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PARTIAL CROSS-SECTIONS IN HIGH-ENERGY NUCLEAR REACTIONS, AND ASTROPHYSICAL APPLICATIONS.
I. TARGETS WITH $Z \leq 28$

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ABSTRACT

Cross-sections for nuclear breakup reactions at high energies are essential for interpreting important observations in cosmic-ray physics, astrophysics, and lunar and planetary science. Compared with the many cross-sections required, the variety and number actually measured thus far are very limited. Accordingly, we have devised new empirical formulae for high-energy cross-sections, using measured yields of proton interactions with various target nuclei ($3 \leq Z_t \leq 28$). They are applicable at energies >100 MeV to all products with $A \geq 6$ for targets ranging from Li to Ni. The new relations are useful in regions where earlier empirical formulae break down completely. Moreover, they give significantly better estimates of cross-sections than those previously available. Thus, for $6 \leq Z_t \leq 20$, the relative standard deviation of a calculated cross-section (from a measured value where the latter is available) is typically 20 to 30 percent at energies above 150 MeV. Above 2 GeV, for $21 \leq Z_t \leq 26$, this deviation is roughly 40 percent; with decreasing energy, the ratio $\sigma_{\rm calc}/\sigma_{\rm exp}$ rises up to $1(+1, -0.5)$ at 150 MeV. The new equations give estimates that are somewhat more accurate than Monte Carlo calculations; yet computations are much simpler and faster by orders of magnitude.

Subject heading: nuclear reactions

I. INTRODUCTION

The cross-sections of high-energy nuclear reactions are of considerable astrophysical interest. They are useful for:

1. Predicting the changes in composition undergone by cosmic radiation due to collisions with the interstellar gas and deducing the primordial composition of cosmic rays on leaving the source regions, the mean path length in interstellar matter, and the confinement time of cosmic rays in galactic magnetic fields before leaking out from the galaxy due to random walk of magnetic field lines and/or scattering by magnetic irregularities (Shapiro and Silberberg 1970 and references therein). A good knowledge of cross-sections is essential for the determination of the following quantities:

a) The confinement time of cosmic rays in the Galaxy, estimating it from the ratios Be/B, Mn/Cr, Al/Na, and Cl/K, which vary according to the survival or decay of the long-lived radioactive spallation products ^{10}Be, ^{53}Mn, ^{26}Al, and ^{36}Cl.

b) The abundances of Na, Al, Ar, and Ca at cosmic-ray sources (a considerable fraction of these nuclei are produced after leaving the source regions by the nuclear breakup of heavier cosmic rays).

c) The determination of the distribution function of cosmic-ray path lengths from a comparison of the ratio $(3 \leq Z \leq 5)/(6 \leq Z \leq 8)$ and $(17 \leq Z \leq 25)/(Z = 26)$; this is essential for checking the various theories of cosmic-ray propagation and leakage from the Galaxy.

2. Determining the production rates of various isotopes in meteorites or on the lunar surface due to bombardment by cosmic rays and high-energy solar particles (Honda and Arnold 1964; Stoenner, Lyman, and Davis 1970).

3. Setting constraints on the processes of nucleosynthesis in stars from the calculated primordial composition of the cosmic rays at the source regions (Silberberg 1972).

4. Explaining the abundances of Li, Be, and B in the solar system; these nuclei had a likely origin in spallation reactions either due to bombardment of the interstellar gas by the cosmic rays prior to the formation of the solar system or due to collisions of the flare particles during the violent early stages of the formation of the Sun (Reeves, Fowler, and Hoyle 1970).

5. Estimating the background radioactivity in spacecraft and at accelerator facilities (Gabriel and Santoro 1971).

This is the first of two papers dealing with partial cross-sections for spallation, stripping, and other nuclear reactions. It is devoted to targets with atomic numbers ranging from $3 \leq Z_t \leq 28$, while the second paper deals with heavier targets. Empirical equations are proposed for calculating nucleon-nucleus cross-sections for various sets of targets and products at energies exceeding 100 MeV. These yield accurate results; the calculation is also relatively simple and need not rely on high-speed digital computers.

The previously available semiempirical estimates are inapplicable for the targets considered in this paper when the difference ΔA between target and product masses exceeds 40 (or when $\Delta A \geqslant 30$ for $E < 200$ MeV). If the latter semiempirical relations are extrapolated into inapplicable regions, they yield predictions that deviate systematically by as much as two orders of magnitude at high energies and up to eight orders of magnitude at 130 MeV. The Monte Carlo calculations, on the other hand, become very time consuming when ΔA is large and the energy is low; for energies below 400 MeV and $\Delta A > 30$, no calculations have thus far been published. The methods outlined in this paper thus extend the calculations of cross-sections into regions where none existed previously.

Available measurements of cross-sections are tabulated (Silberberg and Tsao 1972) for the convenience of those who may be interested in further developing a general theoretical description of nuclear spallation reactions at high energies. In general, the yields of stable isotopes have not been measured.

Our new empirical equations and tables of cross-sections of astrophysical interest are presented in § II. The results and the relative merits of various techniques for calculating cross-sections are discussed in § III.

II. THE NEW SEMIEMPIRICAL ESTIMATES OF CROSS-SECTIONS

In an attempt to use analytical relations, Rudstam (1955, 1956, 1966, 1969) has devised a semiempirical cross-section formula that is particularly useful for targets heavier than calcium. It should not, however, be applied to light product nuclei as pointed out by Rudstam himself, particularly at low energies. For example, the calculated cross-section for the production of ^{7}Be from Ni at 130 MeV is too small by a factor of $\sim 10^8$.

