Early universe in quantum gravity

Leonardo Modesto[1, *] and Gianluca Calcagni[2, †]

[1] *Department of Physics, Southern University of Science and Technology, Shenzhen 518055, China*
[2] *Instituto de Estructura de la Materia, CSIC, Serrano 121, 28006 Madrid, Spain*

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We present a new, testable picture of the early universe in finite nonlocal quantum gravity, which is conformally invariant at the classical and quantum levels. The high-energy regime of the theory consists of two phases, a conformally invariant trans-Planckian phase and a sub-Planckian or Higgs phase described by an action quadratic in the Ricci tensor and where the cosmos evolves according to the standard radiation-dominated model. In the first phase, all the issues of the hot big bang such as the singularity, flatness, and horizon problems find a universal and simple non-inflationary solution by means of Weyl conformal invariance, regardless of the microscopic details of the theory. In the second phase, once conformal symmetry is spontaneously broken, logarithmic quantum corrections to the action make both the primordial tensor spectrum (from graviton fluctuations) and the scalar spectrum (from thermal fluctuations) quasi scale invariant. Although nonlocal quantum gravity is an explicit realization of this scenario, any finite conformal theory with quadratic limit would yield the same results. In particular, the value of the scalar spectral index only depends on two parameters and is consistent with observations. The theory also predicts a positive tensor spectral index $n_{\rm t}=1-n_{\rm s}$, a fixed tensor-to-scalar ratio $r_{0.05}\gtrsim 0.01$, and a blue-tilted stochastic gravitational-wave background, all universal predictions testable in the immediate or near future.

CONTENTS

I. INTRODUCTION

From 2011 to the present day, a new weakly nonlocal action principle has been proposed and extensively studied in order to overcome the renormalizability issue of the Einstein–Hilbert gravitational theory [1–3]. At the classical level, the dynamics is well defined and the Cauchy problem requires a finite number of initial conditions [4]. At the quantum level, the new theory was first introduced

* lmodesto@sustech.edu.cn
† g.calcagni@csic.es

by Krasnikov in 1987 [5], but the first version fully consistent at the quantum level was due to Kuz'min in 1989 [6]. Indeed, the exponential nonlocal operators employed in [5] are not consistent with power-counting renormalizability and an asymptotically polynomial nonlocality is needed instead [6]. Finiteness at the quantum level for any spacetime dimension was achieved in 2014, when the theory was completed with extra local operators at least cubic in the Ricci tensor [7]. Tree-level unitarity was first pointed out in [5], but proved at any perturbative order in [8, 9]. There, the Cutkosky rules [10] were derived defining loop amplitudes by integrating on purely imaginary internal and external energies. Afterwards, an analytic continuation to real external energies defines the amplitudes in Lorentzian signature, consistently with S-matrix unitarity. The theory cannot be defined directly in Lorentzian signature [11].

Having at our disposal a candidate for an ultraviolet (UV) complete theory of quantum gravity, we now turn to its cosmology and provide a derivation of the quasi scale-invariant primordial tensor and scalar spectra, without introducing any inflationary era driven by an extra scalar degree of freedom. It deserves to be recalled that a UV completion of the Starobinsky inflationary model in nonlocal gravity has been proposed and extensively studied [12–17]. However, a careful look at the perturbative expansion of the dynamics [16] suggests a serious tension between renormalizability and stability, since the form factors (nonlocal operators) of that model were designed to guarantee stability on a de Sitter background, but not on others such as, for example, Starobinsky's quasi-de Sitter solution or generic Friedmann–Lemaître–Robertson–Walker (FLRW) metrics.

We will see that, abandoning the efforts to embed inflation in nonlocal quantum gravity and just using the symmetries of the theory, the well-known problems of the classical hot-big-bang model, namely, the flatness, the horizon, the monopole, the trans-Planckian, and the singularity problems are elegantly and naturally solved by the Weyl conformal invariance of the theory, regardless of the details of the theory beyond the Planck energy. Indeed, what really matters is not the specific model of quantum gravity, or the type of action principle, but the universality class that collects together all the finite theories at the quantum level. The absence of Weyl anomaly turns out to be an extremely powerful tool to solve the problems of classical cosmology mentioned above. When the symmetry is spontaneously broken and the dilaton takes an expectation value equal to the Planck mass, thermal fluctuations generate a primordial scalar spectrum consistent with the low-multipole cosmic microwave background (CMB) spectrum. The primordial scalar and tensor spectra are nearly but not exactly scale invariant thanks to logarithmic quantum-gravity corrections to the correlation functions. Once again, here the conformal symmetry is anomaly-free and, thus, it is manifest at the classical as well as at the quantum level, but spontaneously broken at the Planck scale. Therefore, such symmetry offers a natural solution to the above problems regardless of the details of the Lagrangian, and the physics of the cosmos we will discuss in this paper takes a universal character. In particular, the tensor spectrum turns out to have interesting properties which depend only very weakly on the details of the theory: it is blue-tilted and its amplitude is large enough to be detectable by upgrades of present-generation experiments.

Let us mention that conformal or scale invariance has already been invoked in the literature as an alternative to inflation [18–22]. These cosmological models are quite different from the one presented in this paper, and the main points of departure are in the premiss and the conclusion. In our case, behind conformal invariance we do have an explicit theory of gravity and the other fundamental interactions. In turn, this leads to sharper and unexplored predictions in the tensor sector.

The paper is organized as follows. In Sec. II, we review a simple version of nonlocal quantum gravity with matter and the role of finiteness in enforcing Weyl conformal symmetry. We also introduce for the first time the details of an often-used form factor, here simplified further (Sec. II A), and discuss the scales (Sec. II B) and the one-loop quantum corrections (Sec. II C) characterizing the theory. In Sec. III, we describe the conformal trans-Planckian phase and the Higgs or sub-Planckian phase. In Sec. IV, we show how conformal symmetry solves the problems of the standard hot-big-bang model in the trans-Planckian phase. Cosmological background solutions and their relation with the flatness and horizon problems are presented in Sec. V. In Sec. VI, we compute the primordial tensor and scalar spectra generated during the sub-Planckian phase, their indices, and the tensor-to-scalar ratio. Conclusions are drawn in Sec. VII, where future extensions of the results are also discussed. Several appendices contain technical material. A compact presentation of these findings can be found in a shorter companion paper more focused on primordial gravitational waves [23].

II. NONLOCAL QUANTUM GRAVITY

A nonlocal gravitational theory coupled to matter has been introduced in [24, 25], consistently with the following properties: (i) unitarity at any perturbative order in the loop expansion [24], (ii) same linear and nonlinear stability properties and same tree-level scattering amplitudes as for the Einstein–Hilbert theory [26, 27], (iii) same macro-causality properties [28], and (iv) super-renormalizability or finiteness at the quantum level [29].[1] Therefore, and although we are only beginning to tap its

[1] Other nonlocal models for gauge theories or extending the Standard Model of particle physics were proposed in [30] and [31], respectively.

potential, the proposal in [24] has the right requirements to be a unified theory of all fundamental interactions.

We work in $(-,+,\cdots,+)$ signature and define the reduced Planck mass

$$M_{\rm Pl}^2 := \frac{1}{8\pi G}\,, \tag{1}$$

where G is Newton's constant.

The full action in D topological dimensions reads

$$S[\Phi_i] = \int d^D x \sqrt{|g|}\left[\mathcal{L}_{\rm loc} + E_i\, F^{ij}(\boldsymbol{\Delta})\, E_j + V(E_i)\right], \tag{2}$$

$$S_{\rm loc} = \int d^D x \sqrt{|g|}\,\mathcal{L}_{\rm loc}\,, \tag{3}$$

$$\mathcal{L}_{\rm loc} = \frac{M_{\rm Pl}^2}{2} R + \mathcal{L}_{\rm m}(\Phi_i)\,, \tag{4}$$

$$E_i := \frac{\delta S_{\rm loc}}{\delta \Phi_i(x)}\,, \tag{5}$$

$$\boldsymbol{\Delta}_{ki} := \frac{\delta E_i}{\delta \Phi_k} = \frac{\delta^2 S_{\rm loc}}{\delta\Phi_k \delta\Phi_i}\,, \tag{6}$$

where the "potential" $V(E_i)$ is a collection of local operators at least cubic in the local "equations of motion" E_i and Φ_i is any field in the theory, including the spacetime metric $g_{\mu\nu}$, the dilaton Φ, fermions ψ and gauge fields A_μ:

$$\Phi_i \in \{g_{\mu\nu}, \Phi, \psi, A_\mu, \dots\}\,. \tag{7}$$

The functional variations $\delta/\delta\Phi_i$ contain a weight factor $1/\sqrt{|g|}$, so that $\delta\Phi_j(x)/\delta\Phi_i(y) = \delta_{ij}\delta^D(x-y)/\sqrt{|g|}$. For the local Einstein–Hilbert Lagrangian (4) with local matter Lagrangian $\mathcal{L}_{\rm m}$, the local equations of motion for the metric are

$$E_{\mu\nu} = \frac{1}{2}\left(M_{\rm Pl}^2 G_{\mu\nu} - T_{\mu\nu}\right) = 0\,, \tag{8}$$

where $T_{\mu\nu} := -2\delta S_{\rm m}/\delta g^{\mu\nu}$ is the energy-momentum tensor.

In order to make explicit the hidden conformal symmetry of the theory (2), we make the following replacements in the action:

$$g_{\mu\nu} =: \phi^2\,\hat g_{\mu\nu}\,,\quad \Phi = \frac{\hat\Phi}{\phi}\,,\quad \psi = \frac{\hat\psi}{\phi^{\frac{3}{2}}}\,,\quad A_\mu = \hat A_\mu\,. \tag{9}$$

All the fields $\hat g_{\mu\nu}$, $\hat\Phi$, $\hat\psi$, and $\hat A_\mu$ are rescaled by the dimensionless field ϕ in a such way that combined terms take an invariant form under Weyl transformations when ϕ is treated like a scalar,

$$\hat g'_{\mu\nu} = \Omega^2(x)\,\hat g_{\mu\nu}\,,\qquad \phi' = \Omega^{-1}(x)\,\phi\,,\qquad \dots\,. \tag{10}$$

Once the fields (9) are replaced in the action (2) and after the rescaling $\phi \to \phi/M_{\rm Pl}$, so that the dilaton acquires dimensionality $[\phi] = (D-2)/2$, all the coupling constants of the theory become dimensionless.

The potential $V(E_{\mu\nu})$ is needed to guarantee the finiteness of the theory in even dimensions where the presence of the nonlocal form factor is sufficient to make any loop amplitude finite except at the one-loop level. On the other hand, in odd dimensions the theory has no one-loop divergences as a particular feature of the dimensional regularization scheme. All the details are provided in [7] for the vacuum theory and in [29] with matter fields. Notice that, due to the presence of the dilaton ϕ, the theory does not contain any mass scale and, hence, Eq. (2) is conformally invariant classically [24]. Moreover, since the theory is finite at the quantum level, the beta functions vanish and the Weyl symmetry is anomaly free.

Finally, the form factor $F(\boldsymbol{\Delta})$ is defined in terms of an entire analytic function $\mathrm{H}(\boldsymbol{\Delta})$ of the Hessian operator $\boldsymbol{\Delta}$,

$$2\boldsymbol{\Delta}F(\boldsymbol{\Delta}) := e^{\bar{\mathrm{H}}(\boldsymbol{\Delta})} - 1\,,\qquad \bar{\mathrm{H}}(\boldsymbol{\Delta}) := \mathrm{H}(\boldsymbol{\Delta}) - \mathrm{H}(0)\,. \tag{11}$$

Thanks to Eq. (11), the gravitational equations of motions of the nonlocal theory (2) can be written as

$$\left[e^{\bar{\mathrm{H}}(\boldsymbol{\Delta})}\right]_{\mu\nu}^{\ \ \sigma\tau} E_{\sigma\tau} + O(E_{\mu\nu}^2) = 0\,. \tag{12}$$

Since, by construction, the form factor (11) does not possess any poles or zeros in the whole complex plane at finite distance, any solution of the local theory (8) is also a solution of Eq. (12).

The function H actually depends on the dimensionless quantity $z \propto \boldsymbol{\Delta}/(M_{\rm Pl}^2 M_*^2)$, where M_* is an energy scale introduced to compute the dimensionality of the operator $\boldsymbol{\Delta}$.[2] However, according to the discussion above, the theory (2) is conformally invariant and the scale Λ_* can be reabsorbed in the dilaton field ϕ. With a slight but innocuous abuse of notation, we will freely interchange $\mathrm{H}(\boldsymbol{\Delta})$ with $\mathrm{H}(z)$ without further notice. The definition of H is

$$\mathrm{H}(z) = \int_0^{p(z)} dw\, \frac{1-e^{-w}}{w} = \gamma_{\rm E} + \Gamma[0, p(z)] + \ln p(z)\,, \tag{13}$$

where $\gamma_{\rm E} \approx 0.577$ is the Euler–Mascheroni constant, Γ is the upper incomplete gamma function, and $p(z)$ is a polynomial of degree $n+1$ in the variable z:

$$p(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_{n+1} z^{n+1}\,,\quad a_i \in \mathbb{R}\,. \tag{14}$$

The form factor (13) is actually a known special function, dubbed $\mathrm{Ein}(z)$ and called complementary exponential integral [32, formula 6.2.3]. The logarithmic divergence at $z=0$ is exactly canceled out by the divergence of $\Gamma(0,z)$ and, indeed, H is finite and real on the whole real axis. Likewise, its exponential

$$e^{\mathrm{H}(z)} = e^{\gamma_{\rm E} + \Gamma[0,p(z)]}\, p(z) \tag{15}$$

[2] The cosmologist should not confuse z with the redshift (which we will never use in this paper) or H with the Hubble parameter H.

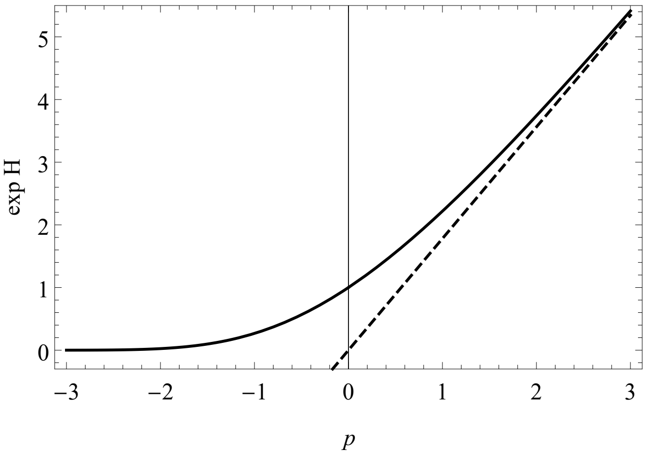

FIG. 1. Form factor (15) (solid curve) and its asymptotic limit (17).

is well defined and positive definite everywhere on the real axis (Fig. 1). Since $\exp \mathrm{H} > 0$, propagators do not change sign and there is no unitarity issue.

The only constraint we place on the polynomial is that its real part be positive definite when $z \to \pm\infty$:

$$\mathrm{Re}\, p(z \to \pm\infty) > 0\,. \tag{16}$$

In this case, in the UV limit ($z \gg 1$)

$$e^{\mathrm{H}(z)} \overset{\mathrm{UV}}{\simeq} e^{\gamma_{\mathrm{E}}} p(z)\,, \tag{17}$$

which explains the name *asymptotically polynomial* usually given to this class of form factors. On the other hand, in the infrared (IR; $z \ll 1$), the analyticity of the form factor provides an effective expansion of the action in higher-derivative operators.

The form factor (13) includes those proposed by Kuz'min [6], Tomboulis [33], and Modesto [1], but it is presented here in a simpler way, with fewer parameters and a more minimalistic constraint (16).

A. Choice of form factor

For the sake of simplicity, we will work with a quartic polynomial:

$$p(z) = a_0 + a_1 z + a_4 z^4\,, \qquad a_0,\, a_1,\, a_4 \in \mathbb{R}\,. \tag{18}$$

Other quartic polynomials are possible, e.g., $p(z) = (a_0 + a_1 z + a_2 z^2)^2$, but Eq. (18) will suffice. This simple expression suits the minimal super-renormalizable (or finite) theory in $D = 4$ dimensions, for which the polynomial has degree four. We removed the monomials z^2 and z^3 since they only affect the transient regime between the UV and the IR and, therefore, cannot change the results in this paper qualitatively.

