted and can account quite well for existing cosmic ray observations. Its remaining major difficulty is to account for the observed intensity and isotropy of very energetic particles whose trajectories have radii of curvature in the galactic magnetic field which

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approach or even exceed the thickness of the galactic disk. The 'leaky galaxy' model is forced to postulate a second independent cosmic ray component which has an extragalactic origin and which is supposed to supply most of the high energy particles responsible for the so-called extensive airshowers. The universal radiation field places an upper limit on the energy of nuclei, protons and electrons which can survive the transit between galaxies. For nuclei and protons this energy limit is approached very closely by the largest cosmic ray events observed.

Another difficulty of the leaky galaxy model is that it requires a very large energy input from sources inside the galaxy in order to compensate for the loss of particles to the rest of the universe and maintain a steady state.

As an alternative to the 'leaky' galaxy model of cosmic rays there still remains the 'closed' galaxy model. Here cosmic ray nuclei are supposed to be in equilibrium with their break-up products and fill the galactic volume which includes a large halo surrounding the disk. The fact that the observed nuclear composition does not correspond to an equilibrium distribution must then be attributed to the proximity of the Solar System to a region which contains cosmic ray sources.

It seems timely to investigate the closed galaxy model again and more thoroughly in the light of the new information, which has become available during the last few decades. Particularly relevant is that diffuse electromagnetic radiation produced by cosmic rays in the radio-, X-ray and γ-ray bands has now been observed which permits estimates of cosmic ray intensity in regions far from the Solar System and also that the position of the Solar System within the galactic structure has become more precise.

The most critical tests of the closed galaxy model will be the confrontation with the experimentally established limits on anisotropy and on secular variations and with the recently observed changes in chemical composition with energy and with the by now quite well established irregularities in the spectrum of extensive airshowers.

2. The Closed Galaxy Model

It has recently been shown by Rasmussen and Peters (1975) that the observed nuclear composition can be reproduced in a model where galactic magnetic fields prevent the escape of cosmic rays, provided that the majority of complex nuclei arriving in the Solar System originate in a source which is in the 'neighborhood' of the Solar System. 'Neighborhood' is to be understood not in terms of distance but in terms of transit time or the amount of interstellar gas traversed by the particles which reach the Solar System.

The authors did not discuss where 'neighboring' sources may be located physically, and therefore the question whether the appropriate configuration was likely to occur in our galaxy was left open. We list the principal questions connected with the model which remain to be elucidated.

(a) Is it plausible that 'neighboring' sources exist with a luminosity high enough to account for the observed elemental and isotropic composition?

(b) Is it probable that there are enough such sources so that no strong fluctuations in nuclear composition on a geological time scale are to be expected?

(c) Does the model predict the existing high degree of spatial isotropy of the cosmic ray flux and is it plausible that this isotropy extends even to the very energetic primaries, which initiate the large airshowers?

(d) Can the model account for the observed energy dependence of the chemical composition?

(e) Can the model account for the observed variations in slope of the size distribution of extensive airshowers?

(f) Can the model account for the intensity of the galactic γ-flux and its distribution parallel and perpendicular to the galactic plane?

(g) Can the model account for the intensity of the electron–positron component and its energy spectrum as well as for the spatial distribution of the diffuse galactic X-ray and radio background to which it gives rise?

(h) What is the energy output required in a steady state?

The model we wish to investigate is defined as follows:

(α) Cosmic rays are emitted by individual sources which may or may not differ in intensity, composition and energy spectra.

(β) The sources are more or less evenly distributed along the galactic spiral arms and new sources arise at intervals which are brief when measured in terms of the average lifetime of the particles in the galaxy.

(γ) The galactic magnetic field inside the spiral arms is assumed to permit a rapid drift of particles back and forth along the arms.

(δ) The drift of particles across the arms is slow compared to the longitudinal drift.

(ε) Outside the spiral arms diffusion is relatively fast and particles disperse rapidly throughout the galactic disk and the surrounding halo.

(ζ) In the halo, cosmic ray particles of all energies are retained by an appropriate configuration of large-scale magnetic fields.

(θ) The solar system is situated somewhere inside a spiral arm.

The model then implies that in the Solar System one observes two populations of cosmic ray particles: (1) the 'young' particles which are on their way out of the spiral arm where they were born, and (2) the 'old' particles which pervade the entire galactic containment volume.

3. The Nuclear Composition

The transport of nuclei from the interior to the surface of the spiral arms permits the definition of a transverse drift velocity

$$u = \frac{a_s}{\tau_s}, \tag{1}$$

where a_s is a measure of the thickness of the arms and τ_s the mean residence time in the arms for particles of given magnetic rigidity.

To this transverse drift velocity u there corresponds an average amount of gas traversed by the particles in the spiral arms, given by

$$x_s = c\tau_s n_s m_p = \frac{a_s n_s}{200(u/c)} \text{ g cm}^{-2}, \tag{2}$$

where n_s is the mean gas density in the spiral arms in proton masses m_p cm^{-3}, and where a_s is expressed in kpc.

The transverse drift velocity u is a monotonically increasing function of the radius of curvature of the particle R, and approaches the light velocity c as R approaches the thickness a_s of the spiral arm. The precise functional form depends on the unknown transport mechanism responsible for the drift and will be approximated here by

$$\frac{u}{c} = \begin{cases} (R/a_s)^\alpha & \text{for } R < a_s, \\ 1 & \text{for } R > a_s. \end{cases} \tag{3}$$

The rigidity dependence of the average amount of matter traversed by particles before leaving the spiral arm then becomes

$$x_s = \begin{cases} x_0(\varrho_0/\varrho)^\alpha & \text{for } \varrho < \varrho_0, \\ x_0 & \text{for } \varrho > \varrho_0; \end{cases} \tag{4}$$

where

$$x_0 = \frac{a_s n_s}{200} \text{ g cm}^{-2} \tag{5}$$

and

$$\varrho_0 = 9 \times 10^8 H_{\parallel} a_s \text{ GV c}^{-1}. \tag{6}$$

The magnetic rigidity $\varrho = (A/Z)p$ is expressed in GV c^{-1}, p is the momentum/nucleon, the longitudinal magnetic field $H_{\parallel}$ is expressed in microgauss, and a_s in kpc.

It will be shown in Section 5 that in order to account for the nuclear composition at rigidities $\varrho > 4$ GV c^{-1}, without coming into conflict with astronomical evidence on the field- and gas-density in the spiral arms the exponent α must have the value

$$\alpha = 0.5 \pm 0.1,$$

for the rest of the discussion we shall use $\alpha = \frac{1}{2}$, but either come back to this question.

Consider first the density of the 'young' component, those nucleons, free and bound, which have not yet left the spiral arms where they were accelerated.

The density is proportional to the source strength and inversely proportional to the total volume V_s of the spiral arms. It is also proportional to the residence time τ_s, as long as the latter is small compared to the lifetime of the nucleons in the ambient gas: i.e.,

$$Y_N \approx \frac{\tau_s S_N}{V_s} = \frac{x_s}{cM_s} S_N, \tag{7}$$

where M_s is the mass of gas contained in the spiral arms.

In general, for a particular nuclear component, one may write

$$Y_i = \frac{\varphi}{cM_s} \sum_{j \geq i} \omega_{ij}(x_s) S_j;$$

or, in vector notation,

$$\mathbf{Y} = \frac{\varphi}{cM_s} \underline{\underline{\omega}} \cdot \mathbf{S}. \tag{8}$$

In this vector equation the components represent different nuclear components, Y_i is the density of component i, S_j is the number of nuclei of component j injected by all cosmic ray sources per unit time, ω_{ij} is an element of the matrix $\underline{\underline{\omega}}$, which describes the nuclear transformation leading to the destruction and production of component i in transit from the sources to the Solar System. It is a function of the various relevant spallation cross-sections (which can now be measured in the laboratory) and of x_s.

Strictly speaking, ω_{ij} depends on

x_s the average amount of matter traversed by the particles when escaping from the arms, and on

$\langle x \rangle$ the average amount of matter traversed by the particles when they reach the Solar System, i.e. before they escape.

