On the problem of initial conditions for inflation[1]

Andrei Linde

SITP and Department of Physics, Stanford University, Stanford, CA 94305 USA

Abstract

I review the present status of the problem of initial conditions for inflation and describe several ways to solve this problem for many popular inflationary models, including the recent generation of the models with plateau potentials favored by cosmological observations.

[1]Based on an invited talk at the conference "Black Holes, Gravitational Waves and Spacetime Singularities," Specola Vaticana 9-12 May 2017

Contents

1 Initial conditions for inflation: A brief review

The theory of initial conditions in the early universe remains one of the most debated issues of modern cosmology. One can approach it at many different levels, e.g. in the context of quantum cosmology, or eternal inflation in string theory landscape, but it is always useful to go back to basics and check what is the status of this problem in the simplest models of inflation with one or two classical scalar fields. And even this may lead to misunderstandings since different people still have different ideas of what is inflation, and what is the meaning of the words "initial conditions". For example, for experts in numerical methods in GR, initial conditions can be imposed at any time, whereas for people who want to understand the origin of the universe, initial conditions are related to the first moment when the classical description of the universe becomes possible.

To clarify these issues, let us remember what was the main problem with the hot Big Bang theory in this respect. In that model, the universe was born in the cosmological singularity, but it became possible to describe it in terms of classical space-time only when time was greater than the Planck time $t_p \sim 1$. At that epoch, the temperature of matter was given by the Planck temperature $T_p \sim 1$, and the density of the universe was given by the Planck density $\rho_p \sim 1$. The size of the causally connected part of the universe at the Planck time was $ct_p \sim 1$. Each such part contained a single particle with the Planck temperature. The subsequent evolution was supposed to be nearly adiabatic, which means that the total number of particles in a comoving volume was approximately conserved. Thus this number from the very beginning was supposed to be greater than the total number of particles in the observable part of the universe, $n \sim 10^{90}$. This means that the universe at the Planck time consisted of 10^{90} causally disconnected parts. The probability that all of these independent parts emerged from singularity at the same time with the same energy density and pressure is smaller than $e^{-10^{90}}$, and it is even more complicated to explain why these parts would have the same density with an accuracy better than 10^{-4}.

Similar problem persisted in old [1] and new inflation [2], where the universe was supposed to be large and hot from the very beginning, consisting of many causally disconnected parts. In this sense, these two models did not solve the problem of initial conditions.

The situation changed dramatically with the invention of the chaotic inflation [3]. The main condition required in the simplest models of chaotic inflation was the existence of a single Planck size domain where the kinetic and gradient energy of the scalar field is few times smaller than its potential energy $V(\phi) \sim 1$. For sufficiently flat potentials, it leads to inflation, so the whole universe appears as a result of expansion of a single Planck size domain. According to [3–5], the probability of this process is not exponentially suppressed. After that, the universe described by the chaotic inflation scenario enters an eternal process of self-reproduction [6].

This solution works for the simplest versions of the chaotic inflation scenario where inflation may start at the densities comparable with the Planck density. However, recent observational data [7] favor inflationary models with plateau potentials, with the height of the plateau $V \sim 10^{-10}$. Such models include the GL model [8], in the Starobinsky model [9], the Higgs inflation model [10,11], and the broad class of the cosmological attractor models [12–17], which generalize most of the previously proposed models with plateau potentials. Thus, one could wonder [18] whether it is possible to solve the problem of initial conditions for inflationary models of this type.

The answer to this question is two-fold. First of all, cosmological observations give us information only about the very last stages of inflation, and they tell us nothing about the beginning of inflation. I will describe several simple inflationary models where inflation begins at the Planck density with $V(\phi) \sim 1$ and ends by a slow-roll driven by a plateau potential with $V \sim 10^{-10}$.

Moreover, in this paper I will show, following [19–24], that one can easily solve the problem of initial conditions for inflationary models favored by the recent cosmological observations even if inflation is possible there only at $V \sim 10^{-10}$.

2 Models with a short plateau

The simplest version of the chaotic inflation starting at $V \sim 1$ has the quadratic potential

$$V(\phi) = \frac{m^2\phi^2}{2} \ . \tag{1}$$

One can make a trivial modification of this model, by adding to it small terms proportional to ϕ^3 and ϕ^4.

$$V(\phi) = \frac{m^2\phi^2}{2}\Big(1 - a\phi(1 + b\,a\,\phi)\Big), \tag{2}$$

For come values of parameters of this model, the potential acquires a short plateau, see Fig. 1. This helps to match the observational data, while still allows inflation to begin at $V = O(1)$ in Plank mass units, thus solving the problem of initial conditions for inflation.

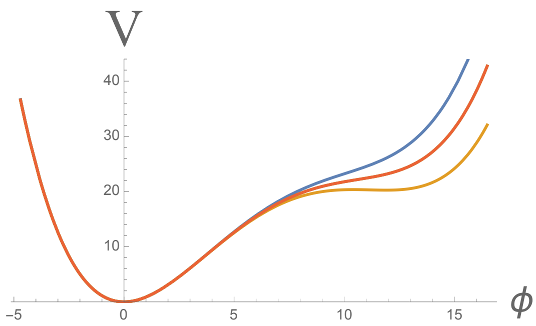

Figure 1: The potential $V(\phi) = \frac{m^2\phi^2}{2}\left(1 - a\phi + a^2 b\,\phi^2\right)$ for $a = 0.12$ and $b = 0.30$ (upper curve), $b = 0.29$ (middle), and $b = 0.28$ (lower curve). The potential is shown in units of m^2, with ϕ in Planck units. For $b = 0.29$ (the middle curve), at the moment corresponding to $N = 58$ e-folding from the end of inflation one has $n_s = 0.965$ and $r = 0.012$, perfectly matching the Planck data.