In order to satisfy the condition (16), $a_4 > 0$. We fix this coefficient to $a_4 = 1$ noting that the mass scale M_* in z can be redefined as $\Lambda_* := M_*/a_4^{1/4}$. For later convenience, we redefine $a_0 =: b_0$ and $b := -a_1 > 0$, where the sign of b is chosen on physical grounds that will become clear later. The polynomial (18) is rewritten as

$$p(z) = b_0 - b\, z + z^4\,. \tag{19}$$

A most important property we would like to obtain is to have an intermediate energy regime (still in the UV) where the linear term in Eq. (19) dominates over the other two. In the UV limit, this happens when

$$\frac{e^{\mathrm{H}(z)}}{e^{\mathrm{H}(0)}} \overset{\mathrm{UV}}{\simeq} e^{\tilde{\gamma}_{\mathrm{E}}} p(z) \overset{(19)}{=} e^{\tilde{\gamma}_{\mathrm{E}}}(b_0 - b\, z + z^4)$$
$$\simeq e^{\tilde{\gamma}_{\mathrm{E}}}(-b\, z)\,, \tag{20}$$
$$\tilde{\gamma}_{\mathrm{E}} := \gamma_{\mathrm{E}} - \mathrm{H}(0)\,, \tag{21}$$

which holds for $b_0 \ll b\,|z|$ and $b\,|z| \gg |z^4|$. For consistency, when $b_0 < 1$, the first inequality must be supplemented by the condition that we are still in the asymptotically polynomial UV regime (17), i.e.,

$$\frac{\max(1, b_0)}{b} \ll |z| \ll b^{\frac{1}{3}}. \tag{22}$$

This condition is satisfied, for instance, when $1 < b_0 \ll b$, or when $b_0 = 0$, $b \gtrsim O(1)$, and $z \gtrsim O(1)$. In fact, one can even relax the strong inequalities in (22) and consider values of z near the upper and lower bounds. In the worst case, $b_0 = O(bz) = z^4$ and Eq. (20) would acquire a factor of 3. However, this factor is not very important because, later on, we will see that b should be large in order to satisfy CMB observations. In other words, any breaking of the approximation (20) would not lead to any inconsistency but it would alter the relation between the theoretical value and the experimental value of the free parameter b, without changing its order of magnitude. We will comment on physical explanations of large values of b in the conclusions.

B. Scales and parameters

Let us now comment on the energy scale Λ_* in the polynomial. More generally, a sort of naturalness principle would require the theory to feature multiple scales through the coefficients in the polynomial (14). Moreover, *a priori* all the monomials in Eq. (14) should be present. In order to make the above scale dependence explicit, we can rewrite Eq. (14) in the form

$$p(\Box) = b_0 + b_1 \frac{\Box}{\Lambda_1^2} + \cdots + \left(b_n \frac{\Box}{\Lambda_n^2}\right)^n\,,$$
$$\Lambda_1 \ll \Lambda_2 \ll \cdots \ll \Lambda_n\,, \tag{23}$$

where each power of the Laplace–Beltrami operator $\Box$ contributes at a different scale and $b_i = O(1)$ are dimensionless positive or negative constants. As said above, the theory is conformally invariant, so that all the above scales Λ_i are proportional to the expectation value of the dilaton field multiplied by dimensionless constants related to the coefficients a_i in Eq. (14), which can take any value in order to introduce a hierarchy of scales. In Eq. (23), the $\Box$ operator can be replaced by the Hessian $\boldsymbol{\Delta}$ without affecting the arguments presented here.

However, in $D=4$ dimensions all the physical implications of the theory in the trans-Planckian as well as in the sub-Planckian regime are captured by the simple polynomial (19), which we can rewrite as

$$\begin{aligned} p(\Box) &= b_0 + b_1\frac{\Box}{\Lambda_1^2} + \left(b_4\frac{\Box}{\Lambda_4^2}\right)^4 \\ &=: b_0 - b\frac{\Box}{\Lambda_0^2} + \left(\frac{\Box}{\Lambda_*^2}\right)^4 , \end{aligned}$$

where now Λ_0 and Λ_* can be of the same order of magnitude and b can be large. As it will be clear later on, in $D=4$ dimensions the quartic term is responsible for exact scale invariance (Harrison–Zel'dovich spectra), while the observed quasi-scale-invariant scalar spectrum requires the presence of the monomial linear in $\Box$ and with negative coefficient.

We see that the theory (2) contains three scales: $M_{\rm Pl}$, Λ_0, and Λ_*. However, if $b\gg 1$ we can make the identification $\Lambda_0=\Lambda_*$ without any loss of generality. Indeed, b simply defines the ratio between Λ_0 and Λ_*:

$$\boxed{p(\Box) = b_0 - b\frac{\Box}{\Lambda_*^2} + \left(\frac{\Box}{\Lambda_*^2}\right)^4 .} \tag{24}$$

In the case of gravity, one should make the replacement $\Box\to 4\boldsymbol{\Delta}/M_{\rm Pl}^2$:

$$p(\boldsymbol{\Delta}) = b_0 - b\frac{4\boldsymbol{\Delta}}{M_{\rm Pl}^2\Lambda_*^2} + \left(\frac{4\boldsymbol{\Delta}}{M_{\rm Pl}^2\Lambda_*^2}\right)^4 , \tag{25}$$

where $[\boldsymbol{\Delta}]=4$. Therefore, the most general theory has two mass scales and one dimensionless free parameter:

$$M_{\rm Pl}\,,\qquad \Lambda_*\,,\qquad b\,. \tag{26}$$

We will come back on the values of Λ_* and b later on, but for the time being we will leave them unspecified.

We can gain further understanding of the role of each term in the action in $D>4$ dimensions. After the spontaneous symmetry breaking of conformal invariance, the effective theory amounts to a collection of higher-derivative operators characterizing the following extra-dimensional renormalizable generalization of Stelle gravity [34, 35]:

$$\mathcal{L}\sim R + \frac{\mathcal{R}^2}{\Lambda_0^2} + \mathcal{R}\frac{\Box}{\Lambda_2^4}\mathcal{R} + \dots + \mathcal{R}\frac{\Box^{\frac{D-4}{2}}}{(\Lambda_{D-3})^{D-4}}\mathcal{R}\,, \tag{27}$$

where $\mathcal{R}$ stands for $R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}$, while the ellipsis indicates $O(\mathcal{R}^3)$ operators which appear when $D>4$. When $D=4$, only the first two terms contribute with, respectively, two and four derivatives, as in Eqs. (24) and (25). Therefore, the observed quasi-scale-invariant spectrum can be traced back to the renormalizable local theory hidden inside the finite theory (2). It deserves to be noticed that, out of all the operators in Eq. (27), only the one of dimension D matters for reproducing a scale-invariant spectrum, which essentially stems from a propagator $\sim 1/k^D$. Operators of lower dimensionality are relevant in the IR regime in order to reproduce Newton's potential, while operators of higher dimensionality are crucial for the finiteness of the theory and conformal invariance. A small deviation from exact scale invariance will come from quantum logarithmic corrections.

We conclude this subsection by noting that the theory is defined by a finite number of parameters. This is one of the reasons why it is under control and predictive both at the classical and the quantum level despite having an infinite number of derivatives.

C. Quantum corrections

It has become progressively clear that even the best behaved nonlocal quantum field theories have problems if formulated directly in Lorentzian signature [11]. If, instead, one defines the Feynman diagrams with imaginary external and internal energies (Euclidean signature) and then analytically continues external momenta to real energies after integrating, then loop integrals can be performed consistently and the theory admits a unique Lorentzian limit [8, 9, 36]. Here we will not need this sophisticated procedure because we will be mainly interested in the UV limit of the theory, where it is asymptotically local. We mention it nevertheless to reassure the reader that the quantum theory is under control at all scales.

According to the results of Sec. II A, we can ignore the constant term in the form factor when the energy $E:=k^0\gtrsim\Lambda_{\rm hd}$, where

$$\Lambda_{\rm hd} := \sqrt{\frac{\max(1,b_0)}{b}}\,\Lambda_*\,. \tag{28}$$

At this scale, the action (2) reduces to a particular local theory quadratic in the curvature which, in vacuum, is Stelle gravity with a special choice of coefficients (Appendix A 1). The subscript "hd" is a reminder that we are working in this higher-derivative limit. Then, one can show that, in the intermediate UV regime

$$\boxed{\Lambda_{\rm hd}\lesssim E\lesssim\Lambda_*\,,} \tag{29}$$

[which comes from Eq. (22) where the left inequality is weak for the reasons argued above, while the right inequality is weak because $b\gtrsim O(1)$] the nonlocal Lagrangian in Eq. (2) acquires logarithmic quantum corrections at one loop which (Appendix A 2):

$$\mathcal{L}=\frac{M_{\rm Pl}^2}{2}R+\mathcal{L}_{\rm m}+E_{\mu\nu}F^{\mu\nu\sigma\tau}E_{\sigma\tau}+V(E_{\mu\nu})+\mathcal{L}_Q\,, \tag{30a}$$

$$\mathcal{L}_Q=\beta_R R\ln\left(-\frac{\Box}{\Lambda_*^2\delta_0^2}\right)R+\beta_{\rm Ric}R_{\mu\nu}\ln\left(-\frac{\Box}{\Lambda_*^2}\right)R^{\mu\nu}, \tag{30b}$$

which are exactly the same quantum corrections of Stelle's theory upon the identification of the cut-off scale $\Lambda_{\rm UV}$ of Stelle quantum gravity with the fundamental scale of the nonlocal theory, $\Lambda_{\rm UV}\propto\Lambda_*$. Indeed, the higher-derivative operators [depending both on the potential $V(E_{\mu\nu})$ and on the leading terms in the polynomial $p(z)$] can be seen as regulators for quadratic gravity, and, in the limit $E\ll\Lambda_*$, the larger nonlocality scale Λ_* turns out to be proportional to the usual quantum-field-theory cut-off $\Lambda_{\rm UV}$. Here we set $\Lambda_{\rm UV}=\Lambda_*\delta_0$ for the R-R term and $\Lambda_{\rm UV}=\Lambda_*$ for the $R_{\mu\nu}$-$R_{\mu\nu}$ term, where δ_0 is a dimensionless constant that should be fixed comparing the renormalization-group-invariant scales of Stelle's theory with the nonlocal scale Λ_*. We will not need to perform this explicit calculation to get our main results. Note that the logarithmic nonanalyticity of the quantum corrections is exclusively due to the Landau singularities of the one-loop amplitude, which are the same of the local theory.

Finally, the coefficients β_R and $\beta_{\rm Ric}$ are nothing but the finite numerical constants coming in front of the quantum corrections. *They are not beta functions*, since the quantum nonlocal theory is divergence-free and there are no counterterms.

The tree-level graviton propagator for the theory (2) with form factor (15) and polynomial (25) can be found in Appendix B 2, while the graviton propagator for the quantum effective Lagrangian (30) is calculated in Appendix B 3.

For the theory (2), the term (30b) is correct only in the intermediate regime (29), while at higher energies $E\gtrsim\Lambda_*$ quantum corrections differ substantially from Stelle's theory. Some scalar-field examples are given in [37–39]. In particular, at energy scales larger than any scale present in the theory, preliminary results show that it should be possible to choose $V(E_{\mu\nu})$ in order to have amplitudes that fall off safely to zero in the UV after resummation [39]. The other option is a strongly interacting theory in the UV but, since in the deep UV regime we are actually in the conformal phase, high-energy physics will be affected only marginally by the details of the potential $V(E_{\mu\nu})$.

III. GRAVITY AT HIGH ENERGY

The physics of the early Universe consists of a conformally invariant trans-Planckian phase and a Higgs Planckian phase in which Weyl conformal symmetry is spontaneously broken. The transition between the two phases is driven by the dilaton field ϕ in Eq. (9), which we can rewrite as

$$\phi\equiv\frac{M_{\rm Pl}}{\sqrt{2}}+\varphi\,, \tag{31}$$

where φ is the Goldstone boson associated with the spontaneous symmetry breaking of conformal invariance. For $\varphi\gg M_{\rm Pl}$ (high energy $E\gg M_{\rm Pl}$), $\phi\simeq\varphi$ and the action is Weyl invariant under the transformation (10), while for $\varphi\ll M_{\rm Pl}$, $\phi\simeq M_{\rm Pl}/\sqrt{2}$ and the conformal symmetry is spontaneously broken by the vacuum solution.

A. Trans-Planckian conformal phase

The trans-Planckian regime is described by a conformally invariant phase. The theory is manifestly Weyl invariant, the dilaton ϕ has an exactly constant potential $U(\phi)=0$, and the correlation functions of all fields are identically zero as a consequence of conformal symmetry. Indeed, consider an arbitrary set of operators $O_i(x)$ that transform under Weyl symmetry with conformal weight Δ_i,

$$O_i'(x)=\Omega^{\Delta_i}(x)\,O_i(x).$$

The vacuum partition function (Euclidean path integral)

$$Z_0:=\int[\mathcal{D}O]\,\exp(-S[O])$$

is invariant under a conformal transformation, since both the action $S[O_1,O_2,\dots]$ and the functional measure $[\mathcal{D}O]=\prod_i[dO_i]$ are invariant by construction: $S[O']=S[O]$, $[\mathcal{D}O']=[\mathcal{D}O]$. Define the n-point correlation function as

$$\langle O_1(x_1)\dots O_n(x_n)\rangle:=\frac{1}{Z_0}\int[\mathcal{D}O]\,O_1(x_1)\dots O_n(x_n)\times e^{-S[O]}, \tag{32}$$

so that $Z_0=\langle 1\rangle$. Renaming first the dummy integration variables $O_i\to O_i'$, then performing a change of variables via the conformal transformation $O_i'(x)=\Omega^{\Delta_i}(x)\,O_i(x)$, and finally using the property of conformal invariance for the integration measure and the action, one has [40, Sec. 2.4.3]

$$\begin{aligned}\langle O_1(x_1)\dots O_n(x_n)\rangle&\\ &\stackrel{O_i\to O_i'}{=}\frac{1}{Z_0}\int[\mathcal{D}O']\,O_1'(x_1)\dots O_n'(x_n)\,e^{-S[O']}\\ &\stackrel{O_i'=\Omega^{\Delta_i}O_i}{=}\frac{1}{Z_0}\int[\mathcal{D}O']\,O_1'(x_1)\dots O_n'(x_n)\,e^{-S[O']}\\ &=\frac{1}{Z_0}\int[\mathcal{D}O]\,O_1'(x_1)\dots O_n'(x_n)\,e^{-S[O]}\\ &=\langle O_1'(x_1)\dots O_n'(x_n)\rangle\\ &=\Omega^{\Delta_1}(x_1)\dots\Omega^{\Delta_n}(x_n)\langle O_1(x_1)\dots O_n(x_n)\rangle\,,\end{aligned} \tag{33}$$

where in the last line we used the fact that $\Omega(x) \neq \Omega[O(x)]$ is a function of the coordinates but not of the fields. Since Ω is not a constant, Eq. (33) implies

$$\langle O_1(x_1)\dots O_n(x_n)\rangle = 0, \quad \text{for} \quad \Delta_1 \neq 0\,,\dots,\,\Delta_n \neq 0\,. \tag{34}$$

According to Eq. (10), for the metric and the graviton $\Delta_{\hat{\boldsymbol{g}},\hat{\boldsymbol{h}}} = 2$, while for the dilaton $\Delta_\phi = -1$. Here, boldface symbols stand for rank-2 tensors. On the other hand, correlation functions involving only fields with $\Delta_i = 0$ (for example, photons or gauge bosons) can be nonzero.

Notice that the quantum corrections do not affect the propagator in the conformal phase because the logarithmic operators in (30b) are of the form

$$\ln(-\mathrm{f}^{-2}\Box_{\boldsymbol{g}})\big|_{\boldsymbol{g}=\phi^2\hat{\boldsymbol{g}}}\,, \qquad \mathrm{f} = \frac{\sqrt{2}\Lambda_*}{M_{\rm Pl}}\,, \tag{35}$$

and they appear together with the dilaton in the conformal phase. Therefore, operators like

$$\mathcal{R}(\boldsymbol{g})\ln(-\mathrm{f}^{-2}\Box_{\boldsymbol{g}})\,\mathcal{R}(\boldsymbol{g})\Big|_{\boldsymbol{g}=\phi^2\hat{\boldsymbol{g}}}$$

contribute to three-point functions (three gravitons or two gravitons and a dilaton) but are not involved in perturbative two-point functions. Only when the Weyl symmetry is broken spontaneously in the Higgs phase do quantum corrections quadratic in the curvature tensors take part in the graviton two-point function (see next subsection).

How high can energies be in this phase? As explained in Eq. (26), in this theory there are in total three scales: $M_{\rm Pl}$, $\Lambda_*/\sqrt{b}$, and Λ_*. Λ_* is a free scale. $M_{\rm Pl}$ is another scale independent of Λ_* and related to the expectation value of the dilaton field $\langle\phi\rangle = M_{\rm Pl}/\sqrt{2}$, which is uniquely fixed after conformal symmetry is spontaneously broken and is given by the measured Newton's constant $G = (8\pi M_{\rm Pl}^2)^{-1}$ when reading it off from the nonminimally coupled action, obtained from Eq. (4) after the transformation (9) ($M_{\rm Pl}^2 R/2 \leftrightarrow \phi^2 R$). However, in the conformal phase no scale larger than the Planck mass can be probed because, in order to observe such scale, we would need a test particle (or a classical perturbation) having a wavelength λ shorter than the scale at which the Weyl symmetry is spontaneously broken, in this case $\ell_{\rm Pl} \coloneqq M_{\rm Pl}^{-1}$. However, in the conformal phase, when $\lambda < \ell_{\rm Pl}$ we can always make it larger by means of a conformal rescaling. In other words, any wavelength shorter than the Planck length can be stretched to the Planck length or above with a conformal transformation. This argument is qualitative and does not fix an absolute upper bound on the energy. Nevertheless, taking as a reference the expectation value of ϕ at the scale of symmetry breaking, we see that this absolute upper bound should be of order of $M_{\rm Pl}$:

$$\Lambda_* \leqslant M_{\rm Pl}\,, \qquad \ell_* \geqslant \ell_{\rm Pl}\,, \tag{36}$$

where $\ell_* \coloneqq \Lambda_*^{-1}$. We will get a weaker relation outside the conformal phase in Sec. VI A, consistently with perturbative quantum field theory. However, we will remain as general as possible and fix Λ_* at or near the Planck mass only at the end of the paper.

