Bouncing Cosmologies: Progress and Problems

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We review the status of bouncing cosmologies as alternatives to cosmological inflation for providing a description of the very early universe, and a source for the cosmological perturbations which are observed today. We focus on the motivation for considering bouncing cosmologies, the origin of fluctuations in these models, and the challenges which various implementations face.

I. MOTIVATION

The inflationary scenario [1] is the current paradigm of early universe cosmology. Inflation solves several problems of Standard Big Bang cosmology, and it gives rise to a causal theory of structure formation [2] (see also [3]) which made a number of predictions for cosmological observations which were subsequently successfully verified. In spite of the phenomenological success, inflation faces a number of conceptual challenges (see e.g. [4] for a review of these problems) which motivate the exploration of alternative early universe scenarios. Before mentioning some of these challenges we must begin with a lightning review of inflationary Universe cosmology.

According to the inflationary scenario, the universe underwent a period of almost exponential expansion at some very early time. As a consequence, the horizon expanded exponentially and became larger than our past light cone - both evaluated at the time of recombination - provided that the period of accelerated expansion was sufficiently long. During this period, spatial curvature was also diluted. Any wavelength of fluctuation was stretched quasi-exponentially during the period of inflation so that the wavelengths corresponding to scales which are being observed today in cosmological experiments were smaller than the Hubble radius $H^{-1}(t)$ at the beginning of inflation, where $H(t)$ is the expansion rate of space. The space-time geometry of inflationary cosmology is sketched in Fig. 1. In this figure, the vertical axis is time t, with $t = t_{\rm i}$ denoting the beginning of the inflationary phase, and $t = t_{\rm R}$ the end; the horizontal axis represents physical spatial distance. The Hubble radius is almost constant between $t_{\rm i}$ and $t_{\rm R}$, and increases linearly before $t_{\rm i}$ and after $t_{\rm R}$. The horizon is shown as the dashed curve which equals the Hubble radius at the beginning of the period of inflation but increases exponentially until $t_{\rm R}$. The curve labeled λ indicates the physical wavelength of a cosmological fluctuation mode.

Assuming that the perturbations begin as quantum vacuum fluctuations [2] deep inside the Hubble radius, one can compute the amplitude of the resulting fluctuations today [2] (see [5, 6] for reviews of the theory of cosmological perturbations). One finds that the induced spectrum of fluctuations is approximately scale-invariant and that the observed amplitude of fluctuations is achieved if the Hubble expansion rate during the period of inflation was of the order $H \sim 10^{13}$GeV, which corresponds to an energy density during the inflationary period which is of the order $\eta \sim 10^{16}$GeV, the scale of particle physics "Grand Unification". With this value of H, it turns out that the period of accelerated expansion has to last for at least 50 e-foldings in order for inflation to be able to solve the horizon and flatness problems.

One key challenge for inflation is the singularity problem. If inflation is realized by the dynamics of scalar matter fields coupled to Einstein gravity, then the Hawking-Penrose singularity theorems [7] can be extended [8] to show that an inflationary universe is geodesically past incomplete. Thus, there necessarily is a singularity before the onset of inflation. Hence, the inflationary scenario cannot yield the complete history of the very early universe. A bouncing cosmological scenario naturally avoids this singularity problem, although at the cost of having to introduce new physics to obtain the bounce[1].

A second challenge which the inflationary scenario faces is the *trans-Planckian* problem for fluctuations. If the period of accelerated expansion lasts only slightly longer than the minimal amount of time which it has to last in order to solve the problems of Standard Big Bang cosmology, then the wavelengths of all scales of cosmological interest today originate at sub-Planckian values at the beginning of inflation. Hence, the origin and early evolution of cosmological fluctuations took place in the

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[1] We shall not here consider those mixed models in which a contracting phase followed by a bounce leads to an inflationary era. Such models enjoy the benefits of both paradigms, but also imply a higher level of sophistication which, at the present time, may not be required by the data; Occam's razor demands they should be introduced only at a later stage if needed.

trans-Planckian regime where General Relativity as the description of space-time and quantum field theory as the description of matter break down. As demonstrated in [9], modifications of the physics in the trans-Planckian region may lead to important modifications of the predicted spectrum of cosmological perturbations. In Fig. 1, we exhibit the trans-Planckian problem for cosmological fluctuations: the shaded region for small distances represents the "trans-Planckian zone of ignorance". The curve labeled λ indicates the wavelength of a cosmological fluctuation mode, and shows that it emerges from the zone of ignorance. Hence, in the absence of an embedding of inflation into a consistent theory of quantum gravity, we cannot trust the standard computations in inflation without implicitly assuming features of physics beyond the Planck scale. There has been a lot of debate on how reasonable it is to make assumptions which leave the standard computations unchanged (see e.g. [10]), or suggestions of how to impose initial conditions on time-like "new physics" hypersurfaces which exclude the trans-Planckian zone of ignorance [11], leading to oscillatory features in the predicted power spectra of fluctuations. For a recent review of this problem, the reader is refered to [12].

Bouncing cosmologies naturally avoid this trans-Planckian problem since the length scale of fluctuations we observe remain many orders of magnitude larger than the Planck length. To be specific, if the energy scale of the bounce corresponds to the same energy scale as in typical inflation models, then the wavelengths of scales coresponding to observed cosmic microwave background (CMB) anisotropies were always larger than 1 mm. This is illustrated in the space-time sketch of Fig. 2. The vertical axis is time, with $t = 0$ being the bounce time, the time when the spatial volume is minimal. This is thus also the time when the wavelengths of cosmological perturbations are minimal, and their values correspond to those in inflationary cosmology at the end of the period of inflation, i.e. at the time t_{R}. The horizontal axis again represents physical spatial distance. What is universal in bounce alternatives to inflation is that the evolution after the bounce is the same as that in standard and inflationary cosmology after the time t_{R}. Different classes of bouncing models predict different contracting phases. The example sketched in Fig. 2 with a contracting phase which is the time reverse of the standard cosmology phase of expansion corresponds to a symmetric bounce as in the *matter bounce* scenario [13, 14].

As is evident from Fig. 2, in bouncing cosmologies scales of cosmological interest originate on sub-Hubble scales at early times in the contracting phase, in the same way that they originate on sub-Hubble scales early in the accelerating phase in inflationary cosmology. Hence, it is possible to have a causal generation mechanism for fluctuations, as in inflationary cosmology. The nature of this structure formation mechanism depends on the specific bouncing model being considered. The origin of the fluctuations is often (e.g. in the *Pre-Big-Bang* [15], the Ekpyrotic [16] or matter bounce [13] scenarios) also taken to be quantum vacuum perturbations, as in inflation, but this is not always the case, *string gas cosmology* [17] being an example in which the initial fluctuations are of thermal nature [18].

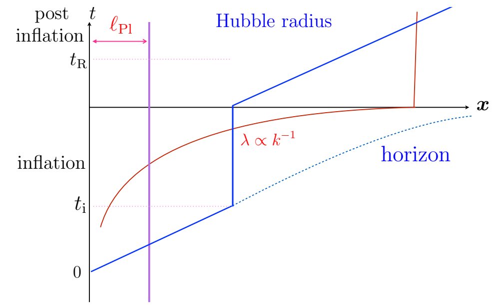

FIG. 1: Space-time sketch of inflationary cosmology. The vertical axis is time t, the period of accelerated expansion being between t_{i} and t_{R} and the horizontal axis shows the relevant physical distances. The horizon is represented by the dotted curve and the Hubble radius by the full one. The physical wavelength of a fluctuation mode is labeled by λ, which is inversely proportional to the wave number k. The vertical line shows the Planck length ℓ_{Pl}, emphasizing the trans-Planckian zone of ignorance corresponding to length scales smaller than ℓ_{Pl}. As is apparent, scales of cosmological interest today originate in the zone of ignorance.

A further conceptual problem of scalar field-driven inflation is the basic sensitivity of the mechanism of obtaining exponential expansion to the unknown solution of the cosmological constant problem. The inflationary expansion of space is generated by the almost constant potential energy density of the matter scalar field. However, quantum matter has a field-independent vacuum energy which is much larger than the upper bound on the current value of the cosmological constant, given by the observed dark energy density (see e.g. [19]). There must be some unknown mechanism which renders quantum vacuum energy gravitationally inert. The challenge for inflationary cosmology is to show that this mechanism does not also render the part of a scalar field potential which has the equation of state of a cosmological constant gravitationally inert (and thus eliminates the possibility of scalar field-mediated inflation). For an interesting at-

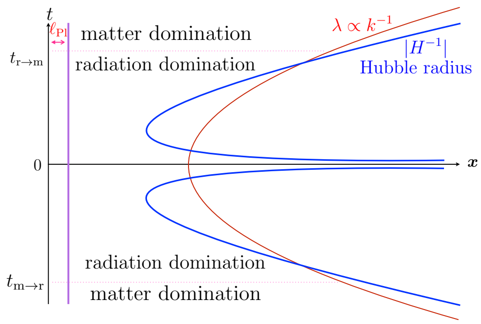

FIG. 2: Space-time sketch of a specific bouncing cosmology. The axes are as in Figure 1. The bounce point corresponds to $t = 0$ at which point the Hubble radius $H^{-1}(t)$, also shown, diverges ($H \to 0$) with the wavelength $\lambda = a(t)/k$ of a typical mode depicted as well. The figure also shows clearly why the trans-Planckian problem is not an issue in a bouncing framework.

tack on this problem in the context of the "degravitation" proposal [20] to address the cosmological constant problem, see [21].

Alternatives to cosmological inflation do not solve the cosmological constant problem. However, the dynamics of space-time expansion is then *a priori* not sensitive to what solves the problem (except if a mechanism similar to that used for the inflationary phase is invoked for the bouncing phase).

There is a completely different reason to be interested in alternatives to inflation. The successful predictions which inflation has made for the spectrum of cosmological perturbations are not specific to inflation. In fact, a decade before the development of inflationary cosmology, it was realized by Sunyaev and Zel'dovich [22] and by Peebles and Yu [23] that a roughly scale invariant spectrum of adiabatic perturbations which at the time of equal matter and radiation are present on super-Hubble scales will lead to the Sachs-Wolfe scale-invariant large angle tail and to acoustic oscillations on degree scales in the angular power spectrum of the CMB, it will lead to a power spectrum of primordial density fluctuations which is scale invariant and has superimposed small amplitude baryon acoustic oscillations, all features which have now been observed. The inflationary scenario is the first model which from first principles predicts such a primordial spectrum of cosmological fluctuations, but it is not the only one. Therefore, even if one believes that inflation is the correct paradigm for the early universe, and that it gives the correct origin of cosmic structures, it is important to work out alternative scenarios and to see in which way these alternatives yield predictions (other than those mentioned above) with which they can be differentiated from inflation. For example, whereas simply observing gravitational waves on cosmological scales from B-mode polarization is not at all a unique prediction of inflation (see e.g. [24] for an elaboration on this point), measuring a slight red tilt would be a rather distinctive prediction since it would allow us to rule out some alternatives such as string gas cosmology which predicts a slightly blue spectrum [25]. Working out distinctive predictions of alternative models will provide new tests for inflation - tests which, if passed will put inflation on a firmer footing, and if failed will allow us to falsify the paradigm.

In the following, we will present three classes of bouncing cosmologies, the *matter bounce* scenario [13], models of *Pre-Big-Bang* [15] or Ekpyrotic [16] type, and *string gas cosmology* [17, 18]. In the next section we will briefly introduce these scenarios and explain how to obtain a spectrum of cosmological perturbations in agreement with current observations. New physics, i.e. physics which goes beyond General Relativity and/or standard matter theory (i.e. models of matter which obey the Null Energy Condition) is required. In Section 3 we discuss some realizations of cosmological bounces using extensions of gravity and/or the matter sector. In Section 4 we illustrate some specific observational signatures of these bouncing cosmology scenarios, signatures with which they can be differentiated from the predictions of inflationary models. We conclude with a discussion of some key challenges for bouncing cosmologies.

Note that the goal of this article is to be pedagogical rather than complete. We are not giving a complete list of alternatives to inflation, nor are we discussing all bouncing scenarios (for a more complete survey or older models see [26], and [27] for a more recent review).

In the following we use the "mostly negative" convention for the metric [see Eq. (1)]. Greek letters denote space-time indices whereas Latin ones stand for spatial only indices.

II. ORIGIN OF SCALE-INVARIANCE OF COSMOLOGICAL FLUCTUATIONS

A. Cosmological Perturbations

In this section we will introduce four bouncing scenarios which have been widely discussed in the literature, and we will show how to obtain a scale-invariant spectrum of cosmological fluctuations at late times. We must begin with a brief review of the basics of cosmological perturbations. For more details the reader is referred to

[5, 6, 28].

The Friedman-Lemaître-Robertson-Walker (FLRW) metric of a homogeneous and isotropic background space-time will be written as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\boldsymbol{x}^2 = a^2(t)\left(\mathrm{d}\eta^2 - \mathrm{d}\boldsymbol{x}^2\right), \tag{1}$$

where t is the cosmic time, η is conformal time, $\boldsymbol{x}$ are the comoving spatial coordinates and $a(t)$ is the scale factor, in terms of which the Hubble expansion rate is given by

$$H(t) \equiv \frac{\dot{a}}{a} = \frac{1}{a}\left(\frac{a'}{a}\right) \equiv \frac{\mathcal{H}}{a}, \tag{2}$$

an overdot and a prime denoting respectively derivatives with respect to t and η. In most cosmological models (string gas cosmology being an exception), matter is modelled in terms of a scalar field φ with a non-trivial background dynamics $\varphi_0(t)$.