Note that inflationary observations give us 3 main parameters: the amplitude of perturbations A_s, the spectral index n_s, and the tensor to scalar ratio r. By adjusting 3 parameters m, a and b in the modified potential one can easily describe the recent data describing the 3 parameters A_s, n_s and r. This is the simplest solution of the problem of initial conditions for inflation, compatible with the latest observational data.

In what follows, we will show how one can go much further and solve the problem of initial conditions for a very broad class of models with plateau potentials, α-attractors [14, 16]. These models are very generic and economical; they can describe all presently available inflation-related observational data using a single parameter m controlling the inflaton mass.

3 α-attractors

There are many different ways to introduce the theory of α-attractors, see [12–17]. On a purely phenomenological level, the main features of inflation in all of these models can be represented in terms of a single-field toy model with the Lagrangian [16, 17]

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{R}{2} - \frac{(\partial_\mu\phi)^2}{2(1-\frac{\phi^2}{6\alpha})^2} - V(\phi)\,. \tag{3}$$

Here $\phi(x)$ is the scalar field, the inflaton. The origin of the pole in the kinetic term can be explained in the context of hyperbolic geometry in supergravity and string theory. The parameter α can take any positive value. In the limit $\alpha \to \infty$ this model coincides with the standard chaotic inflation with a canonically normalized field ϕ and the inflaton potential $V(\phi)$ [3]. However, for any finite values of α, the field ϕ in (3) is not canonically normalized. It must satisfy the condition $\phi^2 < 6\alpha$, for the sign of the inflaton kinetic term to remain positive.

We will assume that the potential $V(\phi)$ and its derivatives are non-singular for $\phi^2 \le 6\alpha$. Instead of the variable ϕ, one can use a canonically normalized field φ by solving the equation $\frac{\partial\phi}{1-\frac{\phi^2}{6\alpha}} = \partial\varphi$, which yields

$$\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}\,. \tag{4}$$

The full theory, in terms of the canonical variables, becomes

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{R}{2} - \frac{(\partial_\mu\varphi)^2}{2} - V(\sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}})\,. \tag{5}$$

Note that in the limit $\phi \to 0$ the variables ϕ and φ coincide; the main difference appears in the limit $\phi \to \sqrt{6\alpha}$: In terms of the new variables, a tiny vicinity of the boundary of the moduli space at $\phi = \sqrt{6\alpha}$ stretches and extends to infinitely large φ. As a result, generic potentials $V(\phi) = V(\sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}})$ at large φ approach an infinitely long dS inflationary plateau with the height corresponding to the value of $V(\phi)$ at the boundary:

$$V_0 = V(\phi)|_{\phi=\pm\sqrt{6\alpha}}\ . \tag{6}$$

To understand what is going on in this theory, let us consider, for definiteness, positive values of ϕ and study a small vicinity of the point $\phi = \sqrt{6\alpha}$, which becomes stretched to infinitely large values of the canonical field φ upon the change of variables $\phi \to \varphi$. If the potential $V(\phi)$ is non-singular at the boundary $\phi = \sqrt{6\alpha}$, we can expand it in series with respect to the distance from the boundary:

$$V(\phi) = V_0 + (\phi - \sqrt{6\alpha})\,V_0' + O(\phi - \sqrt{6\alpha})^2\ . \tag{7}$$

where we denote $V_0' = \partial_\phi V|_{\phi=\sqrt{6\alpha}}$.

In the vicinity of the boundary $\phi = \sqrt{6\alpha}$, the relation (4) between the original field variable ϕ and the canonically normalized inflaton field φ is given by

$$\phi = \sqrt{6\alpha}\,\left(1 - 2e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\right)\ , \tag{8}$$

up to the higher order terms $O(e^{-2\sqrt{\frac{2}{3\alpha}}\varphi})$. At $\varphi \gg \sqrt{\alpha}$, these terms are exponentially small as compared to the terms $\sim e^{-\sqrt{\frac{2}{3\alpha}}\varphi}$, and the potential acquires the following asymptotic form:

$$V(\varphi) = V_0 - 2\sqrt{6\alpha}\,V_0'\ e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\ . \tag{9}$$

Note that the constant $2\sqrt{6\alpha}\,V_0'$ in this expression can be absorbed into a redefinition (shift) of the field φ. This implies that if inflation occurs at large $\varphi \gg \sqrt{\alpha}$, all inflationary predictions in this class of models of the potential $V(\phi)$ are determined only by the value of the potential V_0 at the boundary and the constant α. For any values of $\alpha \lesssim 10$, the amplitude of inflationary perturbations, the prediction for the spectral index n_s and the tensor to scalar ratio r match observational data under a single nearly model-independent condition

$$\frac{V_0}{\alpha} \sim m^2 \sim 10^{-10}\ . \tag{10}$$

Thus the only parameter that is required to fit the present observational data is the parameter $m \sim 10^{-5}$ controlling the amplitude of the scalar perturbations of metric [25].