B. Sub-Planckian Higgs phase

The sub-Planckian phase takes place when the magnitude of the Goldstone boson is much smaller than the Planck mass. In this phase, the action is obtained simply replacing Eq. (31) into (2) for $\varphi \ll M_{\rm Pl}$ or, equivalently, imposing the unitary gauge to fix $\varphi = 0$. The dilaton potential is flat and there are an infinite number of vacua. Symmetry breaking occurs when one such vacuum is selected and the field takes the expectation value $\langle\phi\rangle = M_{\rm Pl}/\sqrt{2}$. This specific value, which has no special meaning during the conformal phase, is determined *a posteriori* by experiments.[3]

In this phase, we are in the energy regime (29) and the first monomial in Eq. (25) dominates in the action (2), which in $D = 4$ turns into (Appendix A 2)

$$S_{\Lambda_{\rm hd}} = -\frac{e^{\tilde{\gamma}_{\rm E}} M_{\rm Pl}^2\, b}{2\Lambda_*^2}\int d^D x\sqrt{|g|}\,R_{\mu\nu}R^{\mu\nu} + S_{\rm m}\,, \tag{37}$$

which must be augmented by the quantum corrections terms (30b). In Eq. (37), $S_{\rm m}$ represents the four-derivative matter-matter and matter-gravity contributions coming from Eq. (2).

As mentioned in Sec. III A, the logarithmic quantum corrections contribute to the two-point correlation function of the graviton when the conformal symmetry is spontaneously broken, i.e., for $\langle\varphi\rangle \lesssim M_{\rm Pl}$. Indeed, expanding the quantum log corrections for small $\varphi/M_{\rm Pl}$ we get

$$\begin{aligned}\ln\left(-\mathrm{f}^{-2}\Box_{\phi^2\hat{\boldsymbol{g}}}\right) &= \ln\left[-\mathrm{f}^{-2}\Box_{(M_{\rm Pl}/\sqrt{2}+\varphi)^2\hat{\boldsymbol{g}}}\right]\\ &= \ln\left(\frac{-2\Box_{\hat{\boldsymbol{g}}}}{\mathrm{f}^2 M_{\rm Pl}^2}\right)\left[1+O\left(\frac{\varphi}{\mathrm{f}M_{\rm Pl}}\right)\right]\\ &= \ln\left(\frac{-\Box_{\hat{\boldsymbol{g}}}}{\Lambda_*^2}\right)+O\left(\frac{\varphi}{\Lambda_*}\right),\end{aligned} \tag{38}$$

[3] Sometimes it has been claimed that setting $\phi = \phi_0 = $ const in conformal gravity might not correspond to a spontaneous breaking of Weyl symmetry because one can perform a conformal transformation $\Omega = (\phi/\phi_0)^w$ (where w is a constant) such that the Lagrangian with $\phi = \phi_0$ is transformed back to a conformally invariant Lagrangian [41, 42]. According to this reasoning, $\phi = \phi_0$ would be just a gauge choice not affecting the symmetry of the system in any meaningful way. Although this is mathematically correct, it misses the elementary particle-physics point that, by definition, spontaneous symmetry breaking entails a vacuum choice after which no symmetry transformation is possible. A classic example is the choice of vacuum in the $U(1)$–invariant set of vacua of the Mexican-hat potential for the Higgs field $\phi = \Phi\exp(i\theta)$, corresponding to a choice of phase θ. More precisely, the total Lagrangian does retain the symmetry but such symmetry is broken for perturbations around the vacuum. The argument of [41, 42] does not take these perturbations φ of the dilaton into account.

which coincides with the log operator in Eq. (30b) up to terms $O(\varphi/\Lambda_*)$ that do not enter the two-point function of the graviton. In Eq. (38), the first contribution, independent of the Goldstone boson, will give rise to the quasi-scale invariant CMB scalar spectrum, thus providing an authentic quantum origin of primordial perturbations (Sec. VI).

IV. SOLUTIONS TO THE PROBLEMS OF THE HOT BIG BANG

The conformal phase described in Sec. III A can serve as an alternative to inflation to solve all the main problems of the standard hot-big-bang model of the early universe: the big-bang or singularity problem, the flatness problem, the horizon problem, and the monopole problem, plus the trans-Planckian problem. Since perturbations are generated after the conformal phase, the reason why CMB temperature anisotropies exist and are as small as $\delta T/T \sim 10^{-5}$ finds an answer in the Higgs phase described in Sec. III B, as we will see later. In this section, we will discuss the first stage of this two-step scenario alternative to inflation.

A. Singularity problem and past completeness

In the conformal phase, the theory (2) in terms of the fields (9) is invariant under the Weyl transformation (10). Therefore, for any specific transformation (10) with given rescaling $\Omega^2 = S(x)$, both the pair $(\hat{g}_{\mu\nu}, \phi)$ and the rescaled fields

$$\hat{g}^*_{\mu\nu} := S(x)\,\hat{g}_{\mu\nu}\,, \qquad \phi^* := S^{-\frac{1}{2}}(x)\,\phi\,, \tag{39}$$

solve the equations of motion exactly. In particular, as evident from Eq. (12), the pair $(\hat{g}_{\mu\nu}, \phi) = (\hat{g}^{\rm FLRW}_{\mu\nu}, M_{\rm Pl}/\sqrt{2})$ is an exact solution of the equations of motion for the theory (2), where $\hat{g}^{\rm FLRW}_{\mu\nu}$ is the flat FLRW metric in conformal time $\tau = \int dt/a(t)$ (where t is proper time) defined by the line element

$$\begin{aligned} d\hat{s}^2_{\rm FLRW} &= \hat{g}^{\rm FLRW}_{\mu\nu} dx^\mu dx^\nu \\ &= a^2(\tau)\left[-d\tau^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2\right], \end{aligned} \tag{40}$$

where a is the cosmological scale factor. If we take the rescaling $S(x) = S(\tau) = 1/a^2(\tau)$, the solution (39) reads

$$\hat{g}^*_{\mu\nu} = \eta_{\mu\nu}\,, \qquad \phi^* = \frac{M_{\rm Pl}}{\sqrt{2}}\,a(\tau)\,, \tag{41}$$

which is Minkowski spacetime in the presence of a nontrivial dilaton profile. In other words, in a conformally invariant theory the cosmic expansion can be reinterpreted as a manifestation of conformal invariance and, in particular, as the dynamics of the dilaton field, while leaving physical predictions unaltered.[4]

However, as proved in Appendix C, the dilaton decouples from the geodesic equation of massless particles. Therefore, photons and any other massless particles move in Minkowski spacetime, which is geodesically complete (Appendix C 2). Indeed, calling λ the affine parameter along the geodesic and ξ_0 a conserved scalar quantity made of one of the Killing vectors, the main geodesic equation

$$\xi_0 = -\frac{d\tau}{d\lambda} \tag{42}$$

is well defined for $\tau \in (-\infty, +\infty)$ and $\lambda \in (-\infty, +\infty)$, contrary to the analog geodesic equation (C19) in the FLRW background in Einstein gravity.

It is important to stress that the solution of the singularity problem was illustrated with a particular background but it is not, and in fact cannot, be based on any background at all, thanks to conformal invariance. To understand this point, let us recall that the singularity issue is not whether a particular background is geodesically complete or not, but whether broad classes of spacetimes (for instance, with a cosmological expansion or with spherical symmetry) are geodesically complete, either in general or in a given theory of gravitation. In particular, one can discover examples of bouncing cosmologies in any otherwise pathological theory, and the question is how typical these solutions are. In the case of Einstein gravity, geodesic incompleteness is established through several sophisticated theorems, the simplest and most powerful being the Borde–Guth–Vilenkin (BGV) theorem [46]. According to it, any universe that expanded in average during most of its history has past-incomplete worldlines. This is not enough to establish the existence of a global singularity such as the big bang, for which past-incompleteness should hold for all observers at the same time. However, it is strong evidence in favor of such a conclusion, since it is based on a background-independent proof which does not rely on any specific metric (such as FLRW), nor on any specific dynamical theory, nor on inflation.

In the case of conformal gravity, the BGV theorem does not apply because, in the trans-Planckian phase, it is not even possible to talk about expanding backgrounds. In other words, the average expansion condition at the core of the BGV theorem cannot be met in a theory where the dynamics is conformally invariant. The expansion condition is not a conformally invariant statement and, vice versa, a conformal transformation can always make an expanding background static or even contracting. To put it simply, the singularity issue becomes meaningless.

This mechanism for solving the big-bang problem, and actually the wider singularity problem in general relativity, is independent of the details of the theory, but we

[4] This fact was already noted in classical scalar-tensor models concerning the equivalence of the Einstein and the Jordan frame [43–45].

stress the necessity to have a finite theory to get exact conformal invariance in the deep UV.

B. Flatness problem

In this section, we show how the flatness problem is solved simply and universally in conformal gravity. The proof consists of two mini theorems, the first of which is well known.

Theorem 1. *Any FLRW line element with intrinsic curvature* $\textsc{k}$*,*

$$\begin{aligned} d\hat{s}^2 &= \hat{g}_{\mu\nu} dx^\mu dx^\nu \qquad (43)\\ &= -dt^2 + a^2(t)\left[\frac{dr^2}{1-\textsc{k}\, r^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right],\\ \textsc{k} &= 0, +1, -1\,, \end{aligned}$$

(where r*,* θ*, and* φ *are the spherical coordinates on the spatial section in four dimensions) is conformally flat.*

Proof. For all values of $\textsc{k}$, there exists a coordinate transformation $(t, r, \theta, \varphi) \to (T, \rho, \theta, \varphi)$ such that the line element (43) turns into

$$\begin{aligned} d\hat{s}^2 &= \omega^2(x)\left[-dT^2 + d\rho^2 + \rho^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right],\\ \omega(x) &= \begin{cases} \omega(T) & \text{for} \quad \textsc{k} = 0 \\ \omega(T,\rho) & \text{for} \quad \textsc{k} = \pm 1 \end{cases}, \qquad (44) \end{aligned}$$

where:

$$T = \int \frac{dt}{\omega} \quad \text{and} \quad \rho = \exp \int \frac{dr}{r\sqrt{1-\textsc{k}\, r^2}}.$$

Therefore, the spacetime is conformally equivalent to Minkowski but, in general, it is not homogeneous because $\omega(x) = a(t)\, r/\rho(r)$ may depend on the radial coordinate as well as on the time coordinate. □

For instance, the coordinate transformation for the case $\textsc{k} = +1$ can be found in [47]. The case $\textsc{k} = -1$ can be worked out similarly.

Another way to prove the theorem is to notice that the Weyl tensor vanishes exactly for the metric (43), $C_{\mu\nu\rho\sigma} = 0$.

In the previous theorem, we did not call upon conformal invariance and the above result applies to any diffeomorphism (Diff) invariant theory. In general, conformal equivalence of the two metrics does not mean physical equivalence. Let us consider now a conformally invariant gravitational theory, i.e., a theory invariant under the symmetry Weyl×Diff.

Theorem 2. *In a Weyl×Diff invariant theory, any FLRW solution (43) with* $\textsc{k} = 0, \pm 1$ *is physically equivalent to the FLRW spacetime with* $\textsc{k} = 0$*.*

Proof. According to Theorem 1, we can first make a coordinate transformation to map Eq. (43) into the inhomogeneous, conformally flat line element (44) and, afterwards, we can make a conformal transformation to a homogeneous and spatially flat FLRW spacetime. The explicit conformal transformation reads

$$\hat{g}'_{\mu\nu} = \Omega^2 \hat{g}_{\mu\nu}\,, \qquad \Omega^2 = \frac{a^2(T)}{\omega^2(T,\rho)}\,, \qquad (45)$$

where $\hat{g}_{\mu\nu}$ is given in Eq. (44). Therefore, in a conformally invariant theory any FLRW solution is equivalent to a spatially flat ($\textsc{k} = 0$) FLRW solution. □

The details of both theorems are independent of the underlying theory except for the requirement of finiteness, without which one cannot invoke exact conformal invariance at the classical as well as at the quantum level.

Conformal invariance resolves the fine tuning at Planckian scales, where, according to the standard hot-big-bang model, the curvature density parameter is $\Omega_\textsc{k} = 10^{-64}$. However, it does not explain the lesser but still present fine tuning at lower energy scales (for instance, at a cosmic temperature $T = 1\,\text{TeV}$, one has $\Omega_\textsc{k} = 10^{-32}$; at radiation-matter equality, $\Omega_\textsc{k} = 10^{-6}$). After conformal symmetry is broken, one must pick a metric as a background and one selects FLRW (flat or curved) on empirical grounds, just like we must pick $\phi \propto M_\text{Pl}$ from the measurement of Newton's constant. Can one choose an FLRW metric with $\textsc{k} = \pm 1$?

An argument against this possibility is the following. Before the symmetry breaking, we know that Minkowski spacetime is the representative metric of the system with no residual gauge invariance. Thanks to the Weyl symmetry rescaling the metric, we can also consider a flat FLRW background ($\textsc{k} = 0$), which is another representative but with some residual gauge. Then, one can solve the equations of motion on this homogeneous isotropic background with a homogeneous dilaton $\phi(t)$. At this point, we can do either of two things:

A. Simply wait until Weyl invariance is spontaneously broken. At that point, the metric remains fixed and spatial flatness is preserved. The flat FLRW solution, which was equivalent to Minkowski before the symmetry was broken, becomes now physically meaningful because we cannot use the Weyl symmetry anymore. In order to be consistent with observations, the FLRW metric must be the known one for radiation and matter at scales larger than the Planck scale.

B. During the conformal phase, make a Diff transformation in order to get a FLRW metric with $\textsc{k} = 1$ or $\textsc{k} = -1$ and, afterwards, a conformal rescaling Eq. (45) in order to make the metric homogeneous again. Now the dilaton is not homogeneous because it transforms according to Eq. (10). However, since the gravitational sector is homogeneous, the inhomogeneous dilaton profile is a solution of the equations of motion only in the case of a constant, which is proportional to M_Pl after symmetry breaking. Therefore, the effective conformal factor must be $\Omega = 1$, leading back to the $\textsc{k} = 0$ FLRW metric.

> To put it differently, the dilaton couples to matter, hence such inhomogeneity affects the dynamics of matter in the universe. Therefore, the Cosmological Principle of homogeneity of the large-scale universe is broken by the presence of the dilaton and, once the Weyl symmetry is broken, we end up with a configuration violating empirical data. This forces us to take $\kappa = 0$ (homogeneous Weyl rescaling).

In both cases, the conclusion is that, after symmetry breaking, we must select an exactly flat FLRW background. Therefore, the flatness problem is solved also at energies above the Planck scale.[5]

C. Horizon problem

In our model for the early Universe, the horizon problem is solved simply because, in the conformal phase, spacetime distances do not have any physical meaning: large and small distances are actually the same.

Such a causal structure can be explicitly shown in asymptotically flat or bouncing backgrounds such as those considered in Sec. V, which may be selected after breaking conformal symmetry. In the examples (50) or (58a), the scale factor $a(t)$ is almost always smaller than the Hubble radius H^{-1}. For the solution (50), $a \leqslant H^{-1}$, while for the solution (58a) it is always possible to find an instant $\bar{t}$ before which the scale factor $a(t)$ is smaller than the Hubble horizon.

Hence, for t small enough, the distance between particles in all the solutions presented below is always inside the Hubble radius and the particles in the Universe can interact with one another without violating causality. Notice that we can talk about causality only after the symmetry is spontaneously broken.

D. Monopole problem

Since we have at our disposal a well-defined theory at any energy scale, we do not need to extend the Standard Model of particle physics to a grand unified theory (GUT). Indeed, the theory (2) is complete regardless of the form of the local action (3) (provided $\mathcal{L}_{\rm loc}$ does not have instabilities), which we can assume to be the Standard Model. In other words, the theory (2) is already a unified theory of all fundamental interactions. Here, the concept of unification is understood as a finiteness regime in which all interactions vanish. Indeed, according to a preliminary study [39], in the finite theory (2) all the scattering amplitudes $\mathcal{A}$ are suppressed as the inverse of a polynomial in the UV regime, $\mathcal{A} \sim 1/k^{2n}$ ($n \in \mathbb{N}$). Therefore, there is no unification strictly speaking, but something closer to the scenario presented in [49], in which all the couplings run to zero in the UV regime.

