IMPROVED SEMIEMPirical ESTIMATES OF CROSS SECTIONS

C. H. Tsao and R. Silberberg
Laboratory for Cosmic Ray Physics
Naval Research Laboratory
Washington, D. C. 20375, U.S.A.

ABSTRACT

New experimental data have become available since the Plovdiv Conference, especially from the group of Heckman, et al. on spallation of iron. Since most of the sub-iron nuclides in cosmic rays (17 ≤ A ≤ 25) are generated by fragmentation of iron, these data are important for calculations of cosmic-ray propagation. Based on these measurements, our previously calculated yields of K, Ca and Sc from the iron-group elements are revised upward. Likewise, the yields of Mn$^{55}$ and Mn$^{54}$ from Fe$^{56}$ are now calculated to be higher. An examination of the cross sections of various elements from Fe to U versus energy reveals some energy dependence in the vicinity of the energy $E_0$ above which the cross sections previously were assumed to become constant. We have constructed corresponding adjustment factors.

1. Introduction.

As new and more precise experimental data become available, one can improve the semiempirical equations that we have developed for calculating partial cross sections (Silberberg and Tsao 1973a and b). We previously provided a set of improved parameters and equations at the Plovdiv cosmic-ray conference (Silberberg and Tsao 1977). Meanwhile new experimental data on the spallation of Fe have become available from the group of Heckman (Westfall et al., 1979). A detailed study of the energy dependence of cross sections suggest minor modifications that at certain energies can change the estimates by up to 20%. In particular, instead of a sharp transition to an energy independence above an energy of $E_0$, there is a slow approach to the asymptotic value between $E_0$ and 2 $E_0$.

There is also a ∼10% uncertainty in the total inelastic cross sections. Our recent formulation (Silberberg et al., 1976) relied largely on Renberg et al., (1972). Some recent data discussed below suggest values that are ∼10% higher. In general, such a small uncertainty does not affect the results of cosmic-ray propagation calculations. However, it does affect the calculation of some features of the distribution of path lengths from the L/M and (sub-Fe)/Fe ratios, e.g. it raises the question: does the exponential distribution of path lengths have to be modified so as to introduce a dearth of short path lengths?

2. Modified Parameters for Cross Section Calculations

The recent measurements of the fragmentation of relativistic $^{56}$Fe by Westfall et al. (1979) imply that the production cross sections of K, Ca and Sc
from iron-group nuclei is normal, and should not be suppressed by the factor \( \Omega = 0.7 \) (Silberberg and Tsao, 1973 and 1977) as earlier data by some observers had suggested. Thus we now set \( \Omega = 1.0 \) for the production of K, Ca and Sc by spallation reactions from targets with 21 \( \leq Z_t \leq 28 \). (For other targets, and for peripheral reactions, \( \Omega = 1.0 \) was already used earlier.)

When the measurements of Perron (1976) became available, we raised the yields \( \sigma (^{56}\text{Fe} \rightarrow ^{54}\text{Mn}) \) by 1.5, and of \( \sigma (^{56}\text{Fe} \rightarrow ^{55}\text{Mn}) \) by 1.1 (Silberberg and Tsao, 1977). In the light of the measurements of Westfall et al. (1979), these yields have to be raised further. Now, \( \sigma (^{56}\text{Fe} \rightarrow ^{54}\text{Mn}) = 1.7 \) times as large as and \( \sigma (^{56}\text{Fe} \rightarrow ^{55}\text{Mn}) = 1.25 \) times the old semi-empirical values.

Our semi-empirical calculations of the breakup of Ca and Sc had a discontinuity; the products of Ca were calculated from the parameters of Table 1C of Silberberg and Tsao (1973) and Sc from Table 1D. To remove the discontinuity, we propose the use of the geometric mean of Tables 1C and 1D for calculating the breakup of both Ca and Sc:

\[
\sigma (\text{Ca or Sc} \rightarrow X) = \left[ \sigma (\text{Table 1C}) \right]^{1/3} \left[ \sigma (\text{Table 1D}) \right]^{1/3}
\]

The abundance of \(^{19}\text{F}\) in cosmic rays is rather low; we suggest that the calculated production rate of \(^{19}\text{Ne}\) be reduced from \( \Omega (^{19}\text{Ne}) = 0.8 \) to the new value \( \Omega (^{19}\text{Ne}) = 0.6 \). The nuclide \(^{19}\text{Ne}\) has few particle-stable states; it should be suppressed analogously to \(^{9}\text{Be}\) and \(^{13}\text{N}\).

