MODIFICATION OF SEMIEMPIRICAL EQUATIONS FOR CALCULATING PARTIAL CROSS SECTIONS OF p-NUCLEUS REACTIONS

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Recent experimental data on partial cross sections of p-nucleus reactions have been used to bring up-to-date our semiempirical methods and parameters. The major improvements are: consideration of alpha-particle structure of the lighter elements ($Z \leq 10$), modification of treatmemt of evaporation products from targets with mass numbers $34 \leq A_t \leq 63$, and new values of the nuclear structure factor Ω. The numerical values of the parameters that were modified will be presented.

1. Introduction. Recent experimental data have permitted us to improve some of the parameters of our semiempirical equations (Silberberg and Tsao, 1973a and b, henceforth called ST-I and ST-II) for calculating the partial cross sections of proton-nucleus reactions. These data are from Lindstrom et al. (1975) who measured all the cross sections of ^{12}C and ^{16}O, Yiou et al. (1973) and Yiou and Raisbeck (1972) who measured the isotopic yields of Be and Li from various light elements, and Radin et al. (1974) who measured the yields of ^{7}Be from several targets. Furthermore, the measurements of the spallation of Fe by Perron (1975) and Lagarde-Simonoff et al. (1975), of Niby Raisbeck et al. (1975) and of Au by Kaufman et al. (1976) have permitted additional improvements.

2. Modifications for Targets with $Z_t \leq 28$. The Ω-values in Table 2 of paper ST-I are partially revised. The new values (revised ones only) are shown in Table 1.

Table 1. New Values for the Nuclear Structure Factor Ω

Product	^{7}Li	^{7}Be	^{9}Be	^{10}B	^{11}B	K	Sc
Ω	1.8	0.95	0.65	0.7	1.7	0.7	0.7

For targets with $Z_t \leq 7$, the cross sections of (p,3p) reactions become 1.5 times larger.

In the case of breakup of ^{12}C, ^{14}N, and ^{16}O, the cross sections for (p,p α) + (p,3p2n) and (p,pd) + (p,2pn) reactions are enhanced by a factor of 1.8; those of (p,2p) reactions by 1.5, unless the product nuclide is of the type $(x\,\alpha + n)$, e.g., ^{13}C, where x = 3. In the latter case, due to the relatively high probability of neutron evaporation, the cross section is instead reduced by the factor 0.5. Since ^{16}O is a very stable nuclide, the reaction ^{20}Ne [(p,p α) + (p,3p2n)] ^{16}O is assumed to be enhanced by a factor of 1.5.

Only for product nuclei with one or a few particle-stable energy levels (i.e., for ^{13}N and ^{19}Ne), are the cross sections $\sigma_{p,pn}(E_o)$ and $\sigma_{p,2p}(E_o)$ in Eqs. 24 and 26 of ST-II multiplied by Ω , and of the light products in Table 1A of paper ST-I, only ^{9}Be is now multiplied by Ω/η.

For light nuclei ($Z_t \leq 5$) with a low p/n ratio ($N_p/N_n < 1$), $\sigma_{p,2p}(E_o)$ is multiplied by $(N_p-2)/(N_n-2)$ where N_p and N_n are respectively the numbers of protons and neutrons in the target nucleus. The constant 2 is based on assuming a ^{4}He core, which does not participate in peripheral interactions.

The enhancement factors ξ of Table 3, paper ST-I,for light evaporation products from targets with $34 \leq A_t \leq 63$ are now:

$$3[1 + 0.02(A_t - 34)] \quad \text{for } ^{6}\text{He}$$

$$1 + 0.02(A_t - 34) \quad \text{for } ^{6}\text{Li}$$

and

$$1 + 0.01(A_t - 34) \quad \text{for } ^{7}\text{Li and } ^{7}\text{Be}.$$

To reduce the discontinuity between Z_t = 20 and 21, the non-peripheral products of Ca and Sc that have $Z \geq 6$ are calculated by taking the geometrical mean of cross sections calculated from the parameters of Tables 1E and 1D of paper ST-1.

For the production of boron, except by (p,pn) and (p,2p) reactions, the geometric mean of calculations based on Table 1A combined with 1B, 1C and 1D or paper ST-I is to be used for target elements Z_t = 6 to 16, 17 to 20, and 21 to 28, respectively. (Previously the type of mean to be used was not specified.)

In the case of spallation of ^{56}Fe, the yields of products ^{49}V, ^{50}V and ^{51}V are multiplied by 0.8, 0.7 and 0.5 respectively, ^{54}Cr by 0.7, ^{54}Mn and ^{55}Mn by 1.5 and 1.1 respectively, ^{53}Fe by 2, Cl by 0.9. We also assume that the yields of ^{52}V and ^{53}V should be multiplied by 0.4.

For interactions of radioactive secondaries (e.g., those produced in air) that are neutron-deficient ($A_t/Z_t < 2.0$), the parameters S of Table 1A and 1B are respectively:

$$S = 0.54 + 0.32(2 - A_t/Z_t)^{1.4}$$

$$S = 0.502 + 0.26(2 - A_t/Z_t)^{1.4}$$

For $35 \leq A_t \leq 70$, the (p,3p), (p,4p) and (p,5p) reaction cross sections are calculated from equation (21) of paper ST-II, with $\sigma(E_o) = \sigma_s(E_o)$, and $H(E) = \{1 - \exp[-(E/35)^4]\}(E/200) \leq 1$. (For σ_s, see Eq. (2a) of ST-II.)