Other semiempirical estimates have been deduced for lighter target nuclei. Bernas *et al.* (1967) derived a relationship between the breakup cross-sections for ^{12}C, ^{14}N, and ^{16}O and the quantity $(\Delta I, \Delta I_z)$, where ΔI and ΔI_z are, respectively, the difference in the isospins between target and product nuclei and that of their z components. Audouze, Epherre, and Reeves (1967) proposed a modification of Rudstam's equation incorporating the $(\Delta I, \Delta I_z)$ rules of Bernas *et al.* (1967) and introduced an extensive set of parameters for various targets, products, and energies. Their equation is applicable in the region $9 \leqslant A_t \leqslant 34$. Beck and Yiou (1968) proposed a set of cross-sections for energies greater than about 2 GeV. These were largely based on the calculational techniques of Audouze *et al.* (1967). The above methods have inherent limits to their usage. The following relationship was devised for a more general application.

a) *The General Equation for Proton-Nucleus Interactions*

The experimental cross-sections from references listed in the Appendix, compiled by Silberberg and Tsao (1972), were used for constructing a semiempirical equation resembling Rudstam's (1966)

$$\sigma = \sigma_0 f(A) f(E) e^{-P\Delta A} \exp\left(-R|Z - SA + TA^2|^{\nu}\right)\Omega\eta\xi\,. \qquad (1)$$

It is applicable for calculating cross-sections of targets having mass numbers in the range $9 \leq A_t \leq 209$ and products with $6 \leqslant A \leqslant 200$, except for very large and small values of ΔA (i.e., of $A_t - A$).

The correction factors $f(A)$ and $f(E)$ in equation (1) are applicable to products from heavy targets with $Z_t > 30$, when ΔA is very large as in the case of fission, fragmentation, and evaporation. In these reactions the cross-sections for the production of nuclei with a large neutron-to-proton ratio are greatly enhanced. In the present paper $f(A)$ and $f(E)$ are both set equal to unity.

The factor $\exp(-P\Delta A)$ describes the diminution of cross-sections as the difference of target and product mass, ΔA, increases. It is closely related to the distribution of excitation energies discussed in the first paper of Metropolis *et al.* (1958). Recent experimental data permit a quantitative estimate (Silberberg 1969; Silberberg and Tsao 1973 [Paper II]) of the target mass dependence of P. This is given in table 1D.

The factor $\exp(-R|Z - SA + TA^2|^{\nu})$ describes the distribution of cross-sections for the production of various isotopes of an element of atomic number Z, as illustrated by the curves of figure 1. It is related to the statistical nature of the evaporation process (Dostrovsky, Rabinowitz, and Bivins 1958). The width of the distribution of cross-sections is represented by the parameter R. The parameter S describes the location of the peaks of these distribution curves for small values of the product mass number A. The parameter T describes the shift of the distribution curves toward greater neutron excess as the atomic number of the product increases. These parameters are defined in tables 1A–D.

TABLE 1A

PARAMETERS OF EQUATION (1) FOR TARGETS HAVING $Z_t \leq 28$ AND PRODUCTS $(2, 6) \leq (Z, A) \leq (4, 12)$

σ_0	$13 f_3 \exp\left[1.15\left(1 - \frac{E}{1250}\right)\right]$	$E < 1250$
	13	$E \geq 1250$
P	$0.16\left(1 - \frac{E}{1250}\right)$	$E < 1250$
	0	$E \geq 1250$
R	$10.7E^{-0.25}$	$E < 1250$ and $Z_t \leq 20$
	1.8	$E \geq 1250$ or $Z_t > 20$
S	$0.54 - 0.32\left(\frac{A_t}{Z_t} - 2\right)^{1.4}$	...
T	0.003	...
ν	2	...
η	1, 1, 1, 1*	...

* There are not enough experimental data for determining the pairing factors for the lightest nuclei; deviations from unity have been incorporated into the factor Ω of table 2.

TABLE 1B

PARAMETERS OF EQUATION (1) FOR TARGETS $6 \le Z_t \le 16$ AND PRODUCTS $Z \ge 6$

σ_0..............	$\dfrac{28(A_t^{2/3} - 1)[1 - 0.3 \ln (\frac{1}{20}A_t)]PR^{1/2}}{1 - \exp(-PA_t)}$	...
P..............	$2.6E^{-0.5}$	$E < 1250$*
	0.075*	$E \ge 1250$*
R..............	$10.2E^{-0.26}$	$E < 1250$
	1.6	$E \ge 1250$
S..............	$0.502 - 0.26\left(\dfrac{A_t}{Z_t} - 2\right)^{1.4}$	...
T..............	0.0005	...
ν..............	2	...
η..............	1.15, 1.15, 0.9, 0.8	...

* When $E_0 > 1250$ MeV, replace 1250 by E_0 and 0.075 by $0.77A_t^{-2/3}$.

TABLE 1C

PARAMETERS OF EQUATION (1) FOR TARGETS $17 \le Z_t \le 20$ AND PRODUCTS $Z \ge 6$

σ_0..............	$\dfrac{28f'_2(A_t^{2/3} - 1)[1 - 0.3 \ln (\frac{1}{20}A_t)]PR^{1/2}}{1 - \exp(-PA_t)}$	...
P..............	$20E^{-0.77}$	$E < E_0$
	$0.77A_t^{-2/3}$	$E \ge E_0$
R..............	$10.2E^{-0.26}$	$E < 1250$
	1.6	$E \ge 1250$
S..............	$0.502 - 0.08\left(\dfrac{A_t}{Z_t} - 2\right)$	...
T..............	0.0005	...
ν..............	2	...
η..............	1.25, 0.9, 1, 0.85	...

TABLE 1D

PARAMETERS OF EQUATION (1) FOR TARGETS $21 \le Z_t \le 28$ AND PRODUCTS $Z \ge 6$ AND FOR SPALLATION OF TARGETS WITH $63 \le A_t \le 209$

σ_0.............	$\dfrac{144f_1f_2PA_t^{0.367}}{1 - 0.3/PA_t - (0.7 - 0.3/PA_t)\exp(-PA_t)}$	...
P.............	$20E^{-0.77}C_p$	$E < E_0$
	$0.77A_t^{-2/3}$	$E \ge E_0$
R.............	$1.29A^{0.15}$	$A < 40$
	$11.8A^{-0.45}$	$A \ge 40$
S.............	$0.486 - 0.06(A_t - \bar{A}_t)/Z_t$	...
T.............	0.00038	...
ν.............	1.5	...
η.............	1.25, 0.9, 1, 0.85	...

$$C_p = \begin{cases} 1 - 0.32 \exp\left[-\left(\dfrac{E - 100}{100}\right)^2\right] & \text{for } 21 \le Z_t \le 30^* \\ 1 - 1.5 \times 10^{-5}(A_t - 100)\left(\dfrac{E_0 + 150}{E + 150}\right) & \text{for } A_t > 100 \\ 1 & \text{otherwise} \end{cases}$$

* The restriction to $Z_t \le 30$ is not firmly established; it is largely based on the spallation cross-sections of ^{75}As, measured by Rudstam (1956) at 100 MeV.