However, x_s and $\langle x \rangle$ are uniquely related once a particular path length distribution has been assumed. We consider the two extreme cases:

(a) that the escape from the spiral arm is a completely random process. In this case the age distribution of the particles leaving the spiral arm and also the age distribution of the particles entering the Solar System is exponential and that

$$\langle x \rangle = x_s;$$

(b) that the escape from the spiral arms is controlled by the kinematics of particle motion in a quasi-static field. In this case the age distribution of particles leaving the spiral arms can approach a δ-function distribution (slab approximation). The particles entering the Solar System then have a uniform age distribution between zero and the age at escape and one finds that

$$\langle x \rangle = \frac{x_s}{2}.$$

The matrices $\underline{\underline{\omega}}$ for these two extreme assumptions have been calculated in the appendix; intermediate cases, for instance the case where the age at escape is normally distributed around a mean value have not been considered.

The factor φ which appears in Equation (8), takes into account that the density of young particles in the Solar System may differ somewhat from the average value in the spiral arms. This factor will be close to unity as long as density gradients are small. And this will be the case if the longitudinal drift velocity is large compared to the transverse drift velocity u and as long as $u/c \ll 1$.

Turning now to the 'old' particles which are in equilibrium with their fragmentation products and which are uniformly distributed throughout the containment volume, the matrix which describes transformations in transit may be written as the product of two matrices $\underline{\underline{\Omega}}\cdot\underline{\underline{\varepsilon}}$.

The total number of particles injected into the containment volume is given by

$$\mathbf{S}' = \underline{\underline{\varepsilon}}(x_s)\cdot\mathbf{S}, \tag{9}$$

where $\underline{\underline{\varepsilon}}(x_s)$ is the matrix which describes transformations preceding escape from the arms.

The density of old particles (which are uniformly distributed throughout the closed galaxy, including the halo, the disk, and the arms themselves) is

$$\boldsymbol{\eta} = \frac{\underline{\underline{\Omega}}\cdot\mathbf{S}'}{cM_G} = \frac{\underline{\underline{\Omega}}\cdot\underline{\underline{\varepsilon}}(x_s)\cdot\mathbf{S}}{cM_G}, \tag{10}$$

where M_G is the total mass of gas in the galaxy.

$\underline{\underline{\Omega}}$ is a simple matrix which characterizes the transformation of the injected source composition $\mathbf{S}'$ into an equilibrium distribution. Like $\underline{\underline{\omega}}$ and $\underline{\underline{\varepsilon}}$, the matrix $\underline{\underline{\Omega}}$ depends on interaction cross-sections and spallation probabilities, but unlike the others, it does not depend on x_s nor on any other parameter characterizing the mode of propagation. The total flux observable in the solar system is then

$$\mathbf{F}(p) = \frac{c}{4\pi}(\mathbf{Y} + \boldsymbol{\eta}) = \frac{1}{4\pi M_G}[K\underline{\underline{\omega}}(x_s) + \underline{\underline{\Omega}}\cdot\underline{\underline{\varepsilon}}(x_s)]\cdot\mathbf{S}(p), \tag{11}$$

where

$$K = \frac{M_G}{M_s}\varphi \tag{12}$$

is (apart from a factor of order unity) a measure of the fraction of galactic gas located in the spiral arms.

The vector equation represented by formula (11) connects the flux of particles of momentum/nucleon, p, with the source emission of particles with the same value p. Changes of velocity in transit have been neglected whether due to nuclear or to electromagnetic interactions. This is an adequate approximation for complex nuclei with momentum/nucleon $p > 2$ GeV c^{-1}, i.e. when the energy is large compared to nuclear binding energy and when the mean free path leading to fragmentation is short compared to the ionization range.

When calculating the flux of the proton component, however, energy loss in transit due to meson production must be taken into account in this model, even when ionization losses can be neglected, because there is no loss by fragmentation and meson production is the dominant process which limits the lifetime of the particles in the containment volume. These losses can be included quite simply and adequately when calculating the relevant matrix element (see Appendix) such that Equation (11) remains valid also for protons. The matrix elements which connect the proton flux with source emission differ from the rest only in that they contain, instead of a nuclear collision mean free path, an attenuation mean free path (which is a function not only of nuclear parameters, but also of the spectral slope of the nucleon component and, therefore, of the source spectra).

In the energy region $p > 2$ GeV c^{-1} in which the formula is valid, most spallation cross-sections have reached their asymptotic values, so that the particle energy enters into calculation of the matrix elements only via the assumed magnetic rigidity dependence of x_s (Equation 4). For a given particle velocity the matrix elements depend therefore on the charge-to-mass ratio of the nuclear species.

For particles of a given velocity, and therefore also of a given rigidity ϱ, the number of unknowns in Equation (11) exceeds the number of equations: in addition to the unknown source abundances there appear the combination of parameters $x_0\sqrt{\varrho_0}$ (Equation 5) which characterizes the spiral arms, and K (Equation 12), an astrophysical parameter which characterizes the distribution of interstellar gas.

The system of equations becomes solvable for the source composition if one introduces an additional widely accepted assumption. It seems safe to postulate that the source strength is zero (or very small) for certain nuclear species, in particular for those which are thermally unstable in ordinary stars. The nuclei whose abundance at the source are likely to be negligibly small are ^{2}H, the various isotopes of Li, Be and B and possibly some other nuclides of higher atomic number.

With these assumptions Equation (11) has been solved for two extreme path length distributions in the spiral arms and for different values of the constant K. The constant $x_0\sqrt{\varrho_0}$ in Equation (5) has been chosen to minimize the difference between the abundance ratio

$$\frac{\mathrm{Li} + \mathrm{Be} + \mathrm{B}}{\mathrm{C} + \mathrm{N} + \mathrm{O}} = \frac{L}{M}$$

observed in the Solar System and the ratio calculated from Equation (11). For the exponential escape probability we get

$$x_0\sqrt{\frac{\varrho_0}{\varrho}} \equiv x_s = \langle x\rangle = 15\varrho^{-1/2}\ \mathrm{g\ cm^{-2}}; \tag{13}$$

and for the δ-function type escape probability (slab approximation)

$$x_s = 2\langle x\rangle = 21\varrho^{-1/2}\ \mathrm{g\ cm^{-2}}, \tag{14}$$

where ϱ is given in GV c^{-1} and $\langle x\rangle$ and x_s are the mean amount of matter traversed when reaching the Solar System and when escaping from the spiral arms respectively. Both give an equally good fit with the experimental abundance ratios, but the variation of nuclear composition with rigidity ϱ is more marked in the slab approximation.

3A. Variation of Composition with Rigidity

A lower limit on the constant K can be obtained most easily from the observed variation of the ratio L/M with energy. In Figure 1a this ratio is plotted for different

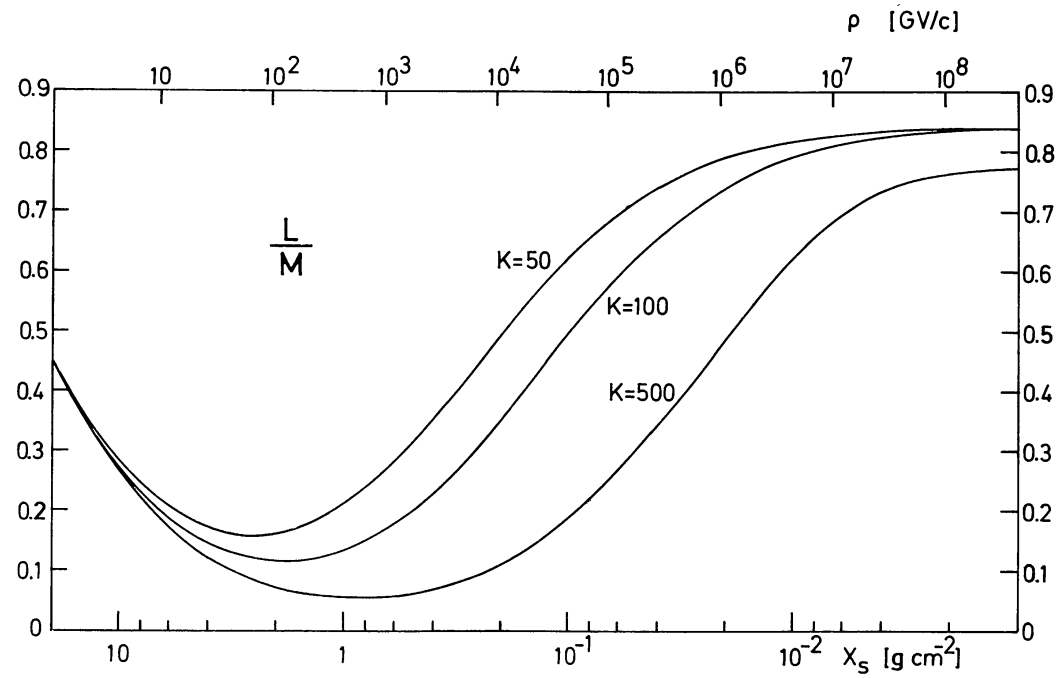

Fig. 1a. The ratio L/M for particles with a magnetic rigidity larger than ϱ as a function of x_s or ϱ for 3 different choices of K. The relation between x_s, the average amount of matter traversed in the spiral arms, and ϱ is $x_s = 21\varrho^{-1/2}$ g cm^{-2} when ϱ is measured in GV c^{-1}.

values of K and for the slab approximation. The initial decrease in the ratio is due to the fact that with increasing energy less matter is traversed by the young particles before they escape from the spiral arms. This is not a unique feature of our model and can also be incorporated into the leaky galaxy model, as shown by Cowsik and Wilson (1973).

