Diffusion and drift of very high energy cosmic rays in galactic magnetic fields

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Abstract. We consider diffusive propagation of galactic cosmic rays with energies up to $10^{18}$ eV. It is shown that drift (Hall diffusion) of particles in the global regular magnetic field of the Galaxy may be responsible for the observed "knee" in the cosmic ray energy spectrum at $3 \times 10^{15}$ eV. The model implies weak energy dependence of cosmic ray diffusion along the magnetic field lines $D \propto E^{-n}$, $n = 0.15 - 0.20$, for a unique power law in the entire energy range $10^9 - 10^{17}$ eV.

Key words: cosmic rays – interstellar medium: magnetic fields – Galaxy: halo

1. Introduction

The "knee" (the point at which the power law spectrum gets sharply steeper) at $E_k = 3 \times 10^{15}$ eV is a well known feature of the cosmic ray spectrum (Kulikov & Kristiansen 1959; Nagano et al. 1984; Fomin et al. 1991). The differential cosmic-ray energy spectrum $N(E) \propto E^{-\gamma}$ has a power law exponent $\gamma = 2.6 - 2.7$ at $10^{10}$ eV $\leq E \leq 3 \times 10^{15}$ eV and exponent $\gamma = 3 - 3.1$ at $3 \times 10^{15}$ eV $\leq E \leq 3 \times 10^{18}$ eV. Actually, some flattening of the spectrum before the steepening is not excluded with $\gamma = 2.5$ at $3 \times 10^{15} - 3 \times 10^{16}$ eV, see Fichtel & Linsley (1986), Kristiansen (1987), and Kifune (1990) for review.

The steepening of the spectrum is usually explained by changes in the conditions of cosmic-ray leakage from the Galaxy (Syrovatsky 1971; Wdowczyk & Wolfendale 1984). Changes of the accelerated-particle spectrum at the cosmic ray sources proper (Peters 1961; Karakula et al. 1990; see also Hillas 1979) or superposition of two populations of sources with different energy spectra (Fichtel & Linsley 1986; Jokipii & Morfill 1986) are also possible. Another process assumes a two-step acceleration process, namely diffusive shock acceleration up to $10^{15} - 10^{16}$ eV at the outer shock front of the supernova remnant and the subsequent re-acceleration in the inner part of the pulsar-driven remnant (Bell 1991) or in the volume of the Galaxy as a whole (Axford 1991).

In the present work, we consider the propagation of cosmic rays in galactic magnetic fields. The random field in the galactic halo is assumed to exist simultaneously with the regular magnetic field, which is mainly toroidal and whose value is comparable with the value of the field in the gas disk. Particle diffusion in the magnetically-active medium is well-known to have a tensor character. The Hall diffusion (determined by the antisymmetrical part of the cosmic-ray diffusion tensor) is not effective at small energies, but it may determine the leakage of cosmic rays out of the Galaxy at $E > E_k = 3 \times 10^{15}$ eV. It is then possible to obtain the knee in the cosmic-ray power law spectrum without making any assumptions concerning the occurrence of a feature in the source spectrum or changes in the energy dependence of the diffusion coefficient along the magnetic field lines. Particle transport by Hall diffusion can also be described as a particle drift in a large-scale inhomogeneous magnetic field, see e.g. the paper by Isenberg & Jokipii (1979), where cosmic ray diffusion in interplanetary magnetic fields is discussed.

2. Cosmic ray transport in galactic magnetic fields

Cosmic ray propagation in the Galaxy is usually described in terms of diffusion, see Berezhinskii et al. (1990). Semipirical diffusion models make it possible to systematize and explain the set of experimental data on cosmic ray composition and anisotropy, at least in the energy range $10^8 - 10^{14}$ eV and perhaps for higher energies. This model is confirmed by the "microscopic" theory of motion for relativistic charged particles in a magnetic field.

The steady-state diffusion equation for cosmic ray density $N(r)$ is (we neglect here by nuclei fragmentation and any energy losses)

$$- \nabla \cdot D_{ij}(r) \nabla N(r) = Q(r).$$

(1)

Here $Q(r)$ is a source term, and $D_{ij}(r)$ is the cosmic ray diffusion tensor. This tensor may be presented as

$$D_{ij} = (D_u - D_\perp) b_i b_j + D_\perp \delta_{ij} + D_\perp e_{ijn} b_n.$$  

(2)

Here $b_i = H_{0i}/H_0$ is a unit vector along the regular galactic magnetic field; $D_u, D_\perp, D_\perp$ are parallel (field-aligned), perpendicular (transverse), and antisymmetric (Hall) diffusion coefficients respectively, $e_{ijn}$ is the absolute antisymmetric tensor.

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The cosmic ray flux $j$ and the cosmic ray anisotropy $\delta$ are given by the expressions

$$j_x = -D_{\text{sm}} V_m N, \quad \delta_x = 3 j_x / (v N). \quad (3)$$

Hall diffusion gives rise to a particle flux perpendicular to the cosmic ray gradient and to the regular field direction.

Coefficients $D_1, D_2$ and $D_3$ are determined by the level of turbulence in the interstellar medium. The random magnetic field with the scale $k^{-1} \leq r_H$ ($k$ is the wave number of the random field; $r_H$ is the particle gyroradius) provides resonant scattering and local diffusion of cosmic rays. Long-wave wandering of the magnetic field lines with $k^{-1} \gg r_H$ give rise to isotropization of the highly anisotropic local diffusion. On the scales larger then the basic scale of the interstellar turbulence $L \approx 100$ pc, the combined action of random magnetic fields with extended scale spectrum results in a global diffusion with diffusion tensor $D_{ij}$. The function $N(r)$ in Eq. (1) is the cosmic ray density averaged over regions with dimensions larger then $L$.