Linear fluctuations of geometry and matter about the background can be classified according to how they transform under spatial rotations. There are scalar modes, vector modes and tensor modes (gravitational waves). There are ten metric degrees of freedom plus the number of degrees of freedom in the matter sector (one if the model contains only a single scalar field like prototypical models of inflation). Four of the metric degrees of freedom are scalar, four are vector, and the remaining two are the two polarization modes of gravitational waves. There are also four gauge modes which represent the invariance under linearized space-time coordinate transformations. Two of the gauge modes are scalar and two are vector. In the following we will not consider vector modes since they decay in the expanding phase (however, they do grow in the contracting phase [29]).

We can choose a gauge in which the scalar metric fluctuations are diagonal. In this gauge, the metric including linear fluctuations takes the form

$$\mathrm{d}s^2 = a^2\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Psi)\,\delta_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}, \tag{3}$$

where $\Phi(\boldsymbol{x},t)$ and $\Psi(\boldsymbol{x},t)$ denote the two scalar metric fluctuations, and the transverse traceless tensor h_{ij} (which also depends on space and time) represents the gravitational waves. At linear order, the equations for scalar metric perturbations and gravitational waves decouple. We will first consider the scalar modes.

If the gravitational action is the Einstein-Hilbert action and if matter contains no anisotropic stress, then it follows from the off-diagonal spatial Einstein equations that the two scalar gravitational potentials Φ and Ψ coincide. The matter field can also be linearized about its background:

$$\varphi(\boldsymbol{x},t) = \varphi_0(t) + \delta\varphi(\boldsymbol{x},t). \tag{4}$$

The Einstein constraint equations relate the two scalar fluctuations Φ and $\delta\varphi$. Physically this is easy to understand: a matter perturbation has a gravitational effect on the metric and leads to a metric perturbation. As a consequence of this contraint there is a single scalar metric degree of freedom.

The equations of motion for cosmological fluctuations can be obtained by linearizing the full Einstein equations about the background metric. More easily, the equations can be obtained by expanding the action of matter plus gravity to second order about the background action. Since the background satisfies the equations of motion, terms linear in cosmological fluctuations cancel out in the action, leaving the quadratic terms as the leading fluctuation terms. Working in terms of the action allows the identification of the variable $v(\boldsymbol{x},t)$ in terms of which the action has canonical kinetic terms [30, 31]. This variable is

$$v = a\left(\delta\varphi + \frac{z}{a}\Phi\right), \tag{5}$$

where

$$z = \frac{a\varphi_0'}{\mathcal{H}}. \tag{6}$$

It turns out that the canonical variable v has an important physical meaning: it is proportional to the curvature fluctuation ζ in comoving gauge (gauge in which $\delta\varphi = 0$):

$$\zeta \equiv \frac{v}{z}. \tag{7}$$

It is this variable which determines the late time curvature fluctuations. Hence, in every early universe model, we need to be able to compute the power spectrum of ζ which is defined via

$$P_\zeta(k) = k^3|\zeta(k)|^2, \tag{8}$$

with $\zeta(k)$ the Fourier transform of ζ [2]. Conventionally, one introduces a scalar index n_{s} to describe the slope of the spectrum, namely

$$P(k) \sim k^{n_{\mathrm{s}}-1}. \tag{9}$$

The canonical variable v evolves like a scalar field with a time-dependent mass, the time dependence being set by the dynamics of the cosmological background

$$v'' - \nabla^2 v - \frac{z''}{z}v = 0, \tag{10}$$

where the ∇ operator corresponds to the gradient operator with respect to the comoving coordinates $\boldsymbol{x}$. At the linear level, we can expand $v(\boldsymbol{x},t)$ into spatial Fourier modes which all evolve independently. In Fourier space, the equation of motion becomes

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0, \tag{11}$$

[2] Here we are using a convention for the Fourier transform in which the Fourier modes have mass dimension $-3/2$ plus the mass dimension of the position space quantity.

where k labels the comoving wave vectors. For vacuum initial conditions imposed at some time $t_{\rm i}$, the initial value of v_k is given by

$$v_k(t_{\rm i}) \;=\; \frac{1}{\sqrt{2k}}\,. \tag{12}$$

Note that the corresponding power spectrum is blue, i.e. there is more power on short wavelengths. The value of the index $n_{\rm s}=1$ corresponds to a *scale-invariant* spectrum, i.e. a power spectrum which is independent of scale k.

If the equation of state (i.e. the ratio between pressure and energy density) of the cosmological background is constant in time, then $z(\eta) \sim a(\eta)$ and it then follows that z''/z is equal (up to a constant of order 1) to $\mathcal{H}^2$. Thus, from Eq. (11) it follows that the "squeezing term" z''/z is negligible compared to the k^2 term for wavelengths smaller than the Hubble radius. This implies that the variable v will undergo oscillations with constant amplitude on sub-Hubble scales. On super-Hubble scales it is the k^2 term which is negligible compared to the squeezing term. Hence, we conclude that as a length scale crosses the Hubble radius (e.g. during the inflationary phase), the fluctuations will stop to oscillate (they "freeze out"), and the amplitude will begin to increase as a consequence of the squeezing term (the fluctuations are "squeezed"). The equation of motion for v_k has two fundamental solutions. In the case of an expanding background, the dominant solution on super-Hubble scales is given by

$$v_k(\eta) \;\sim\; z(\eta)\,. \tag{13}$$

Let us first apply this formalism and show that a period of exponential expansion leads to a scale-invariant spectrum of cosmological perturbations. The time $t_H(k)$ when the mode with wavenumber k crosses the Hubble radius is given by

$$a^{-1}\left[t_H\,(k)\right]k \;=\; H\,. \tag{14}$$

Before $t_H(k)$, the amplitude of v_k is constant, afterwards it increases in proportion to z. Hence, the power spectrum of ζ at some late time t is given

$$\begin{aligned} P_\zeta(k,\eta) &= k^3 z^{-2}(\eta)|v_k(\eta)|^2 \\ &\simeq k^3 z^{-2}(\eta)\left\{\frac{z(\eta)}{z\left[\eta_H(k)\right]}\right\}^2 |v_k(t_{\rm i})|^2 \\ &\simeq \frac{1}{2}\left\{\frac{a\left[t_H(k)\right]}{z\left[t_H(k)\right]}\right\}^2 H^2 \end{aligned} \tag{15}$$

where in the final step we have used (12) and (14). The k-dependence has cancelled out and we have a scale-invariant power spectrum. Long wavelength modes are squeezed more than short wavelength ones. It is this effect which leads to the conversion of an initial vacuum spectrum into a scale-invariant one.

Exponential expansion is not the only expansion history which is able to convert a vacuum spectrum into a scale-invariant one. As we will see in the following subsection, a matter-dominated contraction phase leads to exactly the same preferential squeezing of long wavelength modes, compared to short wavelength ones, and which is able to convert a vacuum spectrum into a scale-invariant one [32].

In the case of more than one matter field there are extra scalar degrees of freedom which are usually called *entropy modes*. Entropy fluctuations induce a growing curvature mode. Let us consider the case when a scalar field φ dominates the background, and a second scalar field χ is the entropy mode with energy density fluctuation $\delta\rho_\chi$. In this case, the induced curvature mode is given by the following equation (see e.g. [34, 35])

$$\dot{\zeta} \;=\; H\left(c_\varphi^2 - c_\chi^2\right)\frac{\delta\rho_\chi}{\rho_\varphi + p_\varphi}\,, \tag{16}$$

where ρ_φ and p_φ are energy density and pressure in the φ field, $\delta\rho_\chi$ is the energy density fluctuation of the χ field, and c_φ and c_χ are the speeds of sound of the φ and χ fluids, respectively.

In cosmological models in which the adiabatic mode (the fluctuations in the dominant matter component) has a blue spectrum (as in the Pre-Big-Bang and Ekpyrotic scenarios), it is possible to obtain a scale-invariant spectrum of curvature fluctuations at late times via the conversion equation (16) if the entropy fluctuations acquires a scale-invariant spectrum.

To conclude this subsection let us turn to an analysis of gravitational waves. The transverse traceless tensor h_{ij} for the tensor modes [see Eq. (3)] can be expanded in terms of the two polarization states

$$h_{ij}(\boldsymbol{x},\eta) \;=\; h_+(\boldsymbol{x},\eta)\epsilon_{ij}^+ + h_\times(\boldsymbol{x},\eta)\epsilon_{ij}^\times\,, \tag{17}$$

where ϵ_{ij}^+ and $\epsilon_{ij}^\times$ are two fixed polarization tensors, and h^+ and $h^\times$ are the amplitude functions for these modes. Each of these two modes evolves independently at linear order.

Inserting the ansatz (17) for an individual polarization state into the Einstein action, we find that the variable in terms of which the perturbed action has canonical kinetic term is

$$\mu(\boldsymbol{x},\eta) \;\equiv\; a(\eta)h(\boldsymbol{x},\eta)\,, \tag{18}$$

resulting in the following equation of motion for each Fourier mode

$$\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k \;=\; 0\,. \tag{19}$$

Comparing the equations of scalar and tensor modes, we see that they are very similar except that the squeezing function z''/z is replaced by a''/a in the case of gravitational waves. Both gravitational waves and scalar fluctuations oscillate on sub-Hubble scales, both freeze out at

Hubble radius crossing (more precisely when k^2 becomes equal to the squeezing function), and both are squeezed on super-Hubble scales. If the equation of state of matter is independent of time, then $z \propto a$ and the squeezing functions are identical.

B. Matter Bounce Scenario

The "matter bounce" scenario [13] is based on the duality [32] (see also [33]) between the evolution of the canonical fluctuation variables in an exponentially expanding period and in a contracting phase with pressure $p = 0$. In an expanding universe, the growing mode of v_k is the mode which is proportional to $z(\eta)$ and the second mode is decaying. In contrast, in a contracting universe the role of the modes is exchanged: the mode proportional to z is decaying, and it is the second mode which is growing. In a matter dominated phase of contraction we have $a(\eta) \sim \eta^2$, and the solution of the mode equation (11) on super-Hubble scales is

$$v_k(\eta) \, = \, c_1 \eta^2 + c_2 \eta^{-1} \, , \tag{20}$$

where the constants c_1 and c_2 are set by the initial conditions. The second mode is the growing one. It is the fact that the equation of state is that of cold matter ($p = 0$) which leads to the characteristic scaling $v_k \sim \eta^{-1}$ which, as we show below, leads to the conversion of a vacuum spectrum to a scale-invariant one on super-Hubble scales.

Given the scaling $v_k \sim \eta^{-1}$, it is easy to show that an initial vacuum spectrum on sub-Hubble scales early in the contracting phase is converted into a scale-invariant one. The calculation follows the one done in the previous subsection in the case of an exponentially expanding background:

$$\begin{aligned} P_\zeta(k, \eta) \, &= k^3 z^{-2}(\eta) |v_k(\eta)|^2 \\ &\simeq k^3 z^{-2}(\eta) \left[\frac{\eta_H(k)}{\eta} \right]^2 |v_k(t_{\rm i})|^2 \, . \end{aligned} \tag{21}$$

Since $\mathcal{H} \sim \eta^{-1}$ and since Hubble radius crossing is given by $k^2 = \mathcal{H}^2$, we find that

$$\eta_H(k) \, \sim \, k^{-1} \, . \tag{22}$$

Inserting this result into (21) and making use of vacuum initial conditions leads to a scale-invariant spectrum of perturbations on super-Hubble scales, as in the case of inflationary cosmology.

In the case of scalar field-driven inflation, the spectrum of cosmological perturbations is not exactly scale-invariant, but it has a slight red tilt. The same is true in the matter bounce scenario: if we add a component to matter which corresponds to the current dark energy (e.g. a small cosmological constant), then a slight red tilt results [36].

Since in the matter bounce scenario the universe begins large and cold, it is reasonable to consider vacuum initial fluctuations. It is, however, also possible to consider thermal initial fluctuations. In this case, the resulting spectrum of curvature fluctuations after the bounce is not scale-invariant. There is, however, a particular equation of state in a contracting phase [37] which allows one to transform a thermal particle spectrum into a scale-invariant one.

C. Pre-Big-Bang Scenario

The Pre-Big-Bang scenario [15] (see [38] for an in-depth review) is an approach to superstring cosmology. If superstring theory is indeed the correct theory which unifies all four forces of nature and provides a quantum theory of gravity, then we should expect that stringy effects will be important in the very early universe. One aspect of string theory is that the graviton is not the only massless mode. In addition, there is a dilaton and an antisymmetric tensor field. It is usually assumed that the dilaton is fixed at late times, but in the very early universe we should expect it to be dynamical.