As the energy increases further, however, the contribution of young particles to the observed flux becomes small, the old particles begin to dominate, and the L/M ratio rises and approaches the equilibrium value. The figure shows that an initial decrease with energy in the L/M ratio by a factor ~ 3 requires values of $K > 100$. Such a choice would, therefore, lead to agreement with existing observations (cf. Juliusson, 1974; Webber *et al.*, 1972). An upper limit on K is derived in Section 4.

The odd-Z elements K, Sc, V, Mn may also be assumed to be rare at the source and their observed flux is presumably due entirely to the spallation of iron nuclei in transit. The predicted behavior of the ratio (K+Sc+V+Mn)/Fe is similar to that of the ratio L/M. It is in qualitative agreement with observations (Juliusson, 1974; Webber *et al.*, 1972) and is shown in Figure 1b.

The model predicts an energy dependence also for those abundance ratios, in which both elements are mainly of primary origin. The effect is then caused by differences in destruction cross-sections.

Figure 2 shows the rigidity dependence of the abundance ratio with respect to oxygen for various elements. The ratios are normalized to unity for $\varrho = 4$ GV c^{-1}.

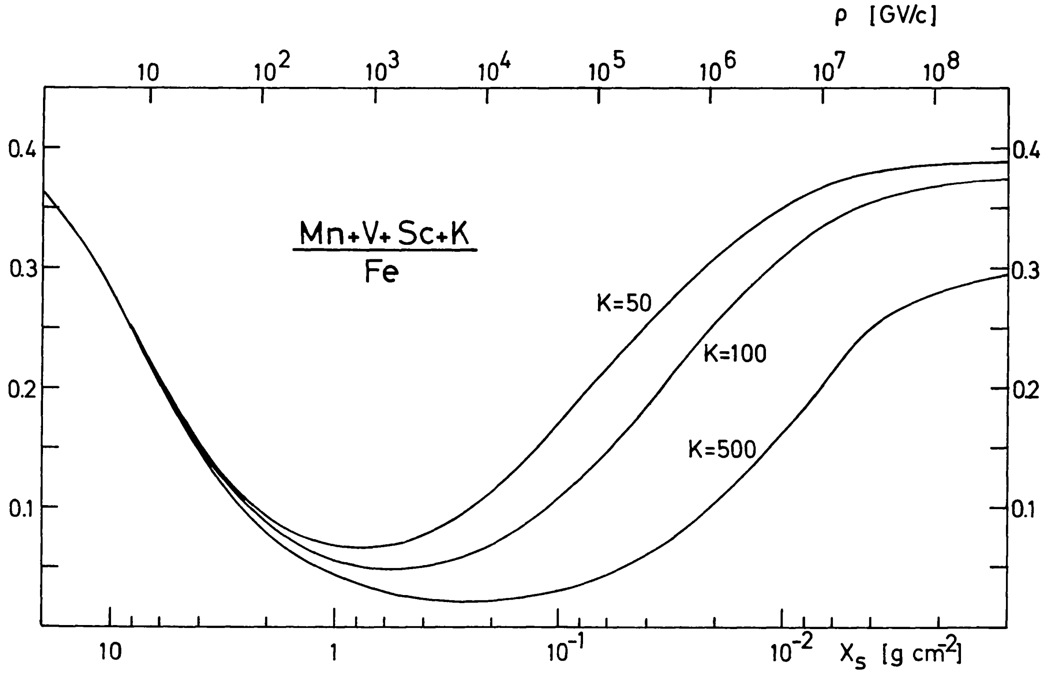

Fig. 1b. The same as Figure 1a but for the ratio $(Mn+V+Sc+K)/\mathrm{Fe}$.

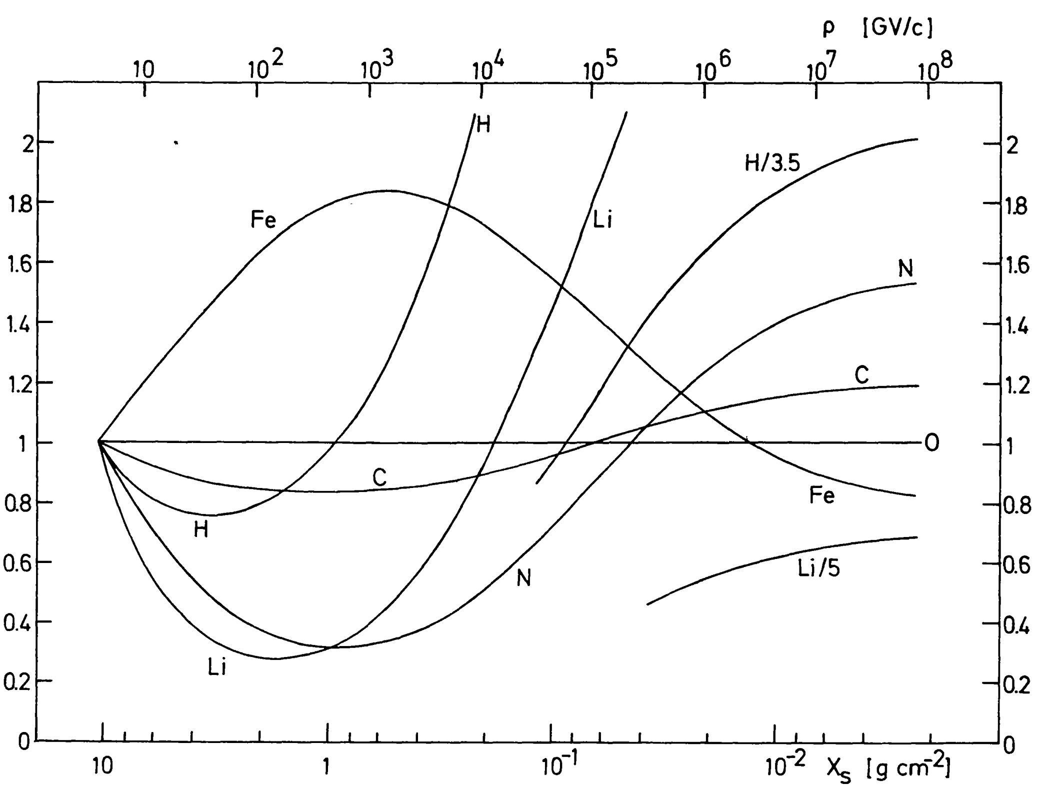

Fig. 2. The ratio of the flux of various elements to that of oxygen as functions of ϱ, normalized at $\varrho=4$ GV c^{-1} ($K=100$, $x_s=21\varrho^{-1/2}$ g cm^{-2}).

In drawing this figure it has been assumed that all primaries emitted by the source have the same simple power-law rigidity spectrum with an exponent $\gamma_s = 1.4$, which leads to a proton spectrum with index 1.70 in the rigidity region between 10 and 100 GV c^{-1}. Equations (11) and (4) permit all spectra to be calculated and therefore the rigidity dependence of the abundance ratios observed in the Solar System.