However, if we insist the action (2) to describe a UV finite and conformally invariant completion of a GUT, we must deal with the monopole problem. By construction, monopoles are exact solutions of the theory (2) whenever they are so for the local theory (3). In a grand unified model, monopoles originate from the spontaneous symmetry breaking of the gauge group at some scale M_X because the expectation value of the Higgs field Φ is, in general, different in causally disconnected domains and can take the value $\langle\Phi\rangle = 0$ in some regions and $\langle\Phi\rangle = v \neq 0$ in other regions. However, in nonlocal quantum gravity the GUT scale lies in the trans-Planckian regime where the theory is conformally invariant and there are no causally disconnected domains. Therefore, $\langle\Phi\rangle$ will be the same in the whole Universe in the conformal phase.

E. Trans-Planckian problem

Finally, we make a comment on the cosmological trans-Planckian problem [50, 51] in nonlocal quantum gravity.

In standard cosmology, in order to get nearly-Gaussian scale-invariant primordial perturbations, it is essential to set the initial state for the perturbations to be the Bunch–Davis vacuum. The latter coincides with the Minkowski solution in the infinite past, when the perturbative modes are deep inside the horizon. Therefore, sufficiently back in the past, the equations for perturbations reduce to the ones in Minkowski spacetime. However, in our theory the cosmological metric is only conformally equivalent to the Minkowski metric, hence, the wavelength $\lambda \sim a(t)$ of the perturbations will be sub-Planckian at some instant t in the past, namely, $\lambda \lesssim \ell_{\rm Pl}$ for $t \lesssim t_{\rm Pl}$. From the effective field theory point of view, it is hard to explain why higher-derivative operators, at the linear and the nonlinear level in the perturbations, do not affect the observed spectrum when $\lambda \simeq O(\ell_{\rm Pl})$. Indeed, operators like

$$E_i(\Phi_i)\frac{\Box^n}{M_{\rm Pl}^{2n}}E_i(\Phi_i)\,, \tag{46}$$

will be of the same order of magnitude as the Einstein–Hilbert term R at the Planck energy scale.

This puzzle has a simple solution in nonlocal gravity (2), whose classical equations of motion have the structure (12). The terms $O(E_i^2)$ are subleading because all

[5] Another way to reach the same conclusion is to regard the spontaneous breaking of the continuous Weyl symmetry as associated to a phase transition (e.g., [48, Chap. 8]). The facts that the solutions of the equations of motion do not have any discontinuity at the time $t_{\rm Pl}$ of symmetry breaking and that, as we will see later, the scaling of two-point functions also does not change abruptly suggest that we are in the presence of a second-order phase transition. In this case, the change in the metric cannot be abrupt either, and one cannot suddenly pass from a class of flat backgrounds to a nonflat one.

the scattering amplitudes vanish at the quantum level in the UV regime, while the solutions of the linear part of the equations of motion (12),

$$e^{\bar{\mathrm{H}}(\boldsymbol{\Delta})^{(0)}} E_i^{(1)} = 0\,, \tag{47}$$

where ${}^{(0)}$ and ${}^{(1)}$ denote, respectively, the background and the first order in the perturbation, are the same of the local Einstein theory coupled to matter. Nonlinear operators in the perturbations are suppressed because the n-point amplitudes vanish in the UV regime, as we saw in Sec. III A. Therefore, no classical higher-derivative operators such as (46), which are quadratic in the curvature in the action, contribute to the linearized classical equations of motion (47).

The only corrections to the equations of motion that we have to take into account at the linear order are quantum and will be crucial later in order to explain the deviation of the primordial spectra from exact scale invariance. Indeed, at the linear level Eq. (47) will be replaced by (B36), which reduces to the linearization (47) when the quantum corrections γ_2^Q and γ_0^Q are sent to zero.

V. BACKGROUND SOLUTIONS

In the trans-Planckian regime, all conformally equivalent background solutions are physically equivalent. Indeed, at such scales all the conformal solutions are equivalent, while timelike and spacelike distances do no have any physical meaning. However, for future usage in the sub-Planckian phase, we describe possible choices of the conformal solution. In particular, any rescaling of the FLRW metric is also an exact solution of the theory and is physically equivalent to one another. Since the metric in the conformal phase is described by the Minkowski background (41), we can get a new FLRW solution simply making a conformal rescaling of $\eta_{\mu\nu}$. We here propose two possible scenarios: (I) an asymptotically flat scenario and (II) a bouncing scenario. For the scenario (I), we identify three subcases that we label (Ia), (Ib), and (Ic).

A. Asymptotically flat backgrounds

In scenario (Ia), the metric in conformal time is obtained making a rescaling (10) with the choice [52, Sec. 3.2]

$$\Omega^2 = a^2(\tau) = A + B\tanh\frac{\tau}{\tau_{\rm Pl}}\,, \tag{48}$$

where A and B are constants, so that the line element with the metric (41) turns into

$$\begin{aligned} ds^{*2} &= -\left(A + B\tanh\frac{\tau}{\tau_{\rm Pl}}\right)\left(-d\tau^2 + d\boldsymbol{x}^2\right),\\ \lim_{\tau\to\pm\infty} a^2 &= A \pm B\,, \qquad A \geqslant B\,, \end{aligned} \tag{49}$$

where we assumed $A \geqslant B$ in order to have $a^2(\tau) > 0$ for all times. Therefore, the metric in Eq. (49) interpolates between two asymptotic Minkowski spacetimes for $\tau \to \pm\infty$.[6] In particular, our Universe was not born from a big bang but from a conformal phase in which geometry is decribed by Minkowski spacetime. See also Sec. IV B about the uniqueness of Minkowski spacetime in conformal gravity.

Another metric similar to Eq. (49) that characterizes scenario (Ib) can be directly defined in proper time,

$$\begin{aligned} ds^{*2} &= -dt^2 + a^2(t)\,d\boldsymbol{x}^2\,,\\ a &= A + B\tanh\frac{t}{t_{\rm Pl}},\\ \lim_{t\to\pm\infty} a &= A \pm B\,, \qquad A \geqslant B\,. \end{aligned} \tag{50}$$

In order to show that in a universe described by the interpolating metric (50) we avoid the horizon problem, we compute the Hubble radius R_H and the particle radius $R_{\rm p}$, or their comoving counterparts r_H and $r_{\rm p}$:

$$\begin{aligned} R_H :=& a\,r_H := \frac{1}{H} = \frac{a}{\dot a}\\ =& t_{\rm Pl}\cosh^2\left(\frac{t}{t_{\rm Pl}}\right)\left(\frac{A}{B} + \tanh\frac{t}{t_{\rm Pl}}\right), \end{aligned} \tag{51}$$

$$\begin{aligned} R_{\rm p} :=& a\,r_{\rm p}\\ :=& a(t)\int_{t_{\rm i}}^{t}\frac{dt'}{a(t')} = (\tau - \tau_{\rm i})a\,, \end{aligned} \tag{52}$$

$$\begin{aligned} \tau =& \int\frac{dt}{a(t)}\\ =& \frac{At - Bt_{\rm Pl}\ln\left[A\cosh\left(\frac{t}{t_{\rm Pl}}\right) + B\sinh\left(\frac{t}{t_{\rm Pl}}\right)\right]}{A^2 - B^2}, \end{aligned} \tag{53}$$

where the latter expression is valid for $A \neq B$. The particle radius is the actual definition of a causal region comprising all points in contact with the observer through light signals since an initial time $t = t_{\rm i}$. However, depending on the background it may be ill defined, as is the case here for $t < 0$, and the Hubble radius provides an alternative criterion for causal contact.

The functions $a(t)$ and $R_H(t)$ are tangent at $t = 0$, while $a < R_H$ for all $t \neq 0$. In particular, for $A > B$ the Hubble radius goes to infinity for $t \to \pm\infty$ (Fig. 2), while for $A = B$ one has a degenerate spacetime in the asymptotic past (Fig. 3),

$$\lim_{t\to+\infty} R_H = +\infty\,, \qquad \lim_{t\to-\infty} R_H = \frac{t_{\rm Pl}}{2}\,. \tag{54}$$

At the instant t, the wavelength of a perturbation is

$$\lambda = a(t)\,\lambda_{\rm com}\,, \qquad \lambda_{\rm com} = \frac{2\pi}{|\boldsymbol{k}|}\,, \tag{55}$$

[6] Another metric interpolating between two Minkowski spacetimes in the infinite past and in the infinite future can be defined in terms of the error function, $a^2(\tau) = A + B\,{\rm erf}(\tau/\tau_{\rm Pl})$.

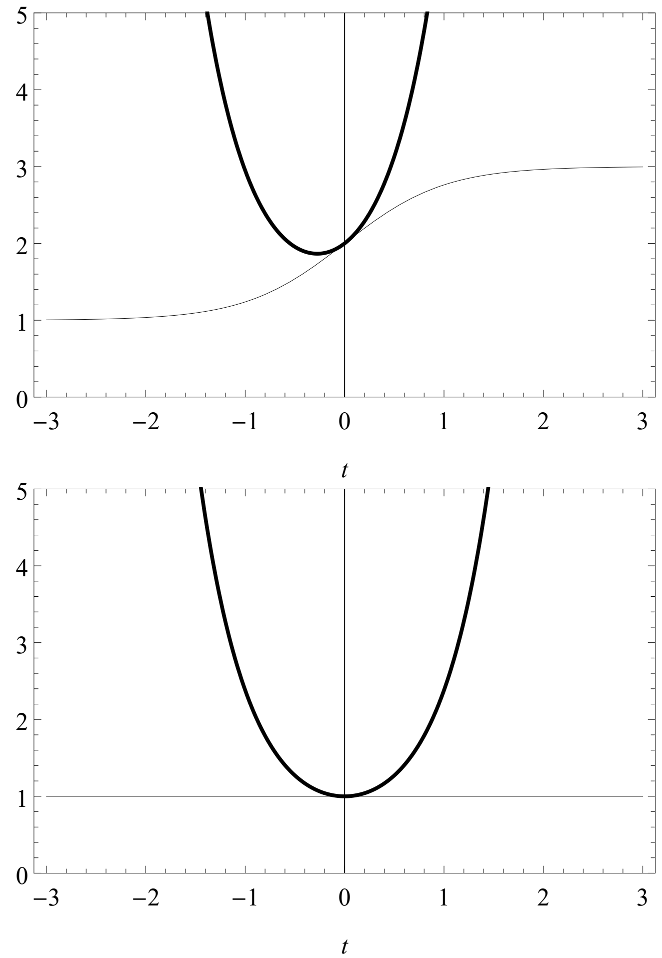

FIG. 2. Top: Scale factor (proper wavelength) $a(t)$ (50) (solid thin curve) and Hubble radius R_H (51) (solid thick curve) for $A > B$ and $t_{\rm Pl} = 1$. Bottom: Same as in the top panel for the comoving wavelength $\lambda_{\rm com} = 1$ and the comoving Hubble radius r_H.

where $\lambda_{\rm com}$ is the comoving wavelength and $|\boldsymbol{k}|$ is the comoving wavenumber. Therefore, the scale factor can be regarded as the proper wavelength with $\lambda_{\rm com} = 1$: any perturbation with wavelength within the horizon would stay in the causally connected patch. Therefore, using the relative size with respect to the Hubble radius as a criterion, the distance between causally connected points is always smaller than the Hubble radius. Taking instead the particle radius with an initial time in the infinite past, one reaches the same conclusion, since in that case $R_{\rm p} = +\infty$. Therefore, all the matter in the Universe is located inside a casually connected domain.

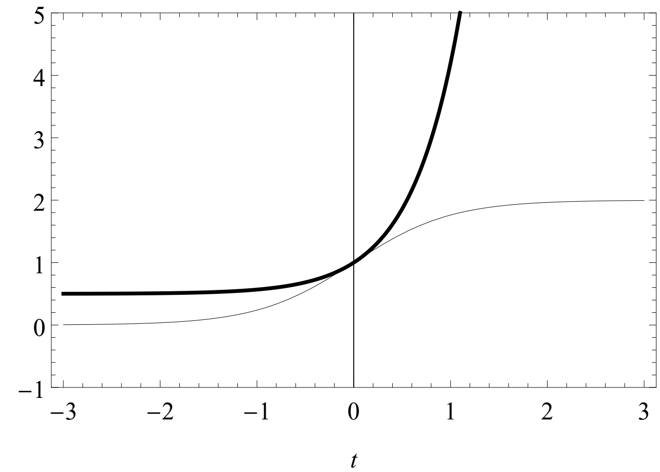

FIG. 3. Scale factor (proper wavelength) $a(t)$ (50) (solid thin curve) and Hubble radius R_H (51) (solid thick curve) for $A = B$ and $t_{\rm Pl} = 1$. The plot for comoving distances is similar to the one in the bottom panel of Fig. 2.

Finally, in scenario (Ic) the scale factor is

$$a = \sqrt{A + \frac{t}{t_0}\, e^{\Gamma(0,t/t_{\rm Pl})}}\,, \tag{56a}$$

$$\lim_{t\to+\infty} a = \sqrt{\frac{t}{t_0}}\,, \tag{56b}$$

$$\lim_{t\to-\infty} a = \sqrt{A}\,, \tag{56c}$$

for which the Hubble radius is

$$R_H = 2\frac{A e^{-\Gamma(0,t/t_{\rm Pl})} + t}{1 - e^{-t/t_{\rm Pl}}} > a \qquad \forall t\,. \tag{57}$$

When $A = 0$, the metric interpolates between a degenerate spacetime in the past and the solution of Einstein's theory for radiation (Fig. 4), while for $A > 0$ the metric approaches Minkowski spacetime in the past (Fig. 5).

B. Bouncing backgrounds

Scenario (II) is defined by a simple and quite general proposal for a bouncing Universe in proper time:

$$a = \left[\left(\frac{t}{t_0}\right)^{2n} + A^{2n}\right]^{\frac{1}{4n}}, \qquad n \in \mathbb{N}^+, \tag{58a}$$

$$\lim_{t\to\pm\infty} a = \sqrt{\frac{t}{t_0}}\,, \tag{58b}$$

$$\lim_{t\to 0} a = \sqrt{A}\,. \tag{58c}$$

Also in the bouncing Universe there is no horizon problem. For the metric (58a), the Hubble radius is

$$R_H = 2t\left[1 + \left(\frac{At}{t_0}\right)^{2n}\right], \tag{59}$$

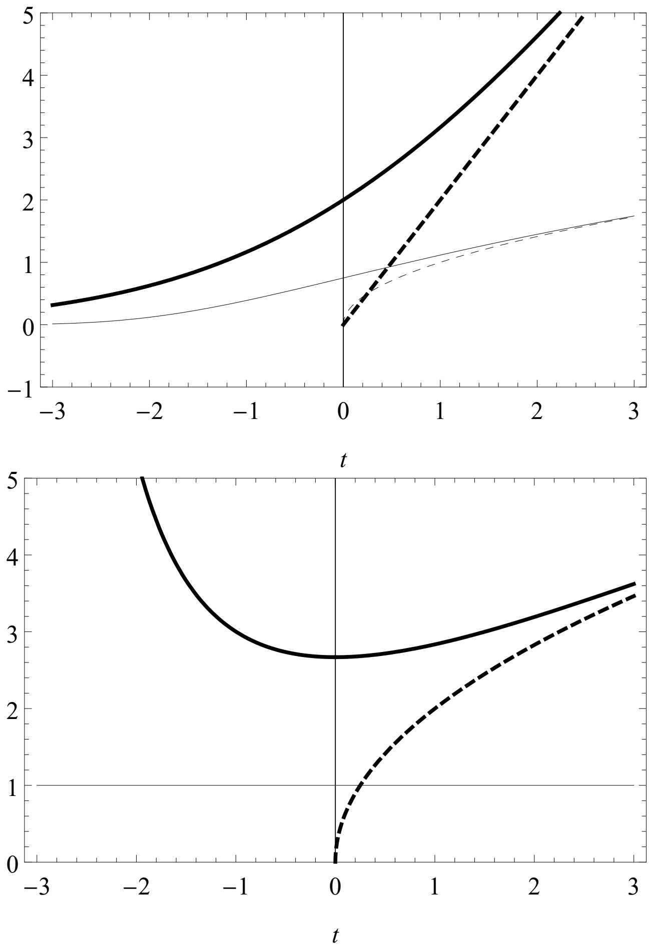

FIG. 4. Top: Scale factor (proper wavelength) $a(t)$ (56a) (solid thin curve) and Hubble radius R_H (57) (solid thick curve) for $A = 0$ and $t_0 = t_{\rm Pl} = 1$. The Einstein-gravity radiation-dominated solution (56b) and the corresponding $R_H = 2t$ are shown as well (dashed thin and thick curves, respectively). Bottom: Same as in the top panel for the comoving wavelength $\lambda_{\rm com} = 1$ and the comoving Hubble radius r_H.