A minus sign was omitted in equation (23) of paper ST-II. It should read: $Y(A_t, Z_t) = 2 - \exp[-d(A_t - \bar{A}_t)/Z_t]$.

3. Modifications for Targets $Z_t > 28$. The revised values of the parameter Ω given in Section 2 are to be used. The enhancement factors ξ for the evaporation of light nuclei, given in Table 3 of ST-I are now changed to the values shown in Table 2.

Table 2. Enhancement Factors ξ for Light Evaporation Products

Product	$64 \leq A_t \not\leq 104$	$A_t > 104$
^{6}He	$4.8 + 0.045\ (A_t - 64)$	6.6
^{6}Li	$1.6 + 0.015\ (A_t - 64)$	2.2
^{7}Li	$1.3 + 0.015\ (A_t - 64)$	1.9
^{7}Be	$1.3 + 0.0105\ (A_t - 64)$	1.7
^{8}Li	$1.0 + 0.0225\ (A_t - 64)$	1.9
^{9}Li	$1.0 + 0.0125\ (A_t - 64)$	1.5
^{9}Be	$1.0 + 0.01\ (A_t - 64)$	1.4
^{10}Be, ^{10}B, ^{11}B	$1.0 + 0.005\ (A_t - 64)$	1.2

The yields of boron from nuclei with $Z_t = 29$ and 30 are obtained from the geometric mean of calculations based on Tables 1A and 1B of pages 317 and 318 (regions ε and SH) of paper ST-I. For $Z_t > 30$, the yields of boron are estimated from the geometric mean of calculations based on equation (1) and Table 1A of page 317 of paper ST-I, and equation (12) of page 351 of ST-II. The evaporation factor ξ is to be introduced into equation (12) in the calculation of boron yields. The parameters a and b of equation 13 are respectively 0.02 and 0.6 for boron, and remain unchanged for heavier products.

It is pointed out at the bottom of page 338 of paper ST-II that for $10 \leqslant \Delta A \leqslant 20$, the restriction $\sigma_s \leq \sigma_s(E_0)$ is to be imposed on heavy nuclei for $E > 1$ GeV. This restriction applies also for values of ΔA somewhat less than 10, down to the boundary between spallation and peripheral cross sections.

The remark at the end of Section 2 (correction of an error appearing in equation (23) for $Y(A_t, Z_t)$ of paper ST-II, applies also to targets with $Z_t > 28$.

The expression x_{max} of equation (35) is to be truncated like other expressions for x_{max}. For the (p,3pxn) reactions of neutron-rich heavy targets (with $A_t > 70$) there is a discontinuity at the boundary of peripheral (P) and spallation (S) cross sections. The spallation cross sections σ_s, defined by equation (2a) of paper ST-II, are too small near the boundary. The recommended procedure for eliminating the discontinuity is: First one determines the value x_o for which the spallation cross section (for the appropriate value of Z) is largest. The expression x_o is defined by:

$$x_o = (A_t - A_o - 2) \text{ truncated},$$

where A_o is obtained from the factor $Z - SA + TA^2$ of equation (2a) of paper ST-II and is approximated by:

$$A_o \approx \left\{ [S - (S^2 - 4TZ)^{\frac{1}{2}}]/2T \right\} - M.$$

The expression M is zero in general, except for the values given in paragraph 5 of page 338 of paper ST-II. Then for $x_{max} < x < x_o$, the cross sections are calculated from:

$$\sigma_{sp} = \sigma_p(x_{max}) + \left(\frac{x - x_{max}}{x_o - x_{max}} \right) [\sigma_s(x_o) - \sigma_p(x_{max})]$$

where $\sigma_p(x_{max})$ is defined by equations (21), (34) and (35) and $\sigma_s(x_o)$ (obtained from equation (2a) of ST-II) by:

$$\sigma_s(x_o) = \sigma_o \exp[-P(A_t - A_o)]$$

The constraints on σ_s (i.e., on σ_o and P) discussed at the bottom of page 338 of paper ST-II are to be applied. Possible discontinuities for (p,pxn) and (p,2pxn) reactions at the boundary of peripheral and spallation cross sections are to be smoothed out, too.

The prescription for calculating the cross sections of (p,4p) and (p,5p) reactions of targets with $35 \leq A_t \leq 70$, given in Section 2, applies also to nuclei with $Z_t > 28$.

For $70 < A_t \leq 209$, $x \leq x_{max}$ and $E \geq 2$ GeV, the cross sections of (p,4pxn) and (p,5pxn) reactions are calculated from the relation:

$$\sigma(p,4pxn)/\sigma(p,3pxn) \approx \sigma(p,5pxn)/\sigma(p,4pxm) \approx 0.1$$

(No prescription is yet available for $E < 2$ GeV.)

The normalization factor $\varphi(A_t,E)$ of page 460 of paper ST-II is not to be applied to (p,pn) reactions.

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