These results were explained in [12, 14] and formulated in a particularly general way in [16]: The kinetic term in this class of models has a pole at the boundary of the moduli space. If inflation occurs in a vicinity of such a pole, and the potential near the pole has a finite and positive first derivative, all other details of the potential and of the kinetic term far away from the pole (from the boundary of the moduli space) become unimportant for making cosmological predictions. In particular, the spectral index depends solely on the order of the pole, and the tensor-to-scalar ratio relies on the residue [16]. All the rest is practically irrelevant, as long as the field after inflation falls into a stable minimum of the potential, with a tiny value of the vacuum energy, and stays there. Stability of the inflationary predictions with respect to even very strong modifications of the shape of the potential outside a small vicinity of the boundary of the moduli space is the reason why these models are called cosmological attractors.

This new class of models accomplishes for inflationary theory something similar to what inflation does for cosmology. Inflation stretches the universe making it flat and homogeneous, and the structure of the observable part of the universe becomes very stable with respect to the choice of initial conditions in the early universe. Similarly, stretching of the moduli space near its boundary upon transition to canonical variables makes inflationary potentials very flat and results in predictions which are very stable with respect to the choice of the inflaton potential.

This addresses the often presented argument that the shape of the inflationary potential in large-field inflation and its cosmological predictions must be unstable with respect to higher order corrections to the potential at super-Planckian values of the field. In the new class of models, the range of the original field variables ϕ for $\alpha \lesssim 1$ is sub-Planckian, and the shape of the potential in terms of the canonical inflaton field is very stable with respect to the choice of the original potential (9), all the way to infinitely large values of the inflaton field.

The simplest example of such theory is given by the model with $V(\phi) = m^2\phi^2$. In terms of the canonically normalized field φ, the potential is given by

$$V(\varphi) = 3\alpha m^2 \tanh^2 \frac{\varphi}{\sqrt{6\alpha}}. \tag{11}$$

This is the simplest representative of the so-called T-models, with the T-shaped potential shown in Fig. 2

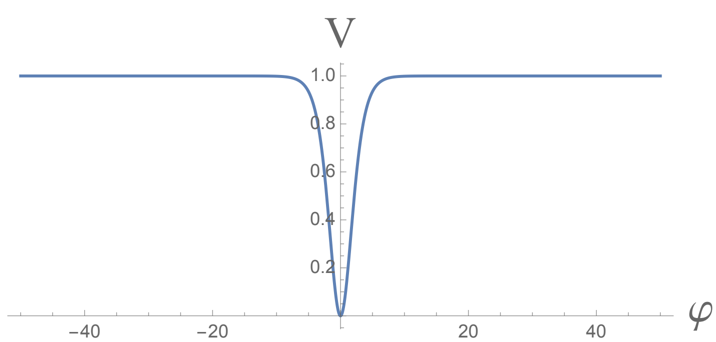

Figure 2: The potential $V(\varphi) = 3\alpha m^2 \tanh^2 \frac{\varphi}{\sqrt{6\alpha}}$ for $\alpha = 1$, shown in units of $3m^2$, with φ in Planck units. For $1/3 < \alpha < 10$ one has $n_s \sim 0.965$ and the tensor to scalar ratio r is in the range from 3×10^{-2} to 10^{-3}, providing good match to the Planck data.

4 α-attractors with two fields

Let us now consider an extended version of the α-attractor model, adding to it a scalar field σ with a canonically normalized kinetic term:

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2(1-\frac{\phi^2}{6\alpha})^2} - \frac{(\partial_\mu \sigma)^2}{2} - \frac{m^2}{2}\phi^2 - \frac{g^2}{2}\phi^2\sigma^2 - \frac{M^2}{2}\sigma^2. \tag{12}$$

The inflaton potential becomes

$$V(\varphi,\sigma) = 3\alpha(m^2+g^2\sigma^2)\tanh^2\frac{\varphi}{\sqrt{6\alpha}} + \frac{M^2}{2}\sigma^2. \tag{13}$$

Its shape is shown in Fig. 3.

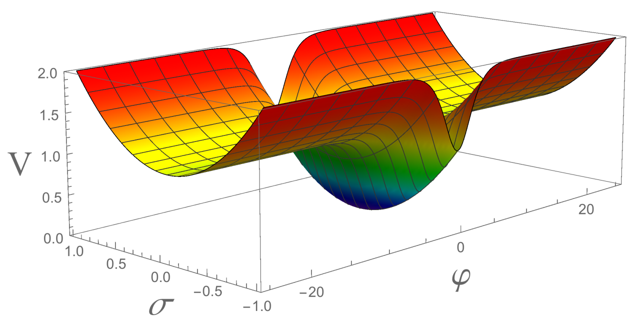

Figure 3: The potential $V(\varphi,\sigma)$ for $\alpha = 1$, shown in units of $3m^2$, with φ and σ in Planck units. The shape of the potential along the valley $\sigma = 0$ is shown in Fig. 2.

