Eternal inflation without metaphysics

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In the usual account of eternal inflation the universe is supposed to be a deSitter background in which pocket universes nucleate at a steady rate. However this is metaphysics because there is no way this mosaic structure can be observed. We don't see the whole universe but only a nearly homogeneous region within our past light cone. We show that we can use the no-boundary wave function to calculate small departures from homogeneity within our past light cone despite the possibility of much larger fluctuations on super horizon scales. We find that the dominant contribution comes from the history that exits eternal inflation at the lowest value of the potential and predict, in a certain class of landscape models, a tensor to scalar ratio of about 10%. In this way the no-boundary wave function defines a measure for the prediction of local cosmological observations.

The string landscape is thought to contain a vast number of vacua, including some that have four large dimensions, our small positive value of the cosmological constant, and the Standard Model. But the landscape does not explain which vacuum in this class we are in. For that one has to turn to cosmology and to a theory of the quantum state of the universe.

A quantum state specifies amplitudes for different geometry and field configurations on a spacelike surface. We have shown that the no-boundary wave function (NBWF) [1] in the saddle point approximation predicts a large amplitude for configurations that behave classically when the universe is large and have an early period of inflation [2]. The NBWF thus acts as a vacuum selection principle in the class described above, selecting regions in field space where the landscape potential admits one or more directions of inflation. The landscape then essentially becomes an ensemble of different models of inflation, weighted by NBWF probabilities. For the rest of this paper we assume this ensemble.

We are interested in the probabilities of different values of local observables such as those referring to the cosmic microwave background (CMB), in a nearly homogeneous Hubble volume and conditioned on our observational situation. Since we do not know where our Hubble volume is, we must sum over its possible locations. Furthermore, as observers we are physical systems within the universe, described by data D which occur only with a very small probability in any Hubble volume. Top-down (TD) probabilities take this in account, by conditioning the bottom-up (BU) NBWF probabilities on the requirement that D exists somewhere [3, 4].

We will find that significant contributions to TD probabilities come only from landscape regions that admit a regime of eternal inflation where $V > \epsilon$. Further it turns out that the dominant contribution to TD probabilities comes from the region(s) where the threshold for eternal inflation lies at the lowest value of the potential, independently of its shape above this value.

In the usual approach to eternal inflation it is argued the universe develops large inhomogeneities that lead to a mosaic structure on super-horizon scales, consisting of (possibly infinitely many) nearly homogeneous patches separated by inflating regions [5]. The probability distributions for local observables can be different in different homogeneous patches, each of which itself can become arbitrarily large. This has led to a challenge, known as the measure problem, for the prediction of local observations in one Hubble volume. To resolve this a cutoff is imposed on the spacetime in order to regulate infinities. The expected number of Hubble volumes of different kinds can then be calculated and used to define the probabilities for the observations of a typical observer.

A very different approach to eternal inflation is based on the measure defined by the universe's quantum state. This paper builds on a series of papers (e.g. [2, 6]) in which we have investigated the implications of the no-boundary quantum state measure. The semiclassical NBWF does not predict a single classical spacetime. Rather it predicts BU probabilities for an ensemble of alternative spacetimes. In models of eternal inflation the TD probabilities for large long wavelength perturbations are high [6]. However, in contrast to the usual approach, we find that the very large scale structure of the eternally inflating histories in the ensemble is irrelevant for the probabilities of observables in our Hubble volume. Instead, the latter depend only on alternatives in our past light cone. To calculate the probabilities of different configurations inside one Hubble volume one sums (coarse grains) over everything outside the past light cone. This results in well-defined probabilities for observations without the need for further ad hoc regularization.

By 'metaphysics' one might mean parts of a theoretical picture that do not directly refer to, explain or influence our observations within a Hubble volume. If so, then this is a predictive discussion of eternal inflation without metaphysics.

The No-Boundary Measure

A quantum state of the universe is specified by a wave function Ψ on the superspace of geometries $(h_{ij}(x))$ and matter field configurations $(\chi(x))$ on a closed spacelike three-surface Σ. Schematically we write $\Psi = \Psi[h,\chi]$. We assume the no-boundary wave function as a model of this state [1]. The NBWF is given by a sum over histories of geometry g and fields ϕ on a four-manifold with one boundary Σ. The contributing histories match the values (h,χ) on Σ and are otherwise regular. They are weighted by $\exp(-I/\hbar)$ where $I[g,\phi]$ is the Euclidean action.

In some regions of superspace the path integral can be approximated by the method of steepest descents. Then the NBWF will be approximately given by a sum of terms of the form

$$\Psi[h,\chi] \approx \exp\{(-I_R[h,\chi] + iS[h,\chi])/\hbar\}, \qquad (1)$$

one term for each complex extremum. Here $I_R[h,\chi]$ and $-S[h,\chi]$ are the real and imaginary parts of the Euclidean action, evaluated at the extremum.