3. Modification of the Energy Dependence of Cross Section Calculations

A detailed look at the energy dependence of cross sections shows that there is a slight energy dependence even for energies higher than the previously assumed plateau threshold energy \( E_0 \). The change in cross sections beyond \( E_0 \) is small, \( \sim 10 \%, \) but is substantiated by several investigations: Husain et al. (1973) for V, Raisbeck et al. (1975) for Fe, Hudis et al. for Cu, Katcoff et al. (1968) for Ag, Kaufman et al. (1976) for Au, and Chang et al. (1974) for U. For large values of \( \Delta A \), the target-product mass difference, the yields beyond \( E_0 \) increase, and for small values of \( \Delta A \) they decrease.

For the yields of light evaporation products (2 \( \leq Z \leq 4 \), with 6 \( \leq A \leq 12 \)) at energy \( E \) from targets with 21 \( \leq Z_t \leq 28 \), the old semiempirical value \( \sigma_0 \) is modified by the correction factor \( f_1 \):

\[
\sigma = f_1 \sigma_0
\]

Here

\[
f_1 = 1 - 0.6 [1 - \left( \frac{E'}{1000} \right)^2] e^{-\left( \frac{E'}{2000} \right)^2} + 0.2 [1 - e^{-\left( \frac{E'}{3000} \right)^2}]
\]

and \( E' = E \) for \( E < E_0 \).

\[
f_1 = \left[ f_1(E) f_1(E') \right]^{1/2}
\]

for \( E_0 \leq E \leq 2500 \text{ MeV} \) and \( E' = 2262 (E/E_0) \) for \( E > 2500 \text{ MeV} \).
For targets with \( 21 \leq Z_t \leq 28 \) and \( \Delta A > \Delta A_c(E_o) \),

\[
\sigma = f_2 \sigma_o.
\]

Also for targets with \( Z_t > 28 \) and products with \( A \leq 56 \) that have
\( \Delta A > \Delta A_c(E_o) \),

\[
\sigma = f_2 \sigma_o.
\]

For targets with \( Z_t \geq 21 \) and \( \Delta A > 7 \) with \( \Delta A \leq \Delta A_c(E_o) - 13 \),

\[
\sigma = f_3 \sigma_o.
\]

For targets with \( Z_t \geq 90, A > 56 \) and \( \Delta A \geq 7 \),

\[
\sigma = f_4 \sigma_o.
\]

Here

\[
f_2 = 1 - 0.4\left[ 1 - \exp\left(-\frac{E'}{2000}\right)^2 \right] e^{-\left(\frac{E'-1800}{1800}\right)^2} + 0.17\left[ 1 - e^{-\left(\frac{E'}{2000}\right)^2} \right]
\]

\[
f_3 = 1 + 0.25\left[ 1 - \exp\left(-\frac{E'}{1500}\right)^2 \right] e^{-\left(\frac{E'-1500}{1800}\right)^2} - 0.05\left[ 1 - e^{-\left(\frac{E'}{2000}\right)^2} \right]
\]

\[
f_4 = 1 - 0.1\left[ 1 - e^{-\left(\frac{E'}{4000}\right)^2} \right].
\]

and \( E' = 2262 \left( E/E_o \right) \).

4. Total Inelastic Cross Sections

There is some uncertainty in estimates of the total inelastic cross sections. The formulation used by us (Silberberg et al., 1976) depends strongly on the values of Renberg et al. (1972). Some recent measurements (Bellettini et al., 1966, Schimmerling et al., 1973, and Lindstrom, 1978) suggest values that are \( \sim 10\% \) higher.
5. References