3B. The ratio of young to old particles

The fraction of young particles in the spiral arm population depends on their residence time and therefore on particle rigidity and on the amount of material traversed. It is shown in Figure 3 for a number of components, using $K = 100$. The young particle

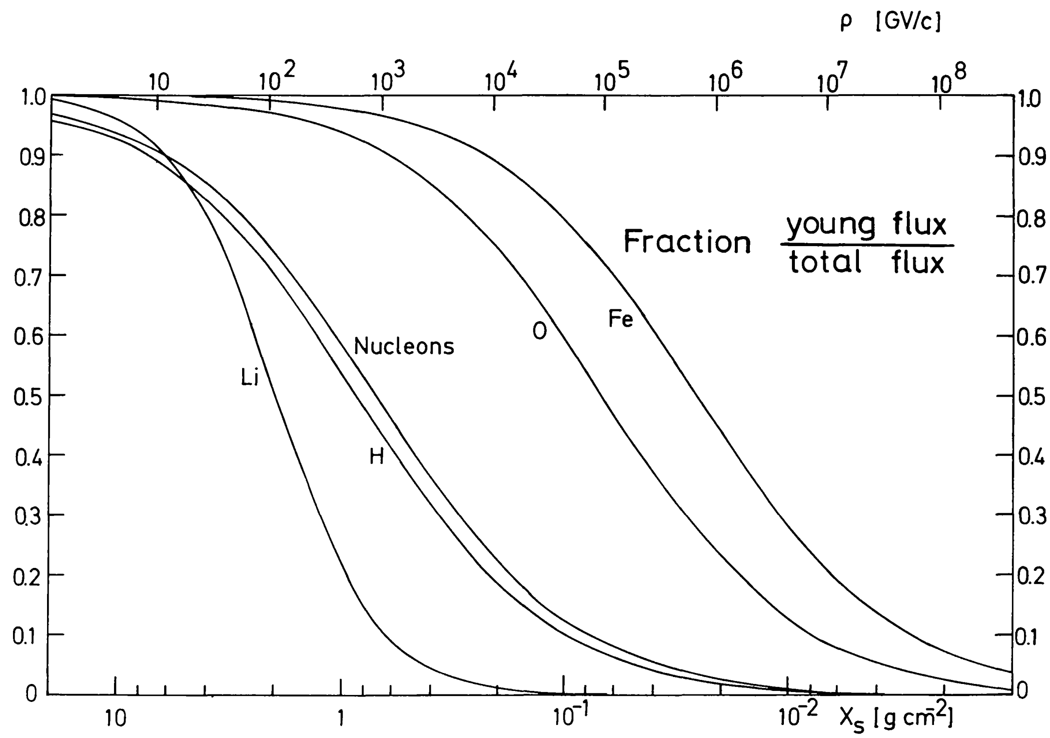

Fig. 3. The fraction of young particles of various elements in the cosmic rays in the spiral arms as a function of ϱ ($K = 100$, $x_s = 21\varrho^{-1/2}$ g cm^{-2}).

population dominates in the geomagnetically sensitive part of the cosmic ray spectrum. For hydrogen and helium, the old or background component begins to become important at relatively low energies. Only for elements as heavy as iron does this dominance extend into the energy region where extensive airshowers are formed.

3C. The composition and the energy spectrum of the primaries of extensive airshowers

The abundance of iron and of other heavy elements in the spiral arms with energies above 10^{14} eV per nucleon is so high that they make a considerable contribution to the primaries of extensive airshowers and their gradual fading out with increasing energy produces variations in the energy spectrum of showers quite similar to those observed.

In Figure 4 we have shown the energy spectrum of airshowers and of the principal airshower primaries on the assumption that the source spectra for all components continue unchanged as a power law (with the same exponent $\gamma_s = 1.4$ which fits the observations at lower energies) but that all emission ends abruptly at a rigidity $\varrho = 10^{18}$ V c^{-1}. The figure shows that under these circumstances, iron nuclei represent *ca.* 30% of the airshower primaries with total energy in the interval 10^{14}–10^{15} eV, less than 10% between 10^{15} and 10^{18} eV and then dominate again at still higher energies between 10^{18} eV and 2.6×10^{19} eV.

The abrupt cut-off at $\varrho = 10^{18}$ V c^{-1} should of course be replaced by more realistic assumptions such as a gradual rigidity dependent steepening of the overall source

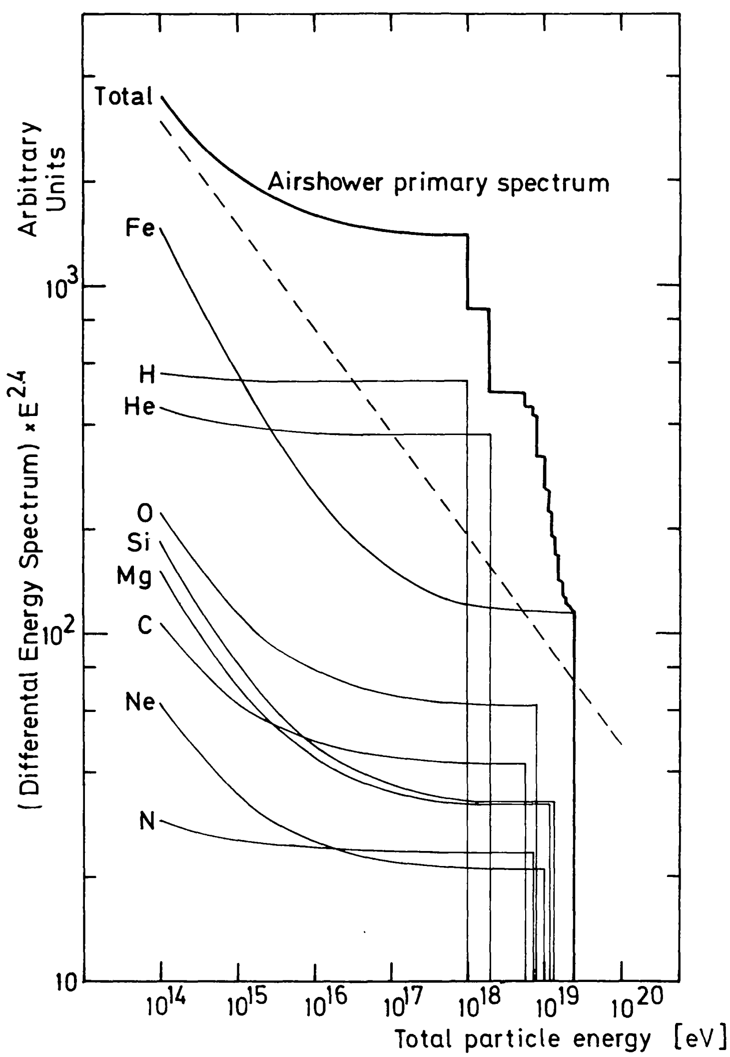

Fig. 4. The differential energy spectrum and composition of the airshower primaries. A sharp rigidity cut-off at $\varrho = 10^{18}$ V c^{-1} is assumed. The broken line gives the slope of the H-spectrum in the range 10–100 GeV. All the spectra are divided by the differential source spectrum $E^{-2.4}$.

