Hawking temperature of the cosmological horizon in a FRW universe

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It is well known that there is a Hawking temperature on the cosmological horizon of the de-sitter spacetime, and the de-sitter spacetime can be a special case of a FRW universe. Therefore, there may be a corresponding Hawking temperature in a FRW universe. Indeed, there have been several clues showing that there is a Hawking temperature on the apparent horizon of a FRW universe. In our paper, however, after finding the corresponding cosmological horizon of a FRW universe, and then investigating the behavior of a Klein-Gordon field near the cosmological horizon, we find that there is a Hawking temperature on the cosmological horizon. Moreover, we also find that the Hawking temperature on the apparent horizon of a FRW universe in some previous work is just a special case in our results, where the variation rate of cosmological horizon $\dot{r}_H$ is zero.

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I. INTRODUCTION

Since Hawking found that there was a thermal radiation like a black body in a black hole, it has been further found that the radiation is in fact due to the existence of event horizon [1]. The event horizon playing a key point can also be seen from the Unruh effect where an uniformly accelerating observer with acceleration a in the Minkowskian spacetime can detect a thermal spectrum with temperature $T = a/2\pi$ [2]. Here the Unruh radiation is closely related to the existence of Rindler causal horizon for the observer. Obviously, the Hawking temperature which is proportional to its surface gravity on the event horizon can give some insight on the deep relationship between gravity and thermodynamics. Furthermore, the thermodynamics of black hole has been constructed with the Bekenstein entropy of a black hole [3–6]. Note that, the Hawking temperature is usually obtained on the event horizon of a stationary black hole. In fact, it can also be obtained on the cosmological horizon of a spacetime such as the cosmological horizon of de Sitter spacetime [7, 8].

Event horizon and cosmological horizon are both global concepts [9, 10]. Strictly speaking, locally it is not known whether there is an event horizon or cosmological horizon associated with a certain dynamical spacetime at some time. Thus this causes the difficulty to discuss Hawking radiation for a dynamic spacetime. However, by using the the null property of event horizon or cosmological horizon and the intrinsic symmetry of a dynamic spacetime, we can find a corresponding hypersurface which can reduce the event horizon or cosmological horizon in the stationary case. Because of this, we also call this corresponding hypersurface as the event horizon or cosmological horizon for a dynamic spacetime in our paper [11–15]. In spite of that, another situation appears. This is, the event horizon (the above corresponding hypersurface) and apparent horizon for a dynamic spacetime are usually different, while they are consistent for a stationary spacetime. Therefore, the Hawking radiation from which horizon is still an open question. Recently, Hayward and other authors have attacked this question [15–17]. By using the quasi-local Misner-Sharp energy [18–20], the so-called unified first law can be deduced from the Einstein equation in a spherical symmetric spacetime [21–24]. And they argued that the Hawking radiation might come from the apparent horizon for a dynamic spherical symmetric black hole spacetime, because after projecting the unified first law on the apparent horizon of a dynamic spherical symmetric black hole spacetime, one can obtain an analogy of the first law of thermodynamics of stationary black hole. In addition, one could use the Hamilton-Jacobi equation of particles to make a simple proof [17, 25]. However, in spite of those works, there are other works showing that the Hawking radiation can come from the event horizon of dynamic black hole spacetime by investigating the behavior of the quantum filed near the event horizon [11–14].

On the other hand, the Friedmann-Robertson-Walker (FRW) universe is a dynamical spacetime. And the de Sitter spacetime can be its special case. Therefore, it may also exists Hawking radiation in a FRW universe. By considering that the FRW universe is also a spherical symmetric spacetime and with an apparent horizon, thus the above discussion on the apparent horizon of dynamic spherical symmetric black hole spacetime can be generalized to the FRW universe. Following this way, there have been many interesting works on it [26–32]. And it has been proved that the Hawking temperature of the apparent horizon in a FRW universe is $T = 1/2\pi r_A$, where the temperature is measured by the corresponding Kodama observer [33] and r_A is the radius of apparent horizon [31, 32]. In particular, we would like to mention here that if we assume the entropy of apparent horizon S satisfying $S = A/4$, where A is the area of the apparent horizon, one is able to derive Friedmann equations of the FRW universe with any spatial curvature by applying the Clausius relation to apparent horizon [34, 35]. However, there is the same situation as the dynamic black hole spactime that the cosmological horizon of a FRW universe is not usually consistent with its apparent horizon. Therefore, what the result is if we investigate the behavior of the quantum filed near the cosmological horizon of FRW universe is one of our motivations.