Neglecting, for the moment, the antisymmetric tensor field, the massless sector of string theory to which the graviton belongs is given by dilaton gravity. In the string frame the action is

$$S \, = \, \int \mathrm{d}^{d+1} x \sqrt{-g} \mathrm{e}^{-\phi} \left(R + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) \, , \tag{23}$$

where R is the Ricci scalar of the string frame metric, g is its determinant, ϕ is the string frame dilaton, and d is the number of spatial dimensions.

A conformal transformation takes the action into the Einstein frame. In the case of a homogeneous and isotropic space-time metric, this transformation takes the form

$$\begin{aligned} \tilde{a} \, &= a \mathrm{e}^{-\phi/(d-1)} \\ \tilde{\phi} \, &= \phi \sqrt{\frac{2}{d-1}} , \end{aligned} \tag{24}$$

where tilde quantities are in the Einstein frame. The Einstein frame action is

$$S \, = \, \int \mathrm{d}^{d+1} x \sqrt{-\tilde{g}} \left(\tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} \right) \, . \tag{25}$$

The dilaton gravity action has time reflection symmetry, but it also has a *scale factor duality symmetry* which is related to the T-duality symmetry of string theory discussed in more detail in the subsection on string gas cosmology. In the case of a homogeneous and isotropic metric, this transformation is

$$\begin{aligned} a \, &\rightarrow \, a^{-1} \\ \bar{\phi} \, &\rightarrow \, \bar{\phi} \, , \end{aligned} \tag{26}$$

where $\bar{\phi}$ is the "invariant" dilaton defined via

$$\bar{\phi} \, = \, \phi - 2d \ln a \, . \tag{27}$$

If we minimally couple perfect fluid matter to space-time in the Einstein frame, then in the case of $d = 3$ and an equation of state of radiation, we obtain the usual expanding radiation-dominated solution

$$\begin{aligned} a(t) &\sim t^{1/2} \\ \phi(t) &= \text{const}\,. \end{aligned} \tag{28}$$

This is the post-big-bang expanding solution. If we first perform a time reflection transformation followed by a duality transformation, we obtain a super-exponentially expanding solution with

$$\begin{aligned} a(t) &\sim (-t)^{-1/2} \\ \phi(t) &\sim -3\ln(-t) \end{aligned} \tag{29}$$

which has growing string coupling constant and corresponds to matter with a stringy equation of state $p = -\frac{1}{3}\rho$. This is the so-called "Pre-Big-Bang" branch. In the Einstein frame, the Pre-Big-Bang solution is a contracting one. In both the Einstein and string frames, it is easy to verify that constant comoving scales exit the Hubble radius in the Pre-Big-Bang phase, as they do in the matter bounce scenario.

Note that both of the above solutions are singular at $t = 0$. The idea of the Pre-Big-Bang scenario is that new physics takes over near $t = 0$ and smoothly connects the Pre-Big-Bang and post-big-bang branches. In the Einstein frame, this thus leads to a bouncing cosmology.

Also for other equations of state of matter we can find a Pre-Big-Bang branch of an expanding solution with decreasing curvature. In the case of pure dilaton gravity, the contracting branch is given by the Einstein frame metric scaling as [38]

$$a(\eta) \sim \eta^{1/2}\,, \tag{30}$$

where η is approaching 0 from negative values. In this case, the mode equation (11) for scalar cosmological fluctuations has super-Hubble solutions

$$v_k(\eta) \sim |\eta|^{1/2} \tag{31}$$

and

$$v_k(\eta) \sim |\eta|^{1/2}\ln(k|\eta|)\,. \tag{32}$$

The second solution is the dominant one as $\eta \to 0$.

From (31) and (32), it is easy to see that in contrast to the matter bounce case, fluctuations are damped on super-Hubble scales. Hence, an initial vacuum spectrum is not tilted towards the red, but towards the blue. The power spectrum is (up to logarithmic corrections)

$$P_k(v) \sim k^3 |v_k\left[\eta_H(k)\right]|^2 \frac{\eta}{\eta_H(k)} \tag{33}$$

Assuming an initial vacuum spectrum, the second term on the right hand side scales as k^{-1}, but since $\eta_H(k) \sim k^{-1}$ the third term scales as k, and hence we get an $n_{\rm s} = 4$ spectrum

$$P_k(v) \sim k^3\,. \tag{34}$$

The tensor modes acquire a spectrum with the same slope.

In order to obtain a scale-invariant spectrum of fluctuations in the Pre-Big-Bang scenario, we need to either dramatically deform the background or else invoke the entropy mechanism given by (16). In the Pre-Big-Bang setup, there is a natural candidate for the entropy field χ of (16): the axion related to the antisymmetric tensor field H appearing in the massless spectrum of string theory. A careful analysis [39] shows that, in the presence of dynamical extra dimension, it is possible to obtain a scale-invariant spectrum of axion energy density fluctuations which, via (16), will seed a scale-invariant spectrum of curvature fluctuations.

D. Ekpyrotic Scenario

The Ekpyrotic scenario [16] is also motivated by superstring theory, and more specifically by Horava-Witten theory [40], a specific realization of M-theory in which space-time is 11 dimensional. Six spatial dimensions are compactified with a fixed radius, but there is one other spatial dimensions which is orbifolded, i.e. it has the structure $S_1/\mathcal{Z}_2$. At the orbifold fixed points there are boundary branes, one of them being our space-time, the other containing hidden sector fields which only couple gravitationally to our world. This model was introduced for reasons of superstring phenomenology, and the mass scale of the orbifold direction must be of the order of the scale of particle physics Grand Unification.

The authors of [16] add a negative exponential potential which causes the two boundary branes to approach each other. In the Einstein frame, the period when the two branes are approaching each other is a contracting phase. The time when the two branes collide corresponds to a singular bounce from the point of view of the low energy effective field theory.

The low energy effective field theory of the Ekpyrotic scenario is given by Einstein gravity minimally coupled to a scalar field ϕ with a negative exponential potential

$$V(\phi) = -|V_0|\exp\left(\sqrt{\frac{2}{p}}\frac{\phi}{m_{\rm Pl}}\right) \tag{35}$$

with index $p \ll 1$, where $m_{\rm Pl}$ is the Planck mass. The field ϕ is related to the separation of the branes via $\phi \sim \ln r$. The key feature of the Ekpyrotic scenario is that the equation of state parameter w is much larger than 1 during the contracting phase. This is a consequence of the potential being a negative exponential. This results in the potential energy partially canceling the kinetic energy, whereas both kinetic and potential terms yield positive contributions to the pressure.

An equation of state $w \gg 1$ corresponds to very slow contraction since

$$(1+w) \,=\, \frac{2}{3\beta} \tag{36}$$

if the scale factor $a(t)$ scales as

$$a(t) \,\sim\, (-t)^{\beta}\,. \tag{37}$$

In conformal time we have

$$a(\eta) \,\sim\, (-\eta)^{\alpha} \tag{38}$$

with

$$\alpha \,=\, \frac{\beta}{1-\beta}\,. \tag{39}$$

Hence, as $w \gg 1$ we have $\alpha \sim 0$. Returning to the mode equation (11) – and to the corresponding equation for gravitational waves (19) –, we see that the spectrum of v is not changed on super-Hubble scales. Thus, if we start with vacuum initial conditions, then the spectrum of both curvature fluctuations and gravitational waves remains a vacuum spectrum

$$P_k(v) \,\sim\, k^2\,, \tag{40}$$

i.e. a $n_{\rm s} = 3$ spectrum.

As realized in [41], a spectator scalar field χ evolving in a negative exponential potential acquires fluctuations with a scale-invariant spectrum. This can easily be seen in the absence of gravity, where the equation of motion for the linearized fluctuation modes becomes

$$\ddot{\delta}\chi + \left(k^2 - \frac{2}{t^2}\right)\delta\chi \,=\, 0\,. \tag{41}$$

This is the same equation which the curvature fluctuations in inflationary cosmology obey (in conformal time), and hence the same analysis which was done in the case of inflationary fluctuations shows that initial vacuum perturbations on sub-Hubble scales acquire a scale-invariant spectrum on super-Hubble scales.

Thus, one way to obtain a scale-invariant spectrum of curvature perturbations in the Ekpyrotic scenario is to posit the existence of a second scalar field χ which evolves in a similar negative exponential potential as the field ϕ which generates the Ekpyrotic contraction. In order for scale-invariant χ field fluctuations (as opposed to χ density fluctuations) to lead to a scale-invariant spectrum of curvature fluctuations, a coupling between the two fields ϕ and χ at the background level is required (see e.g. [34]).

There is, however, another way to generate a scale-invariant spectrum of curvature fluctuations at late times in the expanding phase. Note that, up to now, we have been speaking about the curvature fluctuations on super-Hubble scales in the contracting phase. To connect these fluctuations to curvature fluctuations in the expanding phase, we need to match the perturbations at the bounce by imposing the analog of the Israel matching conditions [42] on a space-like surface [43, 44]. As discussed in detail in [45, 46], the result depends very sensitively on the choice of the matching surface. Choosing the matching surface to be the constant time surface yields the result that the spectrum of the canonical variable v is unchanged across the bounce [47]. However, any other matching surface will allow the contracting phase dominant mode of the metric fluctuation variable Φ [see (3)] to seed the expanding phase dominant mode of Φ which then seeds the dominant mode of v. This means that this method, depending so heavily on the matching surface and conditions, can yield any desired spectrum and thus lacks predictability.

The equation of motion for the variable

$$u_k \,\equiv\, \frac{a}{\phi'}\Phi_k \tag{42}$$

is

$$u_k'' + \left[k^2 - \frac{p}{\left(1-p\right)^2\eta^2}\right]u_k \,=\, 0\,. \tag{43}$$

For $p \ll 1$ this equation implies that the spectrum of u (and hence of Φ) in unchanged on super-Hubble scales. Vacuum initial conditions for the canonical variable v imply, via the constraint equations, that the initial spectrum of u is

$$u_k(t_i) \,\sim\, k^{-3/2} \tag{44}$$

which leads to a scale-invariant spectrum of u and Φ fluctuations on super-Hubble scales in the contracting phase, as initially expected in [48]. Taking the higher-dimensional background of Ekpyrotic cosmology into account, one can show that a mechanism to convey the scale-invariance of the spectrum of Φ in the contracting phase to that of v in the expanding phase naturally arises [49] (see also [50–53] for other studies of how to obtain a scale-invariant spectrum of fluctuations in Ekpyrotic cosmology).

A new variant of the Ekpyrotic scenario [54] was recently proposed which combines features of the original Ekpyrotic scenario with standard inflation. This model, called the *anamorphic universe*, has as a new ingredient a time dependence of masses (in this respect there are similarities with the "varying speed of light" scenario of [55, 56]). As a consequence, the model looks like standard inflation for cosmological fluctuations, whereas matter feels an Ekpyrotic bounce.

E. String Gas Cosmology

String Gas Cosmology [17] is based on key fundamental principles of superstring theory, specifically on degrees of freedom and symmetries which are absent in an effective

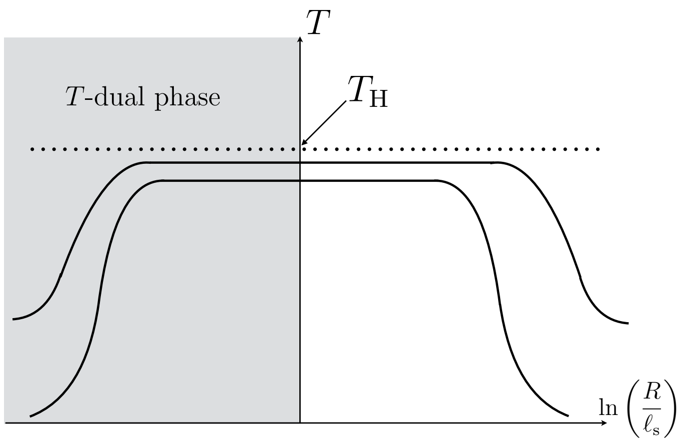

FIG. 3: Temperature T of a gas of closed strings in a box of radius R as a function of radius. The temperature never exceeds the Hagedorn temperature $T_{\rm H}$. The extent of the plateau of the $T(R)$ curve depends on the total entropy of the system - the larger the entropy the wider the plateau.

field theory approach to early universe cosmology. At the outset the assumption is made that all spatial sections are finite. We will in fact assume that space is a nine-dimensional torus.

The first key input is the Hagedorn spectrum of string states, which leads to a maximal temperature which a gas of strings in thermal equilibrium can achieve, the "Hagedorn temperature" $T_{\rm H}$ [57]. Closed strings in fact contain three types of modes: momentum modes which label the center of mass motion of strings, winding modes which label the number of times which the string winds space. Finally there are the oscillatory modes of the strings which have a Hagedorn spectrum. In an effective field theory description, one only keeps the momentum modes.

Let us now consider a box of strings in thermal equilibrium and initially with large radius. Since the energy of the momentum modes is quantized in units of $1/R$, where R is the radius of a toroidal section, the momentum modes are light and most of the energy of the thermal bath will be in these modes. As we decrease the radius R, the momentum modes become heavier. In thermal equilibrium, the energy will gradually flow into the oscillatory modes (whose energy is independent of R) and into the winding modes whose energy is quantized in units of R and hence become light as R decreases. The temperature initially increases as R decreases, but then levels off as the Hagedorn temperature T_H is approached. Once R becomes very small, the energy all flows into the winding modes and the temperature $T(R)$ decreases again (see Fig. 3).