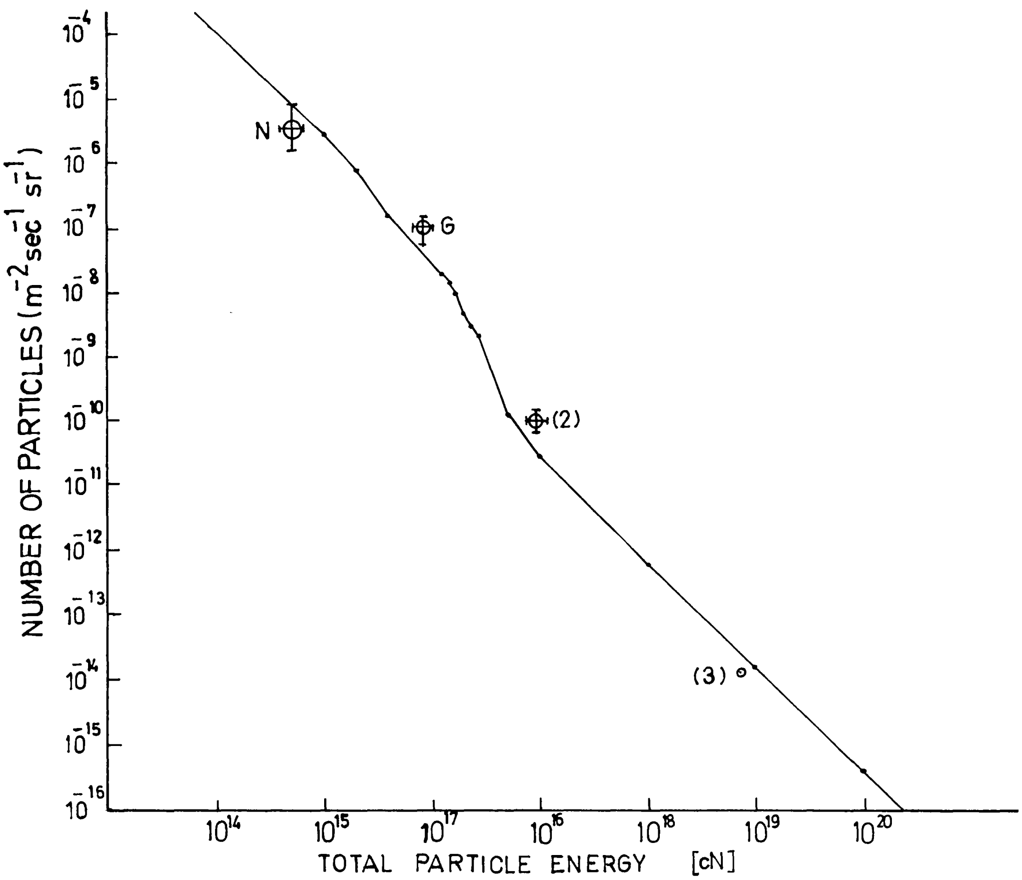

Fig. 5. The integral airshower spectrum on the assumption that all but 3% of the sources have a sharp rigidity cut-off at $\varrho_c = 2 \times 10^{15}$ V c^{-1} and that of the remaining sources a fraction $(\varrho_c/\varrho)^{0.2}$ have a rigidity cut-off at $\varrho > \varrho_c$. The experimental points are those quoted by Hayakawa (1969).

spectrum caused by differences in the cut-off of the individual contributing sources. Evidently such assumptions can be made *ad hoc* to reproduce the existing data on the airshower energy spectrum.

An example is given in Figure 5, where we have made the assumption that all but 3% of the sources have a sharp cut-off at $\varrho_c = 2 \times 10^{15}$ V c^{-1} and that of the rest the fraction of sources which have a cut-off at $\varrho > \varrho_c$ diminishes as $(\varrho_c/\varrho)^{0.2}$. Such an arbitrary but not unreasonable assumption reproduces the experimental data on the airshower spectra fairly well (Hayakawa, 1969).

3D. The source composition

Table I gives the calculated average source composition, based on the slab approximation in the spiral arms using the values $x_s = 2\langle x \rangle = 21\varrho^{-1/2}$ g cm^{-2} and $K = 100$. (An exponential path length distribution using $x_s = \langle x \rangle = 15\varrho^{-1/2}$ g cm^{-2} gives quite similar results.) The various columns are

(1) the source composition which comes closest to reproducing the observed abundances for particles with momentum/nucleon $p > 2$ GeV c^{-1};

(2) the energy per nucleon at which the contribution of old particles begins to exceed that of the young particles of local origin;

(3) the *calculated* total flux of particles with momentum/nucleon $p>2$ GeV c^{-1} (normalized at $C=100$);

(4) the *observed* flux for particles with momentum/nucleon $p>2$ GeV c^{-1} (normalized at $C=100$).

The source composition is very similar to that derived from the leaky box model. At low energy all but a few percent of the observed particles and *ca.* 88% of the protons belong to the young component. The energy at which the contribution of the two components becomes equal varies greatly for the different nuclear species and is given in column 2. It depends on the cross-section for destruction and on whether the particles come primarily from fragmentations or directly from the source. At high energy all particles belong to the old component. The observed fluxes in column 4 are those given in Shapiro *et al.* (1973) except for the Li abundance which, as shown recently by Byrnak (1976), has been systematically underestimated in much of the earlier work.

TABLE I

Element	Source composition	Turn-over energy* (TeV)	Calculated composition $p>2$ GeV c^{-1}	Measured composition $p>2$ GeV c^{-1}
H	28 000	0.2	64 000	64 000
He	2 940	4.4	4 300	4 300
Li	–	0.4	20.7	23
Be	–	0.7	10.9	10
B	–	1.3	30.6	28
C	100	240	100	100
N	4.0	8.1	24.8	25
O	112.5	770	91.1	91
F	–	4.8	2.7	1.7
Ne	15.2	440	15.9	16.0
Na	–	8.5	2.7	2.7
Mg	25.6	1 600	19.0	19.0
Al	0.6	43	2.7	2.8
Si	21.1	2 600	14.0	14.0
P	–	11	0.7	0.6
S	2.3	480	3.0	3.0
Cl	–	18	1.0	0.5
Ar	0.4	97	1.5	1.5
K	–	22	0.8	0.8
Ca	1.34	520	2.2	2.2
Sc	–	33	0.5	0.4
Ti	–	43	1.8	1.7
V	–	56	0.6	0.7
Cr	–	68	1.6	1.5
Mn	–	88	1.3	0.9
Fe	26.2	19 000	11.0	11.0

* Total particle energy where the young and old flux are equal.

4. Anisotropy in the Cosmic Ray Flux

The anisotropy which one must expect in the flux observed in the solar system is proportional to the transverse drift velocity u and to the fraction of young particles. The fraction of young particles increases with their residence time in the spiral arms and is approximately inversely proportional to the drift velocity u. The anisotropy which depends on the product is then nearly independent of cosmic ray energy.

Furthermore the fraction of young particles is proportional to K (Equation 11) and, therefore, upper limits on anisotropy correspond to upper limits on K.

At moderate energies ($<10^{14}$ eV) the most accurate experiments on anisotropy carried out so far are based on the flux of muons and of neutrons on the Earth's surface. These depend on the flux of all nucleons N incident on the atmosphere, regardless of whether they arrive as protons or as complex nuclei. We shall make use of these measurements for placing an upper limit on K.

As long as the observer can be considered to be at rest with respect to the emitting sources, the anisotropy of the nucleon flux can be written

$$\delta_N = \frac{(u/c)\,Y_N}{Y_N + \eta_N}, \tag{15}$$

which with the help of Equations (2), (4), (5), (8), and (10) can be transformed into

$$\delta_N(\varrho) = \begin{cases} (\varrho/\varrho_0)^{1/2} \dfrac{1}{1 + [(\Omega\varepsilon)_{NN}/K\omega_{NN}]} & \text{for } \varrho < \varrho_0, \\ \delta(\varrho_0) & \text{for } \varrho > \varrho_0. \end{cases} \tag{16}$$

The matrix elements are easily evaluated. One obtains

$$\frac{(\Omega\varepsilon)_{NN}}{\omega_{NN}} = \begin{cases} \dfrac{\Lambda}{x_s} & \text{for an exponential path length distribution,} \\ \dfrac{1}{e^{x_s/\Lambda} - 1} & \text{for a } \delta\text{-function path length distribution,} \end{cases} \tag{17}$$

where $\Lambda \sim 80$ g cm^{-2} is the attenuation length for nucleons in hydrogen (see Appendix). Since $x_s \ll \Lambda$, both expressions have practically the same value.

Insertion of (17) into Equation (16) yields the anisotropy

$$\delta_N(\varrho) = \left[(\varrho_0/\varrho)^{1/2} + \frac{200\Lambda}{Ka_s n_s}\right]^{-1} \quad \text{for } \varrho < \varrho_0. \tag{18}$$

Thus $\delta_N(\varrho) < \delta_N(\varrho_0) \approx 6.3 \times 10^{-5}(Ka_s n_s)$.

All experimental upper limits on anisotropy can be satisfied (cf. Osborne *et al.*, 1976; Thambyahpillai, 1975; Turver, 1973) if

$$Ka_s n_s < 20. \tag{19}$$

In Figure 6 the anisotropy $\delta_N(\varrho)$ is plotted vs rigidity ϱ for $Ka_s n_s = 20$.