When the surfaces Σ are three spheres of radius a with $a^2V(\phi) < 1$, where V is the potential of the scalar matter fields, there is an approximately real Euclidean solution of the field equations, and $S \approx 0$. For large radii a, however, there are only complex solutions, and the wave function oscillates rapidly. When S varies rapidly compared to I_R (as measured by quantitative classicality conditions [2]) the NBWF predicts that the geometry and fields behave classically. The NBWF can then be viewed as predicting a family of classical Lorentzian histories that are the integral curves of S and have probabilities to leading order in $\hbar$ that are proportional to $\exp[-2I_R(h,\chi)]/\hbar$, which is constant along the integral curve.

In [2, 6] we evaluated the semiclassical NBWF for a model consisting of a single scalar field moving in a quadratic potential. We found that for large radii a the NBWF predicts a family of alternative Lorentzian Friedman-Lemaître-Robertson-Walker universes with Gaussian perturbations. The alternative histories can be labeled by the absolute values of the perturbations ζ_0 and of the background scalar field ϕ_0 at the 'South Pole' (SP) of the corresponding saddle point. Classicality requires $\phi_0 \gtrsim 1$ (in Planck units). The relative BU probabilities of the alternative configurations follow approximately from

$$I_R(\phi_0) \approx -\pi/4V(\phi_0) \qquad (2)$$

together with the Gaussian probabilities for fluctuations.

The NBWF has the striking property that all saddle point histories undergo some amount of matter driven slow roll inflation, with a number of e-folds $N(\phi_0) \approx 3\phi_0^2/2$. The NBWF therefore *selects* inflationary classical histories. These exhibit the usual Gaussian spectrum of fluctuation modes ζ_q with expected amplitude $\zeta_q^2 \approx (\pi^2/4)\left(H^2/\epsilon\right)_{\rm exit}$, where $\epsilon \equiv \dot\phi^2/H^2$ and the amplitude is evaluated when the perturbations exit the horizon. Hence saddle points starting below the threshold of

eternal inflation are nearly homogeneous. By contrast, in regions of the landscape where the condition for eternal inflation $V > \epsilon$ is satisfied – which means the scalar fields are effectively in a deSitter background – the probabilities are high for significant perturbations on large scales.

Assuming this holds generally, only configurations that emerge form regions of the landscape that admit inflationary solutions will have significant probability. One expects different inflationary patches of a landscape potential are separated by large potential barriers. Hence the NBWF acts as a vacuum selection principle in the landscape which becomes an ensemble of models of inflation, weighted by their no-boundary probabilities.

Near minima in the landscape with a single inflationary direction in field space the NBWF predicts a one-parameter family of background solutions that can be labeled by ϕ_0. There is an obvious generalization to multifield models involving n inflaton fields ϕ^i and an $n-$dimensional family of solutions.

We now seek to calculate the relative contributions of the different inflationary regions in field space to the TD probabilities for local observations in our Hubble volume.

Probabilities for Observations

Top-Down Weighting: Our observations are confined to one Hubble volume and our data D occur with only a very small probability $p_E(D)$ in any Hubble volume on a constant density surface $\Sigma_s(D,\phi_0,\zeta_0)$. TD probabilities for local observables $\mathcal{O}$ takes this observational situation in account, by weighting the BU probabilities by the probability $p(D^{\geq1})$ that D exists somewhere on the surface [10]. We showed in [4] that this weighting is given by a multiplicative factor

$$1 - [1 - p_E(D)]^{N_h} \leq 1. \qquad (3)$$

Here, N_h is the total number of Hubble volumes in Σ_s. When N_h is sufficiently small so that our data is rare the TD factor is very small and reduces to weighting the BU probabilities by the volume of Σ_s [7, 8]. We have argued [4] this is the case in saddle point histories that start below the threshold of eternal inflation, which predict high amplitudes for nearly homogeneous final configurations with (3) proportional to e^{3N}.

By contrast, saddle points in the regime of eternal inflation predict high amplitudes for configurations that have large long wavelength perturbations [6]. The volume of $\Sigma_s(D,\phi_0,\zeta_0)$ of these perturbed configurations can be exceedingly large or even infinite [9] so that the probability that D exists somewhere is nearly one.

The TD weighting has a significant effect on the BU distributions in models that admit a regime of eternal inflation. The NBWF BU probabilities favor histories starting at a low value of the potential followed by only a few e-folds of slow roll inflation [cf. (2)]. However, the TD weighting (3) suppresses the probabilities for such histories, and instead favors saddle points starting in the regime of eternal inflation [11]. In models of eternal inflation, the low BU probability of histories starting above

the threshold of eternal inflation is compensated by the large number of Hubble volumes in the resulting surfaces Σ_s.