Assuming a power-law spectrum for the random magnetic field

$$\delta H^2 (k > k_H) = A^2 H_0^2 \left( k L \right)^{1 + m}, \quad k L \geq 1, \quad m = \text{const}, \quad (4)$$

with constant $A^2 \leq 1$ characterizing a relative value of the random field on the characteristic scale $L$, one can use the approximate expressions

$$D_0 = v / 3, \quad D_1 = g A^2 v / 3, \quad D_A = -r_H v / 3. \quad (5)$$

The numerical coefficients here are calculated for $A^2 \ll 1$; $v \equiv c$ is the particle velocity; the constant $g \approx 0.1 - 0.5$ is not well-determined in the modern theory; the mean free path is estimated as

$$l = A^{-2} L (r_H / L)^m, \quad r_H \leq L; \quad (6)$$

$$l = A^{-2} L^{-1} r_H, \quad r_H > L. \quad (7)$$

The sign of $D_A$ in Eq. (5) corresponds to the diffusion of positively-charged particles $Z > 0$. It changes sign for $Z < 0$. Global diffusion $D_1$ perpendicular to the mean (regular) field is mainly determined by some projection of the local field-aligned diffusion coefficient. Detailed calculations of the diffusion tensor $D_{ij}$ are given by Beresinsky et al. (1990) and by Chyungin & Ptuskin (1990).

The observational data, supported by theoretical considerations, show that the real spectrum of the galactic random magnetic field is possibly a power-law spectrum (4) with exponent $m = 0.3 \pm 0.3$, see Armstrong et al. (1981) and Ruzaikina et al. (1988). The value $m = 1 / 3$ corresponds to a Kolmogorov spectrum, $m = 1 / 2$ to a Kraichnan (1965) hydro-magnetic spectrum, and $m = 0$ to an ensemble of shocks and discontinuities are given, for example, by the model of interstellar turbulence by Bykov & Toplygin (1987).

Equations (6), (7) show that diffusion retainment of cosmic rays in the Galaxy may be effective up to a gyroradius $r_H \leq L$, or up to a maximum energy of about

$$E_{\text{max}} \approx 2 \times 10^{17} (H/2 \ 10^{-6} \mu V) (L/100 \text{ pc}) \text{ eV} \quad (8)$$

at which the gyroradius becomes equal to the characteristic scale of the random field. At $E > E_{\text{max}} (r_H > L)$, the scattering effectiveness decreases as $E^{-2}$. However, the value $L = 100$ pc used above has been inferred from the observational data relating to the vicinity of about 500 pc from the Galactic plane (where the rotation measure for distant radio sources is mainly accumulated). The development of convective and Parker instabilities may give rise to the occurrence of random fields on scales of $1 - 3 \text{ kpc}$ in the galactic halo. Therefore, even if the field intensity decrease at high latitudes is allowed for, the diffusion with mean free path (6) may be assumed to take place in the Galaxy as the whole up to energies of about $E_{\text{max}} \approx 1 - 3 \times 10^{18} \text{ eV}$.

The global structure of the regular magnetic field in the Galaxy is very important, since it determines the orientation of the diffusion tensor (2). We have reliable information on the regular field in the region of the galactic disk (see e.g. Heiles 1987, Ruzaikina et al. 1988). The regular field in the disk is predominantly toroidal. Thus, $H_0$ is the main component of the regular field in galactic cylindrical coordinates (subscription 0 marking the regular field is omitted here). The field may have the form of a bisymmetric spiral with typical ratios of radial and perpendicular components to the azimuthal component on the order of

$$H_r / H_\phi \approx 0.1 - 0.3; \quad H_\phi / H_r \approx 10^{-3} - 3 \times 10^{-2}. \quad (9)$$

Actually, observations are in agreement with the assumption that the field lines of the regular field are closed circles and that the field $H_\phi (r)$ changes sign with radial distance $r$ after each 3 kpc (Rand & Kulkarni 1989). The regular magnetic field has the same direction above and below the galactic plane.

We have scant information on the magnetic field beyond the galactic disk, i.e. at distances $|z| > 500$ pc. The Galaxy possesses an extended magnetic corona of size $3 - 10$ kpc (Sofue et al. 1986; Broadbent et al. 1990; Beck 1990). The orientation of the regular field in the Galactic halo is not known, but the toroidal component $H_\phi$ dominates all appearances.

The galactic wind model (Jokipii & Morfill 1986; Breitschwerdt et al. 1987) predicts a structure of the regular galactic field at large distances from the galactic centre that is similar to the Parker spiral in interplanetary space. The field has asymptotic behavior $H_\phi \propto 1/r$ in this case. The field is blown up from the galactic disk to the halo, and perhaps it conserves structure with interchanging of sign of $H_\phi$. We do not take into account possible advection of cosmic rays by galactic wind flow in this paper.