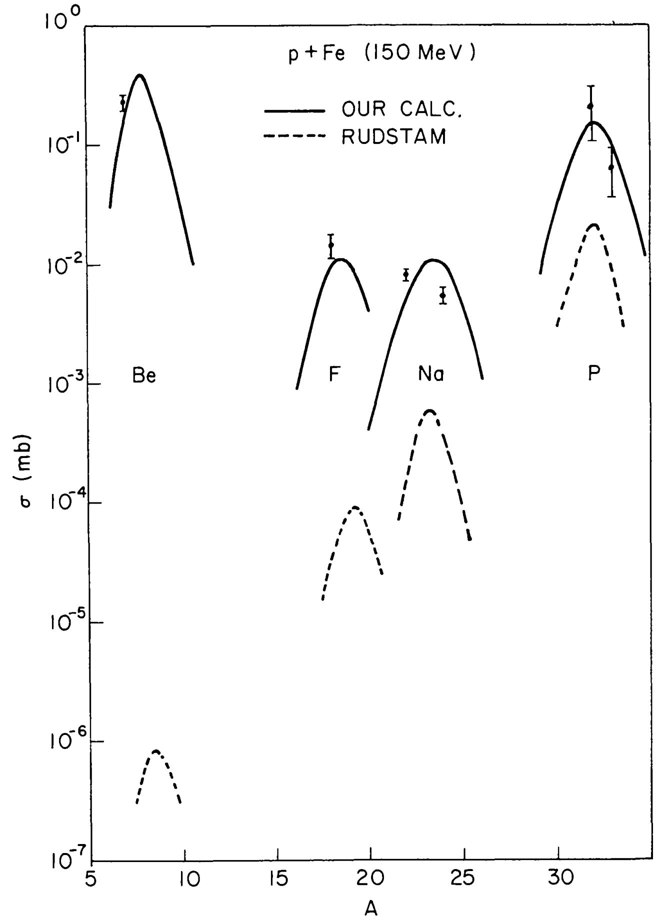

FIG. 1.—Cross-sections for the production of isotopes of Be, F, Na, and P from the spallation of iron by protons at 150 MeV. These are compared with the present calculation (*solid curves*) and with that based on Rudstam's equation (*dashed curves*). Rudstam has pointed out that for light products (like Be and F), his relation is not applicable.

In figure 1 the experimental cross-sections of p + Fe reactions at 150 MeV are compared with those based on equation (1) with the parameters of tables 1, as well as with those based on Rudstam's equation (shown by the dashed line). For iron targets, Rudstam (1966) considered his relation to be applicable only to products with $A \gtrsim 22$. Hence, the dashed curves for Be and F only illustrate the result of extrapolating his relation beyond the applicable region. Figure 1 also shows that the cross-sections cease to decrease with decreasing values of A. Hence, for large values of ΔA, it is substituted in equation (1) by the ΔA_c:

$$\Delta A_c = \begin{Bmatrix} 31.5 + 0.052(A_t - 36)(\ln E - 3.17) & \text{for} & E < E_0 \\ 31.5 + 0.045(A_t - 36)(\ln A_t + 1.23) & \text{for} & E \geq E_0 \end{Bmatrix} \tag{2}$$

The substitution is made if $\Delta A > \Delta A_c$. The energy E is expressed in units of MeV; E_0 is the critical energy above which no significant change in the values of cross-sections is expected. It is derived by combining the high- and low-energy expressions of P of table 1D at $E = E_0$:

$$E_0 = 69 A_t^{0.867} . \tag{3}$$

For very small values of ΔA, peripheral types of reactions play a dominant role. These will be discussed in § II*b*.

The parameter P was derived by minimizing systematic deviations of $\ln(\sigma_{\text{calc}}/\sigma_{\text{exp}})$ as a function of ΔA, for various values of energy, while the parameters R, S, and T were derived by minimizing those deviations as a function of $N - Z$, the neutron excess of product nuclei. They are listed along with other parameters in table 1 for four sets of targets and products.

For products with $(2, 6) \leq (Z, A) \leq (4, 12)$, e.g., for ^{8}Li, ^{9}Li, ^{7}Be, and ^{10}Be, the high-energy cross-sections do not decrease with increasing ΔA; hence, $P = 0$ fits the data for $E \geq 1250$ MeV (see table 1A). For products having $Z_t \geq 17$ and $40 \lesssim A_t \lesssim 65$, the values of P are essentially those of Rudstam (1966), except in the region $E \lesssim 200$ MeV, where a correction factor is applied.

The parameter R (or the width of the distribution of cross-sections for a set of product isotopes) appears to depend on energy for $Z_t \leq 20$ (see tables 1A–C), while for heavier targets it is a function of the mass number of the product nucleus (table 1D). For $Z_t \geq 21$ and $A \geq 40$, the value of R is that of Rudstam (1966), while for $A < 40$, a different function was adopted so as to broaden the distribution curve for cross-sections.

Target nuclei having neutron excess also tend to yield products with neutron excess. The shift in the distribution curve is accomplished by adding to S a term that depends on A_t/Z_t (see tables 1A–C). For $Z_t \geq 21$ (see table 1D), the parameter S is essentially that of Rudstam (1966), with an additional term, however. In table 1D, $\bar{A}_t$ denotes the average mass number corresponding to the "center of gravity" of the stable isotopes of a given charge; $\bar{A}_t = (\Sigma A_i)/n$. (The stable isotopes of Ni, e.g., have $A = 58$, 60, 61, 62, and 64; therefore $\bar{A}_{\text{Ni}} = 61$.) With minor exceptions, the value of A_t can be approximated by $\bar{A}_t = 2Z_t + 0.015Z_t^{1.8}$. The term $\bar{A}_t$ reflects the observation that the neutron deficiency of certain elements relative to their neighbors (e.g., of Ca and Ni) is also reflected (Dostrovsky *et al.* 1965; Goebel, Schultes, and Zähringer 1964; Chackett 1965) in their products. The calculations are to be performed separately for the isotopes of a given element; the product yields for elements are obtained by weighting according to target isotopic abundances.