Once regions in field space for which the condition for eternal inflation holds have been selected, the probabilities of local observations are given by their bottom-up values. It remains to estimate the latter.

Coarse Graining: A wave function $\Psi[h,\chi]$, and the histories constructed from it, specify probabilities for alternatives all across a spacelike surface, to its future, and to its past. To obtain probabilities for observables $\mathcal{O}$ in our Hubble volume requires summing over the probabilities for alternatives not directly relevant for observations. This is coarse graining.

Causality implies that the probabilities of observations in our Hubble volume can depend only on alternatives in our past light cone. Coarse graining over alternatives to the future of Σ_s, including all quantum branching, is immediate. The result is simply that the probabilities for alternatives on Σ_s are given by the wave function.

There remains the coarse graining over alternatives on Σ_s in Hubble volumes outside our own. This can be discussed explicitly in the saddle point approximation (1). We find that a given saddle point yields the same predictions for local observables for all Hubble volumes [12] on Σ_s. Further, all saddle points starting at sufficiently high potential in a particular inflationary direction in field space yield identical predictions. This is the case, for instance, in single-field models of eternal inflation with a sufficient number of e-folds after the exit from eternal inflation. We call regions in the landscape where saddle points produce Hubble volumes with the same distributions for $\mathcal{O}$ eternally inflating *channels*. Probabilities for observables $\mathcal{O}$ depend only on the channel, and we can coarse grain over different eternally inflating histories in any one channel. Labeling the different channels by K we arrive at

$$p(\mathcal{O}|D^{\geq 1}) \approx \sum_K p(\mathcal{O}|K)p(K). \qquad (4)$$

Here, $p(K)$ is the NBWF probability of channel K, and $p(\mathcal{O}|K)$ is the probability for observables $\mathcal{O}$ given that channel. This is a finite and manageable prescription for probabilities for observation in our Hubble volume. The two probabilities involved in (4) can be estimated as follows.

Local observables $\mathcal{O}$ such as those associated with the CMB refer only to short wavelength fluctuations that can be observed in our Hubble volume. This means long-wavelength fluctuations should be coarse grained over to compute joint probability $p(\mathcal{O},K)$. To leading order in $\hbar$, if one coarse grains over all possible values of the long-wavelength fluctuations this sum-over-histories yields one (see however [11]). The probabilities $p(\mathcal{O},K)$ can then be estimated by using saddle points in channel K that are nearly homogeneous everywhere and retain only the small, short-wavelength, observable, fluctuations. Those saddle points can be labeled by $\phi_{K0} \geq \phi_K^{\mathrm{ei}}$,

where ϕ_K^{ei} is the threshold value that marks the onset of the regime of eternal inflation in channel K, and by the values of the short-wavelength perturbations ζ_{K0} at the SP. Since the BU probabilities decrease rapidly with ϕ_0 (cf. (2)) we can approximate $p(K)$ by $\exp(-\pi/(4V_K^{\mathrm{ei}}))$ where $V_K^{\mathrm{ei}} \equiv V(\phi_K^{\mathrm{ei}})$ is the value of the local potential in channel K at the lowest exit from eternal inflation. The conditional probabilities $p(\mathcal{O}|K)$, finally, can be calculated using standard perturbation theory techniques (see e.g. [6]).

Hence, with these approximations we predict that the contributions from different channels in the landscape to the TD probabilities (4) of observables $\mathcal{O}$ in our Hubble volume are approximately given by the homogeneous saddle points with the lowest exits from eternal inflation in each channel. If the landscape has one particular channel where the threshold of eternal inflation is at significantly lower potential than in all others, then this channel provides the dominant contribution to the sum in (4).

Predictions for Observations

Models of Inflation: So far we have concentrated on explaining how predictions for observations can be derived from a quantum state of the universe in a landscape that allows for different regions of eternal inflation. However, one cannot expect this general discussion to yield realistic predictions without further qualification. In particular we have not discussed possible structure on the landscape, nor optimized the class of data D (which can include anthropic constraints) assumed for TD probabilities.

Therefore to illustrate the framework above we now consider a model landscape where the NBWF selects a discrete set of K minima that are separated from each other by steep potential barriers. We further assume that each minimum in the class under discussion has a single inflationary direction ϕ_K in field space, where the local potential $V(\phi_K) = \mu_K \phi_K^{n_K}$ with $n_K \geq 2$, and that otherwise the minima are similar. The threshold values ϕ_K^{ei} that mark the onset of the regime of eternal inflation around the different minima can be calculated from the condition that $V^3 = V_{,\phi}^2$ at ϕ_K^{ei}. Substituting the NBWF probabilities (cf. (2)) with $\phi_0 = \phi_K^{\mathrm{ei}}$ in (4) yields for the TD probabilities

$$p(\mathcal{O}|D^{\geq 1}) \approx \sum_K p(\mathcal{O}|\phi_K^{\mathrm{ei}}) \exp\left(1/\mu_K\right)^{\frac{2}{2+n_K}} \qquad (5)$$

Hence the dominant contribution comes from minima where the scalar field is moving in a quadratic potential, for which $p(\phi_K^{\mathrm{ei}}) \approx \exp(1/m)$ where $m = \sqrt{\mu}$ is the inflaton mass.