A regular magnetic field in the halo could come into existence independently of the disk field (Sokoloff & Shukurov 1990). A dynamo mechanism of field generation works if there are differential rotation and turbulent gas motions with non-zero helicity in the halo. The predominance of magnetic fields parallel to the disk is inherent to dynamo generation in spherical bodies (Krause & Radler 1980). An important feature of this field is its nonpolarity $H_\phi (z) = - H_\phi (-z)$, i.e. the regular field in the halo has opposite directions above and below the galactic disk. The expected ratio of poloidal to toroidal components of the field is $H_p / H_\phi \approx 0.1$ on the average. (It is determined by the small ratio of the characteristic scale of turbulent motions to the height of the halo).

Under the assumption of azimuthal symmetry of the system ($\frac{\partial}{\partial \phi} = 0$), one can write Eq. (1) as

$$\begin{align*}
- \frac{\partial}{\partial r} \left( r D_h b_z^2 + (1 - b_z^2) D_r \right) & \frac{\partial}{\partial r} \\
- \frac{\partial}{\partial z} \left( b_z^2 D_h + (1 - b_z^2) D_z \right) & \frac{\partial}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} \left( b_r b_z (D_h - D_z) \right) \frac{\partial}{\partial r} \\
+ \frac{1}{r} \frac{\partial}{\partial r} \left( r D_h b_z \right) & + \frac{\partial}{\partial z} \left( b_z b_r (D_h - D_z) \right) \frac{\partial}{\partial z} \\
\left\{ N(r, z) = Q(r, z). \right. \quad (10)\end{align*}$$
Further simplification is achieved if one takes into account the dominance of the toroidal field component \( b_2 / b_z < 0.1 \) and uses expressions (5) for the diffusion coefficients \( D_1 \) and \( D_1 (D_2 / D_1) > 3 \times 10^{-2} \). Then one can set \( b_2 = 1 \), \( b_z = b_0 = 0 \), and Eq. (10) reduces to:

\[
- \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho D_1 \frac{\partial}{\partial \rho} = \frac{\partial D_1}{\partial z} \frac{\partial}{\partial z} - \frac{\partial}{\partial z} (D_2 \frac{\partial}{\partial \rho} \theta) + \frac{1}{\rho} \frac{\partial}{\partial \rho} (r D_2 \frac{\partial}{\partial \rho} \theta)
\]

\[
\cdot N(r,z) = Q(r,z).
\]

(11)

We use Eq. (11) to study cosmic ray diffusion in the Galaxy. The components of the cosmic ray anisotropy vector (3) are given now by:

\[
\delta_i = 3 j_i / v N, \quad j_i = - \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho N + D_1 \frac{\partial N}{\partial z},
\]

\[
j_z = - \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho N - D_1 \frac{\partial N}{\partial z}, \quad j_\varphi = 0.
\]

(12)

As mentioned in the Introduction, Hall diffusion may be treated as a particle drift. Let us separate the symmetric and antisymmetric parts in the diffusion tensor (2)

\[
D_{ij} = D_{Sij} + D_2 \epsilon_{ij} b_z,
\]

where \( D_{Sij} = (D_1 - D_1) b_i b_j + D_2 \delta_{ij} \),

and write the basic equation in the form

\[
- \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho N = u_1 \frac{\partial}{\partial \rho} \theta + \frac{\partial}{\partial z} \theta = Q.
\]

(14)

Here

\[
u_i = - \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho N + 3 \frac{p c v}{v_0} \cos \varphi \frac{H}{H_0}.
\]

(15)

is called the drift velocity. A general discussion of the concept of drift for the description of cosmic ray propagation is given by Burger et al. (1985).

Thus, cosmic ray diffusion with the diffusion tensor (2) may be described as diffusion with a symmetrical tensor \( D_{Sij} \) (13) and advective transport with drift velocity \( u_1 \) (15).

The simplified Eq. (11) also contains drift terms, determining the drift in radius and in the \( z \)-coordinate with velocities

\[
u_i = - \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho N, \quad \nu_z = \frac{1}{r} \frac{\partial}{\partial r} (r D_2)\theta.
\]

(16)

It is worth noting that one should use the concept of cosmic ray advection with drift velocity \( u_1 \) with some care. The expressions (3), (12) for cosmic ray flux do not have a term of type \( u_1 N \) as would be the case for particle transport by the flow of a real medium moving with velocity \( u_1 \).

3. Flat halo model

We assume that the propagation region of the cosmic rays in the Galaxy has the form of a cylinder of radius \( R \) and height \( 2h \). The cosmic ray sources are distributed in a thin disc of thickness \( 2h \). Supposing that \( h \ll h \), we use the delta approximation \( Q(r,z) = 2h \rho q(r) \delta(z) \) for the source distribution in the \( z \)-coordinate. At the boundaries of the halo (on the surface \( \Sigma \)) the particles can escape freely into intergalactic space, where the density of cosmic rays is negligible small, i.e. \( N_{\Sigma} = 0 \). We consider models with strongly flattened halo \( h \ll R \) in this section. One can neglect radial diffusion as compared with perpendicular diffusion in this case. Thus, we can omit the first term in Eq. (11) and obtain the transport equation in the form

\[
\left( - \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho D_1 \frac{\partial}{\partial \rho} + \frac{1}{r} \frac{\partial}{\partial r} \rho (r D_2) \frac{\partial}{\partial r} \theta \right) \cdot N(r,z) = 2h q(r) \delta(z)
\]

(17)

For further simplification of the solution, we introduce the special radial dependence

\[
D_1(r) = D_{A0} \left( \frac{r}{r_0} \right)^{-1}, \quad D_1(r,z) = const,
\]

(18)

where \( D_{A0} \) and \( r_0 \) do not depend on radius \( r \). The perpendicular component of the drift velocity (16) \( u_2 \), does not depend on \( r \) in this case.