The potential depends on $|\varphi|$. During inflation at $|\varphi| \gg \sqrt{\alpha}$, one can use the asymptotic equation

$$\tanh^2\frac{|\varphi|}{\sqrt{6\alpha}} = 1 - 4e^{-\sqrt{\frac{2}{3\alpha}}|\varphi|} + O(e^{-2\sqrt{\frac{2}{3\alpha}}|\varphi|}) \ . \tag{14}$$

For notational simplicity, we will study positive values of φ. The potential at $\varphi \gg \sqrt{\alpha}$ is equal to

$$V(\varphi,\sigma) = 3\alpha(m^2+g^2\sigma^2)\,(1 - 4e^{-\sqrt{\frac{2}{3\alpha}}\varphi}) + \frac{M^2}{2}\sigma^2, \tag{15}$$

up to exponentially small higher order terms $3\alpha(m^2+g^2\sigma^2)O(e^{-2\sqrt{\frac{2}{3\alpha}}\varphi})$. The potential $V(\varphi,\sigma)$ has a minimum with respect to σ at $\sigma = 0$. The inflaton potential at $\sigma = 0$ and large φ is

$$V(\varphi) = 3\alpha m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}} \ . \tag{16}$$

During inflation at $\varphi \gg \alpha$, this potential with exponentially good accuracy coincides with the cosmological constant,

$$V(\varphi) \approx 3\alpha\, m^2 \ . \tag{17}$$

Mass squared of the canonically normalized field φ is given by the second derivative of $3\alpha m^2\tanh^2\frac{\varphi}{\sqrt{6\alpha}}$. At $\varphi \ll \sqrt{\alpha}$ one has

$$m^2_\varphi = m^2 \ , \tag{18}$$

but at $\varphi \gg \sqrt{\alpha}$ one finds $m^2_\varphi = -8m^2 e^{-\sqrt{\frac{2}{3\alpha}}\varphi}$. Meanwhile the mass of the field σ at large φ approaches a constant value

$$m^2_\sigma = M^2 + 6\alpha g^2 \ . \tag{19}$$

and the potential asymptotically becomes a sum of the positive cosmological constant $3\alpha m^2$, and a quadratic potential of the field σ:

$$V(\varphi, \sigma) = 3\alpha m^2 + \frac{M^2 + 6\alpha g^2}{2}\sigma^2. \tag{20}$$

The strength of interactions of the inflaton field with itself and with the field σ during inflation at $\sigma = 0$ can be described in terms of the coupling constants of canonically normalized fields, such as $\lambda_{\varphi,\varphi,\varphi,\varphi} = \partial^4_\varphi V(\varphi, \sigma)_{|\sigma=0}$ or $\lambda_{\varphi,\varphi,\sigma,\sigma} = \partial^2_\varphi \partial^2_\sigma V(\varphi, \sigma)_{|\sigma=0}$. As one can easily see, all such couplings are suppressed by the exponentially small coefficient $e^{-\sqrt{\frac{2}{3\alpha}}\varphi}$.

In other words, the inflaton field is "asymptotically free" [26]. By that, we mean the *exponentially small* strength of interactions of the field φ with all other fields at large φ, rather than the *logarithmically small* strength of interactions at large momenta, as in QCD. Our conclusions apply to the models with any potential $V(\phi, \sigma)$ as long as this potential and its derivatives are non-singular at the boundary $\phi = \sqrt{6\alpha}$. These unusual features of the new class of theories lead to stability of the plateau shape of the potential at large φ with respect to quantum corrections [26].

5 Solving the initial conditions problem for simplest single-field α-attractors

Let us study the problem of initial conditions in the model with the α-attractor potential shown in Fig. 2. As we can see, this potential is equal to the cosmological constant $\Lambda = 3\alpha m^2 \sim 10^{-10}$ with exponentially good accuracy *everywhere along the infinitely long plateau from* $-\infty$ *to* $+\infty$, except for a narrow minimum at $|\varphi| \lesssim \sqrt{6\alpha}$. This suggests that if we solve the problem of initial conditions for the exponential dS expansion of the universe containing normal matter and a positive cosmological constant, we will simultaneously solve the problem of initial conditions for inflation in the theories with plateau potentials.

This argument is nearly trivial, and one may wonder why it was not formulated in this simple form many years ago, because it turns the whole problem upside down: One may wonder whether there is any way *to escape* the exponential dS expansion of the universe containing normal matter and a positive cosmological constant?

As an example, one may consider the present stage of the accelerated cosmological expansion. According to the simplest ΛCDM model, 70% of matter in the universe is the positive vacuum energy, the cosmological constant. Because of that, the universe entered the stage of acceleration. There are many extremely large inhomogeneities in the universe, such as galaxies. The size of each of them does not grow at the same rate at the universe. Black holes do not grow as well. But neither galaxies nor the black holes can stop the general quasi-exponential expansion of the universe dominated by the cosmological constant. We have missed the point in time when this expansion could have been stopped. This could happen, for example, if at some stage we would find that we live in a locally overweight part of the universe, with the energy density of matter much greater than the cosmological constant. But this did not happen, and now it is too late: Density of matter decreases, while the cosmological constant does not, and the universe gradually enters the unstoppable dS expansion $a \sim e^{Ht}$.

The same can be expected for the early universe in the theory with the cosmological constant $V \sim 10^{-10}$, corresponding to the height of the inflationary plateau. In an expanding universe with any conventional equation of state, density of normal matter during the epoch of matter domination drops down as t^{-2}. If the expansion continues longer than $t \sim V^{-1/2} \sim (\sqrt{\alpha}m)^{-1} \sim 10^5$ Planck times, i.e. longer than about 10^{-28} seconds, then the universe enters the stage of exponential expansion, after which the field φ slowly rolls down to its minimum while producing quantum fluctuations responsible for the structure formation in our universe.