Here we in fact encounter the second key aspect of string theory, namely the T-duality symmetry of the spectrum of string states which states that the spectrum of string states is unchanged under the transformation

$$R \rightarrow \frac{\ell_{\rm s}^2}{R}\,, \tag{45}$$

where $\ell_{\rm s}$ is the string length scale. This symmetry is obeyed by the perturbative string interactions and is generally taken to be a symmetry of non-perturbative string theory (see [58] for a textbook review).

It is now obvious that if $\ln(R/\ell_{\rm s})$ increases from $-\infty$ to $+\infty$, the temperature-time curve will correspond to a temperature bounce. As argued in [17], a physical way to measure distance for $R \gg \ell_{\rm s}$ is in terms of the position operator dual to the light momentum modes, and for $R \ll \ell_{\rm s}$ it is in terms of the position operator dual to the light winding modes. In terms of this physical position operator, we have a bouncing cosmology as the mathematical variable $\ln(R/\ell_{\rm s})$ increases from $-\infty$ to $+\infty$. Thus, string gas cosmology can be viewed as a bouncing cosmology.

Let us consider democratic initial conditions with all nine space dimensions small and wound by the winding modes of the equilibrium string gas at a temperature close to the Hagedorn temperature. The winding modes initially prevent any of the spatial dimensions to grow. In order for space to expand, the winding modes must be able to annihilate. This requires the intersection of string world sheets. Since string worlds sheets - in the absence of long range forces - have vanishing probability to intersect in more than three spatial dimensions, only three dimensions are able to "effectively decompactify" and become large [17]. This argument has been been confirmed by numerical simulations [59], although there are important caveats [60, 61]. Thus, it appears completely natural from the point of view of string theory that there are only three large dimensions of space even in a theory which is defined in nine space dimensions.

The fact that string gas cosmology takes the role of winding modes into account allows a simple stabilization of size [62] and shape [63] moduli. Introducing gaugino condensation also allows the stabilization of the dilaton [64] without destabilizing the size and shape moduli. In addition, gaugino condensation triggers high scale supersymmetry breaking [65].

If the universe contains a large amount of entropy, then the range of values of R for which the temperature is close to the Hagedorn value is wide (see Fig. 3). Hence, it is natural to suppose that the phase with temperature close to the Hagedorn temperature will be a long one, sufficiently long to maintain thermal equilibrium over a distance scale which is larger than the physical length of the comoving scale which corresponds to our current Hubble radius. The time evolution of the effective cosmological scale factor is sketched in Fig. 4. In this figure, the horizontal axis is time, with $t = t_{\rm R}$ being the time of the phase transition when many of the string winding modes annihilate and the equation of state becomes that of a radiation fluid. The vertical axis represents the scale factor.

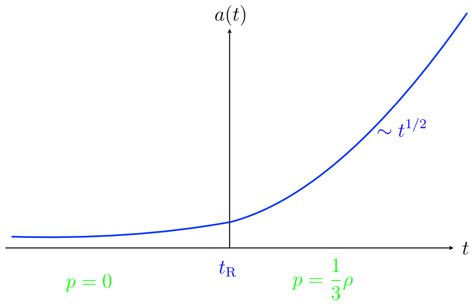

FIG. 4: Sketch of the evolution of the scale factor (vertical axis) as a function of time (horizontal axis) in string gas cosmology. The time t_{R} is the end of the Hagedorn phase and the beginning of the radiation phase of expansion.

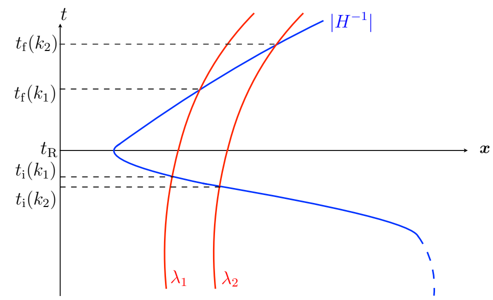

FIG. 5: Space-time sketch of string gas cosmology. The vertical axis is time, with the time t_{R} being the end of the Hagedorn phase. The horizontal axis represents physical distance. Shown are the Hubble radius $H^{-1}(t)$ and the wavelengths of two typical modes (the curves labelled by $\lambda_1 = a/k_1$ and $\lambda_2 = a/k_2$). For each of these modes, the times when the modes exit and re-enter the Hubble radius are indicated.

The corresponding space-time sketch is shown in Fig. 5. Now the vertical axis is time, and the horizontal axis indicates physical length. The initial phase $t < t_{\mathrm{R}}$ is quasi-static and hence the Hubble radius goes to infinity. The physical wavelength of fluctuation modes is constant in this early Hagedorn phase, and smaller than the Hubble radius. Hence, a causal generation mechanism for fluctuations is possible. If the Hagedorn temperature is of the order of the scale of particle physics Grand Unification, then the physical wavelength of fluctuations which are observed today is similar to the length these modes would have at the bounce point in a matter bounce whose energy scale is that of Grand Unification, and also similar to the length they would have at the end of a period of inflation. Hence, the wavelengths are many orders of magnitude larger than the Planck length, and hence far removed from the trans-Planckian zone of ignorance.

The string gas cosmology mechanism of structure formation developed in [18] (see the reviews in [66]) is based on the assumption that our four-dimensional space-time begins in a quasi-static phase of thermal equilibrium. In this setup, thermal fluctuations will be much more important than vacuum perturbations. Hence, it is natural to assume that fluctuations originate as thermal fluctuations of a gas of strings.

The idea of the computation of cosmological fluctuations [18] and gravitational waves [25] in string gas cosmology is to first compute matter correlation functions on sub-Hubble scales in the Hagedorn phase, and to use the Einstein constraint equations to induce the metric perturbations scale by scale at the time when the scale exits the Hubble radius [time $t_{\mathrm{i}}(k)$ in Fig. 5]. In particular, the cosmological fluctuations are given by

$$\langle|\Phi(k)|^2\rangle \, = \, 16\pi^2 G_{\mathrm{N}}^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle \,, \qquad (46)$$

G_{N} being Newton's constant, in terms of the energy density perturbations, and the gravitational waves are given by

$$\langle|h(k)|^2\rangle \, = \, 16\pi^2 G_{\mathrm{N}}^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle \qquad (47)$$

(with $i \neq j$) in terms of the off-diagonal pressure perturbations. In both cases, the brackets indicate thermal expectation values.

For a thermal gas, the energy density fluctuations on a length scale R are given by the specific heat capacity C_V, where V is the volume associated with R:

$$\langle \delta\rho^2 \rangle \, = \, \frac{T^2}{R^6} C_V \,. \qquad (48)$$

From the partition function of closed string thermodynamics (see e.g. [67]) it can be shown that the specific heat capacity is (see [18, 66, 68] for more details)

$$C_V \, \approx \, 2 \frac{R^2/\ell_{\mathrm{s}}^3}{T\,(1 - T/T_{\mathrm{H}})} \,, \qquad (49)$$

where T is the temperature of the gas (which in the Hagedorn phase is slightly lower than the Hagedorn value $T_{\rm H}$). Hence, the power spectrum $\mathcal{P}_\Phi(k)$ of scalar metric fluctuations becomes

$$\begin{aligned}\mathcal{P}_\Phi(k) \equiv& \frac{1}{2\pi^2} k^3 |\Phi(k)|^2 \\ =& 8G_{\rm N}^2 k^{-1} \langle |\delta\rho(k)|^2 \rangle \\ =& 8G_{\rm N}^2 k^2 \langle (\delta M)^2 \rangle_R \\ =& 8G_{\rm N}^2 k^{-4} \langle (\delta\rho)^2 \rangle_R \\ =& 8G_{\rm N}^2 \frac{T}{\ell_{\rm s}^3} \left(1 - \frac{T}{T_{\rm H}}\right)^{-1} ,\end{aligned} \tag{50}$$

where in the first step we have used (46) to replace the expectation value of $|\Phi(k)|^2$ in terms of the correlation function of the energy density, and in the second step we have made the transition to position space. In the above, the temperature T is the temperature at the time when the scale being considered exits the Hubble radius. Since the temperature is approximately constant in the Hagedorn phase, the spectrum of cosmological perturbations is approximately scale-invariant. Looking a bit more closely, we see that smaller length scales exit the Hubble radius slightly later, when the temperature is slightly lower. Hence, the string gas cosmology structure formation scenario automatically generates a small red tilt in the spectrum, like simple inflation models.

As realized in [25], the spectrum of gravitational waves is also approximately scale-invariant. However, a small *blue* tilt is predicted, whereas all inflation models based on General Relativity and matter obeying the NEC predict a small red tilt. The gravitational wave spectrum is given by [25]

$$\mathcal{P}_h(k) \sim \left(\frac{\ell_{\rm Pl}}{\ell_{\rm s}}\right)^4 \left(1 - \frac{T}{T_{\rm H}}\right) \ln^2 \left[\frac{1}{\ell_{\rm s}^2 k^2} \left(1 - \frac{T}{T_{\rm H}}\right)\right] . \tag{51}$$

The predicted blue tilt of the tensor modes comes from the factor $(1 - T/T_{\rm H})$ in (51), in the same way that the red tilt of the scalar modes comes from the factor $(1 - T/T_{\rm H})^{-1}$ in (50).

Note that if the string scale is taken to be the preferred one from heterotic string phenomenology (see [69] for a textbook treatment), then the amplitude of the resulting cosmological perturbations has the right order of magnitude to match observations.

The three key inputs required in order to obtain a scale-invariant spectrum of cosmological perturbations from an early thermal phase are 1) a holographic scaling of the heat capacity, 2) a quasi-static early phase ending with a phase transition to the usual radiation phase of expansion, and 3) the applicability of the perturbed Einstein equations for infrared fluctuation modes.

III. REALIZATIONS OF A BOUNCING PHASE

A. Introductory Remarks

According to the Hawking-Pensose theorems [7], an initial cosmological singularity inevitably arises in a homogeneous and isotropic model if we work in the context of General Relativity and if the matter which couples minimally to gravity obeys the Null Energy Condition (NEC). In order to obtain a bouncing cosmology there are thus several routes. One can work in the context of Einstein gravity but introduce matter which violates the NEC [70]. There are several dangers in taking this route. First, one must avoid instabilites such as ghost [71] and gradient instabilities. Secondly, one faces the challenge of embedding such a model in the framework of an ultraviolet complete theory of matter and gravity [72].

On the other hand, any quantum theory of gravity leads to terms in the effective gravitational action which go beyond the Einstein-Hilbert term. Hence, it may be more promising to attempt to realize bouncing cosmologies in the context of modified theories of gravity. This is the topic of the third subsection.

Ultimately, however, it would best best if a bouncing scenario could be the result of an ultraviolet complete theory of all four forces of nature, such as superstring theory. Attempts in this direction will be briefly reviewed in the final subsection.

Another way to classify bouncing models is whether they are singular from the point of view of an effective field theory (such as the Pre-Big-Bang or initial Ekpyrotic scenarios) or non-singular. We will give examples of both.

B. Bouncing Cosmologies from Modified Matter

The simplest way to obtain a nonsingular bouncing model with modified matter is to introduce a field with opposite sign kinetic energy term, i.e. a "ghost" field, and to arrange that during the contraction phase the absolute value of the ghost field energy density grows relative to that of regular matter. This is the *quintom cosmology* scenario [73]. For example, the regular matter can be described by a perfect fluid [70] or by a massive scalar ϕ field whose time-averaged energy density scales as a^{-3}, and ghost matter by a free scalar field ψ with opposite sign kinetic term whose energy density is dominated by the $\dot{\psi}^2$ term and whose energy density hence scales as

a^{-6} [74][3]. The action is

$$S = \int \left[\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) - \frac{1}{2}\partial^\mu\psi\partial_\mu\psi\right]\sqrt{-g}\,\mathrm{d}^4x\,, \quad (52)$$

with potential

$$V(\phi) \,=\, \frac{1}{2}m^2\phi^2\,. \quad (53)$$

The non-singular bounce takes place when the energy densities of ϕ and ψ become equal.

A specific realization of the quintom cosmology scenario can be obtained in the *Lee-Wick model* [76], a model in which the quadratic divergences in loop amplitudes are canceled by adding a ghost field. Bouncing cosmologies can easily be obtained making use of the Lee-Wick construction [77]. However, the ghost problem does not appear to be solved. The ghost problem is absent, however, in the *conformal cosmology* of [78], a model in which the scalar sector consists of two fields with an indefinite metric on the space of kinetic terms, but in which the ghost is absent because the ghost degree of freedom can be eliminated by a gauge choice. At the homogeneous and isotropic level, the models of [78] have bouncing cosmological solutions.

At the classical level, the background equations of motion can be studied and show the existence of a smooth bounce. Cosmological fluctuations can also be evolved explicitly through the bounce phase, with the result that on large scales (wavelength larger than the duration of the bounce phase) the spectrum of fluctuations before and after the bounce has the same spectral index. In particular, a scale-invariant spectrum of cosmological fluctuations set up during the matter phase of contraction retains its scale-invariant shape after the bounce [74].