Consider now two realizations of the flat halo model with different structures of the regular magnetic field.

3.1. Case A

The model with symmetric field \( H_{E} (z) = H_{E} (-z) \).

Suppose that the regular field \( H_{E} \) has the same sign in the entire system, see Fig. 1. We set \( D_{A0} (z) = const \). Then

\[
u_i = 0, \quad \nu_z = 2 D_{A0} / r_0 = const,
\]

and the Eq. (17) is reduced to the following

\[
\frac{1}{r_0} \frac{\partial}{\partial \rho} \rho N + \frac{1}{r_0} D_{A0} \frac{\partial N}{\partial z} = 2h q(r) \delta(z).
\]

(20)

The solution of this equation with the boundary condition \( N(|z| = h) = 0 \) is

\[
N(r,z) = \frac{h q(r)}{2 D_{A0}} \exp \left( 2h D_{A0} / r_0 D_1 \right) - \frac{1}{2 D_{A0}} \left( 1 - \exp (2h D_{A0} / r_0 D_1) \right), \quad z \geq 0;
\]

\[
N(r,z) = \frac{h q(r)}{2 D_{A0}} \exp \left( -2h D_{A0} / r_0 D_1 \right) \left( 1 - \exp (2h D_{A0} / r_0 D_1) \right), \quad z < 0.
\]

(21)

In accordance with the definition (2), \( D_{A0} > 0 \) for \( Z H_{E} > 0 \), and \( D_{A0} < 0 \) for \( Z H_{E} < 0 \) (\( Z \) is particle charge).

The spatial distribution \( N(z) \), described by Eq. (21), is shown in Fig. 2. The presence of drift leads to the obvious modification of the usual linear fall of cosmic ray density towards the boundaries of the halo, inherent to diffusion without drift.

For the observer at the central plane \( z = 0 \), Eq. (21) gives

\[
N(r,z) = \frac{q(r) h_0}{D_{A0}} \exp \left( \frac{h_0 q(r)}{D_{A0} r_0 D_1} \right).
\]

(22)

For a power law energy spectrum at the sources \( q \propto E^{-\gamma} \), and for power law dependencies \( D \propto E^{\gamma}, \quad D \propto E \) (see Eqs. (5), (6)), we find from Eq. (22) the asymptotic behavior \( N(E) \propto E^{-\gamma \gamma} \) for \( h D_{A0} / r_0 D_1 \ll 1 \) and \( N(E) \propto E^{-\gamma - 1} \) for \( h D_{A0} / r_0 D_1 \gg 1 \). The break in the spectrum \( D_\gamma = 1 - m \) occurs at the energy \( E_\gamma \), corresponding to the fulfillment of the condition \( h D_{A0} / r_0 D_1 \ll 1 \) (see Fig. 3). For smaller energies \( E \ll E_\gamma \), cosmic ray leakage out of the Galaxy is determined by the usual diffusion \( D_1 \). For higher energies \( E \gg E_\gamma \) leakage is determined by the Hall diffusion \( D_{A0} \) (by drift).
3.2. Case B

The model with antisymmetric field $H_{e}(z) = - H_{e}(-z)$.

Let us assume that regular field $H_{e}$ changes sign at the plane $z = 0$, i.e. the field has opposite directions above and below the plane, see Fig. 4. The Hall diffusion coefficient has the form

$$D_{H} = D_{A0} \frac{r}{r_{0}} \text{sign}(z).$$

Then the drift velocity is

$$u_{r} = -D_{A0} \frac{2r}{r_{0}} \delta(z), \quad u_{z} = D_{A0} \frac{2r}{r_{0}} \text{sign}(z).$$

(23)

The radial drift velocity $u_{r}$ (23) does not vanish only at the central plane $z = 0$. The cosmic ray transport Eq. (17) now has the form

$$-D_{\perp} \frac{\partial^{2}N}{\partial z^{2}} - D_{A0} \frac{2r}{r_{0}} \delta(z) \frac{\partial N}{\partial r} + D_{A0} \frac{2}{r_{0}} \text{sign}(z) \frac{\partial N}{\partial z} = 2h_{q}(r) \delta(z).$$

(24)

This may be equivalently presented as the equation for cosmic ray density in the region $0 < z < h$ above the central plane

$$-D_{\perp} \frac{\partial^{2}N}{\partial z^{2}} + D_{A0} \frac{2}{r_{0}} \text{sign}(z) \frac{\partial N}{\partial z} = 0,$$

supplemented with the following condition at $z = 0$

$$\left( -D_{\perp} \frac{\partial N}{\partial z} + D_{A0} \frac{r}{r_{0}} \frac{\partial N}{\partial r} \right)_{z=0} = h_{q}(r),$$

(26)

and with condition $N(z) = N(-z)$, which determines the cosmic ray density at $-h < z < 0$ below the central plane.