There are several methods to investigate the behavior of quantum filed near the horizon of a spacetime [36–38]. In our paper, we mainly use the method first proposed by Damour and Ruffini and then developed by Sannan and Zhao [13, 14, 38, 39]. By using the fact that usually the Klein-Gordon equation in the tortoise coordinates can be reduced to the standard form of wave equation near the cosmological horizon of FRW universe, we can obtain the appropriate parameter κ which can be corresponding to the surface gravity in the stationary case. Moreover, we can find that the ingoing wave of FRW universe is not analytical on the cosmological horizon. And the ingoing wave can be extended by analytical continuation from the inside of cosmological horizon to the outside [13, 14, 38–40]. After doing these, we obtain the Hawking radiation spectrum with the temperature on the cosmological horizon of a FRW universe.

The organization of the paper is as follows. In Sec. II, we first obtain the cosmological horizon in a FRW universe, and then use the Damour- Ruffini method to obtain its Hawking temperature. Sec. III is devoted to the conclusion and discussion. And we particularly discuss some properties of the Hawking temperature. It shows that not only our result is consistent with that in some previous work [31, 32], but also the result in some previous work is a special case just when the variation rate of cosmological horizon $\dot{r}_H$ is zero.

II. THE COSMOLOGICAL HORIZON AND ITS HAWKING TEMPERATURE IN A FRW UNIVERSE

The metric of a FRW universe is

$$ds^2 = -dt^2 + a^2(t)\left(\frac{d\rho^2}{1-k\rho^2} + \rho^2 d\Omega_2^2\right), \tag{2.1}$$

where t is the cosmic time, ρ is the comoving radial coordinate, a is the scale factor, $d\Omega_2^2$ denotes the line element of a 2-dimensional sphere with unit radius, $k = 1$, 0 and -1 represent a closed, flat and open FRW universe, respectively.

For the convenience, we can define $r = a\rho$. Thus, the metric (2.1) can be rewritten

$$ds^2 = -\frac{1-r^2/r_A^2}{1-kr^2/a^2}dt^2 - \frac{2Hr}{1-kr^2/a^2}dtdr + \frac{1}{1-kr^2/a^2}dr^2 + r^2 d\Omega_2^2. \tag{2.2}$$

where $r_A = 1/\sqrt{H^2 + k/a^2}$ is the location of apparent horizon in a FRW universe.

Note that, the metric of the de Sitter spacetime is

$$ds^2 = -\left(1-\frac{r^2}{l^2}\right)dt^2 + \left(1-\frac{r^2}{l^2}\right)^{-1}dr^2 + r^2 d\Omega_2^2. \tag{2.3}$$

and the FRW metric (2.2) can be further rewritten that

$$ds^2 = -\frac{1-r^2/r_A^2}{1-kr^2/a^2}(dt + \frac{Hr}{1-r^2/r_A^2}dr)^2 + \frac{1-kr^2/a^2}{1-r^2/r_A^2}dr^2 + r^2 d\Omega_2^2. \tag{2.4}$$

Thus it can be easily found that the de Sitter spacetime is just a special case of the FRW universe where $k = 0$ and $r_A = H^{-1} = l$ is a constant in (2.4). On the other hand, we know that $r = l$ is the cosmological horizon of the de Sitter spacetime, therefore, there may be a corresponding cosmological horizon in a FRW universe. By using the null property of the cosmological horizon and the spherical symmetry in (2.2), we can indeed obtain that the corresponding cosmological horizon $r = r_H(t)$ which satisfies

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu} = 0, \tag{2.5}$$

is

$$1 - r_H^2/r_A^2 = \dot{r}_H^2 - 2Hr_H\dot{r}_H. \tag{2.6}$$

where $f = r - r_H(t)$. From (2.6), it can be also easily checked that the corresponding cosmological horizon $r_H(t)$ is just the cosmological horizon of the de Sitter spacetime when $k = 0$ and $\dot{r}_H = 0$.

In the following, we will investigate the Hawking temperature of the corresponding cosmological horizon $r = r_H(t)$ in a FRW universe. For the simplicity, we just consider the Klein-Gordon field in a FRW universe. And the Klein-Gordon equation

$$(\Box - m^2)\Phi = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^v})\Phi - m^2\Phi = 0. \tag{2.7}$$

can be rewritten in the FRW coordinates (2.2) such that

$$\begin{aligned}&-\frac{\partial}{\partial t}(\frac{1}{\sqrt{1-\frac{k}{a^2}r^2}}\frac{\partial}{\partial t})\frac{\rho(t,r)}{r} - \frac{\partial}{\partial t}(\frac{Hr}{\sqrt{1-\frac{k}{a^2}r^2}}\frac{\partial}{\partial r})\frac{\rho(t,r)}{r} - \frac{1}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\sqrt{1-\frac{k}{a^2}r^2}}Hr\frac{\partial}{\partial t})\frac{\rho(t,r)}{r}\\ &+\frac{1}{r^2}\frac{\partial}{\partial r}[\frac{r^2}{\sqrt{1-\frac{k}{a^2}r^2}}(1-r^2/r_A^2)\frac{\partial}{\partial r}]\frac{\rho(t,r)}{r} = [m^2 + \frac{l(l+1)}{r^2}]\frac{1}{\sqrt{1-\frac{k}{a^2}r^2}}\frac{\rho(t,r)}{r},\end{aligned} \tag{2.8}$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})Y_{lm}(\theta,\varphi) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}Y_{lm}(\theta,\varphi) + l(l+1)Y_{lm}(\theta,\varphi) = 0. \tag{2.9}$$