The parameter T is larger for lighter products; for $Z_t \geq 21$ and $Z > 5$ (see table 1D), it is identical to that of Rudstam (1966).

The exponent ν in equation (1) is also identical to that of Rudstam (1966) in the above-mentioned case (see table 1D). Otherwise, a value of 2 appears to yield a better fit.

The expression σ_0, in millibarns, is the normalization factor for the cross-sections. For $Z \geq 6$, the values are essentially obtained by integration (see eqs. [13] and [25] of Rudstam [1966]). However, some additional empirical adjustments have been included. The normalization factors f_1 (for $E \leq 750$ MeV) and f_2 were adopted from Rudstam (1966). The factor f_1 is not well known for the range of energies from 750 MeV to E_0; a relation which represents a compromise has therefore been adopted:

$$f_1 = \begin{cases} \exp(-0.25 + 0.0074A_t) \quad \text{for} \quad E \leq 750\ \text{MeV} \\ 1.05 + [\exp(-0.25 + 0.0074A_t) - 1.05]\left(\dfrac{E_0 - E}{E_0 - 750}\right)^2 \\ \text{for} \quad 750\ \text{MeV} < E < E_0 \\ 1.05 \quad \text{for} \quad E \geq E_0\,, \end{cases} \tag{4}$$

$$f_2 = \begin{cases} \exp(1.73 - 0.0071E) \quad \text{for} \quad E < 240\ \text{MeV} \\ 1.0 \quad \text{for} \quad E \geq 240\ \text{MeV}\,. \end{cases} \tag{5}$$

TABLE 2

Nuclear Structure Factor Ω for Various Products*

Product	^{6}He	^{6}Li	^{7}Li	^{8}Li	^{9}Li	^{9}Be	^{10}Be	^{10}B	^{11}B	^{13}N	^{15}O	^{19}Ne	^{21}Ne	^{22}Na	Product	K	Ca	Sc
Ω†......	0.4	1.0	2.0	0.7	1.6	0.5	1.3	0.85	1.4	0.4‡	1.2	0.8	0.8	1.2	Ω§.......	0.6	0.7	0.6

* For other nuclides, Ω = 1. † Applicable in regions defined in tables 1A and 1B.

‡ Applicable also in regions defined in tables 1C and 1D. § Applicable in the region defined in table 1D; not to be used for $Z_t > 28$.

For targets with $17 \leq Z_t \leq 20$ (see table 1C),

$$f_2' = \begin{cases} \exp\,(0.9 - 0.0015E) & \text{for} \quad E < 600\ \text{MeV} \\ 1.0 \quad \text{for} \quad E \geq 600\ \text{MeV}\,. \end{cases} \tag{6}$$

For products $(2, 6) \leq (Z, A) \leq (4, 12)$ (see table 1A),

$$f_3 = \begin{cases} 2\exp\left[-\left(\dfrac{E-650}{720}\right)^2\right] & \text{for} \quad Z_t > 20\,, \quad \text{and} \quad E < 1250\ \text{MeV} \\ 1.0 \quad \text{for} \quad Z_t \leq 20\,, & \text{or} \quad E \geq 1250\ \text{MeV}\,. \end{cases} \tag{7}$$

The nucleon pairing factors η in equation (1) and tables 1 are listed in the order of product nuclides for which (Z, N) are even-even, odd-N, odd-Z, and odd-odd, respectively. For $Z_t \geq 17$, the values are in the same proportions as those of table 12 of Rudstam (1966). For $Z_t \leq 16$, values of η are modified slightly.

Values of the parameter Ω of equation (1) are given in table 2. It is related to the nuclear structure. Nuclei that have only a single particle-stable energy level, such as ^{9}Be and ^{13}N, have small production cross-sections. Also, ^{19}Ne has very few particle-stable levels.

The parameter ξ of equation (1) represents an enhancement factor for light evaporation products. The corresponding values are given in table 3. The values for ^{6}Li and ^{7}Li are extremely uncertain; they are based on interpolating between the experimental measurements of Yiou, Seide, and Bernas (1969) from ^{12}C and ^{16}O targets and Monte Carlo calculations (Dostrovsky *et al.* 1965) of heavy targets.

Due to the lack of adequate data for nuclei with $17 \leq Z_t \leq 20$, table 1C represents a compromise between tables 1B and 1D.

The production of boron nuclei is poorly known. For the present, it is perhaps best to take the average of the results using tables 1A and 1B for $6 \leq Z_t \leq 16$; tables 1A and 1C for $17 \leq Z_t \leq 20$; and tables 1A and 1D for $21 \leq Z_t \leq 28$.

Figure 2 shows a comparison of the calculated cross-sections (using the parameters of tables 1A and 1B) with the experimental values. The data represent the spallation of ^{27}Al at $E > 1.25$ GeV. For clarity, the even and odd elements are shown separately. The curves for Be and N appear peculiar, because ^{9}Be and ^{13}N have anomalously low yields (see the above discussion of table 2). Our calculated curves provide a good fit to the experimental data.

Figure 3 illustrates the dependence of the yields of ^{22}Na on energy and on the target-product mass difference ΔA. Our calculated energy dependence (shown by the curves) is in good agreement with the experimental data, even though the experimental values vary by a factor of 10^5. (At $E = 100$ MeV, and for a copper target, Rudstam's equation would underestimate the yield of ^{22}Na by 3 orders of magnitude; Monte Carlo methods have not been applied for $E < 400$ MeV if $\Delta A > 30$.)