The solution of Eq. (24) [or (25)–(26)] is

$$N(r,z) = \frac{h_{q} r_{0}}{D_{A0}} \left( 1 - \exp\left( -w(1 - |z|/h) \right) \left( 1 - e^{-w} \right)^{-1} \right) \int_{0}^{R/E} dy q(yr) y^{-1} e^{-y^{2}(1 + 2(e^{-w} - 1))^{-1}},$$

for $D_{A0} \geq 0$ ($ZH_{e}(z > 0) < 0$);

$$N(r,z) = \frac{h_{q} r_{0}}{|D_{A0}|} \left( 1 - \exp\left( w(1 - |z|/h) \right) \right) \left( 1 - e^{-w} \right)^{-1} \int_{0}^{R/E} dy q(yr) y^{-1} e^{-y^{2}(1 + 2(e^{-w} - 1))^{-1}},$$

(27)

for $D_{A0} < 0$ ($ZH_{e}(z > 0) > 0$),

where $w = 2h |D_{A0}|/(r_{0} D_{\perp})$. 

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The expressions (27) are illustrated by Figs. 3 and 5. It shows the steepening of the energy spectrum at energy $E_0$, where parameter $D_{A0} = h_D / D_{\perp} r_0 \approx 0.2$. Curve 2, corresponding to $H_0(z > 0) < 0$ in Fig. 3, displays some flattening of the energy spectrum before the steepening at $E_0$. This effect is not strong (for $z \lesssim 2$), but it is in qualitative agreement with the observational data, see e.g. Fichtel & Linsley (1986).

Cosmic rays contain nuclei with different charges $Z$. The predicted form of the spectrum for each kind of nucleus is universal, if all source spectra have the same exponent $\gamma_*$. (Remember that we do not account for nuclear fragmentation and possible energy losses.) This universal spectrum depends on the particle magnetic rigidity, or on the energy per unit charge $E/Z$ for ultra-relativistic particles. Superposition of the spectra on the total energy $E$ for the entire set of cosmic-ray nuclei shows the steepening also, but not so sharply as for a single type of nucleus. The total cosmic ray spectrum expected in the model under consideration is shown in Fig. 6. We adopt the chemical composition at the sources given by Meyer (1985) and do not take into account possible influence of nuclear fragmentation for very high energy cosmic rays.

Let us now estimate the value of the cosmic ray diffusion coefficient. We choose curve 2 of Fig. 3 as describing the form of the energy spectrum for one kind of nucleus.

The measurements of the abundances of secondary nuclei in cosmic rays at energies of a few GeV per nucleon gives the thickness of matter $x_l \approx 10 \text{ g cm}^{-2}$ traversed by particles in the interstellar gas (see e.g. Engelmann et al. 1990). Hall diffusion is not effective at such low energies. The relation of the value of $x_l$ with the parameters of the flat halo diffusion model is (Berezinskii et al. 1990)

$$x_l = \frac{\rho h_D h}{D_{\perp}},$$

(28)

where $\rho$ is the mass density of the gas in the galactic gas disk and $h_D$ the height of the gas disk. The gas density in the halo is set to zero. For the number density of the gas $n = 1 \text{ cm}^{-3}$ and for the height $h_D = 100 \text{ pc}$, one can find from Eq. (28)

$$D_{\perp} = \frac{\rho h_D h}{x_l} = 1.4 \times 10^{28} \left( E/310^9 \text{ eV} \right)^m \left( h/3 \text{ kpc} \right) \text{ cm}^2 \text{ s}^{-1}.$$

(29)

for a typical halo size $h = 3 \text{ kpc}$.

The exponent $m$ may be determined empirically from the dependence of the matter thickness $x_l(E)$ on energy. The interpretation of corresponding data depends on the effectiveness of cosmic ray reacceleration in the interstellar medium (Simon et al. 1986; Cesarsky 1987). This leads to the wide range of possible values of $m = 0.3 \pm 0.3$. Direct measurements of secondary nuclei at the highest available energies $\approx 1 \text{ TeV}/n$ (Swordy et al. 1990) give $x_l(1 \text{ TeV}) \approx 1 - 2 \text{ g cm}^{-2}$, which implies $m = 0.2 - 0.3$ without reacceleration taken into account (reacceleration decreases the value of $m$). Swordy et al. (1990) suggest $m = 0.6$ as compatible with all observations.

The value $m = 0.3 \pm 0.3$ does not contradict the theory of cosmic ray scattering in a random magnetic field, combined with

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the data on the turbulence spectrum in the interstellar medium, see the Eq. (6) and subsequent discussion.

Having in mind the large spread of values allowed for $m$ over the range of energies $10^9$ eV–$10^{17}$ eV, we take $m$ as a free parameter in formula (8).

The value of the Hall diffusion coefficient is

$$D_A = \frac{r_n \lambda}{3} = 8.8 \times 10^{22} \left( \frac{E}{3 \times 10^9 \text{ eV}} \right) \left( 10^{-6} \text{ Oe/H}_0 \right) \text{cm}^2 \text{s}^{-1}. \quad (30)$$

if the magnetic field in the galactic halo is $H_0 = 10^{-6}$ Oe.

Calculations, illustrated by curve 2 in Fig. 3, show the position of the knee at $D_{A0} h / D_L r_0 \approx 0.2$. Then, supposing that the knee corresponds to the energy $E_b = 3 \times 10^{15}$ eV and choosing $r_0 = 10$ kpc, we find $m = 0.16$ from Eqs. (29), (30). The exponent $m$ increases with decreasing $H_0$ in Eq. (30), or with decreasing value of $gh_0$ in Eq. (29).