where m is the rest mass of the Klein-Gordon particle, $Y_{lm}(\theta,\varphi)$ is the usual spherical harmonics and Φ has been separated as

$$\Phi = \frac{1}{r}\rho(t,r)Y_{lm}(\theta,\varphi). \tag{2.10}$$

For the convenience to investigate the behavior of the scalar field near the cosmological horizon, we introduce the generalized tortoise coordinate transformation

$$\begin{aligned} r_* &= r + \frac{1}{2\kappa}\ln[r_H(t) - r],\\ t_* &= t - t_0. \end{aligned} \tag{2.11}$$

where κ is an adjustable constant, and $r_H(t)$ is just the location of the cosmologica.l horizon. Note that, κ can be just the surface gravity of the event horizon or cosmological horizon in the stationary spacetimes.

From (2.11), we can obtain

$$\begin{aligned}
\frac{\partial}{\partial r} &= [1+\frac{1}{2\kappa(r-r_H)}]\frac{\partial}{\partial r_*},\\
\frac{\partial}{\partial t} &= \frac{\partial}{\partial t_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},\\
\frac{\partial^2}{\partial r^2} &= [1+\frac{1}{2\kappa(r-r_H)}]^2\frac{\partial^2}{\partial r_*^2}-\frac{1}{2\kappa(r-r_H)^2}\frac{\partial}{\partial r_*},\\
\frac{\partial^2}{\partial t\partial r} &= [1+\frac{1}{2\kappa(r-r_H)}]\frac{\partial^2}{\partial t_*\partial r_*}-\frac{\dot r_H}{2\kappa(r-r_H)}[1+\frac{1}{2\kappa(r-r_H)}]\frac{\partial^2}{\partial r_*^2}+\frac{\dot r_H}{2\kappa(r-r_H)^2}\frac{\partial}{\partial r_*},\\
\frac{\partial^2}{\partial t^2} &= [\frac{\dot r_H}{2\kappa(r-r_H)}]^2\frac{\partial^2}{\partial r_*^2}-\frac{\dot r_H}{\kappa(r-r_H)}\frac{\partial^2}{\partial t_*\partial r_*}-\frac{\dot r_H^2+\ddot r_H(r-r_H)}{2\kappa(r-r_H)^2}\frac{\partial}{\partial r_*}+\frac{\partial^2}{\partial t_*^2}.
\end{aligned}$$

Thus after using the above differential relations, the radial equation (2.8) can be

$$\begin{aligned}
&\{-\frac{2\kappa(r-r_H)(\dot a^2+k+a\ddot a)}{a[r(2r\kappa-2r_H\kappa+1)\dot a-a\dot r_H]}+\frac{2\left(l^2+l+m^2r^2\right)\kappa a(r-r_H)}{r^2[r(2r\kappa-2r_H\kappa+1)\dot a-a\dot r_H]}\}\rho\\
&+\{\frac{-[\dot r_H^2+(r-r_H)\ddot r_H-1]a^2+[(r+r_H)\dot a\dot r_H+r(r-r_H)(2r\kappa-2r_H\kappa+1)\ddot a]a}{a(r-r_H)[r(2r\kappa-2r_H\kappa+1)\dot a-a\dot r_H]}\\
&+\frac{r[2\kappa r^2+2\kappa r_H^2-(4r\kappa+1)r_H](\dot a^2+k)}{a(r-r_H)[r(2r\kappa-2r_H\kappa+1)\dot a-a\dot r_H]}\}\frac{\partial\rho}{\partial r_*}+\{\frac{(2r\kappa-2r_H\kappa+1)^2(\dot a^2+k)r^2}{2\kappa a(r-r_H)[r(2r\kappa-2r_H\kappa+1)\dot a-a\dot r_H]}\\
&+\frac{-2(2r\kappa-2r_H\kappa+1)\dot a\dot r_H ar+a^2[-(2r\kappa+1)^2+4\kappa r_H(2r\kappa+1)-4\kappa^2r_H^2+\dot r_H^2]}{2\kappa a(r-r_H)[r(2r\kappa-2r_H\kappa+1)\dot a-a\dot r_H]}\}\frac{\partial^2\rho}{\partial r_*^2}\\
&+\frac{2\kappa(r-r_H)\dot a}{r(2r\kappa-2r_H\kappa+1)\dot a-a\dot r_H}\frac{\partial\rho}{\partial t_*}+2\frac{\partial^2\rho}{\partial t_*\partial r_*}+\frac{2\kappa a(r-r_H)}{r(2r\kappa-2r_H\kappa+1)\dot a-a\dot r_H}\frac{\partial^2\rho}{\partial t_*^2}=0. \quad (2.12)
\end{aligned}$$