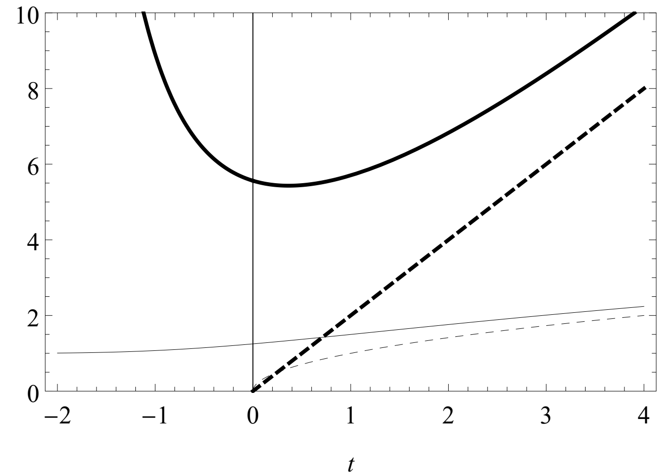

FIG. 5. Scale factor (proper wavelength) $a(t)$ (56a) (solid thin curve) and Hubble radius R_H (57) (solid thick curve) for $A = t_0 = t_{\rm Pl} = 1$. The general-relativity radiation-dominated solution (56b) with corresponding $R_H = 2t$ are shown as well (dashed thin and thick curves, respectively). The plot for comoving distances is similar to the one in the bottom panel of Fig. 4.

which is divergent in $t = 0$,

$$\lim_{t\to 0^+} R_H = +\infty\,. \tag{60}$$

Therefore, $a(t)$ and $R_H(t)$ can cross each other never, once, or twice depending on the values of t_0 and A (Figs. 6 and 7). In the first case, $a < R_H$ for all t; in the second case, $a < R_H$ for all t before a certain point $\bar{t}$; in the third case, $a < R_H$ for $t > \tilde{t}$ or $t < \bar{t}$, where $0 < \bar{t} < \tilde{t}$. We conclude that, in the bouncing Universe, there is always an instant $\bar{t}$ in the past before which particles had a chance to be in causal contact with one another.

From the scale factor (58a), we can reconstruct the conformal time

$$\begin{aligned}\tau &= \int \frac{dt}{a(t)}\\ &= \frac{t}{\sqrt{A}}\,{}_2F_1\left[\frac{1}{4n},\frac{1}{2n};1+\frac{1}{2n};-\left(\frac{t}{At_0}\right)^{2n}\right],\end{aligned} \tag{61}$$

where ${}_2F_1$ is the hypergeometric function.

VI. PRIMORDIAL SPECTRA

While the conformal phase can address the classic problems of the hot-big-bang model, it does not explain the origin of the primordial spectra and their almost scale invariance. It turns out that these are generated during the sub-Planckian phase, where conformal invariance is spontaneously broken. The one-loop quantum corrections to the classical action are directly responsible for the small deviation from Harrison–Zel'dovich spectra.

In this section, after describing some basic approximations needed for technical reasons, we derive the quasi-scale-invariant spectrum of primordial tensor and scalar perturbations.

A. Flat spacetime approximation

According to the discussion in Secs. II and III and the results derived in Appendix B 3, the computations we are going to make in this section rely on two assumptions for the energy regime in which we will consider the theory (2):

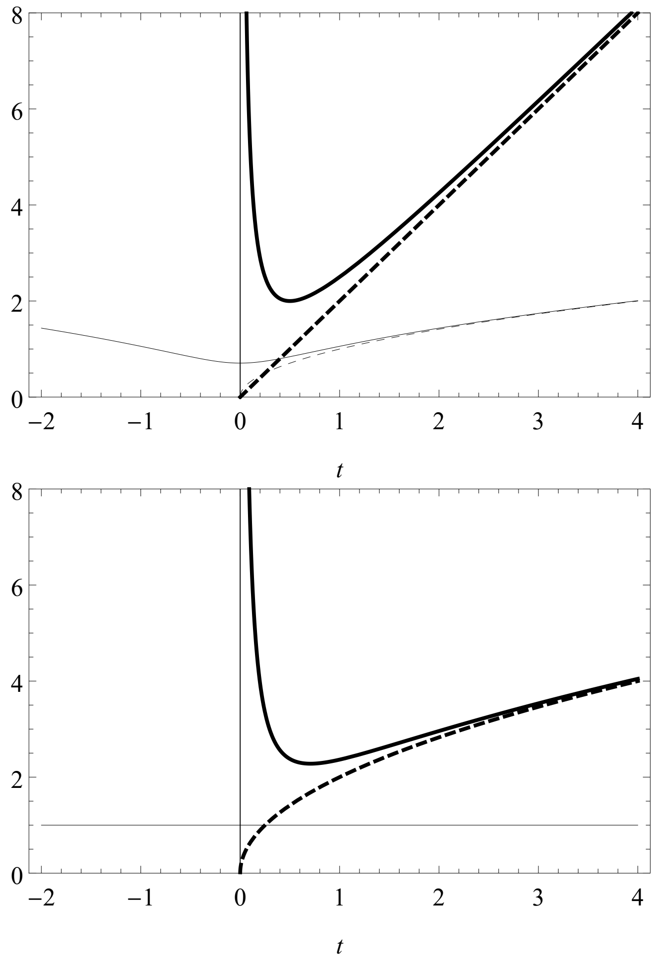

FIG. 6. Top: Scale factor (proper wavelength) $a(t)$ (58a) (solid thin curve) and Hubble radius R_H (59) (solid thick curve) for $A = 0.5$, $n = 1$, and $t_0 = 1$. The general-relativity radiation-dominated solution (58b) with corresponding $R_H = 2t$ are shown as well (dashed thin and thick curves, respectively). In this plot, $a < R_H$ for all t. Bottom: Same as in the top panel but for the comoving wavelength $\lambda_{\rm com} = 1$ and the comoving Hubble radius r_H.

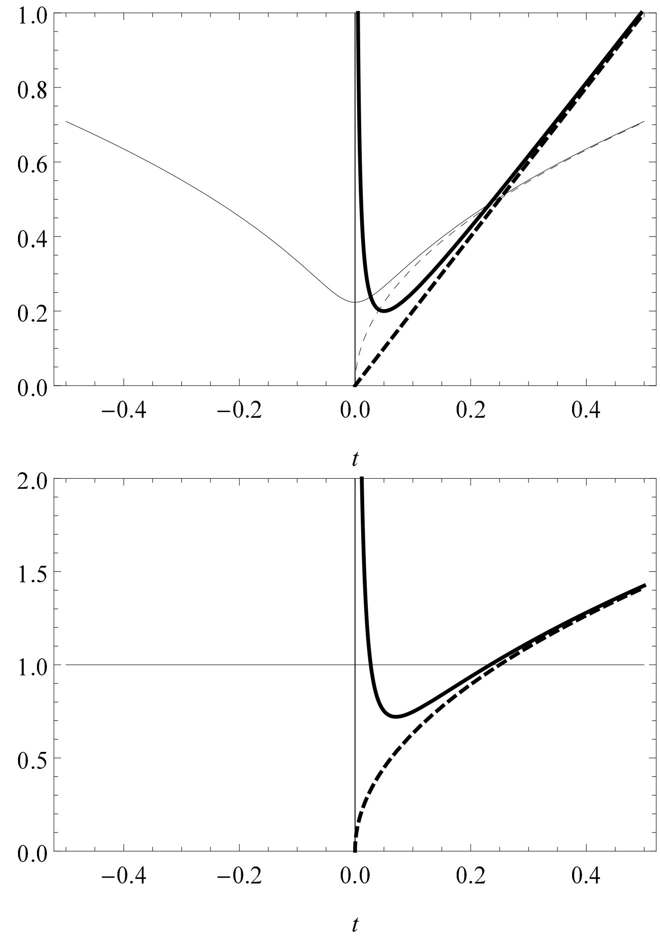

FIG. 7. Top: Scale factor (proper wavelength) $a(t)$ (58a) (solid thin curve) and Hubble radius R_H (59) (solid thick curve) for $A = 0.05$, $n = 1$, and $t_0 = 1$. The Einstein-gravity radiation-dominated solution (58b) with corresponding $R_H = 2t$ are shown as well (dashed thin and thick curves, respectively). In this plot, we have an interval of time in which $a > R_H$. Bottom: Same as in the top panel but for the comoving wavelength $\lambda_{\rm com} = 1$ and the comoving Hubble radius r_H.

(i) Quadratic-gravity or higher-derivative (hd) regime (29), $\Lambda_{\rm hd} \lesssim E \lesssim \Lambda_*$, where $\Lambda_{\rm hd} = \Lambda_*/\sqrt{b}$ for $0 \leqslant b_0 \leqslant 1$ and $\Lambda_{\rm hd} = \Lambda_*\sqrt{b_0/b}$ for $b_0 > 1$. In this range, the theory has the local limit (A4) [Eq. (A5) with matter] which captures its main UV properties. More generally, what follows holds also for Stelle gravity with a nonzero coefficient of the R^2 operator.

(ii) Perturbative (one-loop) approximation (B31), corresponding to $E \gtrsim \Lambda_*/10$. It guarantees that one-loop quantum corrections in the propagator are subdominant with respect to the classical part consistently with the quantum field theory perturbative expansion. This condition also permits some analytic simplifications [Eqs. (B26) and (B27)].

Therefore, if $\max(1, b_0)/b < O(10^{-2})$, then $\Lambda_{\rm hd} < \Lambda_*/10$ and the conditions (i) and (ii) imply that $\Lambda_*/10 \lesssim E \lesssim \Lambda_*$, which boils down to pushing the energy to the lower limit:

$$\boxed{E \simeq \frac{\Lambda_*}{10}\,.} \tag{62}$$

If we make the identification $\Lambda_* = M_{\rm Pl}$, then the regime (62) falls within the sub-Planckian or Higgs phase where conformal invariance is spontaneously broken.

At the end of Sec. III A, we argued that $\Lambda_* \leqslant M_{\rm Pl}$ only using conformal invariance, Eq. (36). Here, we end up with a similar statement using (62) and the fact that the energy of particles propagating in the Higgs phase must be smaller than the Planck mass, $E < M_{\rm Pl}$:

$$\Lambda_* < 10\,M_{\rm Pl}\,,\qquad \ell_* > \frac{\ell_{\rm Pl}}{10}\,. \tag{63}$$

These inequalities are consistent with Eq. (36).

In order to evaluate the correlation functions in flat spacetime and avoid the complications of quantum field theory on curved spaces, we have to verify that we can approximate the FLRW metric with the Minkowski one. Hence, the condition that has to be fulfilled is to consider perturbations within the Hubble horizon,

$$\lambda = \frac{1}{E} \ll R_H\,, \tag{64}$$

where λ is the wavelength of the perturbation. Since we are in the regime (62), i.e, out of the conformally invariant phase, we can consider the radiation-domination scale factor

$$a(t) = \sqrt{\frac{t}{t_0}} \tag{65}$$

and compute for which value of t the perturbation crosses the horizon, $\lambda(t_\star) = R_H(t_\star)$. At the instant t, the wavelength is given by Eq. (55), while $H = \dot a/a = 1/(2t)$ and $R_H = H^{-1} = 2t$. Therefore, at horizon crossing $t = t_\star$ we have

$$\sqrt{\frac{t_\star}{t_0}}\,\lambda_{\rm com} = 2t_\star \implies t_\star = \frac{\lambda_{\rm com}^2}{4t_0}\,. \tag{66}$$

Apart from $t_\star$, there are three more important moments in the evolution of the Universe: the instant t_{Λ_*} when $\lambda = \ell_*$ and the Universe exits the conformal phase, the instant t_λ when $\lambda = 10\ell_*$ [see Eq. (62)], and the instant $t_{\Lambda_{\rm hd}}$ when $\lambda = \Lambda_{\rm hd}^{-1}$ and the theory is approximated by the local quadratic action (A4) [Eq. (A5) with matter]. Let us now find such instants and provide the chronological order of the events.

The instant t_{Λ_*} is given by

$$t_{\Lambda_*}:\ \sqrt{\frac{t_{\Lambda_*}}{t_0}}\,\lambda_{\rm com} = \ell_* \implies t_{\Lambda_*} = \frac{\ell_*^2 t_0}{\lambda_{\rm com}^2} = \frac{\ell_*^2}{4t_\star}\,. \tag{67}$$

The instant t_λ is given by

$$t_\lambda:\ \sqrt{\frac{t_\lambda}{t_0}}\lambda_{\rm com} = 10\,\ell_* \implies t_\lambda = 10^2\,t_{\Lambda_*}\,. \tag{68}$$

The instant $t_{\Lambda_{\rm hd}}$ is given by

$$t_{\Lambda_{\rm hd}}:\ \sqrt{\frac{t_{\Lambda_{\rm hd}}}{t_0}}\,\lambda_{\rm com} = \sqrt{\frac{b}{\max(1,b_0)}}\,\ell_* \implies$$
$$t_{\Lambda_{\rm hd}} = \frac{b}{\max(1,b_0)}\,t_{\Lambda_*}\,. \tag{69}$$

Since $b/\max(1,b_0) \gg 1$, we have the hierarchy

$$\boxed{t_{\Lambda_*} \ll t_\lambda < t_{\Lambda_{\rm hd}}\,,} \tag{70}$$

which is the time analogue of the interval (29).

The condition (64) is satisfied in the regime (62) when

$$\lambda \simeq 10\,\ell_* \ll H^{-1} = 2t_\lambda \implies t_\lambda \gg 5\ell_*\,. \tag{71}$$

Plugging Eq. (66) into (71), we find

$$t_\lambda \gg 5\ell_* = 10\sqrt{t_\star\,t_{\Lambda_*}}\,,$$

in which we now replace $t_{\Lambda_*} = t_\lambda/10^2$:

$$t_\star \ll t_\lambda\,. \tag{72}$$

We will take into account the results of this section in drawing the cosmological model described here.

The conformal and the Higgs phase as well as the scale hierarchy (70) and the local UV limit of the theory are depicted in Fig. 8.

B. Tensor perturbations

After the spontaneous breaking of conformal invariance, distances acquire a meaning, correlation functions become nonvanishing, and one can consider metric perturbations. In this noninflationary early-Universe scenario, tensor perturbations are the subhorizon metric quantum fluctuations corresponding to the graviton, in a regime where one can ignore the curvature of the background and the cosmological expansion. In contrast, in standard inflation metric fluctuations are induced by the quantum fluctuations of a scalar field and the spectrum is calculated at subhorizon scales but evaluated at horizon crossing.

The tensor spectrum in the regime (62) is readily obtained from the graviton propagator (B32) derived in Appendix B 3. The two-point correlation function for transverse and traceless tensors in four dimensions reads [19]

$$\begin{aligned} G^{(2)}_{ijkl}(x,x') &:= \langle h_{ij}(x)h_{kl}(x')\rangle \\ &\overset{\rm (B32)}{=} -{\rm C}\int\frac{d^4k}{(2\pi)^4}\,e^{ik\cdot(x-x')}\,\frac{P^{(2)}_{ijkl}(k)}{k^4\left(\frac{k}{\Lambda_*}\right)^{2\epsilon_2}} \\ &= -{\rm C}\Lambda_*^{2\epsilon_2}\,P^{(2)}_{ijkl}(\partial_x)\int\frac{d^4k}{(2\pi)^4}\,\frac{e^{ik\cdot(x-x')}}{k^{4+2\epsilon_2}}\,, \end{aligned} \tag{73}$$

where Latin indices run over spatial directions, the constants C and ϵ_2 are given by Eqs. (B32) and (B21), respectively,

$${\rm C} := \frac{4\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}M_{\rm Pl}^2}\,,\qquad \epsilon_2 = -\frac{2\beta_{\rm Ric}\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}M_{\rm Pl}^2}\,, \tag{74}$$

and $P^{(2)}_{ijkl}(k)$ are the spatial components of the spin-2 projector in Eq. (B3) [53].