The break of the energy spectrum at the knee is now $\Delta \gamma = 1 - m \approx 0.8$ for one kind of nuclei, and $\Delta \gamma \approx 0.4$ for the total spectrum including all kinds of nuclei.

According to Eqs. (5), (6), the diffusion coefficient (29) with $m = 0.16$, $L = 100$ pc implies $g A^2 \lesssim 0.08$. At $A^2 = 1$, this gives

$$D_L = \frac{D_A}{g A^4} = 1.8 \times 10^{29} \left( \frac{E}{3 \times 10^9 \text{ eV}} ight)^{0.16} \left( \frac{h}{3 \text{ kpc}} \right) \text{cm}^2 \text{s}^{-1} \quad (31)$$

for the diffusion coefficient along the mean magnetic field.

The relatively small value of the exponent $m = 0.16$ results in the weak energy dependence of the grammage $\chi_0 \sim E^{-0.16}$ ($E \gg M c^2$). This dependence is not in contradiction with the data on the secondary nuclei if one assumes the weak reacceleration of cosmic rays in the interstellar medium. It implies the source energy spectrum of high energy cosmic rays close to $E^{-2.5}$.

4. The model with large halo

Some radio astronomical data indicate that our Galaxy has a rather extended halo, with $h \lesssim 10$ kpc (Broadbent et al. 1990). We consider here cosmic ray diffusion and drift in the model with halo size $h$ comparable with the galactic radius $R = 20$ kpc. Let cosmic ray sources be distributed in the disk with thickness $2h_s = 400$ pc. Cosmic ray transport is described by Eq. (11). We suppose that the regular magnetic field $H_g$ has constant sign in the region $h_m \leq z \leq h$ and the opposite sign for $-h \leq z \leq h_m$, $h_m = 500$ pc (see
The cosmic ray transport equation (11) has now the form

\[
\frac{\partial^2 N}{\partial r^2} + \frac{\partial^2 N}{\partial z^2} + \left[ \frac{1}{r} \left( \frac{2r}{r_0} + \frac{2a}{z_0} \right) \text{sign}(z - h_m) + 2a \delta(z - h_m) \right] \frac{\partial N}{\partial r} + \left[ -\frac{2a}{z_0} + \frac{a}{r} \text{sign}(z - h_m) + \frac{2ra}{r_0} \text{sign}(z - h_m) \right] \frac{\partial N}{\partial z} = \frac{q(r) \delta h_m - |z|}{D_{\perp,0}},
\]

where \( a = D_\perp(z > h_m)/D_\perp, \) \( D_{\perp,0} = D_\perp(r = 0, z = 0), \) and \( \delta(z) \) is the step function.

The results of numerical solution of the Eq. (33) for \( h = 10 \) kpc, \( z_0 = 5 \) kpc, \( r_0 = 10 \) kpc are shown in Fig. 8, 9. The Crank-Nicolson approximation was used to discretize the corresponding equations. The resulting system of difference equations was solved by the Liebmann method. Figure 8 illustrates the dependence of the form of particle spectrum on the spatial distribution of cosmic ray sources in the source disk. The position of the observer is at \( r = 10 \) kpc, \( z = 0 \). Figure 9 shows the fit of the model to the real measurements of the cosmic ray spectrum in the vicinity of the knee.

Now we find the diffusion coefficient \( D_\perp \) in the large halo model using the data on the matter thickness \( x_1 \). Taking into account the inequalities \( h_s \ll h, h_s \ll h, \) the source distribution on \( z \)-coordinate may be described approximately the delta function. Then, the transport diffusion equation for cosmic ray nuclei in the GeV energy region (where \( D_\perp \gg 0 \)) is

\[
\left[ -\frac{1}{r} \frac{\partial}{\partial r} r D_\perp \frac{\partial}{\partial r} - \frac{\partial}{\partial z} D_\perp \frac{\partial}{\partial z} + 2n v \sigma h_s \delta(z) \right] \cdot N(r, z) = 2h_s q(r) \delta(z).
\]

Here \( \sigma \) is the fragmentation cross section of cosmic ray nuclei in the interstellar gas.

After integration of Eq. (34) with respect to \( z \) in the vicinity of the plane \( z = 0 \)

\[
D_\perp \propto |D_\perp| \propto \exp \left( \frac{z^2}{z_0^2} + \frac{r^2}{r_0^2} \right)
\]

with constants \( z_0 \) and \( r_0 \).

Fig. 7. Propagation region of cosmic rays in the large halo model

Fig. 8. Proton spectra near the knee in the large halo model for \( H_s(z = 0) > 0 \) (solid curves) and \( H_s(z = 0) < 0 \) (dashed curves). Energy \( E \) is given in arbitrary units. The following radial distributions of the sources are chosen: (1) \( q(r) = \text{const}, \) (2) \( q(r) \propto \delta(r - 6 \text{kpc}), \) (3) \( q(r) \propto \delta(r - 3 \text{kpc}) \)
lim \int \frac{dz}{z}

we find the relation at \( z = 0 \)

\[ D_{\perp 0} \frac{\partial N_0}{\partial z} + n v \sigma h_r z N_0 = h_r q(r), \]  

(36)

where \( N_0 = N(r; z = 0); \frac{\partial N_0}{\partial z} = \frac{\partial N}{\partial z} \bigg|_{z=0} \); \( D_{\perp 0} = D_{\perp}(r; z = 0) \).