when $r\to r_H$ and $t\to t_0$, the radial equation (2.12) can be

$$A\frac{\partial^2\rho}{\partial r_*^2}+2\frac{\partial^2\rho}{\partial t_*\partial r_*}+\alpha_0\frac{\partial\rho}{\partial r_*}=0. \quad (2.13)$$

where we have used the equation (2.6) and

$$A=-\frac{H\dot r_H-(H^2+k/a^2)r_H}{\kappa(Hr_H-\dot r_H)}+2\dot r_H,\ \alpha_0=\frac{(H^2+k/a^2)r_H-H\dot r_H+\ddot r_H-\frac{\ddot a}{a}r_H}{\dot r_H-Hr_H}. \quad (2.14)$$

The solutions of (2.13) are

$$\rho_{out} = e^{-i\omega t_*}, \quad (2.15)$$

$$\rho_{in} = e^{-i\omega t_*+2i\omega r_*/A}e^{-\alpha_0 r_*/A}. \quad (2.16)$$

By using the fact that usually the Klein-Gordon equation in the tortoise coordinates can be reduced to the standard form of wave equation near the horizon

$$\frac{\partial^2\rho}{\partial r_*^2}+2\frac{\partial^2\rho}{\partial t_*\partial r_*}=0, \quad (2.17)$$

we can adjust the parameter κ to make $A = 1$, and

$$\kappa = \frac{H\dot{r}_H - (H^2 + k/a^2)r_H}{(Hr_H - \dot{r}_H)(2\dot{r}_H - 1)}. \tag{2.18}$$

Note that, $A = 1$ can also be implied from the special case, that of the de Sitter spacetime. In this special case, $k = 0$ and $\dot{r}_H = 0$ with $r_A = H^{-1} = l$, the κ in (2.18) is $\kappa = 1/l$ which is just the surface gravity of the cosmological horizon in the de Sitter spacetime.

Therefore, the ingoing wave of the Klein-Gordon filed near the cosmological horizon can be further rewritten

$$\rho_{in} = e^{-i\omega t_* + 2i\omega r_*} e^{-\alpha_0 r_*} = e^{-i\omega t_*} e^{2i\omega r - \alpha_0 r} (r_H - r)^{i\omega/\kappa - \alpha_0/2\kappa}. \tag{2.19}$$

where we have used (2.11). And we can find that the ingoing wave (2.19) is not analytical on the cosmological horizon, thus we can extend it by analytical continuation from the inside of cosmological horizon to its outside [13, 14, 38–40]

$$(r_H - r) \to |r_H - r| e^{i\pi} = (r - r_H) e^{i\pi}, \tag{2.20}$$

and then the ingoing wave (2.19) becomes

$$\rho_{in} \to \tilde{\rho}_{in} = e^{-i\omega t_*} e^{2i\omega r - \alpha_0 r} (r - r_H)^{i\omega/\kappa - \alpha_0/2\kappa} e^{-\frac{i\pi\alpha_0}{2\kappa}} e^{-\frac{\pi\omega}{\kappa}} = e^{-i\omega t_* + 2i\omega r_*} e^{-\alpha_0 r_*} e^{-\frac{i\pi\alpha_0}{2\kappa}} e^{-\frac{\pi\omega}{\kappa}}, r > r_H. \tag{2.21}$$

By using the Heaviside function Y

$$Y(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{2.22}$$

the complete ingoing wave can be

$$\phi^{in}_{\omega} = N_{\omega}[Y(r_H - r)\rho_{in} + Y(r_H - r)\tilde{\rho}_{in}]. \tag{2.23}$$

where N_w is a normalization factor. From (2.23), the scalar product of ϕ^{in}_{ω} is

$$(\phi^{in}_{\omega_1}, \phi^{in}_{\omega_2}) = N_{\omega_1} N_{\omega_2} (\delta_{\omega_1\omega_2} - e^{-\pi(\omega_1+\omega_2)/\kappa} \delta_{\omega_1\omega_2}). \tag{2.24}$$

Note that, if $\kappa < 0$, we can obtain from (2.24)

$$(\phi^{in}_{w}, \phi^{in}_{w}) = -1 = N^2_{\omega}(1 - e^{-2\pi\omega/\kappa}). \tag{2.25}$$

which is just a thermal spectrum with a temperature $T = -\kappa/2\pi$. While if $\kappa > 0$, we can obtain

$$(\phi^{in}_{\omega}, \phi^{in}_{\omega}) = 1 = N^2_{\omega}(1 - e^{-2\pi\omega/\kappa}). \tag{2.26}$$

which is apparently not a thermal spectrum. However, we can redefine the complete ingoing wave in (2.23) that