On the other hand, at the quantum level quintom models suffer from a ghost instability [71] - the vacuum can decay into pairs of regular and ghost particles.

An improved nonsingular bounce can be obtained by the "ghost condensate" mechanism [79] (see also [80]). Ghost condensation is the kinetic sector analog of the Higgs mechanism in the potential sector. In the case of the Higgs field ϕ, the theory has a tachyon when expanded about $\phi = 0$, but not when expanded around the true minimum of the potential. In the ghost condensate scenario, we introduce a non-trivial kinetic term $K(X)$ in the action

$$S \,=\, \int [K(X) - V(\phi)]\sqrt{-g}\,\mathrm{d}^4x\,, \quad (54)$$

where X is the kinetic term

$$X \,=\, \partial^\mu\phi\partial_\mu\phi\,, \quad (55)$$

[3] It should be noted that in [75], a regular scalar field was used in conjunction with positive spatial curvature to avoid the primordial singularity through a bounce: a special state was assumed, with very large occupation number, that was leading to a direct violation of the NEC.

and we choose the kinetic function $K(X)$ such that the Lagrangian is ghost-like when expanded about $X = 0$, but has the regular sign when expanded about the true minimum of $K(X)$. Thus, in the ghost condensate phase there is no perturbative ghost instability.

Applied to bouncing cosmology, we can take [81]

$$K(X) \,=\, \frac{1}{8}\left(X - c^2\right)^2\,, \quad (56)$$

where c is some positive constant. We consider a potential $V(\phi)$ which decays exponentially as $\phi \to \pm\infty$ and is positive at $\phi = 0$. As initial conditions we consider a contracting universe with $\phi = ct$ as $t \to -\infty$, i.e. with the field in the ghost condensate state. As t approaches zero, the potential starts becoming important and slows down the field, thus leading the ghost field to obtain negative kinetic energy. The bounce point is reached when the ghost kinetic energy cancels the potential energy.

At a classical level, the dynamics is well behaved. One can show that, like in the case of the quintom bounce, on large scale a scale-invariant spectrum of cosmological fluctuations passes through the bounce without a change in the spectral index. At a perturbative quantum level the model is also safe, but there is the worry [72] that it is not possible to embed a ghost condensate Lagrangian into an ultraviolet complete theory of physics.

An additional problem is a gradient instability from which the model suffers. This problem can be cured by replacing the ghost condensate Lagrangian by a Galileon Lagrangian [82] and considering Galileon bounces [83].

There are also other avenues of obtaining a bouncing cosmology from modified matter which have been explored. For example, the possibility of obtaining a bounce from a Fermion condensate in the presence of a non-trivial coupling of General Relativity to fermionic fields has been explored in [84]. Another idea is to use coupled scalar tachyons [85]. In a very different spirit, it has been realized that the instability of the Standard Model Higgs field can be used [86] to generate a phase of Ekpyrotic contraction (but one needs an additional ingredient to obtain the actual bounce).

C. Bouncing Cosmologies from Modified Gravity

In any approach to quantum gravity, terms in the gravitational action which are additional to the usual Einstein-Hilbert term are predicted. These terms may be higher derivative in the effective gravitational action [87], and they may be non-local [88]. Assuming that the matter theory is a standard one given in terms of an energy-momentum tensor $T_{\mu\nu}$, then the effective equations of motion including leading quantum correction may be written as

$$\mathcal{D}_{\mu\nu}(g_{\alpha\beta}) \,=\, 8\pi G_{\mathrm{N}} T_{\mu\nu}\,, \quad (57)$$

where $\mathcal{D}_{\mu\nu}$ is an operator which contains the Einstein tensor $G_{\alpha\beta}$ as leading term, but in addition the quantum gravitational higher derivative and/or non-local terms. We can extract the leading term and write

$$\mathcal{D}_{\mu\nu}(g_{\alpha\beta}) \,=\, G_{\mu\nu} + \tilde{\mathcal{D}}_{\mu\nu}(g_{\alpha\beta})\,. \tag{58}$$

Equivalently, we can write (57) as

$$G_{\mu\nu} \,=\, 8\pi G_{\rm N} T^{\rm eff}_{\mu\nu} \tag{59}$$

with

$$T^{\rm eff}_{\mu\nu} \,=\, T_{\mu\nu} - \frac{1}{8\pi G_{\rm N}}\tilde{\mathcal{D}}_{\mu\nu}(g_{\alpha\beta})\,. \tag{60}$$

It is now easy to see that even with a matter energy-momentum tensor which obeys the NEC, it is possible to obtain a bounce if the gravitational contribution $\tilde{\mathcal{D}}_{\mu\nu}(g_{\alpha\beta})$ is such that the effective energy-momentum tensor violates the NEC.

One example where the idea can be realized is Horava-Lifshitz gravity [89], an approach to quantum gravity which gives up general space-time diffeomorphism invariance and maintains only spatial diffeomorphism invariance plus a new space-time anisotropic scaling symmetry

$$t \,\rightarrow\, \ell^3 t \quad \text{and} \quad x^i \,\rightarrow\, \ell x^i\,, \tag{61}$$

where $\ell \in \mathbb{R}$ is a scaling parameter. One can determine the most general gravitational action which is consistent with the above symmetries and which is power-counting renormalizable. The resulting gravitational field equations contain a new gravitational contribution $\tilde{\mathcal{D}}_{\mu\nu}(g_{\alpha\beta})$ which, when taken to the right hand side of the gravitational field equations, has the form of dark radiation with negative effective energy density whose energy density scales as a^{-4} in a contracting FLRW universe. The coefficient of the dark radiation depends on spatial curvature. We only obtain dark radiation for positively or negatively curved spaces, but not in the spatially flat case. Provided that regular matter scales in time less fast than a^{-4}, e.g. if it has a matter-dominated equation of state, then eventually, the contribution of dark radiation will become equal in magnitude to that of regular matter, and a cosmological bounce will occur [90] (see also [91]).

The main worry about extensions of General Relativity is the issue of stability of metrics we know should be stable such as the FLRW or the Schwarzschild metrics. For cosmological backgrounds, we have to worry about ghost-like or tachyonic instabilities of the linearized perturbations. This issue was first studied in [92], and then in the context of a bouncing cosmology in [93]. There are several versions of Horava-Lifshitz gravity. One of them is called the "projectable version", another is the original version [89] extended by terms which were only implicit in the original version - the "healthy extension" [94]. It was found that in the healthy extension of Horava-Lifshitz gravity, there are no disastrous instabilities at linear order [95], but in the projectable version there are [96]. In the healthy version, it can moreover be verified that the spectral index of the cosmological fluctuations is unchanged across the bounce [93].

Bouncing cosmologies have also been studied in the context of $F(R)$ theories of gravity (see e.g. [97]), in models in which the gravitational action is supplemented by a Gauss-Bonnet term (see e.g. [98]) and in $F(T)$ theories of gravity (see e.g. [99]). Bouncing cosmologies can also be obtained in models in which our space-time is a brane moving through a higher-dimensional space-time which is curved (e.g. of black hole type) [100]. Bouncing cosmologies are also an inevitable consequence of the "limited curvature gravitational action" of [101].

D. Bouncing Cosmologies from String Theory

String theory is a candidate ultraviolet complete theory which unifies all four forces of nature at a quantum level. As already seen in the subsection on string gas cosmology, it should be expected that string theory will lead to a very different evolution of the early universe than what is obtained in the effective point particle theory limit. At the present time, however, string gas cosmology is based on ideas from perturbative string theory coupled with thermodynamic arguments. A rigorous non-perturbative treatment is lacking. Hence, it is useful to explore other approaches to string cosmology which are more closely related to non-perturbative physics, and which result in bouncing cosmologies. String gas cosmology is one example, but in this subsection we will focus on a couple of attempts which can be formulated in terms of an effective background field theory.

One such approach is the *S-brane bounce* [102], which is based on a certain class of Type II superstring backgrounds [103] for which the string partition function has a temperature duality

$$T \,\rightarrow\, \frac{T_{\rm c}^2}{T} \tag{62}$$

where $T_{\rm c}$ is a critical temperature whose value is related to the string scale.

We begin the evolution in a contracting universe starting with a very large value of the temperature $T \gg T_{\rm c}$ (which physically corresponds to a very small value of the effective temperature). As the universe contracts, T decreases (which means that the effective temperature increases). Once the effective temperature exceeds the supersymmetry breaking scale, we can describe the background cosmology using dilaton gravity. When the temperature reaches the value $T = T_{\rm c}$, a tower of string states which for $T \neq T_{\rm c}$ are massive (of the string scale) become massless and have to be included in the low energy effective action. Hence, the effective action for the low mass modes of matter contains a term localized on the spatial hypersurface $T = T_{\rm c}$. This object is the space-like analog of a D-brane in string theory, or of the zero width limit of a topological defect in field theory. Hence, we call this term an *S-brane* (see also [104] for another approach to S-branes in string theory).

The equation of state of an S-brane has vanishing energy density $\rho = 0$ and negative pressure $p < 0$ and hence yields a term which violates the NEC, thus allowing for a bouncing solution. As T approaches the value $T = T_c$ from above, the universe is contracting, but after the S-brane is encountered as $T = T_c$ the universe starts to re-expand, with decreasing T, as shown explictly in [102]. The evolution of cosmological perturbations in this background was studied in [105] where it was shown that a scale-invariant spectrum of curvature fluctuations passes through the bounce unchanged.

The second approach which we will mention here is based on the AdS-CFT correspondence [106], a conjecture which relates string theory on an AdS (anti-de-Sitter) space-time to a conformal field theory (CFT) living on its boundary. This proposal can give a non-perturbative definition of string theory on AdS since we have a non-perturbative description of the boundary CFT.

To apply the AdS/CFT correspondence to a bouncing cosmology, we consider a time-dependent deformation of AdS which asymptotically for $t \to \pm\infty$ looks like pure AdS but has growing bulk curvature as $t \to \pm 0$. Mapped onto the CFT living on the boundary, we obtain a time-dependent gauge coupling constant which tends to zero as $t \to \pm 0$. This leads to the hope that the boundary field theory time evolution can be smoothly continued through $t = 0$, and that this continuation can then be used to construct a bulk space-time in the future of $t = 0$ [107] (see also [108] for earlier work). We would thus be able to pass through the bulk gravitational singularity (the singular cosmological bounce point) in a unique way by mapping the physics onto a well-defined boundary theory.

Whereas this procedure works at the level of the background cosmology, singularities on the gauge theory side arise when cosmological fluctuations are introduced. This has recently been studied [109] in the case of a deformation of AdS obtained by a time-dependent dilaton [110] (see also [111]). The singularity in the boundary theory takes the form of a branch cut in one of the fluctuation modes, and in divergent total particle production [109]. Hence, we need to impose time cutoffs at $t = \pm\xi$, where ξ is set, e.g., by the bulk curvature obtaining the Planck value. In this case, the cosmological perturbations can be evolved on the boundary in a more unique way than they could in the bulk, where the ambiguity of the space-like matching surface is a major problem.

Concretely, the application of the AdS/CFT correspondence to cosmological fluctuations is now the following. We begin with bulk fluctuations in the contracting branch of the deformed AdS. When the bulk curvature becomes strongly coupled at the time denoted by $-t_c$, we project the fliuctuations onto the boundary gauge theory. We then evolve the boundary fluctuations through $t = 0$ until the time $+t_c$ when the bulk becomes weakly coupled again. At that time we reconstruct the bulk fluctuations.

The analysis of [109] shows that on large scales, the spectrum of bulk fluctuations has the same slope at $t = -t_c$ and $t = t_c$. The amplitude, however, grows by a factor which diverges as ξ approaches zero.

String gas cosmology is a scenario based on fundamental principles of superstring theory which can yield a bouncing cosmology, as discussed earlier. The Ekpyrotic scenario is, as well, originally motived by ideas from superstring theory. For another recent construction of a bouncing cosmology from string theory see [112].

It is also possible that bouncing cosmologies can arise from alternative approaches to quantum gravity. Loop quantum gravity is the prime example. In fact, in loop quantum cosmology it can be shown that the cosmological singularity can be avoided at the quantum level, and that bouncing cosmologies are possible [113]. For a specific construction of a "matter bounce" in the context of loop quantum cosmology see [114].

IV. OBSERVATIONAL SIGNATURES

Alternative models of early universe cosmology must be able to explain all of the existing data and must be consistent with the current constraints. It is also very important that they make new predictions with which they can be differentiated from the current and widely accepted inflationary paradigm. Given the wealth of data which is expected in the near future from new telescopes this is an ideal time to consider such predictions. They could concern new observational windows such as CMB B-mode polarization maps, 21cm redshift surveys and gravitational wave astronomy, or vastly improved data from radio and optical telescopes. Below we mention a couple of examples of how new observational windows will allow us to differentiate the predictions of various bouncing cosmological models with those of the inflationary scenario. We will discuss higher order effects such as non-Gaussianities, the tensor spectrum and its relations with the observed scalar component, and the running of the scalar spectrum (see also[115] for a discussion of tensor non-Gaussianities in the matter bounce).