TABLE 3

ENHANCEMENT FACTORS ξ FOR LIGHT EVAPORATION PRODUCTS

Product	$14 \leq A_t \leq 34$	$34 < A_t \leq 54$, $54 < A_t \leq 62$
^{6}He.........	$1 + 0.10(A_t - 14)$	$3[1 + 0.04(A_t - 34)]$
^{6}Li..........	1.0	$[1 + 0.04(A_t - 34)]$
^{7}Li..........	1.0	$1 + 0.03(A_t - 34)$
^{7}Be..........	1.0	1.0, $1 + 0.08(A_t - 55)$

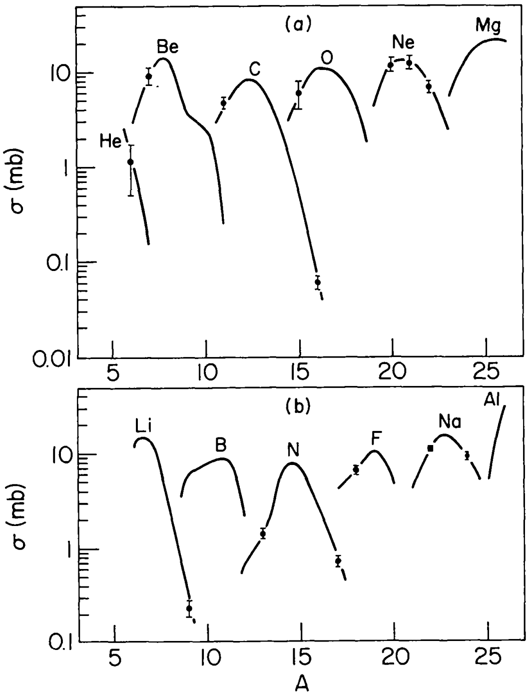

FIG. 2.—Comparison of calculated and experimental cross-sections of proton interactions with ^{27}Al at $E > 1.25$ GeV. For clarity, the even and odd elements are shown separately.

b) *Peripheral Reactions*

For reactions where ΔA is very small, earlier semiempirical relations are no longer applicable; here peripheral interactions and quasi-elastic nucleon-nucleon interactions play an important role. Therefore, quite different empirical relations must be derived.

A knowledge of the cross-sections for peripheral reactions is important in cosmic-ray physics. Much of the elements B, N, F, Na, Al, and Mn in cosmic rays originated in single-nucleon stripping reactions of the next heavier elements. Also, much of ^{10}Be (important for the determination of the galactic confinement time of cosmic rays) originated in the stripping of ^{11}B.

The energy dependence of (p, pn) cross-sections has been illustrated in the upper curve of figure 12 of Shapiro and Silberberg (1970); the outstanding feature is the high peak at lower values of energy. A detailed discussion of peripheral reactions will be given in our second paper.

c) ^{4}He-*Nucleus Collisions*

Still very little is known about these reactions, particularly at energies ≥100 MeV. Hence, no semiempirical equations are proposed here. Some features, when compared with proton-nucleus interactions, are:

1. The energy dependence of cross-sections for reactions in which a single neutron is lost by the target nucleus is markedly different—the reaction mechanisms that contribute to the high peak of (p, pn) reactions at low energy do not do so (Crandall *et al.* 1956; Lindner and Osborne 1953; Radin 1970) in the case of ^{4}He-nucleus collisions.
2. The cross-sections for the production of lithium, and also for beryllium, appear to be enhanced (Fontes *et al.* 1971; Jung *et al.* 1969) more than those of heavier products from targets like C and N.

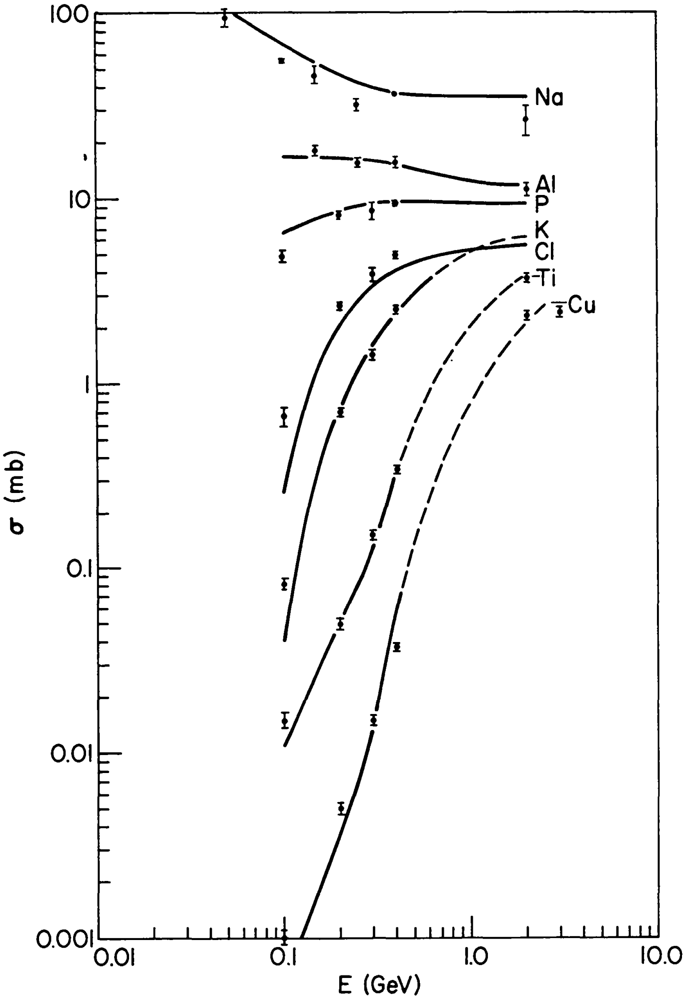

FIG. 3.—Comparison of calculated and experimental yields of ^{22}Na for $E \geq 100$ MeV and targets in the range $11 \leq Z_t \leq 29$.

3. The energy dependence of the proton- and helium-induced partial cross-sections appears similar (Crespo, Alexander, and Hyde 1964; Korteling and Hyde 1964) (though those induced by He are larger by a factor of ~ 2) when considered as a function of kinetic energy per *nucleus* rather than per nucleon of the helium ion. This is to be expected, since the energy deposition from proton and helium ion bombardments is similar at the same kinetic energy per *nucleus*.