$$\phi_\omega^{in'} = e^{\frac{\pi\omega}{\kappa}} N_\omega [Y(r_H - r)\rho_{in} + Y(r_H - r)\tilde{\rho_{in}}], \tag{2.27}$$

thus we can obtain

$$(\phi_\omega^{in'}, \phi_\omega^{in\prime}) = 1 = N_\omega^2 (e^{2\pi\omega/\kappa} - 1). \tag{2.28}$$

which is a thermal spectrum with the temperature $T = \kappa/2\pi$. In words, we can obtain the thermal spectrum in both cases

$$N_\omega^2 = 1/[\exp(\omega/K_B T) - 1], \tag{2.29}$$

where the temperature T is

$$T = \frac{|\kappa|}{2\pi} = |\frac{(H^2 + k/a^2)r_H - H\dot{r}_H}{2\pi(Hr_H - \dot{r}_H)(2\dot{r}_H - 1)}|. \tag{2.30}$$

III. CONCLUSION AND DISCUSSION

Whether there is a Hawking temperature in a FRW universe is a very interesting question. Viewed from the fact that the de Sitter spacetime can be a special case of a FRW universe and there is a Hawking temperature on the cosmological horizon of the de Sitter spacetime, thus it may also have a corresponding Hawking temperature in a FRW universe. Indeed, there have been some clues showing that there is a Hawking temperature on the apparent horizon in a FRW universe. However, in our paper, after first finding the corresponding cosmological horizon of a FRW universe, and then investigating the behavior of a Klein-Gordon field near the cosmological horizon, we obtain that there is a Hawking temperature on the cosmological horizon of a FRW universe.

Some remarks on our results are in order.

(1) The relation between the apparent horizon and cosmological horizon in a FRW universe. From (2.6), we can easily find that these two horizons are usually not consistent. However, they are same when $\dot{r}_H = 0$ or $\dot{r}_H = 2Hr_H$. By using $r_A = 1/\sqrt{H^2 + k/a^2}$, we can further reduce that $H = 0$ or $\dot{H} = k/a^2$ in the $\dot{r}_H = 0$ case, while $2H^2 + \dot{H} + k/a^2 = 0$ in the $\dot{r}_H = 2Hr_H$ case. Note that, the Ricci scalar of a FRW universe is $R = 6(2H^2 + \dot{H} + k/a^2)$, thus the latter case is also equivalent to $R = 0$.

(2) The uniqueness of horizon. Note that, in the generalized tortoise coordinates (2.11), the horizon can apparently be chosen other horizons such as the apparent horizon. However, considered the fact that usually the Klein-Gordon field in the generalized tortoise coordinates near the horizon can be reduced the standard form of wave equation (2.17), the horizon is unique. And it should be the cosmological horizon in (2.6). This can be seen from (2.12) that when $r \to r_H$ and $t \to t_0$

$$\begin{aligned}\lim_{r\to r_H, t\to t_0} \{ & \frac{(2r\kappa - 2r_H\kappa + 1)^2(\dot{a}^2 + k)r^2 - 2(2r\kappa - 2r_H\kappa + 1)\dot{a}\dot{r}_H ar}{2\kappa a(r - r_H)[r(2r\kappa - 2r_H\kappa + 1)\dot{a} - a\dot{r}_H]} \\ & + \frac{a^2[-(2r\kappa + 1)^2 + 4\kappa r_H(2r\kappa + 1) - 4\kappa^2 r_H^2 + \dot{r}_H^2]}{2\kappa a(r - r_H)[r(2r\kappa - 2r_H\kappa + 1)\dot{a} - a\dot{r}_H]}\} = 1. \end{aligned} \quad (3.1)$$

And at first it should be satisfied

$$\begin{aligned}\lim_{r\to r_H, t\to t_0} & \{(2r\kappa - 2r_H\kappa + 1)^2(\dot{a}^2 + k)r^2 - 2(2r\kappa - 2r_H\kappa + 1)\dot{a}\dot{r}_H ar \\ & + a^2[-(2r\kappa + 1)^2 + 4\kappa r_H(2r\kappa + 1) - 4\kappa^2 r_H^2 + \dot{r}_H^2]\} = 0. \end{aligned} \quad (3.2)$$

which is just the location of the cosmological horizon in (2.6).

(3) Another method to obtain κ and r_H. There is a more simple method to determine κ and r_H. From the generalized tortoise coordinate transformation (2.11), we can have

$$\begin{aligned} dr_* &= [1 + \frac{1}{2\kappa(r - r_H)}]dr - \frac{\dot{r}_H}{2\kappa(r - r_H)}dt, \\ dt_* &= dt. \end{aligned} \quad (3.3)$$

Thus

$$\begin{aligned} dr &= \frac{2\kappa(r - r_H)}{2\kappa(r - r_H) + 1}dr_* + \frac{\dot{r}_H}{2\kappa(r - r_H) + 1}dt_*, \\ dt &= dt_*. \end{aligned} \quad (3.4)$$