Can the universe avoid inflation in such models? There are two obvious possibilities. First of all, for some reason the field $|\varphi|$ may be born not on the infinite plateau but in the small vicinity off the minimum, with $|\varphi| < \sqrt{6\alpha}$. Whereas it is possible, in the context of the models with long plateau potentials this possibility seems extremely unlikely because of the infinitely large phase space for the canonical inflaton field with $|\varphi| > \sqrt{6\alpha}$.

Following suggestion by Starobinsky, I would like to illustrate this argument made in [21] by using an analogy with inflation in economy, see Fig. 4. There is an often made statement that if the government drops lots of money from a helicopter, this may cause inflation. Of course, there is always a possibility that the money will be lost on its way. In our case, if we consider a theory with a plateau potential, and the scalar field ϕ drops in an expanding universe down from the state with the Planck energy density, then it is very difficult for it to end up in a narrow minimum of the potential and miss an infinite plateau. And if it falls to the plateau, then, just as in the situation with a positive cosmological constant background, it is difficult to avoid inflation.

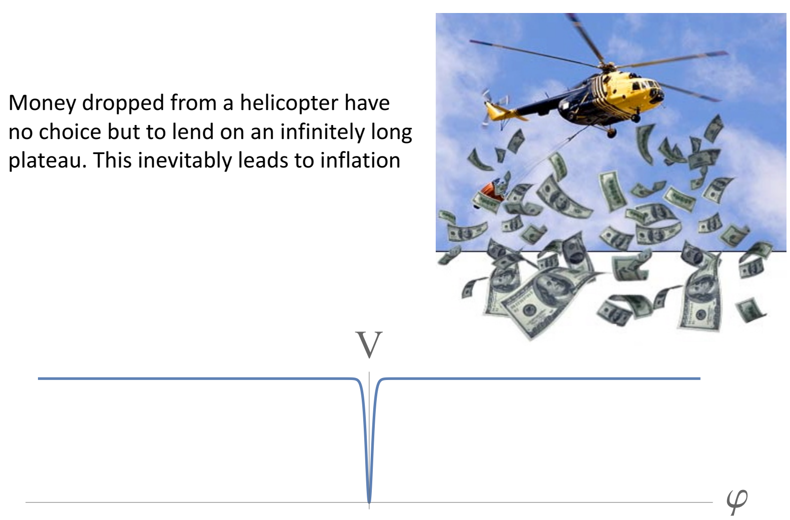

Figure 4: Inflation in economy and in the universe.

The only other way to avoid inflation in this scenario is to assume that the whole universe collapses within 10^{-28} seconds. Indeed, if any part of an expanding universe continues expanding longer than 10^{-28} seconds, the energy density of matter there becomes smaller than V, and it enters the stage of exponential expansion. *This means that all universes, which are described by models with plateau potentials and can be explored by an observer with the lifespan greater than* 10^{-28} *seconds, should have passed the stage of inflation [21].* In other words, only virtual universes with the lifetime smaller than the inverse inflaton mass can avoid inflation.

This argument made in [21] was considerably advanced and strengthened by an analytical investigation and a computer simulation of a universe expanding from the state with Planckian density dominated by a grossly inhomogeneous scalar field φ [22], see also [23, 24]. The simulations used sophisticated GR methods similar to the ones used for investigation of the black hole merger by LIGO.

One of the features commonly used in numerical simulations is periodicity of the boundary conditions. This can be also interpreted as an investigation of a topologically non-trivial universe, like a torus. This is a somewhat unconventional approach, but it offers great advantages in studies of the problem of initial conditions for inflation, especially in the context of an open or flat universe. Indeed, one of the problems with the traditional approach to the Big Bang cosmology is that the open of flat universes were supposed to be infinitely large from the very beginning, with complicated correlations between infinitely many of their causally disconnected parts. No such problems appear if the universe is compact and topologically nontrivial [19, 27–30].

Consider, for example, an inhomogeneous expanding toroidal flat universe of the size of the horizon, which was of the same order as the Planck length $O(1)$ at the Planck time $O(1)$, when its total energy density was $O(1)$ in the Planck units. Since the potential energy density initially was $O(10^{-10}) \ll 1$, the universe was dominated by kinetic and gradient energy density of the scalar field, $\dot{\varphi}^2 \sim (\partial_i \phi)^2 \sim 1$. In that case, the scale factor of the universe, which determined the size of the torus, was growing slower than the scale $H^{-1} \sim t$. And this means that the friction coefficient $\sim H$ rapidly became smaller than the original momenta of the scalar field inhomogeneities. In this situation, the effective equation of state of the inhomogeneities becomes similar to the equation of state of ultra-relativistic matter, which is not supporting the growth of inhomogeneities and black hole formation. Moreover, ultra-relativistic perturbations would travel many times around the small torus during the Hubble time, which would also support homogenization of the universe [19, 28–30].