Hence

\[ N_0 = \frac{q(r) h_r h_{\text{eff}}}{D_{\perp 0}} \left(1 + \sigma n v h_r h_{\text{eff}}/D_{\perp 0}\right)^{-1}, \]

(37)

where

\[ h_{\text{eff}}^{-1} = - \frac{1}{N_0} \frac{\partial N_0}{\partial z}. \]

(38)

The expression (37) has the same form as that for cosmic ray density in the leaky-box model (\( T_{\text{leak}} \) is the leakage time)

\[ N = Q T_{\text{leak}} (1 + \sigma x_t)^{-1}, \]

(39)

see Berezinskii et al. (1990) for discussion.

Thus, for calculations of fragmentation of the stable nuclei, our model is equivalent to the leaky box with the mean matter thickness traversed by cosmic rays

\[ x_t = q v h_r h_{\text{eff}}/D_{\perp 0} \]

(40)

(compare with Eq. (28)).

The value of \( h_{\text{eff}} \) describing the cosmic-ray gradient near the galactic plane is determined numerically by a solution of Eq. (33).

The diffusion coefficient \( D_{\perp 0} \) in the GeV energy range may then be found from Eq. (40).

For the case of uniform source distribution \( q(r) = \text{const} \), the diffusion coefficient \( D_{\perp} \) turns out to be

\[ D_{\perp 0} (r = 10 \text{ kpc}) = 1.8 \times 10^{28} \left( E/3 \times 10^9 \text{ eV} \right)^{0.18} \text{ cm}^2 \text{ s}^{-1} \]

(41)

\( (a > 0, H_0 = 10^{-6} \text{ Oe}) \).

This value is approximately equal to the diffusion coefficient (29) found for the flat halo model with constant \( D_1 \) and with halo size \( h = 4 \text{ kpc} \). The corresponding value of the mean field-aligned diffusion coefficient is [for \( L = 100 \text{ pc}, A^2 = 1 \); see Eqs. (5), (6)]

\[ D_{\| 0} = 2.0 \times 10^{29} \left( E/3 \times 10^9 \text{ eV} \right)^{0.18} \text{ cm}^2 \text{ s}^{-1}, \]

(42)

5. Local anisotropy of cosmic rays

The complicated nature of cosmic ray propagation at energies \( E \lesssim 10^{17} \text{ eV} \) may be roughly presented simply as the superposition of the diffusion of magnetized particles mainly along the magnetic field lines and of the random wandering of the magnetic field lines themselves. The transport equation (1) describes the global diffusion. The quantities \( N(r) \) and \( D_{\|}(r) \) which figure in Eq. (1) are defined as the mean cosmic-ray density and the mean diffusion tensor, averaged over scales greater than the characteristic scale of the random field \( L \approx 100 \text{ pc} \). Spatial variations of cosmic-ray density about the mean value are not larger, and local measurements at the Earth may be considered as representative of the mean density of cosmic rays in the vicinity of about few hundreds parsecs, but the yield of local galactic cosmic-ray sources which are closer than a few hundreds parsecs must be calculated individually. However, local cosmic ray anisotropy may deviate greatly from the mean anisotropy \( \delta_r(r) \) defined by Eq. (3). The deviation is due to the large difference between the mean and local diffusion tensors.

Large-scale wandering of magnetic field lines results in isotropization of the local diffusion tensor. For approximate calculation of the local anisotropy, one can suppose that local diffusion is one-dimensional (along the local magnetic field) with the mean free path given by Eq. (6). The local cosmic ray density and the local cosmic ray gradient may be set equal to the averaged values \( N \) and \( G_\perp = N^{-1} V_\perp N \), respectively. Thus, cosmic ray anisotropy \( a_i \) measured at the Earth is calculated according to the formula

\[ a_i = -|e_i G_j| = |e_i e_g + e_r G_r|, \]

(43)

where \( e_i \) is the unit vector directed along the local magnetic field.

The absolute value of the anisotropy vector (43) is

\[ a = |e_i G_j| = |e_g + e_r G_r|, \]

(44)

(\( G_r = 0 \) in our model). The anisotropy is directed along the local magnetic field.

The maximal possible value of anisotropy \( a_{\text{max}} = |G_\perp(E) \) at each energy \( E \) is achieved when the cosmic ray gradient \( G_\perp(E) \) is parallel to the local field direction \( e_i \). Corresponding values are presented in Table 1.

The direction of the magnetic field on a scale of the order of 100 pc from the Sun is given by observations of polarization of the optical starlight. The data by Matheson and Ford (1970)
Table 1. Maximal local anisotropy of cosmic rays $a_{\text{max}}$ ($z = \pm 50$ pc is the distance of an observer above or below the galactic midplane)

<table>
<thead>
<tr>
<th>Energy (eV)</th>
<th>Flat halo model</th>
<th>Large halo model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z = +50$ pc</td>
<td>$z = -50$ pc</td>
</tr>
<tr>
<td>$3 \times 10^{12}$</td>
<td>$4.7 \times 10^{-3}$</td>
<td>$5.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$3 \times 10^{15}$</td>
<td>$1.1 \times 10^{-2}$</td>
<td>$1.5 \times 10^{-2}$</td>
</tr>
<tr>
<td>$10^{17}$</td>
<td>$1.1 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

indicate the direction $l^{\theta} = 50^\circ$, $|b^{\theta}| \leq 10^\circ$. The uncertainty in the determination of the latitude $b$ is essential. We calculate the cosmic ray anisotropy for the two cases: $b^{\theta} = \pm 10^\circ$ and $b^{\theta} = 0^\circ$. Anisotropy is determined for the position of the observer at $|z| = 50$ pc, above or below the midplane.