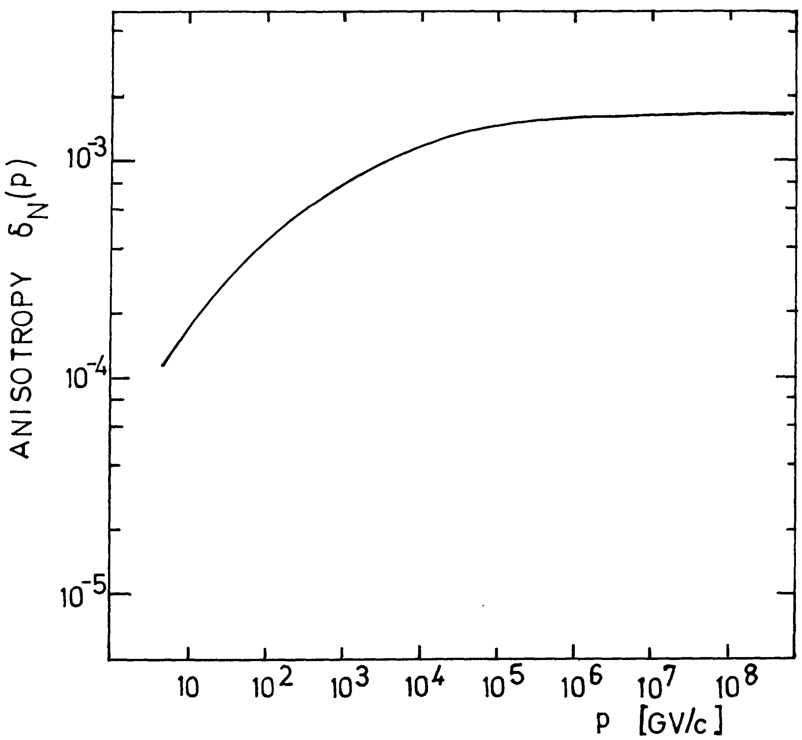

Fig. 6. The anisotropy of the nucleon flux as a function of rigidity ($Ka_s n_s = 20$, $\varrho_0 = 4.4 \times 10^{18}$ GV c^{-1}).

At high energies anisotropy measurements refer to the frequency of extensive air-showers, which depends on the flux of particles of a given total energy $E \approx Z\varrho$. In the case of Fe where no significant portion of the flux can be attributed to fragments from heavier elements, the anisotropy has the same simple form as δ_N with Λ replaced by the interaction mean free path for iron in hydrogen $\lambda_{\mathrm{Fe}} \sim 2.5$ g cm^{-2}. The anisotropy for iron nuclei of energy $E \sim Z\varrho$ is then given by

$$\delta_{\mathrm{Fe}}(E) = \left[\left(\frac{E_0}{E}\right)^{1/2} + \frac{200\lambda_{\mathrm{Fe}}}{Ka_s n_s}\right]^{-1} \quad \text{for } E < E_0, \tag{20}$$

$$\delta_{\mathrm{Fe}}(E) \leqslant \delta_{\mathrm{Fe}}(E_0) = 2 \times 10^{-3}(Ka_s n_s), \tag{21}$$

where $E_0 = Z_{\mathrm{Fe}}\varrho_0 \approx 10^{10}$ GeV (see Section 5).

If $Ka_s n_s = 20$, this suggests a few percent anisotropy in the flux of iron nuclei for $E > 5 \times 10^{15}$ eV and a few per mille variation down to energies $E \approx 3 \times 10^{13}$ eV.

Elements lighter than iron will have smaller anisotropies at high energies and play a less important role in producing airshowers. Therefore as long as the large scale magnetic field of our galaxy does not permit or severely restricts escape into inter-galactic space, the anisotropy even for primaries of the largest airshowers will be small but perhaps observable.

5. Astrophysical Constants Consistent with Nuclear Composition and with Isotropy

The limit $K > 100$ obtained from the observed energy dependence of the L/M ratio and the limit $Ka_s n_s < 20$ obtained from the isotropy at medium and low energies (together with the condition $x_0\sqrt{\varrho_0} = 21(\text{GV c}^{-1})^{1/2}$ g cm^{-2} derived from the nuclear composition, see Equations (4), (14)), yield the following inequalities:

$$a_s n_s \leqslant 0.20 \text{ kpc cm}^{-3}, \qquad a_s H_{\|} \geqslant 0.49 \text{ kpc } \mu\text{gauss},$$
$$\varrho_0 > 4.4 \times 10^8 \text{ GV c}^{-1}.$$

A possible set of parameters which satisfy the cosmic ray data and do not conflict with other astrophysical evidence, is:

fraction of galactic gas in the spiral arms

$$K^{-1} = M_s/\varphi M_G \sim 1\%,$$

thickness of spiral arms

$$2a_s = 0.2 \text{ kpc},$$

gas density in the spiral arms

$$n_s = 2 \text{ cm}^{-3},$$

longitudinal magnetic field

$$H_{\|} = 4.9\ \mu\text{gauss}.$$

With these values Equation (5) yields

$$\varrho_0 = 4.4 \times 10^8 \text{ GeV c}^{-1}.$$

The average amount of material traversed by the particles which reach the Solar System in the closed galaxy model at moderately low energy, when almost all particles belong to the young component, is of course very similar to that obtained in the usual leaky box model.

At $\varrho = 10$ GV c^{-1} one finds (for the slab approximation):

average amount of gas traversed when leaving the arms

$$x_s = 6.6 \text{ g cm}^{-2},$$

average residence time in the arms

$$\tau_s = 2 \text{ My},$$

average amount of gas traversed when reaching the Solar System

$$\langle x \rangle = 3.3 \text{ g cm}^{-2},$$

the transverse drift velocity

$$u/c = 1.5 \times 10^{-4}.$$

If one uses the exponential type of path length distribution instead of the slab approximation, one finds that the nuclear composition, its change with energy and the observed limits on anisotropy can be satisfied by $K^{-1}=1\%$, $n_s=2\ \mathrm{cm}^{-3}$, $H_{\|}=2.5\ \mu$gauss, $\varrho_0=2.25\times 10^8\ \mathrm{GeV\ c}^{-1}$, and at $\varrho=10\ \mathrm{GeV\ c}^{-1}$ one finds that $x_s=4.75\ \mathrm{g\ cm}^{-2}$, $\tau_s=1.4$ My, $\langle x\rangle=4.75\ \mathrm{g\ cm}^{-2}$, $u/c=2\times 10^{-4}$. We now return to Equation (4), where we have arbitrarily set $\alpha=\frac{1}{2}$.

If instead of $\alpha=\frac{1}{2}$ one choses a different exponent for the rigidity dependence of the transverse drift velocity, and at the same time maintains the relation between x_s and ϱ which is dictated by the observed nuclear composition, one must chose different values for the two parameters

$$x_0 = \frac{a_s n_s}{200}\ \mathrm{g\ cm}^{-2} \quad \text{and} \quad \varrho_0 = 9 \times 10^8 H_{\|} a_s\ \mathrm{GV\ c}^{-1}.$$

It seems hardly possible to change the values of the astrophysical constants a_s, n_s and $H_{\|}$ by more than a factor two, either way, from the values given above without getting into conflict with astronomical evidence. Using these restrictions one finds when inserting $x_s=10.5\ \mathrm{g\ cm}^{-2}$ at $\varrho=4\ \mathrm{GV\ c}^{-1}$ into Equation (4) that the exponent is limited to

$$0.4 \leqslant \alpha \leqslant 0.6.$$

Thus one finds that in order to account for the observed nuclear composition at $\varrho \sim 4\ \mathrm{GV\ c}^{-1}$ without violating other astronomical evidence the transverse drift velocity in the spiral arms must increase with the radius of the particle trajectory as $R^{0.5\pm 0.1}$. It can be considered a supporting argument for the model that this same dependence accounts correctly for the observed variation of abundance ratios with energy.

6. Time Variations

Long period time variations in the cosmic ray flux arise when a source flares up in the neighborhood of the solar system. In our model where the particles can diffuse rapidly along the spiral arms, a source may be close to the solar system (in the sense that the particles have traversed only small amounts of gas on arrival), although it is physically distant when measured along the spiral arms. A Supernova which explodes in a tube of field lines passing through the solar system would be a close neighbor. It is not possible to estimate the frequency of occurrence of close neighbors without making assumptions about the size of the cavity which such explosions produce in the surrounding field.

Nevertheless one can make some qualitative statements about time variations in this model.

(a) Major fluctuations are unlikely to occur in the *old* component, because all active cosmic ray sources in the galaxy contribute equally independent of their location. A lower limit can be set for the number of sources contributing to the observed flux at any one time:

$$N \geqslant \tau/T,$$

where τ is the lifetime of nucleons in the galaxy (against attenuation by meson production) and T is the average interval between the appearance of new sources.

(In the limit that the lifetime of each source is very short, $N=\tau/T$ but if the active life of the sources exceeds τ, N can be very much larger.)

The amplitude of expected fluctuations is, therefore,

$$\frac{1}{N} < \frac{T}{\tau}.$$

If one thinks of Supernova outbursts as major cosmic ray sources one may put $T \approx$ 25–100 yr. An attenuation mean-free-path of $\Lambda \approx 80$ g cm^{-2} corresponds to a lifetime of particles in a containment volume with mean gas density $\langle n \rangle$ is given by

$$\tau > \frac{50}{\langle n \rangle} \text{ My}.$$

Thus the ratio T/τ and, therefore, the amplitude of expected fluctuations in the old component must be extremely small.