A. Non-Gaussianities

Bouncing models contain many interaction terms ensuring that the bounce (and the previous phases) take place. These, in turn, lead to an extra contribution to the 3-point function which is given by the commutator

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = i\int_{t_G}^{t} \langle [\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3), \mathcal{L}_{\rm int}(t')]\rangle \mathrm{d}t', \tag{63}$$

where $\mathcal{L}_{\rm int}$ contains the relevant interaction terms, and the curvature perturbations $\zeta(\boldsymbol{k}_i)$ are evaluated at time t; t_G is an initial time at which no non-Gaussianity is yet present [4]. The interaction Lagrangian $\mathcal{L}_{\rm int}$ needs be calculated at least to third order in the perturbation ζ, leading to a complicated function of ζ whose details can be found in [116] where it was first calculated.

One then usually expresses the three-point function as

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^7\,\delta^{(3)}\left(\boldsymbol{k}\right)\frac{\mathcal{P}_\zeta^2}{\prod\limits_i k_i^3}\mathcal{A}, \tag{64}$$

where $\boldsymbol{k} = \sum_i \boldsymbol{k_i}$ and $\mathcal{A}$ is a shape function, depending on the wave vectors $\boldsymbol{k}_i$. In Eq. (64), the quantity $\mathcal{P}_\zeta^2$ is not the actual curvature perturbation spectrum but rather represents a sum of permutations of the 3 different wavenumbers involved, namely

$$\mathcal{P}_\zeta^2 = \mathcal{P}_\zeta\left(k_1\right)\mathcal{P}_\zeta\left(k_2\right) + \mathcal{P}_\zeta\left(k_2\right)\mathcal{P}_\zeta\left(k_3\right) + \mathcal{P}_\zeta\left(k_1\right)\mathcal{P}_\zeta\left(k_3\right). \tag{65}$$

In general, one restricts attention to the so-called non linearity parameter $f_{\rm NL}$, defined in real space by

$$\zeta = \zeta_{\rm gauss}\left(\boldsymbol{x}\right) + \frac{3}{5}f_{\rm NL}\zeta_{\rm gauss}^2\left(\boldsymbol{x}\right), \tag{66}$$

where $\zeta_{\rm gauss}\left(\boldsymbol{x}\right)$ stands for the linear, Gaussian part of the curvature perturbation. One then arrives at

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = \frac{3(2\pi)^7}{10}\delta^3\left(\boldsymbol{k}\right)f_{\rm NL}\mathcal{P}_\zeta^2\frac{\sum_i k_i^3}{\Pi_j k_j^3}. \tag{67}$$

Common configurations include the local shape ($k_3 \ll k_1, k_2$), the equilateral shape ($k_1 = k_2 = k_3$), or the folded or orthogonal shapes [117–119]. Current bounds set by Planck, including temperature and polarization data, are [120],

$$\begin{aligned} f_{\rm NL}^{\rm local} &= 0.8 \pm 5,\\ f_{\rm NL}^{\rm equil} &= -4 \pm 43,\\ f_{\rm NL}^{\rm ortho} &= -26 \pm 21, \end{aligned} \tag{68}$$

(68% CL) which are consistent with a Gaussian (all non-linearity parameters vanish) spectrum. Such suppressed non-Gaussianities are consistent with the prediction in canonical, single field, slow-roll models of inflation [116, 121].

Bouncing cosmologies present many difficulties for the calculation of these non-Gaussianities, as in particular many models induce large contributions. In an inflationary setup, perturbations, and therefore non-Gaussianities too, are produced at one instant of time, at Hubble crossing, and then subsequently propagated mostly unchanged. Many terms in the interaction Lagrangian $\mathcal{L}_{\rm int}$, e.g., those depending on time derivatives of ζ_k, are utterly negligible, leading to a final result which never grows very large, and is in fact largely controlled by the slow-roll parameter ϵ. In a bouncing model however, although the first mechanism is rather similar, the subsequent evolution can be very different for two reasons. First, the initial phase is one of contraction, during which the modes may change with time, producing a more important growth to begin with. This implies that the resulting contraction non-Gaussianities can be much larger than their inflationary counterpart. This stems from the fact that the "slow-roll" parameter, in a contracting universe, is not a small parameter. In the case of the matter bounce, it was found [122] that $f_{\rm NL}^{\rm local} = -\frac{35}{8}$, comparable to the current bounds, and similarly the other $f_{\rm NL}$ are expected to be negative with a magnitude of a few.

The second difference between an inflation phase and a bouncing model is the existence of the bounce itself, which has a tendency to increase any pre-existing non-Gaussianity. For instance, in a simple model consisting of a single scalar field with positive spatial curvature [123], it was realized that the closer the bounce to de Sitter, the higher the production of non-Gaussianities, and at a level which is such as to raise doubts on the validity of the perturbative treatment of the bounce. Of course, one may then argue that the problem stems from the fact that the positive spatial curvature is crucial for the bounce to merely take place, and that therefore this is not necessarily a valid generic result. Granted, but more recently, it was also shown [124], using a ghost-condensate galileon model to perform the bounce in a flat spatial background, that $f_{\rm NL}$ should also grow during the bounce phase. Besides, it was also shown that the increase of the level non-Gaussianity is directly related with that of the overall amplitude of curvature perturbation, leaving the tensor mode unchanged. As a result, the tensor to scalar ratio, discussed in the following section, can remain small and compatible with the current constraints at the expense having too much non-Gaussianities. This is even a possibly fatal blow for single field matter bounce models [124].

The non-Gaussianities in string gas cosmology, on the other hand, are predicted to be negligible on cosmological scales [125]. The reason is the following: thermal fluctuations have a specific correlation length which is microscopic and at which scale the non-Gaussianities are thermal and or order one. However, on larger scales the non-Gaussianities are Poisson suppressed and hence become negligible on scales relevant for cosmological observations. There is a caveat to this argument: if the strings in string gas cosmology are long-lived (see e.g. [126] for a discussion of this point), then a network of cosmic superstrings [127] is predicted to survive to the present time. These strings would approach a scaling solution of the same form as that describing cosmic strings forming in a field theory phase transition. These cosmic strings would leave behind distinctive stringy signals in observations, signals with specific patterns in position space (see [128] for a recent review of these signals).

[4] In this subsection we use the convention that the mass dimension of $\zeta(k)$ is -3 and not $-3/2$ as we did earlier in this review.

B. Running of the Scalar Spectrum

In simple single field inflationary models, the red tilt of the scalar spectrum is due to the fact that the Hubble expansion rate H is very slowly decreasing during the period of inflation. In fact, the rate of decrease of H is accelerating during the slow rolling phase. This leads to the fact that the slope of the spectrum is increasing in magnitude towards smaller wavelength. This implies that the *running* of the spectrum is negative, i.e. the slope of the spectrum is smaller at larger values of the momentum.

In the matter bounce scenario, on the other hand, the tilt of the spectrum of cosmological perturbations is caused by the fact that the contribution of the dark energy component is decreasing as a function of time during the contracting phase. This implies that at large values of k, the spectrum converges to a scale-invariant one, and that thus the running of the scalar spectrum is positive. This point was recently worked out in detail in [129].

Thus, a measurement of the running of the scalar spectrum would allow us to differentiate the canonical single field inflation models from the matter bounce scenario. An analysis of the running of the scalar spectrum in string gas cosmology is not available at this time.

C. Tensor to Scalar Ratio

The tensor to scalar ratio r is defined by

$$r \equiv \frac{\mathcal{P}_\mathcal{T}}{\mathcal{P}_\mathcal{S}}\,, \tag{69}$$

where $\mathcal{P}_\mathcal{T}$ and $\mathcal{P}_\mathcal{S}$ are the power spectra of the tensor and scalar modes, respectively. In the matter bounce scenario, both the tensor and the scalar fluctuations have a scale-invariant spectrum, and the tensor to scalar ratio before the bounce phase is predicted to be of order one since the scalar and tensor modes obey the same equation of motion. During the bounce, it is possible that the scalar modes are enhanced relative to the tensor modes. This issue has recently been studied in [124] in the case of a particular two field scenario. Nevertheless, generically a large value of r is predicted. Hence, the value of r does not provide a good window to differentiate between inflation and the matter bounce.

In string gas cosmology, the value of r is given by the ratio between the energy density and the pressure fluctuations, as discussed in the section on string gas cosmology. In the model of [17, 18] there is a consistency relation which relates r to $1 - n_\mathrm{s}$. Given the observed value of n_s, a value of r below the current observational bounds is predicted (see [130] for a detailed discussion). A measurement of r alone is hence also not a good way to differentiate between string gas cosmology and inflation.

The situation is completely different in the case of the Ekpyrotic scenario. In this case, both the adiabatic scalar spectrum and the tensor spectrum retain their original vacuum slope, which means that the fluctuations are negligible on cosmological scales. The scalar fluctuations which are currently observed must hence be due to a non-standard mechanism such as entropy fluctuations, while there is no possibility of an additional generation mechanism for tensor modes. The quantitative analysis begins by considering the scale factor to decrease following Eqs. (37) and (38), so the mode equation (11) gives

$$\mu = \frac{1}{2}\sqrt{-\pi(\eta-\eta_*)}H^{(1)}_{\frac{1}{2}-\beta}\left[-k(\eta-\eta_*)\right] \tag{70}$$

where $H^{(1)}$ is the Hankel function, and $\beta \ll 1$; η_* is the time at which the ekpyrotic potential would reach negative infinity if it were described at all times by the exponential shape leading to ekpyrotic contraction. The tensor spectrum

$$\mathcal{P}_\mathcal{T} = \frac{k^3}{\pi^2}|h|^2 \tag{71}$$

must then be transfered through the kinetic driven phase, the bounce, and the subsequent expanding phases by means of either matching conditions or numerical evaluation. A value for the tensor-to-scalar ratio of $r \sim 0.1$ implies that, today, the gravitational power spectrum should be of order $\mathcal{P}_\mathcal{T}(r \sim 0.1) \simeq 10^{-10}$ on CMB scales. The most stringent constraint on tensor modes then comes from big-bang nucleosynthesis (BBN), during which it has to be negligible. This implies contraints on the amplitude at high frequencies, i.e. small wavelengths. But the power spectrum being very blue, this also implies that on the much longer wavelengths concerned by the CMB, the tensor contribution should be even smaller. In fact, according to this line of thoughts, any sizeable measurement of r would immediately rule out the Ekpyrotic scenario unless some yet-unknown mechanism is invoked. The actual calculation was done in Ref. [131], from which we reproduce the result in Fig. 6

The value of r is however very highly model dependent in bouncing cosmologies (just as it is very discriminatory in inflation model-building [132]), as illustrated by a simple effective bounce model with two fields: considering two scalar fields ϕ and ψ, the latter evolving in an exponential potential reminiscent of the Ekpyrotic situation (but positive) and the former having a negative definite kinetic energy only, i.e. (52) with the potential (53) replaced by $V = V_0 \exp\left(-\lambda\phi/m_\mathrm{Pl}\right)$, the authors of Ref. [133] showed that during the collapse phase, one finds $r = 8\lambda^2$. Since this is the only available free parameter, it also fixes the scalar spectral index. In order to recover a scale-invariant spectrum, one needs to set $\lambda = \sqrt{3}$, leading to $r = 24$, much in excess of the current observational constraint.

In a matter bounce scenario with a phase of Ekpyrotic contraction used only to wash any primordial anisotropy, the curvature fluctuations can grow relatively large, without spoiling the perturbative nature of the bounce, while the tensor modes remain small at all times. In such a

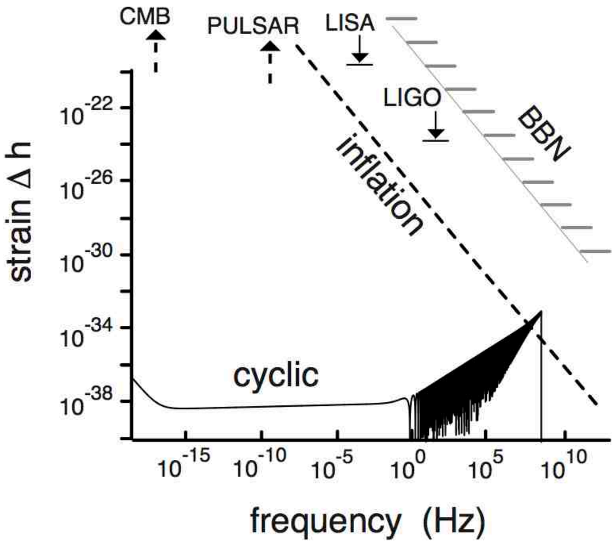

FIG. 6: The ekpyrotic/cyclic prediction for the power spectrum of gravitational waves expressed in terms of the strain $\Delta h \equiv \sqrt{\mathcal{P}_{\mathcal{T}}}$ as a function of the frequency. The normalization is fixed at small wavelengths at which one assumes the tensor contribution to be 4 orders of magnitude below the BBN constraint. Using the blue spectrum to reconstruct this contribution on large wavelengths, one thus finds a value of r that is utterly negligible and that is therefore predicted to be unobservable.

model therefore, the tensor-to-scalar ratio remains under control and can be made compatible with the data [83]. Similar consideration apply to the matter bounce curvaton scenario [134].