Contributions from Different Saddle Points: The NBWF predicts probabilities $p(C_\ell^{\mathrm{obs}}|K)$ for the standard multipole coefficients of the observed CMB two point correlator in any class of backgrounds K [6]. The BU NBWF probabilities for fluctuations are Gaussian to lowest order in their amplitude. The resulting probabilities $p^\ell(C_\ell^{\mathrm{obs}}|K)$ for a given ℓ are therefore essentially a χ^2-distribution specified by a mean $\langle C_\ell^{\mathrm{obs}} \rangle = C_\ell^K$ and

(cosmic) variance $\sigma_\ell^K \equiv 2(C_\ell^K)^2/(2\ell + 1)$ where the C_ℓ^K are the theoretical multipole coefficients that completely characterize Gaussian fluctuations.

If the C_ℓ^K differ significantly we would expect all $C_\ell^{\rm obs}$'s to be within a few σ's of one *or* the other predicted expected values C_ℓ^K. If some $C_\ell^{\rm obs}$'s have been measured, they can be used to make predictions about $C_\ell^{\rm tbo}$'s to be observed by computing the conditional probability $p(C_\ell^{\rm tbo}|C_\ell^{\rm obs})$. Since the $C_\ell^{\rm obs}$'s are independent random variables these turn out to be given by

$$p(C_\ell^{\rm tbo}|C_\ell^{\rm obs}) = \sum_K p(C_\ell^{\rm tbo}|K)p(K|C_\ell^{\rm obs}) \qquad (6)$$

where, using the Bayes relation,

$$p(K|C_\ell^{\rm obs}) = \frac{p(C_\ell^{\rm obs}|K)p(K)}{\sum_K p(C_\ell^{\rm obs}|K)p(K)} \qquad (7)$$

and $p(K)$ is the NBWF probability for channel K. If either the no-boundary probabilities $p(K)$ or the $C_\ell^{\rm obs}$'s are enough to make $p(K|C_\ell^{\rm obs})$ peaked around one K then (6) predicts that further observations will confirm that.

Finally we note that, even though the TD probabilities (4) for the linear fluctuations are a sum of Gaussian distributions from different channels, no non-Gaussianity is predicted for the standard measures of it as discussed in [6].

Conclusion:

As a quantum mechanical system, the universe has a quantum state. A theory of that state such as the NBWF is a necessary part of any final theory. The probabilities following from the state are a measure for prediction in cosmology. Applied to predictions of our local observations the NBWF measure appears to be finite without the need for further ad hoc regularization. We briefly summarize the essential principles behind this.

The state predicts probabilities for different configurations of geometry and field on a spacelike surface. Our observations of the universe are limited to one particular Hubble volume in a much larger universe. Their probabilities are defined by summing (coarse-graining) over unobserved features, for example the location of our past light cone in spacetime, or structure arising from quantum events far outside our past light cone. The saddle point approximation to the wave function incorporates some of this coarse graining. The resulting probabilities for observation are well-defined and depend only on alternatives in our past light cone.

Applying the no-boundary measure to a model landscape we found the dominant contribution to top-down probabilities comes from the region(s) in field space where the threshold for eternal inflation holds at the lowest value of the potential. In a model consisting of isolated minima with polynomial, monotonically increasing directions of inflation, this implies an essentially Gaussian spectrum of microwave fluctuations with a scalar spectral index $n_s \sim .97$ and a tensor to scalar ratio of about 10%.

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[10] We emphasize that the probability that D exists somewhere in the universe is not a probability for an observation of D in one Hubble volume. Rather it refers to a feature of a history, and not to something that is observable from one Hubble volume.

[11] In a landscape $p_E(D)$ can depend on the minimum and on the direction one rolled down from. The TD weighting (3) favors saddle point histories with $N_h \geq 1/p_E$. In the text we work in the approximation that this selects the different regimes of eternal inflation in the landscape. However in a more precise calculation, using that $N_h \geq 1/p_E$, a *subset* of eternally inflating histories would be selected. This would differentiate between regions in field space with a comparable no-boundary weight for eternal inflation but with different values of p_E.

[12] This is because the saddle point approximation builds in a certain amount of coarse graining over quantum events. It means we do not take in account the (exponentially small) contribution, through big quantum jumps, of one regime of eternal inflation to the amplitude of Hubble volumes emerging from a different eternally inflating region in the landscape.