4. If $\Delta A \lesssim 3$ in ^{4}He-nucleus collisions, the cross-sections at high energy (Radin 1970) are only $\sim 1.6x$ rather than $\sim 2x$ larger than in p-nucleus collisions.

d) *Cross-Sections of Interest in Cosmic-Ray Physics*

Among the "heavy" cosmic-ray nuclei incident on the Earth's atmosphere, the most abundant ones presumably are: ^{11}B, ^{12}C, ^{14}N, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, and ^{56}Fe. The spallation cross-sections of these nuclides have been calculated from equation (1) and are shown in tables 4, 5, and 6 for energies of $E \geq 2300$, and at 400 and 150 MeV nucleon^{-1}, respectively, taking into consideration radioactive decays; minor adjustments have been made where experimental data are available. The italicized values refer to yields of which at least 50 percent has been measured. From the small number of italicized values one can see that most of the essential cross-sections are still un-

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measured. It has been assumed that ^{7}Be survives. It can decay only by electron capture, but at energies under consideration the probability of capture in interstellar space is low. The cosmic-ray confinement time in the Galaxy has been assumed sufficiently short for the survival of ^{10}Be. The experimental cross-section for ^{11}B into ^{10}Be has been based on using an averaged half-life estimate of ^{10}Be of 1.6×10^6 years (Yiou and Raisbeck 1972; McMillan 1972).

The total inelastic cross-sections σ_i are given in the last row of table 4; the same values also apply to tables 5 and 6. These have been calculated semiempirically from a compilation of the available experimental measurements (Ashmore, Mather, and Sen 1958; Booth *et al.* 1958*a*, *b*; Chen, Leavitt, and Shapiro 1955; Coor *et al.* 1955; Johansson, Svanberg, and Sundberg 1961; Voss and Wilson 1956). Rather similar values can be obtained from the theoretical expression of Coor *et al.* (1955; their eq. [13]), who took into consideration nuclear transparency and assumed a Gaussian distribution for nuclear density.

Table 7 shows the spallation cross-sections of iron into products heavier than aluminum. It has been assumed that in the cosmic radiation the nuclides ^{37}Ar, ^{44}Ti, ^{49}V, ^{51}Cr, ^{54}Mn, and ^{55}Fe decay by electron capture at $E \leqslant 400$ MeV nucleon^{-1} and survive at higher energies. It has also been assumed that ^{53}Mn survives (Reames 1970) at $E \geq 150$ MeV nucleon^{-1} due to restripping of the captured electron before decay occurs and that ^{41}Ca survives at $E \geqslant 400$ MeV nucleon^{-1}. The numerical values at 150 and 400 MeV nucleon^{-1} have been adjusted slightly to make the sum of the cross-sections equal to the total inelastic cross-section.

Table 8 shows the calculated yields of long-lived nuclides that are produced by cosmic-ray interactions in the interstellar gas, in meteorites, and on the lunar surface.

TABLE 4

PARTIAL CROSS-SECTIONS (IN MILLIBARNS) FOR COLLISION OF VARIOUS "HEAVY NUCLEI" WITH HYDROGEN ($E \geq 2.3$ GeV)

	TARGET							
PRODUCT	^{11}B	^{12}C	^{14}N	^{16}O	^{20}Ne	^{24}Mg	^{28}Si	^{56}Fe
^{6}Li..........	14	*15**	11	*14*	13	13	13	30
^{7}Li..........	20	*13*	13	*14*	13	13	13	20
^{7}Be..........	7	*10*	*12*	*11*	10	*10*	*10*	*8.5*
^{9}Be..........	5.5	6	3	*3.7*	3	3	3	5
^{10}Be.........	*14*	3.5	1.9	*1.0*	1.9	1.9	1.9	4
^{10}B..........	26	16	14	*12*	11	10	9	9
^{11}B..........	...	*51*	25	*25*	18	15	12	9
^{12}C..........	...	...	26	24	18	13	10	7
^{13}C..........	...	...	25	20	14	10	8	5
^{14}N..........	...	...	...	26	18	13	10	6
^{15}N..........	...	...	...	*50*	23	17	13	6
^{16}O..........	...	...	...	...	24	18	13	6
^{17}O..........	...	...	...	...	25	19	14	6
^{18}O..........	...	...	...	...	23	16	*12*	6
^{19}F..........	...	...	...	...	45	19	14	7
^{20,21,22}Ne....	...	...	...	...	...	*69*	*55*	*21*
^{23}Na........	...	...	...	...	...	51	23	9
^{24,25,26}Mg....	...	...	...	...	...	...	77	25
^{27}Al.........	...	...	...	...	...	...	52	10
σ_i...........	195†	205	235	260	315	355	400	676

* Italicized values refer to cross-sections based primarily on experimental information.

† σ_i is the total inelastic cross-section.

TABLE 5

PARTIAL CROSS-SECTIONS (IN MILLIBARNS) FOR COLLISION OF VARIOUS "HEAVY NUCLEI" WITH HYDROGEN (E = 400 MeV)

	TARGET							
PRODUCT	^{11}B	^{12}C	^{14}N	^{16}O	^{20}Ne	^{24}Mg	^{28}Si	^{56}Fe
^{6}Li..........	17	*14**	10	*12*	6	4	2.5	3
^{7}Li..........	25	*12*	10	*11*	5	3	2	3
^{7}Be..........	8	*10*	10	*7*	5	*3.2*	*2*	*1*
^{9}Be..........	9	*4.7*	3	*2.4*	2	1.1	0.7	0.6
^{10}Be.........	*14*	*2.2*	1.3	*0.6*	0.7	0.4	0.3	0.5
^{10}B..........	29	21	17	*12*	9	5	3	0.5
^{11}B..........	...	*57*	30	*25*	15	9	5	0.7
^{12}C..........	...	...	33	*27*	17	11	6	0.3
^{13}C..........	...	...	28	23	14	9	5	0.3
^{14}N..........	...	...	...	26	18	11	7	0.3
^{15}N..........	...	...	...	*54*	25	16	9	0.3
^{16}O..........	...	...	...	...	28	18	11	0.3
^{17}O..........	...	...	...	...	30	19	12	0.3
^{18}O..........	...	...	...	...	24	*23*	*14*	0.3
^{19}F..........	...	...	...	...	50	22	14	0.3
Ne..........	...	...	...	...	...	*80*	*64*	1
Na..........	...	...	...	...	...	56	25	0.3
Mg..........	...	...	...	...	...	...	105	1
Al..........	...	...	...	...	...	...	57	0.5

* Italicized values refer to cross-sections based primarily on experimental information.