In order to explicitly compute Eq. (73), we recall that the two-point correlation function in D-dimensional Euclidean momentum space is obtained through the Fourier

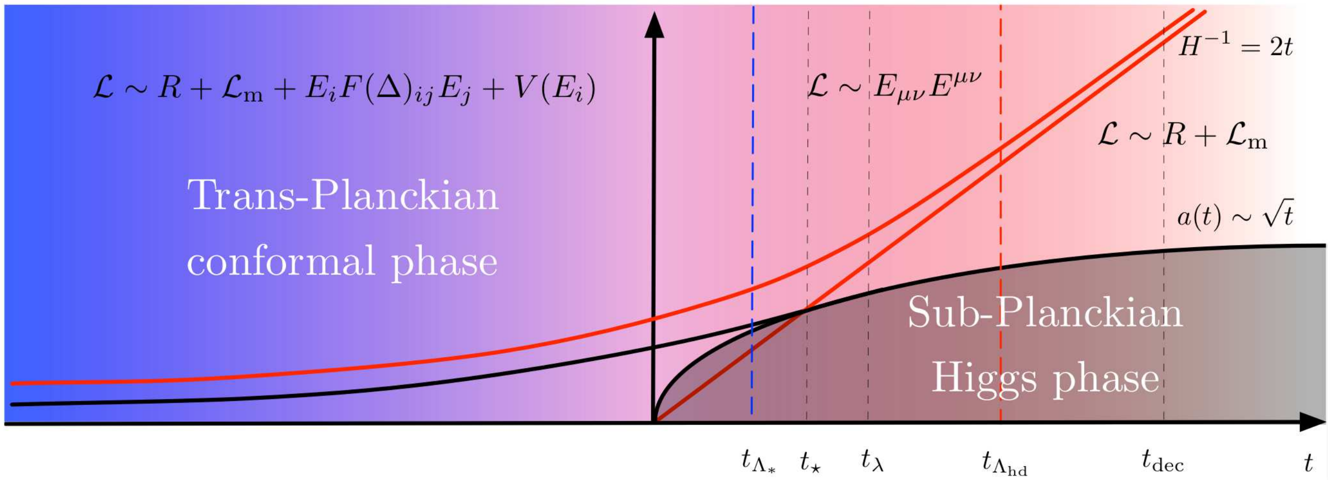

FIG. 8. The cosmological scenario proposed in this article. For $t < t_{\Lambda_*}$, the Universe is in the trans-Planckian conformal phase described by the theory (2). For $t \gtrsim t_{\Lambda_*}$, the conformal symmetry is spontaneously broken. The instant $t_\star$ represents horizon crossing when $k = aH$ for a radiation-dominated Universe, Eq. (66), while t_λ is when the perturbation is evaluated after the spontaneous breaking of the Weyl symmetry, consistently with the perturbative approximation (62). In the epoch $t_\lambda \lesssim t \ll t_{\Lambda_{\rm hd}}$, the Universe is well described by quadratic gravity (Stelle's theory) plus quantum corrections. The CMB is formed at the decoupling time $t_{\rm dec}$.

transform

$$(x^2)^{-s} = B_D(2s) \int \frac{d^D k}{(2\pi)^D} e^{ik\cdot x} (k^2)^{s-\frac{D}{2}}\,, \tag{75a}$$

$$B_D(2s) \coloneqq \frac{2^{D-2s}\pi^{\frac{D}{2}}\Gamma\left(\frac{D}{2}-s\right)}{\Gamma(s)}\,. \tag{75b}$$

Therefore, comparing Eqs. (73) and (75) with $D = 4$, we see that $s = -\epsilon_2$, so that the propagator in position space reads

$$G^{(2)}_{ijkl}(x, x') = -\frac{\mathrm{C}\Lambda_*^{2\epsilon_2}}{B_4(-2\epsilon_2)}\, P^{(2)}_{ijkl}(\partial_x)|x - x'|^{2\epsilon_2}\,. \tag{76}$$

For $x'_0 = x_0$ and using again Eq. (75) with $s = -\epsilon_2$, but now in $D-1$ dimensions,

$$\begin{aligned} G^{(2)}_{ijkl}(\boldsymbol{x}, \boldsymbol{x}') &= -\frac{\mathrm{C}\Lambda_*^{2\epsilon_2}}{B_4(-2\epsilon_2)}\, P^{(2)}_{ijkl}(\partial_x)|\boldsymbol{x} - \boldsymbol{x}'|^{2\epsilon_2} \\ &= -\frac{\mathrm{C}\Lambda_*^{2\epsilon_2} B_3(-2\epsilon_2)}{B_4(-2\epsilon_2)} \\ &\quad \times P^{(2)}_{ijkl}(\partial_x)\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}}{(\boldsymbol{k}^2)^{\epsilon_2+\frac{3}{2}}}\,. \end{aligned}$$

From the second formula in Eq. (75), we can compute the ratio $B_3(-2\epsilon_2)/B_4(-2\epsilon_2)$:

$$\frac{B_{D-1}(2s)}{B_D(2s)} = \frac{1}{2\sqrt{\pi}}\frac{\Gamma\left(\frac{D-1}{2}\right)}{\Gamma\left(\frac{D}{2}\right)} \overset{D=4}{=} \frac{1}{4}\,. \tag{77}$$

Finally, according to the definitions in Appendix D,

$$\begin{aligned} G^{(2)}_{ijkl}(r) &= -\frac{\mathrm{C}\Lambda_*^{2\epsilon_2}}{4} P^{(2)}_{ijkl}(\partial_x)\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{k^{2\epsilon_2+3}} \\ &\overset{\rm (D1)}{=} -P^{(2)}_{ijkl}(\partial_x)\int_0^{+\infty} dk\, \frac{k^2}{2\pi^2}\, P_h(k)\, \frac{\sin(kr)}{kr} \\ &\overset{\rm (D3)}{=} -P^{(2)}_{ijkl}(\partial_x)\int_0^{+\infty} \frac{dk}{k}\, \Delta_h^2(k)\, \frac{\sin(kr)}{kr}\,, \end{aligned} \tag{78}$$

where $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{x}'$, $r = |\boldsymbol{r}|$, $k = |\boldsymbol{k}|$, and

$$P_h(k) = \frac{\mathrm{C}\Lambda_*^{2\epsilon_2}}{4\, k^{2\epsilon_2+3}}\,, \qquad \Delta_h^2(k) = \frac{\mathrm{C}}{8\pi^2}\left(\frac{\Lambda_*}{k}\right)^{2\epsilon_2}. \tag{79}$$

Taking into account the two polarization modes of the graviton, the final result for the tensor power spectrum $\mathcal{P}_{\rm t}$ is

$$\mathcal{P}_{\rm t}(k) \coloneqq 2\Delta_h^2(k) = \frac{4\Lambda_*^2}{b\,(2\pi)^2\, e^{\tilde{\gamma}_{\rm E}}\, M_{\rm Pl}^2}\left(\frac{k}{\Lambda_*}\right)^{-2\epsilon_2}. \tag{80}$$

Therefore, using the definition (B22), the tensor spectrum reads

$$\boxed{\mathcal{P}_{\rm t}(k) = \frac{n_{\rm t}}{(2\pi)^2\,\beta_{\rm Ric}}\,(\ell_* k)^{n_{\rm t}}\,,} \tag{81}$$

where the tensor spectral index is

$$\boxed{n_{\rm t} \coloneqq \frac{d\ln\mathcal{P}_{\rm t}}{d\ln k} = -2\epsilon_2 = \frac{4\beta_{\rm Ric}}{b\, e^{\tilde{\gamma}_{\rm E}}}\frac{\Lambda_*^2}{M_{\rm Pl}^2}\,,} \tag{82}$$

which is positive because $\beta_{\rm Ric} > 0$.

C. Scalar perturbations

Scalar perturbations are thermal fluctuations of the radiation filling the Universe. They are not induced by the quantum fluctuations of a scalar field as in inflation. In order to convert the power spectrum of primordial density perturbations to the spectrum of fluctuations in the CMB at large angular separations, we follow the standard treatment of the Sachs–Wolfe effect relating temperature deviations $\delta T/T$ to the gravitational potential Φ at the last-scattering surface [54, 55]:

$$\frac{\delta T(\boldsymbol{x})}{T}=-\frac{1+3w}{3(1+w)}\,\Phi(\boldsymbol{x})\,, \tag{83}$$

where w is the constant barotropic index defining the equation of state $p=w\rho$ of the perfect fluid assumed to dominate, where p is the pressure and ρ is the energy density. For radiation, $w=1/3$ and the front coefficient in Eq. (83) is $-1/2$.

At the last-scattering surface, the theory (2) is well approximated by Einstein gravity. Therefore, the potential is related to density perturbations by the standard Poisson equation

$$\nabla^2\Phi(\boldsymbol{x})=\frac{1}{2M_{\rm Pl}^2}\delta\rho(\boldsymbol{x})\,. \tag{84}$$

Regarding the potential, at the last-scattering surface the leading contribution to Φ comes from the Einstein–Hilbert action because, at that time, the energy scale is much smaller than the Planck energy [18]. Moreover, we remind the reader that the FLRW solutions of Einstein's equations are exact solutions also of the theory (2).

Equations (83) and (84) imply

$$\frac{\delta T(\boldsymbol{x})}{T}=-\frac{1+3w}{3(1+w)}\,\frac{1}{2M_{\rm Pl}^2}\frac{1}{\nabla^2}\delta\rho(\boldsymbol{x})\,,$$

so that the two-point function of CMB temperature fluctuations reads

$$\left\langle\frac{\delta T(\boldsymbol{x})}{T}\frac{\delta T(\boldsymbol{y})}{T}\right\rangle=\left[\frac{1+3w}{3(1+w)}\right]^2\langle\Phi(\boldsymbol{x})\Phi(\boldsymbol{y})\rangle\,, \tag{85}$$

$$\langle\Phi(\boldsymbol{x})\Phi(\boldsymbol{y})\rangle=\frac{1}{4M_{\rm Pl}^4}\left\langle\frac{1}{\nabla_{\mathbf{x}}^2}\delta\rho(\boldsymbol{x})\frac{1}{\nabla_{\mathbf{y}}^2}\delta\rho(\boldsymbol{y})\right\rangle, \tag{86}$$

where we formally inverted Eq. (84).

On the other hand, both the density perturbations and their two-point correlation function have to be evaluated at the energy scale $E\simeq\Lambda_*/10$ right after the conformal phase. Therefore, we need to express the perturbations of the energy-momentum tensor in terms of the gravitational perturbations through the linearized equations of motion for the theory (2) at the scale (62). Afterwards, such density perturbations affect the fluctuations of the gravitational potential, which satisfies the 00 component (84) of Einstein's equations at the decoupling time $t_{\rm dec}$. In turn, the potential affects the temperature of the CMB. Schematically,

$$\delta\rho\Big|_{E\simeq\frac{\Lambda_*}{10}}\underset{(84)}{\Longrightarrow}\Phi\Big|_{\text{last scattering}}\Longrightarrow\frac{\delta T}{T}\Big|_{\text{last scattering}}.$$

In order to relate the density perturbations to the graviton two-point correlation function, we need the linearized equations of motion for the theory (2) including the quantum corrections, i.e.,

$$\left[e^{\mathrm{H}^{(0)}}\right]_{\mu\nu}^{\sigma\tau}\left[\frac{M_{\rm Pl}^2G_{\sigma\tau}^{(1)}-T_{\sigma\tau}^{(1)}}{2}\right]+E_{\mu\nu}^{Q(1)}=0\,, \tag{87}$$

where E^Q are the quantum corrections to the equations of motion coming from the Lagrangian (30b). The linearized equations are given in Appendix B 4 by Eq. (B36), written in compact notation as (B37).

Now we are ready to evaluate the two-point correlation function of the energy momentum tensor. The details of this tedious computation are given in Appendix E and the final result is Eq. (E2). However, we are only interested in the two-point correlation function of the energy-density perturbation $\delta\rho:=T_{00}^{(1)}$, which is given by Eq. (E9). Let us focus on the first contribution proportional to $5/12$, later showing that the other term is negligible:

$$\begin{aligned}\langle\delta\rho(x)\delta\rho(y)\rangle&\simeq-\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int\frac{d^4k}{(2\pi)^4}\,e^{ik\cdot(x-y)}\,\frac{5}{12}\left(\frac{k}{\Lambda_*}\right)^{2\epsilon_2}\\&\overset{(75)}{=}-\frac{5M_{\rm Pl}^2\Lambda_*^{2(1-\epsilon_2)}}{3b\,e^{\tilde\gamma_{\rm E}}}\,\frac{(|x-y|^2)^{-\epsilon_2-2}}{B_4(2\epsilon_2+4)}\,.\end{aligned} \tag{88}$$

On the spatial section $x_0=y_0$, this becomes

$$\begin{aligned}\langle\delta\rho(\boldsymbol{x})\delta\rho(\boldsymbol{y})\rangle&=-\frac{5M_{\rm Pl}^2\Lambda_*^{2(1-\epsilon_2)}}{3b\,e^{\tilde\gamma_{\rm E}}}\,\frac{(|\boldsymbol{x}-\boldsymbol{y}|^2)^{-\epsilon_2-2}}{B_4(2\epsilon_2+4)}\\&=-\frac{5M_{\rm Pl}^2\Lambda_*^{2(1-\epsilon_2)}}{3b\,e^{\tilde\gamma_{\rm E}}}\,\frac{B_3(2\epsilon_2+4)}{B_4(2\epsilon_2+4)}\\&\quad\times\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}(k^2)^{\epsilon_2+2-\frac{3}{2}},\end{aligned} \tag{89}$$

where in the last equality we used again Eq. (75) for $D=3$ and $s=\epsilon_2+2$. Using the second formula in Eq. (75), we can compute the ratio

$$\frac{B_3(2\epsilon_2+4)}{B_4(2\epsilon_2+4)}=\frac{\Gamma\left(-\epsilon_2-\frac{1}{2}\right)}{2\sqrt{\pi}\Gamma(-\epsilon_2)}\simeq\epsilon_2\,, \tag{90}$$

where we assumed $\epsilon_2\ll1$. Finally,

$$\begin{aligned}\langle\delta\rho(\boldsymbol{x})\delta\rho(\boldsymbol{y})\rangle&\simeq-\frac{5M_{\rm Pl}^2\Lambda_*^{2(1-\epsilon_2)}}{3b\,e^{\tilde\gamma_{\rm E}}}\,\epsilon_2\\&\quad\times\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}k^{2\epsilon_2+1}.\end{aligned} \tag{91}$$

At this point, we can compute Eq. (86) employing the definitions of Appendix D. In particular, we apply Eq. (D1) to $\psi(\boldsymbol{x})=\Phi(\boldsymbol{x})$:

$$\begin{aligned}\langle\Phi(\boldsymbol{x})\Phi(\boldsymbol{y})\rangle &= \frac{1}{4M_{\rm Pl}^4}\left[-\frac{5M_{\rm Pl}^2\Lambda_*^{2(1-\epsilon_2)}}{3b\,e^{\tilde\gamma_{\rm E}}}\right]\epsilon_2\\ &\quad\times\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\,e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}\,\frac{1}{k^4}k^{2\epsilon_2+1}\\ &= -\frac{5\Lambda_*^{2(1-\epsilon_2)}}{12b\,e^{\tilde\gamma_{\rm E}}M_{\rm Pl}^2}\frac{\epsilon_2}{2\pi^2}\int_0^{+\infty}\frac{dk}{k}\frac{\sin(kr)}{kr}k^{2\epsilon_2}\\ &= \int_0^{+\infty}\frac{dk}{k}\Delta_\Phi^2(k)\frac{\sin(kr)}{kr}\,, \end{aligned}\tag{92}$$

where, according to Eq. (D2), we introduced the power spectrum

$$\begin{aligned}\Delta_\Phi^2(k) &= -\frac{5\Lambda_*^2}{6b\,e^{\tilde\gamma_{\rm E}}M_{\rm Pl}^2}\frac{\epsilon_2}{(2\pi)^2}\left(\frac{k}{\Lambda_*}\right)^{2\epsilon_2}\\ &\overset{(74)}{=}\frac{5}{12}\frac{\epsilon_2^2}{(2\pi)^2\beta_{\rm Ric}}\,(\ell_* k)^{2\epsilon_2}\,,\end{aligned}\tag{93}$$

which is positive since $\beta_{\rm Ric}$ is positive.

In order to get the full result including the second term in Eq. (E9) proportional to $5/24$, we simply have to add to Eq. (93) a similar contribution obtained replacing $5/12$ with $5/24$ and ϵ_2 with ϵ_0. Hence, the total contribution to the power spectrum is

$$\begin{aligned}\Delta_\Phi^2(k) &= \frac{5}{12}\frac{\epsilon_2^2}{(2\pi)^2\beta_{\rm Ric}}\,(\ell_* k)^{2\epsilon_2}\\ &\quad+\frac{5}{24}\frac{\epsilon_0^2}{(2\pi)^2(\beta_{\rm Ric}+3\beta_R)}\,(\ell_* k)^{2\epsilon_0}\\ &= \frac{5}{12}\frac{\epsilon_2^2}{(2\pi)^2\beta_{\rm Ric}}\,(\ell_* k)^{2\epsilon_2}\\ &\quad+\frac{5}{24}\frac{\epsilon_2^2(\beta_{\rm Ric}+3\beta_R)}{(2\pi)^2\beta_{\rm Ric}^2}\,(\ell_* k)^{2\epsilon_0}\\ &= \frac{5}{12}\frac{\epsilon_2^2}{(2\pi)^2\beta_{\rm Ric}}\left[(\ell_* k)^{2\epsilon_2}+\frac{\beta_{\rm Ric}+3\beta_R}{2\beta_{\rm Ric}}\,(\ell_* k)^{2\epsilon_0}\right].\end{aligned}\tag{94}$$

However, according to Eq. (B25), the coefficient of the second term is $\ll 1$. Therefore, Eq. (93) is a good approximation of the scalar power spectrum.