After substituting (3.4) into the metric (2.2), we have

$$\begin{aligned} ds^2 &= \{-\frac{1 - r^2/r_A^2}{1 - kr^2/a^2} - \frac{2Hr}{1 - kr^2/a^2}\frac{\dot{r}_H}{2\kappa(r - r_H) + 1} + \frac{1}{1 - kr^2/a^2}[\frac{\dot{r}_H}{2\kappa(r - r_H) + 1}]^2\}dt_*^2 \\ & +\{-\frac{2Hr}{1 - kr^2/a^2}\frac{2\kappa(r - r_H)}{2\kappa(r - r_H) + 1} + \frac{2}{1 - kr^2/a^2}\frac{2\kappa(r - r_H)}{2\kappa(r - r_H) + 1}\frac{\dot{r}_H}{2\kappa(r - r_H) + 1}\}dt_* dr_* \\ & +\frac{1}{1 - kr^2/a^2}[\frac{2\kappa(r - r_H)}{2\kappa(r - r_H) + 1}]^2 dr_*^2 + r^2 d\Omega_2^2 \\ &= \frac{2\kappa(r - r_H)}{(1 - kr^2/a^2)[2\kappa(r - r_H) + 1]}\frac{-Hr[2\kappa(r - r_H) + 1] + \dot{r}_H}{2\kappa(r - r_H) + 1}\{\frac{2\kappa(r - r_H)}{-Hr[2\kappa(r - r_H) + 1] + \dot{r}_H}dr_*^2 \\ & +\frac{-(1 - r^2/r_A^2)[2\kappa(r - r_H) + 1]^2 - 2Hr\dot{r}_H[2\kappa(r - r_H) + 1] + \dot{r}_H^2}{-Hr2\kappa(r - r_H)[2\kappa(r - r_H) + 1] + 2\kappa(r - r_H)\dot{r}_H}dt_*^2 + 2dt_* dr_*\} + r^2 d\Omega_2^2. \end{aligned} \quad (3.5)$$

which we require that it can reduce the following formalism in the limit $r \to r_H, t \to t_0$

$$ds^2 = \Omega^2(-dt_*^2 + 2dt_* dr_*) + r^2 d\Omega_2^2. \tag{3.6}$$

where Ω is the corresponding conformal factor. Therefore, we can obtain the same r_H and κ in (2.6) and (2.18), respectively.

(4) The temperature in some special cases. Here we give some special cases where the cosmological horizon is consistent with the apparent horizon.

In the $\dot{r}_H = 0$ case, the temperature in (2.30) is

$$T = \frac{1}{2\pi H r_A^2}. \tag{3.7}$$

Note that, in reference [31, 32] the temperature is $T = \frac{1}{2\pi r_A}$. And this temperature measured by the Kodama observer has a factor Hr_A in front of the temperature measured by the observer $(\partial/\partial t)^a$ in (2.2). In addition, $\dot{r}_H = 0$ ensures the observer in the coordinates system in (2.11) same as the observer $(\partial/\partial t)^a$ in (2.2). Thus our result is in fact consistent with the result in reference [31, 32]. Furthermore, it also shows that the result in reference [31, 32] is just a special case of our result.

In the $\dot{r}_H = 2Hr_H$ case, the temperature in (2.30) is

$$T = \frac{|\kappa|}{2\pi} = |\frac{2H^2 r_A^2 - 1}{2\pi H r_A^2 (4Hr_A - 1)}|. \tag{3.8}$$

And it contains an interesting case. When $k = 0$, we can further calculate $a(t) = t^{1/2}$ with $H = \frac{1}{2t}$ and $r_A = \frac{1}{H} = 2t$, which can just represent the period of radiation dominated in the early FRW universe. From our result, the temperature in this period is $T = \frac{1}{6\pi r_A}$.

(5) The generalized tortoise coordinate. In our paper, we choose the generalized tortoise coordinate as that in (2.11). In fact, we can also choose the generalized tortoise coordinate just as

$$\begin{aligned} r_* &= \frac{1}{2\kappa} \ln[r_H(t) - r], \\ t_* &= t - t_0. \end{aligned} \tag{3.9}$$

By using the same procedure, we can obtain the same r_H but different κ, which can be seen in appendix A. And the reason can be simply viewed from the fact that the cosmological horizon is independent of observers, while the κ related with the temperature is dependent of observers. In fact, the 4-velocity of the observer in the new coordinate system after the coordinates transformation can be showed in the same coordinate system in (2.2). After the coordinates transformation (2.11), the 4-velocity of the new observer is $(\frac{\partial}{\partial t})^a + \frac{\dot{r}_H}{2\kappa(r-r_H)+1}(\frac{\partial}{\partial r})^a$, while after the coordinates transformation (3.9), it is $(\frac{\partial}{\partial t})^a + \dot{r}_H(\frac{\partial}{\partial r})^a$. Obviously, these two observers are different. And when $\dot{r}_H = 0$ they are same as the observer $(\partial/\partial t)^a$ in (2.2).