TABLE 6

PARTIAL CROSS-SECTIONS (IN MILLIBARNS) FOR COLLISION OF VARIOUS "HEAVY NUCLEI" WITH HYDROGEN (E = 150 MeV)

	TARGET*						
PRODUCT	^{11}B	^{12}C	^{14}N	^{16}O	^{20}Ne	^{24}Mg	^{28}Si
^{6}Li............	18	*11*†	*9*	*10*	5	2.6	1.5
^{7}Li............	25	*9*	8	*8.5*	3	1.9	1.1
^{7}Be............	*10*	*10*	*6*	*5.4*	4	2.5	0.7
^{9}Be............	10	*3.2*	3	*1.7*	1.2	0.7	0.4
^{10}Be...........	*11*	*1.1*	0.65	*0.4*	0.3	0.2	0.1
^{10}B............	38	25	18	*10*	7	3	1.8
^{11}B............	...	*66*	31	*25*	11	5	2.5
^{12}C............	...	...	43	30	14	6	3
^{13}C............	...	...	23	25	12	5	2
^{14}N............	...	...	...	33	17	7	4
^{15}N............	...	...	...	*52*	24	11	5
^{16}O............	...	...	...	...	33	15	7
^{17}O............	...	...	...	...	36	16	8
^{18}O............	...	...	...	...	31	*14*	7
^{19}F............	...	...	...	...	50	22	10
Ne............	...	...	...	...	...	117	53
Na............	...	...	...	...	...	58	*24*
Mg............	...	...	...	...	...	...	129
Al............	...	...	...	...	...	...	60

* Iron has been omitted from the list of targets here, since at ~150 MeV the total production from ^{56}Fe of all the nuclides in this table amounts to < 1 percent of the production of nuclides heavier than aluminum.

† Italicized values refer to cross-sections based primarily on experimental information.

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TABLE 7

PARTIAL CROSS-SECTIONS (mb) FOR COLLISIONS OF IRON WITH HYDROGEN

	ENERGY (MeV)		
PRODUCT	150	400	≥2300
Si	0.3	2	31
P	0.1	1	10
S	1	5	38
Cl	1	7	16
Ar	2	*12**	*41*
K	5	11	28
Ca	16	29	43
Sc	10	15	17
Ti	116	129	85
V	87	91	*49*
Cr	230	176	89
Mn	*168*	*158*	72†
Fe	42	36	*56*

* Italicized values refer to cross-sections based primarily on experimental information.

† Assuming that both ^{53}Mn and ^{54}Mn survive; the survival of ^{54}Mn was recently suggested by Cassé (1971).

The present calculation contains additional refinements not included in a short earlier version (Shapiro and Silberberg 1970). These are: equations (2), (6), and (7); the treatment of boron (end of § II*a*, and Paper II); the parameters P in tables 1B–D and R in table 1A; the factors $H(E)$ and Y (see Paper II); the parameter Ω of ^{7}Li, ^{9}Be, and ^{11}B in table 2 and corresponding changes in the yields of ^{7}Li, ^{9}Be, ^{10}B, ^{11}B, and ^{12}C in tables 4–6. The latest change, based on the interactions of high-energy nitrogen ions (Heckman *et al.* 1972), results in an increase of the parameter Ω for ^{10}B and corresponding changes in tables 4–6; also, Ω and ξ for ^{6}Li and ^{7}Li has been changed (Raisbeck, Lestringuez, and Yiou 1972).

TABLE 8

CALCULATED YIELDS (IN MILLIBARNS) OF LONG-LIVED RADIOACTIVE ISOTOPES

		ENERGY (MeV)		
TARGET	PRODUCT	150	400	≥2300
^{16}O	^{14}C	1.7	2.0	2.3
^{24}Mg	^{22}Na	38	26	17
^{28}Si	^{22}Na	17	16	13
^{28}Si	^{26}Al	30	21	13
^{40}Ca	^{36}Cl	31	24	10
^{56}Fe	^{36}Cl	0.7	2.4	12
	^{39}Ar	1.2	2.8	8.9
	^{41}Ca	0.7	1.3	3.0
	^{44}Ti	0.4	0.6	0.9
	^{49}V	48	36	26
	^{53}Mn	42	30	17
	^{54}Mn	42	30	17
	^{55}Fe	74	56	50

The production cross-sections for ^{2}H, ^{3}H, and ^{3}He in proton-^{4}He collisions also are of great interest for cosmic-ray physics. A good review on this subject, including a compilation of the cross-sections, has recently been presented by Ramaty and Lingenfelter (1969). An additional measurement of ^{3}H production has recently been made by Lebowitz and Miller (1969).

III. DISCUSSION

a) *Comparison with Experimental Data and Other Calculations*

Cross-sections were calculated using equation (1), together with the parameters of tables 1–3 and equations (2)–(7), and compared with the experimental cross-sections from the Appendix table. (The individual cross-sections are given in an NRL report by Silberberg and Tsao [1972].)

The standard deviations of the calculated cross-sections (relative to measured values) are given in table 9A, subdivided according to the different sets of parameters of tables 1. For $Z_t \leqslant 20$, the standard deviation is about 30 percent or less. The deviations are somewhat larger for heavier targets, ranging from $\sigma_{\text{calc}}/\sigma_{\text{exp}} = 1(+0.4, -0.3)$ for $E \geq 2$ GeV to $1(+1, -0.5)$ at $E \simeq 200$ MeV. Figure 4 shows the distribution of the ratio $\sigma_{\text{calc}}/\sigma_{\text{exp}}$ for $6 \leq Z_t \leq 16$ and $Z \geq 6$.

Table 9B shows that our calculated cross-sections for $6 \leq Z_t \leq 16$ agree better with the measured values than do those based on other methods. The Monte Carlo calculations of table 9B have been carried out by Epherre and Gradsztajn (1967), Korteling and Caretto (1970*a*), and Dostrovsky, Gauvin, and Lefort (1968). The latter two works were based on the VEGAS prompt cascade of Chen *et al.* (1968) and the nuclear evaporation calculation of Dostrovsky, Fraenkel, and Friedlander (1959). The semiempirical methods of Rudstam (1966) and Audouze *et al.* (1967) were briefly discussed in § II.