The primordial scalar spectrum $\mathcal{P}_{\rm s}$ is defined by the spectrum of the gauge-invariant curvature perturbation on uniform density slices ζ, $\mathcal{P}_{\rm s}(k):=k^3P_\zeta(k)/(2\pi^2)$. In the presence of one barotropic fluid with constant w and ignoring the anisotropic stress, one can show that the relation between the potential Φ and ζ is (see, e.g., [56, Chap. 8])

$$\Phi\simeq-\frac{3+3w}{5+3w}\,\zeta\,.\tag{95}$$

In particular, for radiation $\Phi=-(2/3)\zeta$, so that the final form for the scalar power spectrum is

$$\boxed{\mathcal{P}_{\rm s}(k)=\frac{9}{4}\Delta_\Phi^2(k)=\frac{15}{64}\frac{(n_{\rm s}-1)^2}{(2\pi)^2\beta_{\rm Ric}}\,(\ell_* k)^{n_{\rm s}-1}\,,}\tag{96}$$

since the scalar spectral index is

$$\boxed{n_{\rm s}-1:=\frac{d\ln\mathcal{P}_{\rm s}}{d\ln k}=2\epsilon_2=-\frac{4\beta_{\rm Ric}\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}M_{\rm Pl}^2}\,,}\tag{97}$$

which is negative definite. Here we used Eq. (74).

The small value of $n_{\rm s}-1$ is related to perturbative quantum field theory via the constant $\beta_{\rm Ric}$, while the existence of the intermediate scale $\Lambda_*/\sqrt{b}<\Lambda_*=O(M_{\rm Pl})$ with $b>1$ could be due to the number of fundamental fields (Appendix B 3) or to the geometric structure of the Universe in the conformal phase (Sec. VII). Therefore, there is no puzzle in having a scale smaller than the Planck mass in our model of perturbative quantum field theory.

Comparing Eqs. (82) and (97), we get the prediction

$$\boxed{n_{\rm t}=1-n_{\rm s}>0\,.}\tag{98}$$

This relation between the tensor and the scalar index is typical of noninflationary scenarios where scalar perturbations are sustained by thermal fluctuations. Examples are string-gas cosmology [57–59][7] and the new ekpyrotic model [62–64].

D. Tensor-to-scalar ratio

Using Eqs. (81) and (96), we can finally compute the tensor-to-scalar ratio at the pivot scale k_0:

$$\boxed{r:=\frac{\mathcal{P}_{\rm t}(k_0)}{\mathcal{P}_{\rm s}(k_0)}=\frac{64}{15(1-n_{\rm s})}\,(\ell_* k_0)^{2(1-n_{\rm s})}\,.}\tag{99}$$

VII. DISCUSSION

In this paper, we proposed a quantum-gravity scenario for the cosmology of the early Universe that passes through a Planckian conformally invariant phase. It is this Weyl conformal symmetry that geometrically solves, in a simple and natural way, the cosmological problems of the standard hot-big-bang model and, at the same time,

[7] A noninflationary model coming from matrix theory may realize string-gas cosmology from first principles [60, 61]. Its cosmological spectra are under construction.

predicts a scale-invariant power spectrum for the CMB at large scales without the need of any inflationary era.

These findings rely on a recently proposed nonlocal field theory for gravity coupled to matter consistent with unitarity, causality, and finiteness at the quantum level. Indeed, our statements above are based on the property of finiteness, that secures Weyl conformal invariance at the classical as well as at the quantum level. Once the conformal symmetry is spontaneously broken, two-point correlation functions get perturbative quantum log-corrections that make primordial spectra deviate from exact scale invariance. The main results of the paper are the tensor spectrum $\mathcal{P}_{\rm t}$ (81), the tensor index $n_{\rm t}$ (82), the scalar spectrum $\mathcal{P}_{\rm s}$ (96), the scalar index $n_{\rm s}$ (97), the relation (98) between the tensor and the scalar index, and the tensor-to-scalar ratio r (99).

In particular, logarithmic quantum corrections provide *unique predictions* for the tensor spectral index $n_{\rm t}$ (positive and equal to $1-n_{\rm s}$) and for the tensor-to-scalar ratio r (dependent only on $n_{\rm s}$ and the scale ℓ_*). Therefore, quantum corrections are the reason for a quasi-scale-invariant spectrum for primordial perturbations, and a genuine evidence of quantum gravity in its realization in the quantum-field-theory framework.

The only free parameters in the model are the constants $\beta_{\rm Ric}$ (or, actually, the number of matter fields in the spectrum of the theory) and b (or, equivalently, $n_{\rm s}$, which is known from observations). However, $\beta_{\rm Ric}$ can only affect the amplitude of the perturbations, but not $n_{\rm t}$ or r, that are completely fixed in our model regardless of the details of the theory in the UV regime. The whole scenario is summarized in Fig. 8.

According to PLANCK observations, at the pivot scale $k_0 = 0.05\,{\rm Mpc}^{-1}$ the scalar index is $n_{\rm s} = 0.9649 \pm 0.0042$ at 68% confidence level, assuming $dn_{\rm s}/d\ln k = 0$ [65]. In particular,

$$\epsilon_2 = \frac{n_{\rm s}-1}{2} \approx -0.01755\,, \qquad k_0 = 0.05\,{\rm Mpc}^{-1}\,. \tag{100}$$

Therefore, $n_{\rm t}$ and r are uniquely specified and we have a real *quantum gravity prediction.* In particular,

$$n_{\rm t} \approx 0.0351\,. \tag{101}$$

According to Eq. (63), the tensor-to-scalar ratio at $k_0 = 0.05\,{\rm Mpc}^{-1}$ is bounded from below by

$$r_{0.05} = 0.009\,, \tag{102}$$

while its value for the natural choice $\ell_* = \ell_{\rm Pl}$ is

$$\boxed{r_{0.05} = 0.011\,,} \tag{103}$$

which is three times larger than the prediction for Starobinsky inflation, $r_{\rm Starobisky} = 0.0037$ for $\mathcal{N} = 57$ e-folds [66, 67]. The lower bound (102) is well within reach of present observations [68, 69], as discussed more in detail in [23].

Let us here focus on the parameter space of the theory compatible with data. Plugging the observed value of $n_{\rm s}$ into Eqs. (81) and (96) at $k_0 = 0.05\,{\rm Mpc}^{-1}$ and taking $\ell_* = \ell_{\rm Pl}$, we find

$$\mathcal{P}_{\rm t}(k_0) \simeq \frac{8.266\times 10^{-6}}{\beta_{\rm Ric}}, \quad \mathcal{P}_{\rm s}(k_0) \simeq \frac{7.867\times 10^{-4}}{\beta_{\rm Ric}}. \tag{104}$$

We provide two possible mechanisms to explain the small values for the amplitudes $\mathcal{P}_{\rm s}$ and $\mathcal{P}_{\rm t}$. In order to be consistent with the observed amplitude of scalar perturbations $\mathcal{P}_{\rm s} \approx 2.2\times 10^{-9}$, one possibility is to assume a very large value for $\beta_{\rm Ric} \approx 3.6\times 10^5$. Since $\beta_{\rm Ric} \propto N_{\rm fields}$ (Appendix B 3 and [70]), where $N_{\rm fields}$ is the total number of fields in the theory, the latter requires a large number of new fundamental particles. This assumption may be consistent with a particle-physics solution of the problem of dark matter, interpreted as the contribution of exotic particles, while at the same time it could also populate the “desert” lying between the mass of the Higgs boson and the Planck scale. The theory does not predict the value of the free mass scale $M_{\rm Pl}/\sqrt{b}$ that appears in $n_{\rm t}$, but we can argue that it takes an intriguing value. From Eqs. (82) and (98) with $\Lambda_* = M_{\rm Pl}$,

$$b = \frac{4\beta_{\rm Ric}}{(1-n_{\rm s})\,e^{\tilde\gamma_{\rm E}}}\,. \tag{105}$$

For $\beta_{\rm Ric} \approx 3.6\times 10^5$ and $\tilde\gamma_{\rm E} = \gamma_{\rm E}$, we get $b \approx 2.3\times 10^7$ and the mass scale $M_{\rm Pl}/\sqrt{b} = 5\times 10^{14}\,{\rm GeV}$, which looks like a low GUT scale and is about one order of magnitude larger than the Starobinsky mass for the curvaton.

The scenario just described is not the only possibility. Indeed, remaining within the Standard Model of particle physics, another reliable mechanism could be rooted in the choice of a different background in the sub-Planckian phase, in particular, the interpolating metric (49). When performing the rescaling from the Minkowski metric (41) to the spacetime (49), the two-point correlation function must be rescaled, too. Since the potential Φ is proportional to the metric perturbation $\boldsymbol{h}$ and $\boldsymbol{h}' = \Omega^2\boldsymbol{h}$, one has $\langle\Phi(\boldsymbol{x})\Phi(\boldsymbol{y})\rangle' = \Omega^4\langle\Phi(\boldsymbol{x})\Phi(\boldsymbol{y})\rangle$ and

$$\mathcal{P}_{\rm s}' = \Omega^4\mathcal{P}_{\rm s}\,. \tag{106}$$

The same rescaling is applied to the amplitude $\mathcal{P}_{\rm t}$, but the ratio r is conformally invariant. Therefore, evaluating Ω^2 at the Planck conformal time, i.e., for $\tau \gtrsim \tau_{\rm Pl}$, $\Omega^2(\tau_{\rm Pl}) = a^2(\tau_{\rm Pl}) \simeq A + B$. We get an amplitude $\mathcal{P}_{\rm s}'(k_0)$ consistent with observations if, for instance, $\beta_{\rm Ric} = 1$ and $A + B = 1.7\times 10^{-3}$, or $\beta_{\rm Ric} = 40$ and $A + B = 10^{-2}$. Therefore, in this case the level of fine tuning is very modest.

In order to complete the cosmological picture and further test the predictions of our model, one should look at other aspects of the evolution of the early as well as of the late universe. In the first case, scenarios based on conformal invariance are known to be able to generate a level of primordial non-Gaussianity larger than the one of inflation but still within the observational bounds [18, 19, 22]. The calculation of the three-point functions in our theory may be quite involved and should deserve a dedicated study. At the level of late-time cosmology, in this paper we have said nothing about the cosmological constant problem, which could be another field of application of the symmetries of the theory. Also, any late-time modification of the ΛCDM model should be able to explain the H_0 and the σ_8 tensions and satisfy certain general conditions outlined in [71–73]. These and other open questions await to be addressed in the future.

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Appendix A: Theory at the scale $\Lambda_{\rm hd}$

In this section, we derive the limit of the classical and the quantum theory in the Higgs phase of the conformal symmetry, i.e., in the energy range (29).

1. Classical theory at the scale $\Lambda_{\rm hd}$

From the Lagrangian (30), the action in the above limit reads

$$S_{\Lambda_{\rm hd}} \simeq \int d^D x\,\sqrt{|g|}\,E_{\mu\nu}F(\mathbf{\Delta})^{\mu\nu}_{\sigma\tau}E^{\sigma\tau}\,, \tag{A1}$$

where the form factor $F(\mathbf{\Delta})$ is defined in Eq. (11) and $\exp \mathrm{H}(\mathbf{\Delta})$ approaches the polynomial (25) in the UV regime. According to the results of Sec. II A, we can ignore the constant term in the form factor when $E \gtrsim \Lambda_{\rm hd}$, where $\Lambda_{\rm hd} = \sqrt{b_0/b}\,\Lambda_*$. Then,

$$\begin{aligned} 2\mathbf{\Delta}F(\mathbf{\Delta}) &= e^{\bar{\mathrm{H}}(\mathbf{\Delta})} - 1 \overset{\rm UV}{\simeq} e^{\tilde{\gamma}_{\rm E}}p(\mathbf{\Delta}) \\ &\overset{E\gtrsim\Lambda_{\rm hd}}{\simeq} e^{\tilde{\gamma}_{\rm E}}\left(-b\,z+z^4\right) = e^{\tilde{\gamma}_{\rm E}}\left[-b\frac{4\mathbf{\Delta}}{M_{\rm Pl}^2\Lambda_*^2}+\left(\frac{4\mathbf{\Delta}}{M_{\rm Pl}^2\Lambda_*^2}\right)^4\right]. \end{aligned} \tag{A2}$$

Therefore,

$$F(\mathbf{\Delta}) = -\frac{2e^{\tilde{\gamma}_{\rm E}}b}{M_{\rm Pl}^2\Lambda_*^2}+\frac{2^7e^{\tilde{\gamma}_{\rm E}}}{(M_{\rm Pl}\Lambda_*)^8}\mathbf{\Delta}^3\,. \tag{A3}$$

Below Λ_*, we can also ignore the $\mathbf{\Delta}^3$ term, so that the gravitational sector of the classical action at the scale $\Lambda_{\rm hd}$ is

$$\begin{aligned} S_{\Lambda_{\rm hd}}^{\rm vacuum} &= -\frac{2e^{\tilde{\gamma}_{\rm E}}b}{M_{\rm Pl}^2\Lambda_*^2}\int d^D x\,\sqrt{|g|}E_{\mu\nu}E^{\mu\nu} \\ &= -\frac{2e^{\tilde{\gamma}_{\rm E}}b}{M_{\rm Pl}^2\Lambda_*^2}\int d^D x\,\sqrt{|g|}\,\frac{M_{\rm Pl}^2}{2}\,G_{\mu\nu}\,\frac{M_{\rm Pl}^2}{2}\,G^{\mu\nu} \\ &= -\frac{e^{\tilde{\gamma}_{\rm E}}M_{\rm Pl}^2 b}{2\Lambda_*^2}\int d^D x\,\sqrt{|g|}\left(R_{\mu\nu}\,R^{\mu\nu}+\frac{D-4}{4}R^2\right). \end{aligned} \tag{A4}$$

The full action consist also of the matter-matter and matter-gravity contributions coming from the operator $E_{\mu\nu}E^{\mu\nu}$ in Eq. (2). The total fourth-order action reads

$$\begin{aligned} S_{\Lambda_{\rm hd}} &= -\frac{2e^{\tilde{\gamma}_{\rm E}}b}{M_{\rm Pl}^2\Lambda_*^2}\int d^D x\,\sqrt{|g|}\,\left(\frac{M_{\rm Pl}^2}{2}\,G_{\mu\nu}-\frac{1}{2}T_{\mu\nu}\right)\left(\frac{M_{\rm Pl}^2}{2}\,G^{\mu\nu}-\frac{1}{2}T^{\mu\nu}\right) \\ &= -\frac{e^{\tilde{\gamma}_{\rm E}}b}{M_{\rm Pl}^2\Lambda_*^2}\int d^D x\,\sqrt{|g|}\left[\frac{M_{\rm Pl}^4}{2}\left(R_{\mu\nu}\,R^{\mu\nu}+\frac{D-4}{4}R^2\right)-M_{\rm Pl}^2\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)T^{\mu\nu}+\frac{1}{2}T_{\mu\nu}T^{\mu\nu}\right]. \end{aligned} \tag{A5}$$

2. Quantum effective action at the scale $\Lambda_{\rm hd}$

The gravitational action at the energy scale $\Lambda_{\rm hd}$ is actually Stelle gravity for a particular choice of the coefficients for the $\mathbf{Ric}^2$ and the R^2 operators in Eq. (A4). According to [70, 74], the divergent part of the quantum effective action in Stelle gravity in $D = 4$ dimensions reads

$$\Gamma^{\rm div} = -\frac{1}{2\varepsilon}\frac{1}{(4\pi)^2}\int d^4\sqrt{|g|}\left(\beta_2 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \beta_3 R^2\right), \qquad \frac{1}{\varepsilon} = \ln\frac{\Lambda_{\rm UV}^2}{\mu^2}, \tag{A6}$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, $\beta_{2,3}$ are beta functions, $\Lambda_{\rm UV}$ is the cut-off energy scale in cut-off regularization, and μ is the renormalization energy scale.