As a result, if the Planck size topologically non-trivial universe does not collapse as a whole within the Planck time, the chances that it will collapse later become small. The results of the computer simulations performed in [22] confirm this conjecture. We found that the original inhomogeneities may lead to a local process of black hole formation shortly after the Planck time. But this process does not affect large part of space, which continues expanding. This eventually results in a subsequent inflationary regime when the energy density of matter in an expanding part of the universe becomes smaller than the value of V on the plateau. This suggests a possible strengthening of our earlier formulation: most of the Planck size topologically nontrivial universes, which are described by the models with plateau potentials and have lifespan greater than the Planck time $\sim 10^{-33}$ seconds, eventually the stage of inflation.

Note that whereas the investigation of the problem of initial conditions was performed in [22] in the context of the models with plateau potentials, the actual computations evolved evolution in a rather limited range of the values of the inflaton field. Thus we expect that the final results should apply to many models of large field inflation.

6 Solving the problem of initial conditions in two-field models

Once we consider models with more than one scalar, many new possibilities to address the problem of initial conditions appear. Consider, for example, the two-field α-attractor model (12) with the potential shown in Fig. 3. Solution of the problem of initial conditions in such models becomes trivial, being reduced to the theory of initial conditions in the simplest chaotic inflation models with a quadratic potential [3–5]. Indeed, according to (20), the potential $V(\varphi, \sigma)$ at $\varphi \gg \sqrt{6\alpha}$ reduces to the quadratic potential with the cosmological constant, $V(\varphi, \sigma) = 3\alpha m^2 + \frac{M^2+6\alpha g^2}{2}\sigma^2$. Inflation in this model can start at the Planck density with $V(\varphi, \sigma) = O(1)$. Its early stages are driven by the field σ with a quadratic potential, as in [3–5]. This solves the problem of initial condition for the first stage of inflation driven by the field σ. When this stages completes, the field σ vanishes, and the second stage of inflation driven by the field φ with the α-attractor potential begins. It is this last stage that describes the latest 60 e-foldings of inflation, in agreement with the Planck data.

Note, that similar mechanisms can easily solve the problem of initial conditions not only for the large field inflation, but for small field inflation as well, see Sect. 10 of my 2013 Les Houches lectures [20] and references therein.

7 Conclusions

In this paper, I described several different ways to solve the problem of initial conditions for a broad class of inflationary models, including the theories with plateau potentials, where the last stage of inflation occurs when the potential can be many orders of magnitude below the Planck scale. This shows, contrary to some recent claims in the literature [18], that the inflationary models favored by the latest observational data, such as the GL model, the Starobinsky model, the Higgs inflation model, and the broad class of α-attractors do not suffer from the problem of initial conditions. Moreover, under certain conditions this problem can be solved not only for large field models discussed in [22], but for small field models as well.

I am very grateful to the organizers of the conference "Black holes, Gravitational Waves and Spacetime Singularities" for their hospitality. This work is supported by SITP and by the US National Science Foundation grant PHY-1720397.

8 Appendix: Quantum creation of universes with non-trivial topology

In the main body of the paper, I was using simple intuitive arguments not requiring familiarity with the tools of quantum cosmology. However, quantum cosmology [31] and the theory of quantum creation of the universe "from nothing" [32–35] allows to look at the problem of initial conditions from a different perspective.

Most of the related investigations described creation of a closed universe. But an initial size of a closed inflationary universe studied in [32–35] should be greater than H^{-1}, which is 5 orders of magnitude greater than the Planck length in the models considered above. In some cases, this may lead to exponential suppression of the probability of quantum creation of such universes, but this problem can be solved using anthropic considerations [35]. There is no exponential suppression of the probability of quantum creation of an open or flat compact universe [19, 27, 30]. I will briefly remind the corresponding results, following [19].

Consider a flat compact universe having the topology of a torus, S_1^3,

$$ds^2 = dt^2 - a_i^2(t)\, dx_i^2 \tag{21}$$

with identification $x_i + 1 = x_i$ for each of the three dimensions. We will assume for simplicity that $a_1 = a_2 = a_3 = a(t)$. In this case the curvature of the universe and the Einstein equations written in terms of $a(t)$ will be the same as in the infinite flat Friedmann universe with metric $ds^2 = dt^2 - a^2(t)\, d\mathbf{x^2}$. In our notation, the scale factor $a(t)$ is equal to the size of the universe in Planck units $M_p^{-1} = 1$.

In order to derive the Wheeler-DeWitt equation [31] for the compact flat toroidal universe, one should first consider the gravitational action

$$S = \int dt\, d^3x\, \sqrt{-g}\, \left(-\frac{1}{2}R + \frac{1}{2}\partial_\mu\phi\, \partial^\mu\phi - V(\phi)\right) \tag{22}$$

and take into account that the volume of the 3D box is equal to a^3. Let us assume for a moment that ϕ is constant, which is the case if the field stays at the top of the potential, or at the dS plateau, as in the models which we discussed here. In this case one can represent the effective Lagrangian for the scale factor as a function of a and $\dot{a}$,

$$L(a) = -3\dot{a}^2 a - a^3 V\ . \tag{23}$$

Finding the corresponding Hamiltonian and using the Hamiltonian constraint $H\Psi(a) = 0$ yields the Wheeler-DeWitt equation