D. Tilt of the Tensor Spectrum

All inflationary models based on Einstein gravity coupled to matter which obeys the NEC predict a red tilt of the tensor spectrum, i.e. $n_{\rm t} < 0$. The reason is that, during inflation, the magnitude of H is a decreasing function of time[5]. The amplitude of the gravitational waves on a particular scale k is, as we have seen in Section 2.1, proportional to the value of H at the time $t_H(k)$ when the scale exits the Hubble radius during inflation. Larger values of k correspond to waves which exit later and hence at smaller values of H.

In the case of simple single field slow-roll inflationary models there is, in fact, a consistency relation which connects the values of the tensor tilt with the tensor to scalar ratio r. It takes the form [136]

$$n_{\rm t} \,=\, -\frac{r}{8}\,. \tag{72}$$

Whereas the sign of $n_{\rm t}$ is a very generic prediction of scalar field inflation, the specific relation between $n_{\rm t}$ and r is not generic since by complicating the matter sector one can change the scalar fluctuations.

In the case of the simple matter bounce model, one obtains the consistency relation

$$n_{\rm t} \,=\, (n_{\rm s} - 1)\,, \tag{73}$$

the reason being that both sets of fluctuations start out as vacuum perturbations, and that the squeezing of the scalar and tensor modes is the same. The same applies for the ekpyrotic contraction, with $n_{\rm t} = 3$ [see Eq. (40)].

On the other hand, in string gas cosmology the origin of fluctuations is thermal as opposed to quantum, and the initial amplitudes of the scalar and tensor modes are very different. The scalar mode amplitude is set by the value of the temperature, which slowly decreases as a function of time, leading to a red tilt. On the other hand, the amplitude of the tensor modes is set by the pressure anisotropies, which are proportional to the pressure, which increases as a function of time at the end of the Hagedorn phase, thus leading to a blue tilt of the tensor modes [25]. A careful analysis shows the following string gas consistency relation between the scalar and tensor modes [130]

$$n_{\rm t} \,\simeq\, -(n_{\rm s} - 1)\,. \tag{74}$$

Hence, measuring the value of r and the tensor tilt would allow us to discriminate between different early universe scenarios.

Tensor modes on cosmological scales leave an imprint on the spectrum of B-mode CMB polarization. Scalar cosmological fluctuations cannot produce B-mode polarization at linear order in perturbation theory. Gravitational waves, on the other hand, do. The challenge for future observations is not only to detect the amplitude of B-mode polarization, but to determine the tilt. A determination of only the amplitude does not yield information supporting the inflationary origin of fluctuations against other possible generation mechanisms[6]. A determination of the tilt, on the other hand, would provide new and powerful information. If observations were to find a small blue tilt, this would rule out the standard inflationary paradigm, and it would confirm a prediction first made in the context of superstring theory. Note that the direct comparison between primordial fluctuations and B-mode polarization is further complicated by

[5] Models of inflation based on matter which violates the NEC can yield a blue spectrum, see e.g. [135].

[6] For an elaboration on this point, see [24].

the fact that there are other sources of B-mode polarization, even on cosmological scales. Quite generically, models with vector mode fluctuations also induce B-mode polarization [137]. For example, cosmic strings can produce direct B-mode polarization [138].

V. CHALLENGES FOR BOUNCING COSMOLOGIES

A. Addressing the Problems of Standard Big Bang Cosmology

Inflationary cosmology is a successful solution to several problems of Standard Big Bang cosmology, most importantly the *horizon, flatness, entropy* and *structure formation* problems. We have already addressed how the various bouncing models considered in this review can provide a structure formation scenario which reproduces the successes of inflation. But how are the other problems addressed [27, 70]?

All bouncing models discussed here solve the *horizon problem.* Since time runs from $-\infty$, the horizon can be infinite and there is no causal obstacle to explaining the isotropy of the cosmic microwave background (the horizon problem).

Let us now turn to the *flatness problem.* In the case of inflationary cosmology it is the accelerated expansion of space which dilutes spatial curvature compared to the dominant contribution to the energy density. The matter bounce scenario is neutral concerning the impact of the spatial curvature term: in the case of a symmetric bounce it decreases during the period of contraction by the same amount that it increases during the expansion phase. In the Pre-Big-Bang and Ekpyrotic scenarios the situation is better: the contribution of spatial curvature decreases faster during the period of contraction than it increases during the expanding phase. String gas cosmology, on the other hand, does not solve the flatness problem.

Inflation also provides an elegant mechanism to produce the large entropy which our current universe contains: the exponential expansion of space leads to an exponential increase in the total energy density of matter. This energy density is converted into radiation during the reheating phase at the end of inflation, producing a large entropy. The entropy problem is only a problem if we assume that the universe begins small (Planck size). The initial starting point in the matter bounce, Pre-Big-Bang and Ekpyrotic cosmologies are completely orthogonal: here it is assumed that the Universe starts large and cold. There is then no problem whatsoever in explaining the current entropy. If string gas cosmology is viewed as part of a bouncing cosmology, the entropy problem is once again absent.

In the same way that inflationary cosmology faces some conceptual problems, the alternatives discussed here also face some difficulties of their own. We present some, deemed relevant, below.

B. Initial conditions

In any cosmological model it is necessary to specify initial conditions. In a successful early universe model the initial conditions should not have to be finely tuned. Whether inflation is successful in this respect is an issue under debate (see e.g. [139, 140] for arguments claiming that the initial conditions for inflation need to be finely tuned). Indeed, it can be shown that the slow-roll trajectory which yields inflation is not a local attractor in phase space in small field inflation models (models in which the inflationary trajectory takes place over field intervals $\delta\varphi < m_{\rm Pl}$ [141]. On the other hand, for large field inflation models the slow-roll trajectory is a local attractor [142], even in the presence of metric fluctuations [143]. For a recent review of this issue the reader is referred to [144]. Let us recall heuristically how this works: one starts at a density somewhat below the Planck density with a large and inhomogeneous universe, with all field modes with energy densities smaller than the Planck scale excited. Space will expand, and modes with wavelengths smaller than the Hubble scale will be redshfited while those with wavelengths larger will remain. In regions where the field values for these modes is strictly positive, inflation will then commence (see [145] for a recent numerical analysis of the dynamics).

Models of bouncing cosmologies must impose initial conditions at a time as far remote in the past as possible, and this corresponds to a moment when the Universe was large and mostly empty. As shown earlier in Fig. 2, all relevant scales are then inside the Hubble radius at the time the initial conditions are set up. Given the cosmological background, it is hence natural to assume quantum vacuum initial conditions for the fluctuations. On the other hand it is not so easy to justify the initial conditions for the background (see e.g. [146] for a discussion of this point in the context of Pre-Big-Bang cosmology).

In string gas cosmology, it is assumed that the universe begins as a hot gas of strings in a quasi-static universe. Hence, thermal initial conditions for the fluctuations are natural. Once again, there is a question as to what produces the initial conditions for the background.

C. Initial Inhomogeneities

A bouncing scenario works opposite to inflation in the sense that one assumes the Universe begins large and then contracts. It would seem that any pre-existing inhomogeneities in a contracting universe will rapidly and automatically collapse, leading to a highly inhomogeneous state. This however is not obviously true, as the contraction may be sufficiently slow compared to the diffusion rate of the primordial constituents. In Ref. [147], conditions were given for an initially large contracting dust-dominated universe satisfying the Weyl curvature hypothesis, to wipe out any primordial inhomogeneity and hence to yield a satisfactory initial state out of which one can settle vacuum initial conditions leading to our universe. It should however be mentioned that, in this framework, the presence, in the contracting phase, of a cosmological constant such as that observed today, could easily destabilize the perturbations produced in such a universe [148], thereby leading to predictions in disagreement with the current data.

D. Anisotropies

The question of shear in a contracting phase followed by a bounce is central to setting constraints on such models. Indeed, in the inflationary context, or even in most expanding cosmological models, the shear, behaving as a^{-6}, rapidly becomes negligible compared to any other component when the scale factor grows, and the Friedman-Lemaître approximation of Eq. (1) can be safely used. The contracting epoch preceding the bouncing phase is the exact opposite and thus may induce a problem [149].

To illustrate this point, let us consider a spatially flat case and, instead of the homogeneous and isotropic Eq. (1), assume spacetime to be well-described by an anisotropic Bianchi I metric (still homogeneous)

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t) \sum_{i=x,y,z} \mathrm{e}^{2\theta_i(t)} \mathrm{d}x_i^2, \tag{75}$$

where $\sum_i \theta_i = 0$. The Einstein equations then read

$$H^2 = \frac{8\pi G_\mathrm{N}}{3}\rho + \frac{1}{6}\sum_i \dot{\theta}_i^2 = \frac{8\pi G_\mathrm{N}}{3}\left(\rho + \rho_\theta\right) \tag{76}$$

and

$$\dot{H} = -4\pi G_\mathrm{N}\left(\rho + p\right) - \underbrace{\frac{1}{2}\sum_i \dot{\theta}_i^2}_{4\pi G_\mathrm{N}\left(\rho_\theta + p_\theta\right)}, \tag{77}$$

leading naturally to

$$\rho_\theta = p_\theta = \frac{\sum_i \dot{\theta}_i^2}{16\pi G_\mathrm{N}} \quad \Longrightarrow \quad \rho_\theta \propto a^{-6} \tag{78}$$

as announced. Eq. (78) implies that the shear component rapidly becomes negligible in a matter or radiation dominated expanding Universe, and of course even more so in the almost exponential case of inflation. During a phase of contraction however, as the scale factor shrinks to zero, the shear can quickly come to dominate over all other components, effectively ruining the FLRW approximation with the risk of transforming a regular bounce into a Kasner singularity through the Belinsky Khalatnikov Lifshitz (BKL) instability [150]. If the initial shear stems from primordial quantum fluctuations of the vector perturbations in a vacuum state, then the resulting anisotropies remain at the same level as the scalar ones. Provided the latter are well-behaved, the former must therefore also remain small. If an initial classical shear is present however, it will subsequently grow uncontrolled, thereby threatening the entire scenario.

The most natural way out of the shear problem consists in postulating an ekpyrotic phase, as discussed above. Indeed, this phase includes a component, the scalar field ϕ with potential (35), whose equation of state is very large, $w \gg 1$, thus providing a contribution $\rho_\phi \propto a^{-3(1+w)}$ which overcomes that of any anisotropy that may have been originally present, hence preserving the FLRW nature of the bounce [151]. Thus, the Ekpyrotic scenario is completely safe from the anisotropy problem, and the matter bounce scenario can be made safe by adding an Ekpyrotic phase of contraction at higher curvatures. Some matter bounce models are in fact even sensitive to the presence of radiation [152]. See also [153] for other approaches to addressing the anisotropy problem.

E. Relics

The early universe, whatever the model (inflation or bounce), reaches extremely high energy densities, close to the Grand Unified (GUT) or even the string or Planck scales. This often provides an interesting means of testing the otherwise unattainable theories supposed to be valid at these scales, thus transforming cosmology into an invaluable tool for high energy physics. Unfortunately, every coin has two faces, and the very same attractive property implies a rather strong caveat in the form of constraints: high energy theories usually predict loads of new objects such as topological defects, exotic particle or even primordial black holes (which we won't mention because estimates of their remnant density differ by many orders of magnitude depending on the model considered), which can be copiously produced during the early stages of the universe, spoiling the subsequent evolution; Ref. [154] contains a short review of these relics.

The best known example stems from supersymmetric theories, and it is the gravitino, i.e. the supersymmetric partner of the graviton. Depending on the values of the coupling constants of the original theory, it can be a stable particle, which may even be useful in cosmology in the form of the missing dark matter. It can also be produced thermally at high enough temperatures in the radiation dominated epoch. It is then an example of a thermal relic. Stable relics may overclose the universe. Unstable relics, on the other hand, could interfere with the process of cosmic nucleosynthesis.

The production of thermal relics depends on the final number density of relics, and hence their contribution as dark matter, depends on particle physics parameters and on the details of the cosmological evolution during the early phases. If the scale of supersymmetry breaking is higher than the maximal temperature in the expanding Big Bang phase (which in the context of an inflationary model is the reheating temperature T_reheat) then gravitinos will not be produced and there are no constraints. On the other hand, for a scale of supersymmetry breaking which would explain the particle physics hierarchy problem and should be not too much higher than 1TeV, the relic particles will be produced. Ref. [154] quotes the constraint $T_{\rm reheat} \lesssim 4\times 10^{10}$Gev for a gravitino mass of about 1TeV which has to be satisfied in order that the relics do not overclose the Universe.

Indeed, SUSY is a key ingredient in either GUT models or string theory, and the natural energy scale associated with the corresponding theories is expected to be much higher than the abovementioned value, and hence the relic problem is an important one. However, the problem affects bouncing cosmologies and simple inflationary models equally.