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Appendix A: Choosing different generalized tortoise coordinate

In this appendix, we just use the more simple method to determine r_H and κ. By using (3.9), we can have

$$\begin{aligned} dr_* &= \frac{1}{2\kappa(r-r_H)}dr - \frac{\dot{r}_H}{2\kappa(r-r_H)}dt, \\ dt_* &= dt. \end{aligned} \tag{A1}$$

Thus

$$\begin{aligned} dr &= 2\kappa(r-r_H)dr_* + \dot{r}_H dt_*, \\ dt &= dt_*. \end{aligned} \tag{A2}$$

After substituting (A2) into the metric (2.2), we have

$$\begin{aligned} ds^2 &= \{-\frac{1-r^2/r_A^2}{1-kr^2/a^2} - \frac{2Hr\dot{r}_H}{1-kr^2/a^2} + \frac{\dot{r}_H^2}{1-kr^2/a^2}\}dt_*^2 + \{-\frac{4Hr\kappa(r-r_H)}{1-kr^2/a^2} \\ &\quad + \frac{4\kappa(r-r_H)\dot{r}_H}{1-kr^2/a^2}\}dt_* dr_* + \frac{[2\kappa(r-r_H)]^2}{1-kr^2/a^2}dr_*^2 + r^2 d\Omega_2^2 \\ &= \frac{2\kappa(r-r_H)(-Hr+\dot{r}_H)}{(1-kr^2/a^2)}\{\frac{2\kappa(r-r_H)}{-Hr+\dot{r}_H}dr_*^2 + \frac{-(1-r^2/r_A^2) - 2Hr\dot{r}_H + \dot{r}_H^2}{2\kappa(r-r_H)(-Hr+\dot{r}_H)}dt_*^2 \\ &\quad + 2dt_* dr_*\} + r^2 d\Omega_2^2. \end{aligned} \tag{A3}$$

Similarly, we require that the (A3) can be reduced the formalism (3.6) in the limit $r \to r_H$ and $t \to t_0$. Therefore,

$$\lim_{r\to r_H, t\to t_0} \frac{(1 - r^2/r_A^2) + 2Hr\dot{r}_H - \dot{r}_H^2}{2\kappa(r - r_H)(-Hr + \dot{r}_H)} = 1. \tag{A4}$$

which reduces

$$(1 - r_H^2/r_A^2) + 2Hr_H\dot{r}_H - \dot{r}_H^2, \tag{A5}$$

and

$$\kappa = \frac{H\dot{r}_H - r_H/r_A^2}{\dot{r}_H - Hr_H}. \tag{A6}$$

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[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975) [Erratum-ibid. **46**, 206 (1976)].

[2] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

[3] J. D. Bekenstein, Lett. Nuovo Cim. **4** (1972) 737.

[4] J. D. Bekenstein, Phys. Rev. D **7**, 949 (1973).

[5] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).

[6] J. M. Bardeen, B. Carter and S. W. Hawking, Commun. Math. Phys. **31**, 161 (1973).

[7] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).

[8] M. K. Parikh, Phys. Lett. B **546**, 189 (2002) [arXiv:hep-th/0204107]; A. J. M. Medved, Phys. Rev. D **66**, 124009 (2002) [arXiv:hep-th/0207247].

[9] S. W. Hawking and G. F. R. Ellis, The Large scale structure of space-time, *Cambridge University Press, Cambridge, 1973*

[10] R. M. Wald, General Relativity, *Chicago, Usa: Univ. Pr. (1984) 491p*

[11] X. Li and Z. Zhao, Phys. Rev. D **62**, 104001 (2000),

[12] W. A. Hiscock, Phys. Rev. D **15** (1977) 3054.

[13] Z. Zhao and X. X. Dai, Mod. Phys. Lett. A **7** (1992) 1771. Z. Zhao, C. Q. Yang and Q. A. Ren, Gen. Rel. Grav. **26**, 1055 (1994).

[14] Z. H. Li, Y. Liang and L. Q. Mi, Int. J. Theor. Phys. **38** (1999) 925.

[15] R. G. Cai, L. M. Cao, Y. P. Hu and S. P. Kim, Phys. Rev. D **78**, 124012 (2008) [arXiv:0810.2610 [hep-th]].

[16] A. B. Nielsen, Gen. Rel. Grav. **41**, 1539 (2009) [arXiv:0809.3850 [hep-th]].

[17] S. A. Hayward, R. Di Criscienzo, L. Vanzo, M. Nadalini and S. Zerbini, arXiv:0806.0014 [gr-qc]; R. Di Criscienzo, M. Nadalini, L. Vanzo, S. Zerbini and G. Zoccatelli, Phys. Lett. B **657**, 107 (2007) [arXiv:0707.4425 [hep-th]], R. Di Criscienzo, S. A. Hayward, M. Nadalini, L. Vanzo and S. Zerbini, Class. Quant. Grav. **27**, 015006 (2010).