(b) By analogy the amplitude of fluctuations in the *young* component would be at most of order T/τ_s, where τ_s is the residence time of the particles in the spiral arms. This ratio is also very small except at very large energy, where the fraction of young particles in the total flux becomes negligible and cannot contribute to the time variations.

One cannot safely conclude from this argument that secular time variations must be very small even at low energies where the young particles dominate. The conclusion is valid only if the dimensions of the cavity created by the explosion of a Supernova is of the order of the transverse dimensions of the spiral arms. If this cavity is very much smaller, then it cannot be ruled out that individual sources could for a time make sizeable contributions to the young component and therefore to the observed flux at not too high energies. This could give rise to measurable intensity fluctuations, as well as to inhomogeneities in nuclear composition.

7. Compatibility of the Closed Galaxy Model with Observations on Cosmic Ray Electron and γ-rays

The electron/positron flux expected from a closed galaxy model has recently been studied by Ramaty and Westergaard (1976). They found fluxes compatible with those measured in the Solar System within the wide margin of uncertainty created by the as yet poorly understood mechanism of solar modulation. The present model differs from that used by these authors in that the containment volume is now divided into two regions with different propagation properties. The effect of this modification is not expected to change the situation appreciably. It is being studied and will be the subject of a separate publication.

The model predicts the nucleon density inside and outside the spiral arms and, therefore, permits a calculation of γ-ray flux in various directions in the galactic plane, if an appropriate gas distribution is assumed. The strong concentration of cosmic ray energy in the spiral arms predicted by our model is in qualitative agreement with the assumptions made by Fichtel *et al.* (1975) in their attempt to account for the γ-ray intensities measured on SAS-2.

Since the model predicts the cosmic ray density in the halo, the observed γ-ray intensity perpendicular to the galactic plane places an upper limit on the product of halo radius and halo gas density. A preliminary estimate leads to reasonable values for this product even if one assumes no extragalactic contribution to this flux. Together with the value of $K \sim M_g/M_s \sim 100$ one can obtain separate estimates for the halo dimensions and the halo gas density. A more detailed analysis will be published soon.

8. Energy Requirements

The energy requirements of the closed galaxy model are an order of magnitude lower than that of the leaky galaxy model in its present form.

If we use the constants of Section 5 and estimate the total length of galactic spiral arms to be

$$L_s = 300 \text{ kpc},$$

the total mass of the gas in the spiral arms is

$$M_s \approx \pi a_s^2 L_s n_s m_p \approx 4 \times 10^8 \, M_\odot$$

and the total mass of the gas in the galaxy is

$$M_G \approx K M_s \approx 4 \times 10^{10} \, M_\odot.$$

The energy Q required to maintain a steady state in the galaxy is given by

$$Q = \frac{c}{\Lambda} (\varepsilon_s M_s + \varepsilon_G M_G), \tag{22}$$

where ε_s is the energy density of cosmic rays in the spiral arms and

$$\varepsilon_G = \varepsilon_s \frac{\eta_N}{\eta_N + Y_N} \tag{23}$$

is the energy density outside the arms. The curves in Figure 3 permit us to estimate the ratio $\varepsilon_G/\varepsilon_s$ averaged over the cosmic ray spectrum. Accordingly we use

$$\varepsilon_s = 1 \text{ eV cm}^{-3}, \qquad \varepsilon_G = 0.2 \text{ eV cm}^{-3}. \tag{24}$$

One finds that

$$Q = 10^{40} \text{ erg s}^{-1}. \tag{25}$$

This corresponds to about 1% of the estimated rate at which energy is released by Supernova explosions in our galaxy.

9. Conclusions

A closed galaxy model can account for the presently established cosmic ray observations, with reasonable assumptions about the dimensions, the structure and the gas- and field-density in our galaxy.

It may also account for most of the diffuse γ-ray background as arising in the galactic halo rather than from extragalactic sources.

It eliminates the need for an extragalactic cosmic ray component.

A single power spectrum in rigidity at the source for all components can account for the observed energy variation of cosmic ray composition.

The observed energy spectrum of airshowers can be reproduced if one assumes that the spectra emitted by different sources terminate at different rigidity cut-off values.

The model predicts a high degree of cosmic ray isotropy up to the highest energies.

It does not preclude that for brief periods individual sources contribute a measurable fraction to the local cosmic ray intensity.

The energy input needed to maintain cosmic ray density at its present level corresponds to about 1% of the estimated rate at which energy is released in our galaxy through the explosion of Supernovae.

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Appendix

1. The propagation equation and its solution

We consider nuclear cosmic rays in a steady state moving with the velocity of light (c) in a volume V. The decrease in density due to escape from the volume is described by an operator $O(\varrho)$ that acts on the density, but does not change the magnetic rigidity (ϱ) of the particles. This decrease is counterbalanced by the increase due to sources and nuclear interactions (spallation) – i.e.,

$$O(\varrho)N_i(p) = \frac{Q_i f(p)}{V} - \frac{N_i(p)}{T_i} + \sum_j nc\sigma_{ji}N_j(p). \tag{A1}$$

The rigidity $\varrho = pA_i/Z_i$ where p is the momentum per nucleon and A_i and Z_i are the mass- and charge-number respectively. N_i is the density of cosmic ray particles of kind 'i'. $S_i(p) = Q_i f(p)$ is the number of particles of momentum p created per unit time in the sources in V. T_i is the mean lifetime in V $[T_i = (nc\sigma_i)^{-1}$ where n is the gas density and σ_i the destruction cross-sections]. σ_{ji} is the partial cross-section for producing particles of kind 'i' from 'j'.

The operator O depends on the rigidity of the particle because the drift of the particles in V is supposed to be governed by magnetic fields. Source strength and particle density are expressed as functions of p because the momentum per nucleon is conserved in spallation reactions, but not rigidity.

We assume that gas density and cosmic ray flux are uniform in V and that above 2 GeV per nucleon the spallation cross-sections are constant.

Equilibrium

If there is no escape, $O(\varrho)N_i(p) = 0$ and Equation (A1) takes the form

$$0 = \frac{Q_i f(p)}{M_V c} - \frac{N_i(p)}{\lambda_i} + \sum_j \frac{\sigma_{ji}}{m_p} N_j(p), \tag{A2}$$

where $M_V = Vnm_p$ is the mass of gas in V (m_p is the proton mass). The mean free path $\lambda_i = T_i ncm_p$. The set of equations can be written in vector form

$$0 = \frac{\mathbf{Q} f(p)}{M_V c} + \underline{\underline{M}}\mathbf{N}(p);$$

$\mathbf{N}$ and $\mathbf{Q}$ are vectors with components N_i and Q_i, respectively. Double underlining designates a matrix. $\underline{\underline{M}}$ is a triangular matrix whose diagonal elements describe the destruction and the off-diagonal elements the production of nuclei.

The solution is

$$\mathbf{N}(p) = \underline{\underline{\Omega}} \frac{\mathbf{Q} f(p)}{M_V c}, \tag{A3}$$

with

$$\underline{\underline{\Omega}} = -\underline{\underline{M}}^{-1}.$$

General Solution

If the gas density n is constant the solution of (A1) can be factored (cf. Ginzburg and Syrovatskii, 1964) into a function $G(p, x)$ and a set of functions $\vec{\mathscr{N}}(x)$ where x is the amount of matter traversed. We have $x = tncm_p$, where t is the time after the acceleration of the particle, $G(p, x)$ and $\vec{\mathscr{N}}(x)$ are determined by the differential equations

$$\frac{\partial}{\partial x} \vec{\mathscr{N}}(x) = \underline{\underline{M}} \vec{\mathscr{N}}(x) \tag{A4}$$

and

$$\frac{O(\varrho)G(p, x)}{nm_p c} + \frac{\partial}{\partial x} G(p, x) = 0;$$

and the boundary conditions

$$\vec{\mathcal{N}}(0) = \frac{\mathbf{Q}}{nm_p c}, \qquad G(p, 0) = \frac{f(p)}{V}. \tag{A5}$$

In principle we should have a different G function for each isotope but the small variation in A/Z for the various isotopes gives only a negligible change in the flux. Only for protons has it a large effect that will be considered later. The cosmic ray density can then be expressed as an integral

$$\mathbf{N}(p) = \int_0^\infty \vec{\mathcal{N}}(x) G(p, x)\, \mathrm{d}x, \tag{A6}$$

where $G(p, x)$ represents the density of particles with momentum p and 'age' x resulting from a source of unit strength in the absence of nuclear interactions.