Topological defects in general are another very common prediction of particle physics models beyond the Standard Model. In particular, in the context of a GUT theory with a high energy symmetry group G which is simply connected, the vacuum manifold, i.e. the set of field values which minimize the potential energy function after the symmetry has broken to the Standard Model, is

$$\mathcal{M} \sim G/[SU_{\rm c}(3)\times U_{\rm em}(1)]\,, \tag{79}$$

and its second homotopy group

$$\pi_2(\mathcal{M}) \sim \pi_1[SU_{\rm c}(3)\times U_{\rm em}(1)] \sim \pi_1[U(1)] \sim \pi_1(S^1) \sim \mathbb{Z} \tag{80}$$

is non trivial. This implies that pointlike topological defects, monopoles, must be produced at some stage of the symmetry breaking scheme. Their production in standard cosmology can be estimated by phase transition arguments, and their number density, given their subsequent evolution, is found much in excess of the closure density, thereby ruining the entire universe's evolution. This problem was part of the reason why inflation was proposed, as a such a phase naturally dilutes the monopole number density in an exponential way: it suffices to ensure, in a GUT implementation of inflation, that the monopole producing phase takes place just before inflation, so that the monopoles are almost instantaneously washed away.

In a bouncing framework however, monopoles will be produced if the maximal temperature is higher than the temperature of the monopole-forming phase transition (in the same way that they are produced in inflation if the reheating temperature is higher than this scale). An easy way for bouncing cosmologies to avoid a potential monopole problem is therefore that the maximal temperature is less than the critical temperature at which symmetry breaking takes place. Then, the symmetry whose breaking produces the dangerous topological relics was thus never restored, and therefore never broken.

Another way to avoid a monopole problem which works independently of the cosmological scenario is to start with a particle physics model which does not at higher energies have a simply connected symmetry group. Whereas this approach goes against the spirit of Grand Unification of the 1970s, it is quite realistic in the context of current string-based particle physics models [155].

Note that a bounce may also leave behind signals of the contracting phase via a different dark matter distribution which is induced during contraction [156].

F. Instabilities

A phase of contraction is subject to many more constraints than a phase of expansion, because many new instabilities can take place. In particular, scalar (curvature) and vector (shear) perturbations can grow too large at the bounce transition. Ref. [27] expands on these questions, which we briefly consider below.

1. Curvature

In many bouncing models, the Bardeen potential develops a constant mode, as in the usual inflationary scenario, but, in contrast to the latter, the second mode is growing, and could pose a threat to the overall treatment of the bounce as a background + perturbations system. In many situations, it was shown that this growing mode can remain under control: there exists a set of conditions for the perturbative series to make sense, and one can almost always find a gauge in which these conditions are satisfied [157]. This also shows that not all gauges represent valid descriptions of cosmological perturbations near the bounce point, as the gauge-fixing conditions can become undefined [7].

Even if the possibly large amplitude of perturbation modes can be tamed down to acceptable levels, their dependence in wavelength can turn an almost scale invariant spectrum into a blue one, thus spoiling the predictions of an otherwise working model. Let us illustrate this phenomenon with a phase of ekpyrotic contraction leading to the bounce. The time evolution of a mode ζ_k corresponding to wavenumber k of the comoving curvature perturbation (7) obeys

$$\zeta_k'' + 2\frac{z'}{z}\zeta_k' + c_{\rm S}^2 k^2 \zeta_k = 0, \tag{81}$$

where the sound velocity $c_{\rm S}$ is given in terms of the pressure $p(X)$ [with $X\equiv \frac{1}{2}\left(\partial\varphi\right)^2$]

$$c_{\rm S}^2 = \frac{p_{,X}}{p_{,X}+2Xp_{,XX}} \tag{82}$$

and $z \equiv a\sqrt{\dot{H}/(c_{\rm S}H)^2}$. The Mukhanov-Sasaki mode

[7] Note that a similar problem arises during reheating in inflationary cosmology.

variable $v_k = z\zeta_k$ satisfies

$$v_k'' + \left(c_\mathrm{S}^2 k_2 - \frac{z''}{z}\right) = 0, \tag{83}$$

which is nothing but the generalization, for a possibly time-varying sound speed, of Eq. (11).

Two problems may then arise. The first concerns the actual spectrum: in the ekpyrotic contraction regime $c_\mathrm{S}^2 \simeq 1$, and for long wavelengths $k^2 \gg z''/z$, the solution reads $\zeta_k \sim k^{-1/2} + \sqrt{k}\int z^{-2}\mathrm{d}\eta + \cdots$, where vacuum initial conditions have been taken into account. The constant solution term is usually that which one considers as eventually producing the spectrum, while the second term, much bluer, is always assumed negligible. However, as shown in [158], using the background equation of motion $\dot{H} = -Xp_{,X}$ and the definition of z, the contribution due to the integral term can be estimated. The ratio between this contribution and the supposedly dominant isocurvature component then reads

$$\frac{\zeta_k^\mathrm{int}}{\zeta_k^\mathrm{iso}} \propto \mathrm{e}^{\mathcal{N}_\mathrm{ekp} - 2\mathcal{N}_k}, \tag{84}$$

with $\mathcal{N}_k$ the number of e-folds of ekpyrosis after the mode v_k has passed the potential z''/z and $\mathcal{N}_\mathrm{ekp}$ the total duration, in e-folds, of the ekpyrotic phase. For modes of cosmological relevance, one has $\mathcal{N}_k \simeq 10$, while the constraint $\mathcal{N}_\mathrm{ekp} \gtrsim 60$, so (84) spoils the overall mechanism, yielding a blue spectrum.

The second problem potentially induced by the evolution of curvature perturbation concerns the bounce itself. Indeed, in order that it actually takes place, a ghost condensate must be used, and therefore, there must exist a finite amount of time around the bounce for which $c_\mathrm{S}^2 \leq 0$. Now if, and this is a very model-dependent if, during that time, there are modes satisfying $|c_\mathrm{S}^2 k^2| > z''/z$, then the solution naturally acquires an exponentially growing term. Bounces such as that can be saved if either the bounce duration in conformal time is sufficiently short, or if $|c_\mathrm{S}^2|$ is sufficiently small (in practice exponentially small) during the transition, so that the resulting exponential growth is not too large.

2. *Vector modes*

Although we did not consider vector modes in this short review, we must mention them in the context of potential problems, as alluded to above. In the usual paradigm of an expanding universe with ordinary matter having no anisotropic stress (perfect fluid or scalar field in practice), the vector modes are not sourced dynamically, and hence are merely constrained to scale as a^{-2}.

With the vector metric perturbation scaling as the inverse square of the scale factor, one finds that for a fluid with constant equation of state w, the velocity perturbation reads $V^i \propto k^2 a^{3w-1}$, which is constant for a radiation dominated universe, and decreasing (resp. growing) for any fluid having $w < \frac{1}{3}$ (resp. $w > \frac{1}{3}$) in an expanding universe.

Assuming they were initially not dominant, this diluting factor renders vector perturbations mostly harmless for standard cosmology and totally irrelevant after even a short period of inflation took place. Obviously, in a contracting universe, the situation can change drastically [29]. In the Pre Big-Bang scenario for instance, the breakdown of perturbation theory seems unavoidable. It was however argued in [159] that if a primordial component of vector perturbation were sourced by some dynamical vector field beginning with vacuum initial conditions, their subsequent evolution, although growing, should remain comparable to that of scalar perturbations, and thus should not spoil the perturbation expansion.

It was also proposed [29] to reverse this potential catastrophe into a window of opportunity: assuming the growth to be somehow controlled by, say, non linear effects, during the bouncing phase, one could use the resulting relatively large amplitude vector contribution to generate a large enough primordial magnetic field that would explain the otherwise mysterious value necessary to understand current data [160].

G. Curvature and the Null Energy Condition

Many bouncing models assume, to begin with, that the curvature is to be neglected at all times, including at the bounce itself. The argument for assuming flat spatial sections in a bouncing setup is that the curvature term in the Friedman equation, $\mathcal{K}/a^2$, where $\mathcal{K}$ is the curvature constant, will be effectively negligible with respect to any other constituent that will necessarily be present in the model, such as matter ($\rho_\mathrm{mat} \propto a^{-3}$) or radiation ($\rho_\mathrm{rad} \propto a^{-4}$), in the limit where the scale factor shrinks indefinitely $a \to 0$. In many cases of interest, this argument is valid, but it may not be as generic as one would spontaneously think.

First of all, in the context of general relativity, a regular bounce with flat spatial sections can only take place provided the Null Energy Condition $\rho + p \geq 0$ is violated. This is the main reason for implementing bounces by means of negative energy scalar fields, ghost condensates [161], conformal galileon [162] and the like [163], sometimes leading to instabilities that have to be dealt with [164].

At the bouncing point, the Hubble length diverges, $H \to 0$, implying that the curvature contribution, if any, ought to be exactly canceled by the sum of all positive and negative energy contributions. Even though this requirement might sound like fine-tuning, it actually is not: the constraint then provides an algebraic equation giving the value of the scale factor, a_B say, at which the bounce takes place, as a function of the relative proportions of the various components involved as well as on the curvature radius at that moment.

Another challenge induced by curvature is related to the question of shear. Indeed, although the ekpyrotic phase permits to avoid the BKL instability in the flat case [151], the presence of curvature could spoil the picture through a mixmaster phenomenon. Space would expand and contract in different directions, leading to a highly inhomogeneous and anisotropic universe [165].

VI. CONCLUSIONS

Bouncing cosmologies still have a long way to go before they can be considered as sound as the inflationary paradigm. Whereas inflationary models are self-consistent at the level of an effective field theory coupled to Einstein gravity, the same is not true of bouncing models. On the other hand, the aim of bouncing cosmologies is more ambitious in that one of their goals is to address the singularity problem of our current cosmological models. To do this, one has to go beyond a theory which is based on General Relativity coupled to particle matter which obeys the usual energy conditions. Another point to keep in mind is that inflationary models which look self-consistent from the point of view of effective field theory may not be consistent from the point of view of the complete ultraviolet theory (see e.g. [166] for a discussion of this problem).

Having built in a resolution of the primordial singularity, a contracting phase followed by a bounce provides a natural extension of the usual standard model of cosmology. There are various scenarios in which a scale-invariant spectrum of cosmological perturbations emerges which can explain all of the current data. We have discussed the *matter bounce* model, the *Pre-Big-Bang* and *Ekpyrotic* scenarios, and *string gas cosmology.*

Theoretically, implementing a bouncing phase after a contraction era is not so simple in the context of general relativity, as it entails a violation of the Null Energy Condition (except in the unlikely event that the spatial curvature somehow plays a crucial role). This requires unusual scalar fields like a ghost condensate, with possibly many resulting instabilities. Other implementations of a bouncing scenario, like Pre-Big-Bang cosmology or the Ekpyrotic scenario, invoke stringy effects to resolve the singularity and to yield a bounce. String gas cosmology is based on fundamental principles of superstring theory, but at the moment has no good implementation in terms of an effective action. Thus, none of the bouncing cosmologies considered here are at the present time fully understood. Some may argue that inflation, demanding merely a simple scalar field with a potential having a plateau [167], is perhaps simpler to achieve, and thus privileged from the Occam's razor point of view. On the other hand, this philosophical standpoint, useful as it is for providing guidance to write down underlying theories, may not be valid for actual physical phenomena such as the evolution of the entire universe.

From the point of view of cosmological observations, there is for the moment no need to go beyond inflationary cosmology: simple single scalar field models are consistent will all cosmological observations. On the other hand, many of the successful predictions of inflation depend only on having a mechanism which produces an almost scale-invariant spectrum of curvature fluctuations on scales which are super-Hubble at early times, and thus the current observations cannot be interpreted as favoring inflation. We have shown that even the slight red tilt of the spectrum of curvature fluctuations (excluding purely scale-invariant perturbations at the 5σ level) is not a unique prediction of inflation, but is naturally obtained in several bouncing models, in particular in string gas cosmology and in the matter bounce. Similarly, the absence of measurable amounts of either non-Gaussianities or tensor modes is also obtained naturally in string gas cosmology, although a simple matter bounce is in tension with the data, whereas the data are in perfect agreement with inflationary predictions of simple inflation models.

We have discussed ways in which to distinguish between pure inflation and pure bouncing cosmologies using future observational results. Of particular importance is the measurement of the amplitude and slope of the tensor modes, since this will allow us to distinguish between the consistency relations amongst observables predicted by simple inflationary models on the one hand and bouncing cosmologies (string gas cosmology in particular) on the other.

Of course, nothing prevents that the actual history of our universe contains both an inflationary phase and a preceding bounce. At least at the level of scalar field toy models for matter, most mechanisms for constructing a cosmological bounce allow for the inclusion of an inflationary phase after the bounce. At the time of writing, it is not clear if, in such a scenario, one will ever be able to find truly discriminating measurements.

In this review we have studied bouncing cosmologies without ever mentioning cyclic models. As we have seen, in the bouncing models we have studied, a vacuum spectrum is transformed into a scale-invariant one on scales which exit the Hubble radius during the contracting period. In a model which is cyclic from the four space-time dimensional point of view, the initial spectrum at the beginning of the second phase of contraction would be scale-invariant, and it would be transformed into a spectrum with index $n_{\rm s} = -1$ before the second bounce. This processing of cosmological fluctuations [168] makes four-dimensional cyclic cosmologies unpredictive. The cyclic Ekpyrotic scenarios [169] avoid this problem since it is new scales which are probed in each cycle (the model is not strictly cyclic from the four-dimensional point of view).

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