[18] C. W. Misner and D. H. Sharp, Phys. Rev. **136**, B571 (1964).

[19] H. Maeda and M. Nozawa, Phys. Rev. D **77**, 064031 (2008)[arXiv:0709.1199 [hep-th]].

[20] R. G. Cai, L. M. Cao, Y. P. Hu and N. Ohta, Phys. Rev. D **80**, 104016 (2009) [arXiv:0910.2387 [hep-th]].

[21] S. A. Hayward, Phys. Rev. D **49**, 6467 (1994).

[22] S. A. Hayward, Phys. Rev. D **53**, 1938 (1996) [arXiv:gr-qc/9408002].

[23] S. A. Hayward, Class. Quant. Grav. **15**, 3147 (1998) [arXiv:gr-qc/9710089]. S. Mukohyama and S. A. Hayward, Class. Quant. Grav. **17**, 2153 (2000) [arXiv:gr-qc/9905085].

[24] S. A. Hayward, S. Mukohyama and M. C. Ashworth, Phys. Lett. A **256**, 347 (1999) [arXiv:gr-qc/9810006].

[25] K. Srinivasan and T. Padmanabhan, Phys. Rev. D **60**, 024007 (1999) [arXiv:gr-qc/9812028]; S. Shankaranarayanan, T. Padmanabhan and K. Srinivasan, Class. Quant. Grav. **19**, 2671 (2002) [arXiv:gr-qc/0010042]; M. Angheben, M. Nadalini, L. Vanzo and S. Zerbini, JHEP **0505**, 014 (2005) [arXiv:hep-th/0503081].

[26] R. G. Cai and L. M. Cao, Phys. Rev. D **75**, 064008 (2007) [arXiv:gr-qc/0611071].

[27] M. Akbar and R. G. Cai, Phys. Rev. D **75**, 084003 (2007) [arXiv:hep-th/0609128].

[28] R. G. Cai and L. M. Cao, Nucl. Phys. B **785**, 135 (2007) [arXiv:hep-th/0612144]; A. Sheykhi, B. Wang and R. G. Cai, Nucl. Phys. B **779**, 1 (2007) [arXiv:hep-th/0701198]; A. Sheykhi, B. Wang and R. G. Cai, Phys. Rev. D **76**, 023515 (2007) [arXiv:hep-th/0701261].

[29] R. G. Cai, Prog. Theor. Phys. Suppl. **172**, 100 (2008) [arXiv:0712.2142 [hep-th]].

[30] X. H. Ge, Phys. Lett. B **651**, 49 (2007) [arXiv:hep-th/0703253]; Y. Gong and A. Wang, Phys. Rev. Lett. **99**, 211301 (2007) [arXiv:0704.0793 [hep-th]]. S. F. Wu, G. H. Yang and P. M. Zhang, arXiv:0710.5394 [hep-th]. S. F. Wu, B. Wang and G. H. Yang, Nucl. Phys. B **799**, 330 (2008) [arXiv:0711.1209 [hep-th]; S. F. Wu, B. Wang, G. H. Yang and P. M. Zhang, arXiv:0801.2688 [hep-th]; T. Zhu, J. R. Ren and S. F. Mo, arXiv:0805.1162 [gr-qc]; M. Akbar, arXiv:0808.0169 [gr-qc].

[31] R. G. Cai, L. M. Cao and Y. P. Hu, Class. Quant. Grav. **26**, 155018 (2009) [arXiv:0809.1554

[hep-th]].

[32] R. Li, J. R. Ren and D. F. Shi, Phys. Lett. B **670**, 446 (2009) [arXiv:0812.4217 [gr-qc]].

[33] H. Kodama, Prog. Theor. Phys. **63**, 1217 (1980), M. Minamitsuji and M. Sasaki, Phys. Rev. D **70**, 044021 (2004) [arXiv:gr-qc/0312109].

[34] R. G. Cai and S. P. Kim, JHEP **0502**, 050 (2005)[arXiv:hep-th/0501055].

[35] R. G. Cai, L. M. Cao and Y. P. Hu, JHEP **0808**, 090 (2008) [arXiv:0807.1232 [hep-th]].

[36] K. D. Kokkotas and B. G. Schmidt, Living Rev. Rel. **2**, 2 (1999) [arXiv:gr-qc/9909058].

[37] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **95**, 011303 (2005) [arXiv:gr-qc/0502074];

[38] T. Damour and R. Ruffini, Phys. Rev. D **14**, 332 (1976). S. Sannan, Gen. Rel. Grav. **20** (1988) 239.

[39] Y. P. Hu, G. H. Tian and Z. Zhao, Mod. Phys. Lett. A **24**, 229 (2009) [arXiv:gr-qc/0610108].

[40] R. Penrose, Int. J. Theor. Phys. **1**, 61 (1968).