$$\left[\frac{d^2}{da^2} + 12a^4 V\right]\Psi(a) = 0 \tag{24}$$

For large a, the solution of Eq. (24) can be easily obtained in the WKB (semiclassical) approximation, $\Psi \sim a^{-1}\exp[\pm i\frac{2a^3\sqrt{V}}{\sqrt{3}}]$; positive sign corresponds to an expanding universe. This approximation breaks down at $a \lesssim V^{-1/6}$. At that time the size of the universe is much greater than the Planck scale, but much smaller than the Hubble scale $H^{-1} \sim V^{-1/2}$. The meaning of this result, to be discussed below in a more detailed way, is that at $a \gg V^{-1/6}$ the effective action corresponding to the expanding universe is very large, and the universe can be described in terms of classical space and time. Meanwhile at $a \lesssim V^{-1/6}$, the effective action becomes small, the classical description breaks down, and quantum uncertainty becomes large. In other words, contrary to the usual expectations, at $a \lesssim V^{-1/6}$ one cannot describe the universe in terms of a classical space-time even though the size of the universe at $a \sim V^{-1/6}$ is much greater than the Planck size, and the density of matter as well as the curvature scalar in this regime remains small, $R = 4V \ll 1$.

The general solution for Eq. (24) can be represented as a sum of two Bessel functions:

$$\Psi(a) = \beta\sqrt{a}\Big(\mathrm{J}_{-\frac{1}{6}}\Big(\frac{2\sqrt{V}a^3}{\sqrt{3}}\Big) + \gamma\,\mathrm{J}_{\frac{1}{6}}\Big(\frac{2\sqrt{V}a^3}{\sqrt{3}}\Big)\Big), \tag{25}$$

where β and γ are some complex constants, see Fig. 5.

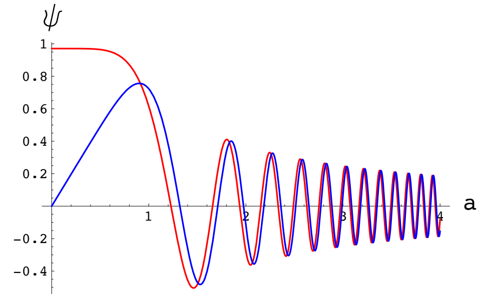

Figure 5: Two eigenmodes of the Wheeler-DeWitt equation for the wave function of the flat compact toroidal universe. The scale factor a is shown in units of $V^{-1/6}$.

Figure 5 confirms that the WKB approximation is not valid at small a, and the "cosmic clock" starts ticking only at $a > V^{-1/6}$.

One can provide an alternative interpretation of this result, without invoking the Wheeler-DeWitt equation. By substituting the classical solution $a = e^{Ht}$ into the effective Lagrangian (23), one finds that the total action of the universe is proportional to $\sqrt{V}a^3(t) \sim \sqrt{V}a^3(t)e^{3Ht}$

For $a < V^{-1/6}$, the action is smaller than 1, so the wave function does not oscillate. Not surprisingly, the total energy of the universe at the critical time when a becomes equal to $V^{-1/6}$ is of the same order as Hawking temperature $T_H = O(H)$, which corresponds to the typical energy of a single quantum fluctuation in dS universe. Thus the universe gradually emerges from nothing, and its wave function does not oscillate until its total energy reaches $O(H)$.

Once the universe grows larger than $a \sim V^{-1/6}$, its action rapidly becomes exponentially large, classical description of the new-born universe becomes possible, and its topology becomes irrelevant due to the magic of inflation.

Of course, if it is so easy to create a homogeneous universe, it may not be too difficult to create an inhomogeneous universe as well. The most important conclusion of the investigation performed in [19, 27, 30] is that the probability of quantum creation of a compact homogeneous inflationary universe with non-trivial topology is not exponentially suppressed. Meanwhile the main result obtained recently in [21, 22] is that even if the new-born universe is grossly inhomogeneous, it typically becomes homogeneous at later stages of the cosmological evolution.

References

[1] A. H. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," Phys. Rev. D **23**, 347 (1981).

[2] A. D. Linde, "A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems," Phys. Lett. **108B**, 389 (1982).

[3] A. D. Linde, "Chaotic Inflation," Phys. Lett. B **129**, 177 (1983).

[4] A. D. Linde, "Initial Conditions For Inflation," Phys. Lett. **162B**, 281 (1985).

[5] A. D. Linde, "Particle physics and inflationary cosmology," Contemp. Concepts Phys. **5**, 1 (1990) [hep-th/0503203].

[6] A. D. Linde, "Eternally Existing Self-Reproducing Chaotic Inflationary Universe," Phys. Lett. B **175**, 395 (1986).

[7] P. A. R. Ade *et al.* [Planck Collaboration], "Planck 2015 results. XX. Constraints on inflation," Astron. Astrophys. **594**, A20 (2016) [arXiv:1502.02114 [astro-ph.CO]].

[8] A. B. Goncharov and A. D. Linde, "Chaotic Inflation in Supergravity," Phys. Lett. B **139**, 27 (1984).

[9] A. A. Starobinsky, "A New Type of Isotropic Cosmological Models Without Singularity," Phys. Lett. B **91**, 99 (1980).

[10] D. S. Salopek, J. R. Bond and J. M. Bardeen, "Designing density fluctuation spectra in inflation," Phys. Rev. **D40**, 1753 (1989).

[11] F. L. Bezrukov and M. Shaposhnikov, "The Standard Model Higgs boson as the inflaton," Phys. Lett. B **659**, 703 (2008) [arXiv:0710.3755 [hep-th]].