As in Section 3, Equation (4), $x_s = x_0(\varrho_0/\varrho)^\alpha \approx x_0(\varrho_0/\varrho)^{1/2}$ is the mean amount of matter traversed in V by particles of rigidity ϱ. We define $P(x, x_s)$ as the probability that a particle will traverse more than x g cm^{-2} of matter inside V. It means that a fraction $P(x, x_s)$ of the particles accelerated a time $t = x/(nm_p c)$ ago is still inside V and we have

$$P(x, x_s)\frac{f(p)}{V} = G(p, x);$$

$\vec{\mathcal{N}}(x)\, \mathrm{d}x$ being the number of particles of each kind with an age between x and $x + \mathrm{d}x$ for a zero rate of escape.

The solution for $\vec{\mathcal{N}}(x)$ can be written as

$$\vec{\mathcal{N}}(x) = \underline{\underline{s}}(x) \frac{\mathbf{Q}}{nm_p c}; \tag{A7}$$

where $\underline{\underline{s}}$ is a triangular matrix with elements that are linear combinations of exponentials of $-x/\lambda_i$. It is given in the next section for a case where the only important transformation is $1 \to 2$ (i.e. only $\sigma_{12} \neq 0$).

2. Two component equations

The source volume V_s lies inside the closed galactic volume V_G and the source for the particles in V_G are those that escape from V_s.

By use of Equation (A6), the young particles are found to have a density given by

$$\mathbf{Y}(p) = \int_0^\infty \vec{\mathcal{N}}(x) G(p, x)\, \mathrm{d}x = \frac{f(p)}{V_s} \int_0^\infty \vec{\mathcal{N}}(x) P(x, x_s)\, \mathrm{d}x.$$

This can also be expressed in terms of a matrix $\underline{\underline{\omega}}$ when we define

$$\underline{\underline{\omega}}(\varrho) = \int_0^\infty \underline{\underline{s}}(x) P(x, x_s)\, \mathrm{d}x$$

and use (A7)

$$\mathbf{Y}(p) = \frac{\underline{\underline{\omega}}}{M_s c}\, \mathbf{Q} f(p) = \frac{\underline{\underline{\omega}}\mathbf{S}(p)}{M_s c},$$

where M_s is the mass of the source region $M_s = n_s m_p V_s$ and n_s the density of the medium therein.

The composition of the source for the old component is determined by the age of the particles when they escape from V_s. The probability that a particle escapes after having traversed an amount of matter between x and $x + \mathrm{d}x$ is

$$P(x) - P(x + \mathrm{d}x) = -\frac{\partial}{\partial x} P(x, x_s)\, \mathrm{d}x.$$

This we will call the path-length distribution.

Therefore the source is

$$\mathbf{S}'(p) = -\int_0^\infty \vec{\mathcal{N}}(x) \frac{\partial}{\partial x} P(x, x_s)\, \mathrm{d}x\, f(p) = \underline{\underline{\varepsilon}}\mathbf{S}(p),$$

where

$$\underline{\underline{\varepsilon}}(\varrho) = \int_0^\infty \underline{\underline{s}}(x)\left(-\frac{\partial}{\partial x} P(x, x_s)\right) \mathrm{d}x.$$

Then Equation (A3) is used to get the background density

$$\boldsymbol{\eta}(p) = \underline{\underline{\Omega}}\,\underline{\underline{\varepsilon}}\, \frac{\mathbf{S}(p)}{M_G c},$$

where ε is a function of x_s and therefore of ϱ.

We introduce a factor φ to allow for a deviation of the local young-component density from the average spiral arm density. The total local density is then

$$\mathbf{Y}(p) + \boldsymbol{\eta}(p) = \frac{1}{M_G c}\left(K\underline{\underline{\omega}}(\varrho) + \underline{\underline{\Omega}}\,\underline{\underline{\varepsilon}}(\varrho)\right)\mathbf{S}(p),$$

with $K = \varphi M_G / M_s$.

Examples of Matrices

The general form of the matrices is illustrated by taking a case where only $\sigma_{12} \neq 0$. The $\underline{\underline{s}}$-matrix is given by

$$\underline{\underline{S}} = \begin{pmatrix} \exp(-x/\lambda_i) & 0 \\ \dfrac{\sigma_{12}}{\sigma_2 - \sigma_1}\left[\exp\left(-\dfrac{x}{\lambda_1}\right) - \exp\left(-\dfrac{x}{\lambda_2}\right)\right] & \exp\left(-\dfrac{x}{\lambda_2}\right) \end{pmatrix}$$

For an exponential $P(x, x_s) = \exp(-x/x_s)$,

$$\underline{\underline{\omega}} = \begin{pmatrix} \dfrac{\lambda_1 x_s}{\lambda_1 + x_s} & 0 \\ \dfrac{\sigma_{12}}{\sigma_2 - \sigma_1}\left[\dfrac{\lambda_1 x_s}{\lambda_1 + x_s} - \dfrac{\lambda_2 x_s}{\lambda_2 + x_s}\right] & \dfrac{\lambda_2 x_s}{x_s + \lambda_2} \end{pmatrix}$$

and

$$\underline{\underline{\varepsilon}} = \frac{1}{x_s}\underline{\underline{\omega}}.$$

For the $P(x, x_s)$ used in the calculations

$$P(x, x_s) = \begin{cases} 1 & \text{for } x < x_s \\ 0 & \text{for } x > x_s \end{cases}$$

$$\underline{\underline{\omega}} = \begin{pmatrix} \lambda_1(1 - e^{-x_s/\lambda_1}) & 0 \\ \dfrac{\sigma_{12}}{\sigma_2 - \sigma_1}[\lambda_1(1 - e^{-x_s/\lambda_1}) - \lambda_2(1 - e^{-x_s/\lambda_2})], & \lambda_2(1 - e^{-x_s/\lambda_2}) \end{pmatrix},$$

and

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} e^{-x_s/\lambda_2} & 0 \\ \dfrac{\sigma_{12}}{\sigma_2 - \sigma_1}(e^{-x_s/\lambda_1} - e^{-x_s/\lambda_2}) & e^{-x_s/\lambda_2} \end{pmatrix}.$$

This last choice of $P(x, x_s)$ corresponds to a narrow distribution of lifetimes for particles of a given rigidity in the spiral arms around the value $\tau_s = x_s/n_s c m_p$.

Protons

For heavy nuclei ($A/Z \approx 2$) both p and ϱ are conserved in spallation reactions. Protons produced in fragmentation by a heavy nucleus will retain the momentum but only get about half the rigidity of the parent. Thus secondary protons that are created near the source soon after the acceleration of the parent will drift slower to the boundary of the spiral arm than protons that are created later and farther from the source because the latter are carried along with the heavy particle. The actual drift will correspond to some intermediate value of the rigidity between that of the heavy particle and that of the protons. Here we take the mean of the secondary proton-densities evaluated using the rigidity of the parent and of the daughter. The two numbers differ by less than 20%. The secondary protons contribute less than 25% of the total proton density, and therefore this crude averaging procedure seems adequate.

3. Cross-sections

The destruction cross-sections for nuclei of mass A_i are adequately represented by the formula

$$\sigma_i = 54(A_i^{1/3} - 0.2)^2 \text{ mb}$$

taken from Meyer *et al.* (1975).

For protons that are not destroyed in collision we use an attenuation mean free path of 80 g cm^{-2} (cf. Rasmussen and Peters, 1975).

The partial cross-sections are taken from the work done at Bevalac and Orsay, cf. Lindstrom *et al.* (1975); Raisbeck *et al.* (1971, 1974, 1975); Perron (1975); Yiou *et al.* (1973). These experimentally measured cross-sections are supplemented by results from the semi-empirical formula of Silberberg and Tsao (1973). The unstable isotope ^{10}Be was assumed to have decayed completely.

To get the cross-sections for producing protons and α-nuclei we assume that every fragment that is not a heavy nucleus is a proton with probability $\frac{2}{3}$ and an α-particle with probability $\frac{1}{3}$ and thus

$$\sigma_{ip} = \frac{1}{3}\left(A_i\sigma_i - \sum_{A_j<A_i} A_j\sigma_{ij}\right),$$

$$\sigma_{i\alpha} = \frac{1}{6}\left(A_i\sigma_i - \sum_{A_j<A_i} A_j\sigma_{ij}\right).$$

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