Table 2 illustrates the results of the calculations under the assumption of uniform source distribution $q(r) = \text{const}$. Versions of the flat halo model and of the model with large halo which provide the flattening of the cosmic ray spectrum before the knee are only taking into account in Tables 1, 2. The anisotropy in Table 2 is considerably smaller then $a_{\text{max}}$ (Table 1), since the vector $G_b$ is directed predominantly perpendicular to the galactic plane and the local magnetic field is supposed to lie primarily in the galactic plane.

We are not trying to give here a complete interpretation of measurements of cosmic ray anisotropy. This should include effects of local galactic cosmic ray sources (Dorman et al. 1985). Detailed structure of the magnetic field must be considered for calculation of the second angular harmonic (Kota 1975; Dorman et al. 1983; Ptuskin 1984; Nagashima et al. 1989). This problem will be discussed in a separate paper. Here it is sufficient to say that Tables 2 show the amplitudes of anisotropy that does not exceed the observational values, see Kifune (1990) for review. Relevant observational data give $a(10^{12}-10^{13} \text{eV}) \approx 5 \times 10^{-4}-10^{-3}$ and $a(10^{17} \text{eV}) \approx 3 \times 10^{-2}$.

6. Conclusion

A global toroidal magnetic field disturbs the random walk of cosmic ray particles in stochastic magnetic fields. This process may be described in terms of Hall diffusion $D_A$ (or as systematic drift) superimposed on the usual diffusion $D$. At an energy of $10^9$ eV, the Hall diffusion is insignificant and plays a minor role in cosmic ray leakage from the Galaxy. However, the Hall diffusion coefficient increases rapidly and directly with energy, and at energy $\geq 3 \times 10^{15}$ eV may begin dominating the slowly-increasing usual diffusion. That is why the knee occurs in our model. A power-law generation spectrum with constant exponent is assumed for cosmic ray sources.

The calculations of the cosmic ray energy spectrum, matter thickness traversed by low-energy cosmic rays, and cosmic ray anisotropy have confirmed the realism of the model, but imply that the energy dependence of the diffusion should not be strong ($D \propto E^n$, $m = 0.15-0.20$) in the range of energies $E = 10^9-10^{17}$ eV on the whole. Analytical calculations were made for the case of a flat halo and numerical calculations were done for the case of a large halo. It is shown that an empirical diffusion model is in agreement with the "microscopic" theory of cosmic ray propagation in random and regular magnetic fields. This model does not contradict our knowledge of the structure of galactic magnetic fields. The highest proton energy at which the diffusion approximation is applicable in the Galaxy is $10^{17}-10^{18}$ eV. (Consideration of cosmic ray propagation at higher energies requires trajectory calculations, see Berezhinskii et al. (1990), Smith & Clay (1990), Bereznisky et al. (1991), Giller et al. (1991)).

The model exhibits another interesting feature. If the toroidal regular field in the halo is directed oppositely in the northern and southern galactic hemispheres, the spectrum may get flatter before the knee, seemingly in qualitative agreement with observations. It should be noted that this is the magnetic field a geometry to be expected if the regular field in quasi-spherical halo is generated by a dynamo mechanism, irrespective of the field in the disk.

The calculated cosmic ray anisotropy in our model is not in disagreement with observations. Detailed consideration of the problem of anisotropy of very high energy cosmic rays will be given in a separate paper.

Diffusion and drift of different cosmic ray nuclei at ultrarelativistic energies depend on the energy per unit charge $E/Z$. Thus the position of the knee for each kind of nucleus depends on $E/Z$. For the composite cosmic ray spectrum on the total energy $E$, one expects an increase of the fraction of heavy nuclei after the knee (at $E > E_k$). The observations give no definite conclusion on this subject now, but it is not excluded that the fraction of heavy nuclei decreases with energy at $E \geq E_k$ (see e.g. Fichtel & Linsey 1986). The situation remains highly uncertain, Kifune (1990). To save the model in this case, we could suggest different spatial distributions of cosmic ray sources for protons and heavy nuclei, since the position of the knee and the form of the energy spectrum at $E \geq E_k$ depend on the source distribution $q(r)$, see Fig. 8. For example, the proton sources may be distributed approximately homogeneously and the heavy nuclei (iron) sources may be concentrated at distance $r = 4-6$ kpc, where the largest concentration of pulsars and supernovae resides.

Table 2. Local cosmic ray anisotropy $a$ for different orientations of the local magnetic field ($l = 50^\circ; b = 0^\circ; \pm 10^\circ$) and for different positions of an observer ($r = 10$ kpc; $z = \pm 50$ pc)

<table>
<thead>
<tr>
<th>Energy (eV)</th>
<th>Flat halo model $z = +50$ pc</th>
<th>Large halo model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = 0^\circ$</td>
<td>$b = \pm 10^\circ$</td>
<td>$b = -10^\circ$</td>
</tr>
<tr>
<td>$3 \times 10^{12}$</td>
<td>$&lt; 10^{-5}$</td>
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<tr>
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<td>$8.0 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$10^{17}$</td>
<td>$4.2 \times 10^{-6}$</td>
<td>$1.6 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
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