[12] R. Kallosh and A. Linde, "Universality Class in Conformal Inflation," JCAP **1307**, 002 (2013) [arXiv:1306.5220 [hep-th]].

[13] S. Ferrara, R. Kallosh, A. Linde and M. Porrati, "Minimal Supergravity Models of Inflation," Phys. Rev. D **88**, no. 8, 085038 (2013) [arXiv:1307.7696 [hep-th]].

[14] R. Kallosh, A. Linde and D. Roest, "Superconformal Inflationary α-Attractors," JHEP **1311**, 198 (2013) [arXiv:1311.0472 [hep-th]].

[15] S. Cecotti and R. Kallosh, "Cosmological Attractor Models and Higher Curvature Supergravity," JHEP **1405**, 114 (2014) [arXiv:1403.2932 [hep-th]].

[16] M. Galante, R. Kallosh, A. Linde and D. Roest, "Unity of Cosmological Inflation Attractors," Phys. Rev. Lett. **114**, no. 14, 141302 (2015) [arXiv:1412.3797 [hep-th]].

[17] R. Kallosh and A. Linde, "Escher in the Sky," Comptes Rendus Physique **16**, 914 (2015) [arXiv:1503.06785 [hep-th]].

[18] A. Ijjas, P. J. Steinhardt and A. Loeb, "Inflationary paradigm in trouble after Planck2013," Phys. Lett. B **723**, 261 (2013) [arXiv:1304.2785 [astro-ph.CO]]; A. Ijjas, P. J. Steinhardt and A. Loeb, "Pop goes the universe," Scientific American, **316**, 32 (February 2017).

[19] A. D. Linde, "Creation of a compact topologically nontrivial inflationary universe," JCAP **0410**, 004 (2004) [hep-th/0408164].

[20] A. Linde, "Inflationary Cosmology after Planck 2013," Les Houches lectures: *100e Ecole d'Ete de Physique: Post-Planck Cosmology* 8 Jul - 2 Aug 2013. Les Houches, France arXiv:1402.0526 [hep-th].

[21] J. J. M. Carrasco, R. Kallosh and A. Linde, "Cosmological Attractors and Initial Conditions for Inflation," Phys. Rev. D **92**, no. 6, 063519 (2015) [arXiv:1506.00936 [hep-th]].

[22] W. E. East, M. Kleban, A. Linde and L. Senatore, "Beginning inflation in an inhomogeneous universe," JCAP **1609**, no. 09, 010 (2016) doi:10.1088/1475-7516/2016/09/010 [arXiv:1511.05143 [hep-th]].

[23] M. Kleban and L. Senatore, "Inhomogeneous Anisotropic Cosmology," JCAP **1610**, no. 10, 022 (2016) [arXiv:1602.03520 [hep-th]].

[24] K. Clough, E. A. Lim, B. S. DiNunno, W. Fischler, R. Flauger and S. Paban, "Robustness of Inflation to Inhomogeneous Initial Conditions," JCAP **1709**, no. 09, 025 (2017) [arXiv:1608.04408 [hep-th]].

[25] R. Kallosh and A. Linde, "Planck, LHC, and α-attractors," Phys. Rev. D **91**, 083528 (2015) [arXiv:1502.07733 [astro-ph.CO]].

[26] R. Kallosh and A. Linde, "Cosmological Attractors and Asymptotic Freedom of the Inflaton Field," JCAP **1606**, no. 06, 047 (2016) [arXiv:1604.00444 [hep-th]].

[27] Y. B. Zeldovich and A. A. Starobinsky, "Quantum Creation Of A Universe In A Nontrivial Topology," Sov. Astron. Lett. **10**, 135 (1984).

[28] O. Heckmann & E. Schucking, in *Handbuch der Physik*, ed. S. Flugge (Springer, Berlin, 1959), Vol. 53, p.515; G.F. Ellis, Gen. Rel. Grav. **2**, 7 (1971); J.R. Gott, "Chaotic Cosmologies," Mon. Not. R. Astron. Soc. **193**, 153 (1980); C.N. Lockhart, B.Misra and I. Prigogine, Phys. Rev. **D25**, 921 (1982); H.V. Fagundes, Phys. Rev. Lett. **51**, 517 (1983).

[29] N. J. Cornish, D. N. Spergel and G. D. Starkman, "Does chaotic mixing facilitate $\Omega < 1$ inflation?" Phys. Rev. Lett. **77**, 215 (1996) [arXiv:astro-ph/9601034].

[30] D. H. Coule and J. Martin, "Quantum cosmology and open universes," Phys. Rev. D **61**, 063501 (2000) [arXiv:gr-qc/9905056].

[31] B. S. DeWitt, "Quantum Theory Of Gravity. 1. The Canonical Theory," Phys. Rev. **160** (1967) 1113.

[32] A. Vilenkin, "Creation of Universes from Nothing," Phys. Lett. **117B**, 25 (1982).

[33] J. B. Hartle and S. W. Hawking, "Wave Function Of The Universe," Phys. Rev. D **28**, 2960 (1983).

[34] A. D. Linde, "The Inflationary Universe," Rept. Prog. Phys. **47**, 925 (1984).

[35] A. Vilenkin, "Quantum Creation Of Universes," Phys. Rev. D **30**, 509 (1984).