The measured cross-sections of Korteling and Caretto (1970*a*), Raisbeck and Yiou (1971), and Brodzinski *et al.* (1971) became available after the parameters of this

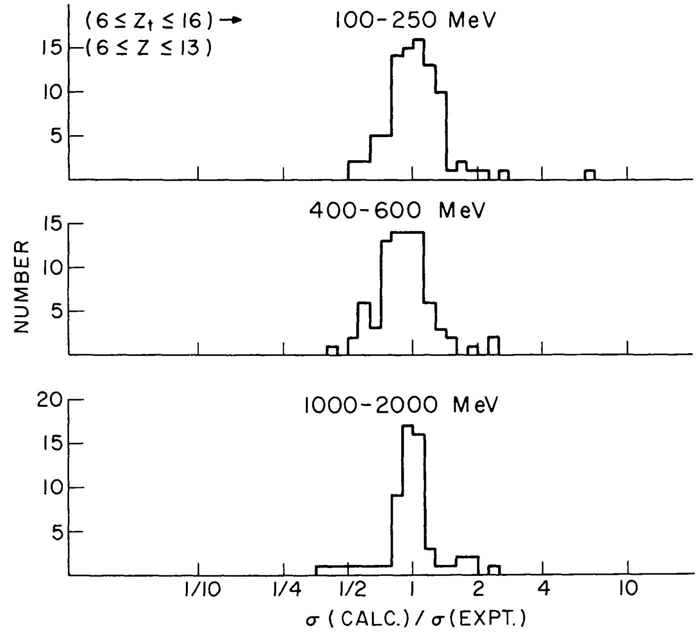

FIG. 4.—Comparison of the calculated and experimental production cross-sections of nuclei with $6 \leq Z \leq 13$ from targets with $6 \leq Z_t \leq 16$, in three energy intervals.

TABLE 9

A. STANDARD DEVIATIONS OF CALCULATED CROSS-SECTIONS

Target	Product	Energy [GeV]	$\sigma_{\text{calc}}/\sigma_{\text{exp}}$
$3 \leq Z_t \leq 28$	$Z \leq 5$	≥ 0.15	$1(+0.3, -0.25)$
$6 \leq Z_t \leq 16$	$Z \geq 6$	0.15–0.6 ≥ 1	$1(+0.3, -0.25)$ 1 ± 0.2
$17 \leq Z_t \leq 20$	$Z \geq 6$	≥ 0.15	$1(+0.3, -0.25)$
$21 \leq Z_t \leq 28$	$Z \geq 6$	0.15–0.25 0.4–0.6 ≥ 2	$1(+1, -0.5)$ $1(+0.7, -0.4)$ $1(+0.4, -0.3)$
$3 \leq Z_t \leq 16$	Peripheral	≥ 0.15	1 ± 0.2
$17 \leq Z_t \leq 28$	Peripheral	≥ 0.15	$1(+0.4, -0.3)$

B. COMPARISON OF OUR CALCULATION WITH OTHER CALCULATIONS FOR $6 \leq Z_t \leq 16$

Other Methods	Ratio of the Other Standard Deviation to Ours
Monte Carlo	2 to 3
Rudstam (1966)	3
Audouze *et al.* (1967)	2

paper had been determined. These data thus provide an experimental test of the equations and parameters of this paper. The standard deviation of the calculated cross-sections from these measured values is consistent with that given in table 9A.

b) *Essential Data for Future Improvements*

The cross-sections for the production of boron isotopes from targets heavier than oxygen have not been measured. Also, the factors Ω for boron isotopes in table 2 are rather uncertain.

The cross-section for the production of ^{10}Be from ^{11}B is large, but still unmeasured at $E > 1$ GeV, and uncertain at lower energies due to statistical errors in the cross-sections. It is important, since much of cosmic-ray ^{10}Be could be due to this reaction, and the long-lived radioactive isotope ^{10}Be can be used to estimate the age of cosmic rays.

The cross-sections for the production of ^{6}Li, ^{7}Li, and ^{9}Be from targets heavier than oxygen are practically unknown. Hence, the factors of table 2 and the enhancement factors ξ of table 3 are highly uncertain for these nuclides.

There are very few experimental data on the spallation cross-sections of targets $17 \leq Z_t \leq 20$. Hence, table 1C was improvised by selecting parameters from tables 1B and 1D until further measurements are made. The factor $\exp(-P\Delta A)$ was taken from table 2D and $\exp(-R|Z - SA + TA^2|^2)$ essentially from table 1B.

It is shown in Paper II (dealing with targets $Z_t \geq 29$) that the VEGAS program of Chen *et al.* (1968), coupled with the evaporation calculations of Dostrovsky *et al.* (1959), reproduces the experimental results equally well as those proposed here with the parameters of tables 1D and with those of Paper II for peripheral reactions. (For $Z_t \lessdot 20$, the present calculation appears superior.) Therefore, it would be highly valuable for cosmic-ray physics to have available the peripheral and spallation cross-sections of ^{56}Fe, calculated with the above Monte Carlo procedure at about 200 and 400 MeV. (Iron is one of the more abundant nuclides among the heavy primary cosmic rays.) The cross-sections of ^{56}Fe calculated at 2 GeV by Porile and Tanaka (1964) with the cascade of Metropolis *et al.* (1958) also are so close to the experimental values that a similar calculation for a more extensive set of product nuclei would be valuable.

The authors wish to thank Dr. M. M. Shapiro for helpful discussions, criticisms, and encouragement.

APPENDIX TABLE

REFERENCES TO EXPERIMENTAL DATA ON CROSS-SECTIONS

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APPENDIX

REFERENCES TO MEASURED CROSS-SECTIONS

The experimental partial cross-sections for high-energy proton-nucleus interactions have been reported in the past by many investigators, listed in the Appendix table. We have compiled extensive tables of cross-sections from these references in an NRL report (Silberberg and Tsao 1972). The cross-sections from these references were used to formulate the parameters of tables 1–3 and to determine the standard deviations of the calculated cross-sections listed in table 9.

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