Replacing the cut-off for ε in Eq. (A6), the quantum effective action including the finite contribution at high energy reads

$$\begin{aligned}\Gamma^{\rm div} + \Gamma^{\rm finite} &= -\frac{1}{2}\ln\frac{\Lambda_{\rm UV}^2}{\mu^2}\frac{1}{(4\pi)^2}\int d^4\sqrt{|g|}\left(\beta_2 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \beta_3 R^2\right)\\ &\quad+\frac{1}{2}\frac{1}{(4\pi)^2}\int d^4x\sqrt{|g|}\left[\beta_2 C_{\mu\nu\rho\sigma}\ln\left(-\frac{\Box}{\mu^2}\right)C^{\mu\nu\rho\sigma} + \beta_3 R\ln\left(-\frac{\Box}{\mu^2}\right)R\right]\\ &= \frac{1}{2}\frac{1}{(4\pi)^2}\int d^4x\sqrt{|g|}\left[\beta_2 C_{\mu\nu\rho\sigma}\ln\left(-\frac{\Box}{\Lambda_{\rm UV}^2}\right)C^{\mu\nu\rho\sigma} + \beta_3 R\ln\left(-\frac{\Box}{\Lambda_{\rm UV}^2}\right)R\right].\end{aligned} \tag{A7}$$

Now we can use the identity [75]

$$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = 2\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) + O(\mathbf{Riem}^3) \tag{A8}$$

to express the divergent and finite contributions to the quantum effective action (A7) in the Einstein basis up to higher curvature operators, i.e.,

$$\begin{aligned}\Gamma^{\rm div} + \Gamma^{\rm finite} &= \frac{1}{2}\frac{1}{(4\pi)^2}\int d^4x\sqrt{|g|}\left[2\beta_2 R_{\mu\nu}\ln\left(-\frac{\Box}{\Lambda_{\rm UV}^2}\right)R^{\mu\nu} - \frac{2}{3}\beta_2 R\ln\left(-\frac{\Box}{\Lambda_{\rm UV}^2}\right)R + \beta_3 R\ln\left(-\frac{\Box}{\Lambda_{\rm UV}^2}\right)R\right]\\ &= \frac{1}{(4\pi)^2}\int d^4x\sqrt{|g|}\left[\beta_2 R_{\mu\nu}\ln\left(-\frac{\Box}{\Lambda_{\rm UV}^2}\right)R^{\mu\nu} + \frac{3\beta_3 - 2\beta_2}{6}R\ln\left(-\frac{\Box}{\Lambda_{\rm UV}^2}\right)R\right].\end{aligned} \tag{A9}$$

This is also the one-loop quantum effective action of the theory (2) after some modifications. In contrast with Stelle's theory, the nonlocal theory considered in this paper is finite. This has two consequences. On one hand, we can identify the cut-off scale in Eq. (A6) with the fundamental scale of theory up to an $O(1)$ rescaling. On the other hand, the scale in $1/\varepsilon$ may differ in the two counterterms in Eq. (A6). In particular, and without loss of generality, we can set

$$C^2\text{ term}: \quad \Lambda_{\rm UV} = \Lambda_*, \qquad\qquad R^2\text{ term}: \quad \Lambda_{\rm UV} = \Lambda_*\,\delta_0, \qquad \delta_0 > 0, \tag{A10}$$

respectively for the Weyl-square and the Ricci-square counterterm.

Finally, comparing Eqs. (A9) and (30b), we get the relation between the one-loop coefficients in the nonlocal theory (2) and the beta functions of Stelle gravity:

$$\beta_{\rm Ric} = \frac{\beta_2}{(4\pi)^2}, \qquad \beta_R = \frac{3\beta_3 - 2\beta_2}{6(4\pi)^2}. \tag{A11}$$

We reiterate that the left-hand sides are not beta functions coming from counterterms.

Appendix B: Tree-level graviton propagator

In this appendix, we calculate the tree-level graviton propagator for the theory (2) in vacuum, after recalling some results in the literature of quantum gravity.

1. Tree-level propagator with generic form factors

Consider a D-dimensional general, purely gravitational local or nonlocal theory quadratic in the Ricci scalar and the Ricci tensor:

$$S = \frac{M_{\rm Pl}^2}{2}\int d^D x\sqrt{|g|}\,[R + R\gamma_0(\Box)R + R_{\mu\nu}\gamma_2(\Box)R^{\mu\nu}]\,, \tag{B1}$$

where $\gamma_0(\Box)$ and $\gamma_0(\Box)$ are generic analytic form factors depending on the Laplace–Beltrami operator $\Box$ and, in principle, on multiple scales Λ_i. The propagator for the gravitational perturbation $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ in the theory (B1) up to gauge-dependent terms reads [76]

$$\mathcal{O}^{-1}(k) = -\frac{4}{M_{\rm Pl}^2}\left(\frac{P^{(2)}}{k^2\,[1-k^2\gamma_2(k^2)]} - \frac{P^{(0)}}{k^2\,\{(D-2)+k^2\,[4(D-1)\gamma_0(k^2)+D\gamma_2(k^2)]\}}\right), \tag{B2}$$

where

$$P^{(2)}_{\mu\nu\rho\sigma} := \frac{1}{2}\left(\Theta_{\mu\rho}\Theta_{\nu\sigma}+\Theta_{\mu\sigma}\Theta_{\nu\rho}\right) - \frac{1}{D-1}\Theta_{\mu\nu}\Theta_{\rho\sigma}\,,$$

$$P^{(0)}_{\mu\nu\rho\sigma} := \frac{1}{D-1}\Theta_{\mu\nu}\,\Theta_{\rho\sigma}\,,\qquad \Theta_{\mu\nu} := \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\,, \tag{B3a}$$

are the Barnes–Rivers projectors [53].

2. Tree-level propagator in the theory (2)

The graviton propagator for the theory (2) has been computed in [24]. Here, we recall the form of the kinetic operator and the gauge-invariant components of the propagator. In D dimensions, the expansion of the vacuum action at second order in the graviton perturbation is

$$\begin{aligned}
S^{(2)}[h] &= \frac{1}{2}\int d^D x\left\{h^{\alpha\beta}\left(\frac{\delta^2 S_{\rm loc}}{\delta h^{\alpha\beta}\delta h^{\gamma\sigma}}\right)\left[e^{\bar{\rm H}(\boldsymbol{\Delta})}\right]^{\gamma\sigma}_{\mu\nu} h^{\mu\nu}\right\}\\
&= \frac{M_{\rm Pl}^2}{8}\int d^D x\, h^{\alpha\beta}\left\{\left[P^{(2)}-(D-2)P^{(0)}\right]\Box\right\}^{\gamma\sigma}_{\alpha\beta}\left\{e^{\bar{\rm H}\left[\frac{P^{(2)}-(D-2)P^{(0)}}{\Lambda_*^2}\Box\right]}\right\}_{\gamma\sigma,\mu\nu} h^{\mu\nu} \qquad (\text{B4})\\
&= \frac{M_{\rm Pl}^2}{8}\int d^D x\, h^{\alpha\beta}\left(\left\{P^{(2)}e^{\bar{\rm H}\left(\frac{\Box}{\Lambda_*^2}\right)} - (D-2)P^{(0)}e^{\bar{\rm H}\left[-\frac{(D-2)\Box}{\Lambda_*^2}\right]}\right\}\Box\right)_{\alpha\beta,\mu\nu} h^{\mu\nu}\\
&=: \frac{M_{\rm Pl}^2}{8}\int d^D x\, h^{\alpha\beta}\left\{\left[P^{(2)}\,e^{{\rm H}_2} - (D-2)P^{(0)}\,e^{{\rm H}_0}\right]\Box\right\}_{\alpha\beta,\mu\nu} h^{\mu\nu}\\
&=: \frac{1}{2}\int d^D x\, h^{\alpha\beta}\,\mathcal{O}^{\rm K}_{\alpha\beta\mu\nu}\,h^{\mu\nu}\,, \qquad (\text{B5})
\end{aligned}$$

where $\bar{\rm H}(z) = {\rm H}(z) - {\rm H}(0)$ was defined in Eq. (11), the entire function ${\rm H}(z)$ is defined in Eq. (13), and in the second and third equalities we wrote explicitly the dimensionless arguments of H.

Therefore, the propagator can be obtained from the propagator of the Einstein–Hilbert theory under the replacements

$$\Box\,P^{(2)} \quad\to\quad \Box\,e^{\bar{\rm H}\left(\frac{\Box}{\Lambda_*^2}\right)}\,P^{(2)} =: \Box\,e^{{\rm H}_2}\,P^{(2)}\,, \tag{B6}$$

$$\Box\,P^{(0)} \quad\to\quad \Box\,e^{\bar{\rm H}\left[-\frac{(D-2)\Box}{\Lambda_*^2}\right]}\,P^{(0)} =: \Box\,e^{{\rm H}_0}\,P^{(0)}\,. \tag{B7}$$

These rules are useful when computing the propagator for the action (2) including the logarithmic one-loop quantum corrections [see Eq. (B12)]. Augmenting the kinetic operator $\mathcal{O}^{\rm K}$ with a gauge-fixing term $\mathcal{O}_{\rm gf}$, from $\mathcal{O} = \mathcal{O}^{\rm K} + \mathcal{O}_{\rm gf}$ we get the tree-level graviton propagator up to gauge-dependent contributions [76]:

$$\mathcal{O}^{-1} = \frac{4}{M_{\rm Pl}^2}\left\{\frac{P^{(2)}}{\Box\,e^{\bar{\rm H}\left(\frac{\Box}{\Lambda_*^2}\right)}} - \frac{P^{(0)}}{\Box\,(D-2)e^{\bar{\rm H}\left[-\frac{(D-2)\Box}{\Lambda_*^2}\right]}}\right\} = \frac{4}{M_{\rm Pl}^2}\left[\frac{P^{(2)}}{\Box\,e^{{\rm H}_2}} - \frac{P^{(0)}}{(D-2)\Box\,e^{{\rm H}_0}}\right]. \tag{B8}$$

According to Eq. (17), in the UV the propagator becomes polynomial:

$$\mathcal{O}^{-1} \overset{\text{UV}}{\simeq} \frac{4}{M_{\text{Pl}}^2 e^{\tilde{\gamma}_{\text{E}}}} \left\{ \frac{P^{(2)}}{\Box\, p\left(\frac{\Box}{\Lambda_*^2}\right)} - \frac{P^{(0)}}{(D-2)\Box\, p\left[-\frac{(D-2)\Box}{\Lambda_*^2}\right]} \right\}, \tag{B9}$$

where $\tilde{\gamma}_{\text{E}}$ is defined in Eq. (21). Writing explicitly the scale in the argument of the polynomial (24),

$$p\left(\frac{\Box}{\Lambda_*^2}\right) = b_0 - b\,\frac{\Box}{\Lambda_*^2} + \left(\frac{\Box}{\Lambda_*^2}\right)^4, \tag{B10}$$

$$p\left[-\frac{(D-2)\Box}{\Lambda_*^2}\right] = b_0 + (D-2)b\frac{\Box}{\Lambda_*^2} + \left[(D-2)\frac{\Box}{\Lambda_*^2}\right]^4. \tag{B11}$$

When $b_0 \neq 0$, this constant can be dropped from the polynomial at the energy scale $E \gtrsim \Lambda_{\text{hd}}$ defined in Eq. (28).

3. One-loop graviton propagator

In this section, we calculate the graviton propagator for the quantum effective Lagrangian (30). Taking the sum of Eq. (B5) and the quadratic expansion in the graviton $h_{\mu\nu}$ of Eq. (30b), the quantum effective action reads

$$\begin{aligned}\Gamma^{(2)}[h] &= \int d^D x\left(\mathcal{L} + \mathcal{L}^Q\right) = \frac{1}{2}\int d^D x\, h_{\mu\nu}\left[\mathcal{O}^{\text{K}} + \mathcal{O}^Q\right]^{\mu\nu\rho\sigma} h_{\rho\sigma}\\ &= \frac{M_{\text{Pl}}^2}{8}\int d^D x\, h_{\mu\nu}\,\Box\left(\left(e^{\text{H}_2} + \Box\gamma_2^Q\right) P^{(2)} - \left\{(D-2)e^{\text{H}_0} - \Box\left[4(D-1)\gamma_0^Q + D\gamma_2^Q\right]\right\} P^{(0)}\right)^{\mu\nu,\rho\sigma} h_{\rho\sigma},\end{aligned} \tag{B12}$$

where γ_2^Q and γ_0^Q are the quantum corrections to the kinetic operator. According to Eq. (30b), the quantum form factors are

$$\gamma_2^Q \coloneqq \frac{2\beta_{\text{Ric}}}{M_{\text{Pl}}^2}\ln\left(-\frac{\Box}{\Lambda_*^2}\right), \qquad \gamma_0^Q \coloneqq \frac{2\beta_R}{M_{\text{Pl}}^2}\ln\left(-\frac{\Box}{\Lambda_*^2\,\delta_0^2}\right). \tag{B13}$$

The linearized equations of motion that we get from Eq. (B12) are

$$\begin{aligned}0 &= \Box\left(\left(e^{\text{H}_2} + \Box\gamma_2^Q\right) P^{(2)} - \left\{(D-2)e^{\text{H}_0} - \Box\left[4(D-1)\gamma_0^Q + D\gamma_2^Q\right]\right\} P^{(0)}\right)_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}\\ &= \Box\left\{e^{\text{H}_2} P^{(2)} - (D-2)e^{\text{H}_0} P^{(0)} + \Box\gamma_2^Q P^{(2)} + \Box\left[4(D-1)\gamma_0^Q + D\gamma_2^Q\right] P^{(0)}\right\}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}\\ &= \Box\left\{e^{\bar{\text{H}}}\left[P^{(2)} - (D-2)P^{(0)}\right] + \Box\gamma_2^Q P^{(2)} + \Box\left[4(D-1)\gamma_0^Q + D\gamma_2^Q\right] P^{(0)}\right\}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}\\ &= \Box\left(e^{\bar{\text{H}}}\left\{\left[P^{(2)} - (D-2)P^{(0)}\right] + e^{-\text{H}_2}\Box\gamma_2^Q P^{(2)} + e^{-\text{H}_0}\Box\left[4(D-1)\gamma_0^Q + D\gamma_2^Q\right] P^{(0)}\right\}\right)_{\mu\nu}^{\rho\sigma} h_{\rho\sigma},\end{aligned} \tag{B14}$$

where the operator $\bar{\text{H}}$ acting on the first term contains the projectors according to Eq. (B4). In the last two steps of Eq. (B14), we used the completeness relations of the projectors [53]:

$$\boldsymbol{P}^{(2)^2} = \boldsymbol{P}^{(2)}, \qquad \boldsymbol{P}^{(0)2} = \boldsymbol{P}^{(0)}, \qquad \boldsymbol{P}^{(2)}\boldsymbol{P}^{(0)} = 0, \tag{B15}$$

where we use a boldface notation to omit spacetime indices. From the action (B12) augmented by the gauge-fixing term, or combining together the results (B8) and (B2), the one-loop one-particle-irreducible propagator for the quantum effective Lagrangian (30) reads

$$\boldsymbol{\mathcal{O}}^{-1}(k) = -\frac{4}{M_{\text{Pl}}^2}\left(\frac{\boldsymbol{P}^{(2)}}{k^2\left[e^{\text{H}_2} - k^2\gamma_2^Q(k^2)\right]} - \frac{\boldsymbol{P}^{(0)}}{k^2\left\{(D-2)e^{\text{H}_0} + k^2\left[4(D-1)\gamma_0^Q(k^2) + D\gamma_2^Q(k^2)\right]\right\}}\right). \tag{B16}$$

In order to get the propagator, we used the identifications (B6) and (B7) to replace "1" in Eq. (B2) with $\exp \text{H}_2$ at the denominator of the spin-two contribution, and "$D-2$" with $(D-2)\exp \text{H}_0$ at the denominator of the spin-zero contribution.

In the high-energy limit, we can replace the form factors with the polynomial as we have done in Eq. (B9), and, thus, the propagator (B16) turns into

$$\boldsymbol{\mathcal{O}}^{-1}\overset{\rm UV}{\simeq}-\frac{4}{M_{\rm Pl}^2}\left(\frac{\boldsymbol{P}^{(2)}}{k^2\left[e^{\tilde{\gamma}_{\rm E}}p\left(-\frac{k^2}{\Lambda_*^2}\right)-k^2\gamma_2^Q(k^2)\right]}-\frac{\boldsymbol{P}^{(0)}}{k^2\left\{(D-2)e^{\tilde{\gamma}_{\rm E}}p\left[\frac{(D-2)k^2}{\Lambda_*^2}\right]+k^2\left[4(D-1)\gamma_0^Q(k^2)+D\gamma_2^Q(k^2)\right]\right\}}\right), \tag{B17}$$

where $\tilde{\gamma}_{\rm E}$ is defined in Eq. (21). Assuming now to be in the energy regime (29), which is still well within what we consider the UV regime, the action reduces to Stelle's theory with a certain choice of coefficients (Appendix A) and the dominant contribution comes from the first monomial in Eqs. (B10) and (B11), since at energy scales $E \gtrsim \Lambda_{\rm hd}$ we can forget the constant b_0 in the polynomial:

$$e^{\tilde{\gamma}_{\rm E}}p\left(-\frac{k^2}{\Lambda_*^2}\right)\simeq\frac{e^{\tilde{\gamma}_{\rm E}}b\,k^2}{\Lambda_*^2}\,,\qquad e^{\tilde{\gamma}_{\rm E}}p\left[\frac{(D-2)k^2}{\Lambda_*^2}\right]\simeq-\frac{e^{\tilde{\gamma}_{\rm E}}(D-2)\,b\,k^2}{\Lambda_*^2}\,. \tag{B18}$$

The propagator turns into:

$$\begin{aligned}\boldsymbol{\mathcal{O}}^{-1}(k)&\simeq-\frac{4}{M_{\rm Pl}^2}\left(\frac{\boldsymbol{P}^{(2)}}{k^4\left[\frac{e^{\tilde{\gamma}_{\rm E}}b}{\Lambda_*^2}-\gamma_2^Q(k^2)\right]}-\frac{\boldsymbol{P}^{(0)}}{k^4\left\{-(D-2)\frac{e^{\tilde{\gamma}_{\rm E}}(D-2)b}{\Lambda_*^2}+\left[4(D-1)\gamma_0^Q(k^2)+D\gamma_2^Q(k^2)\right]\right\}}\right)\\&=-\frac{4}{e^{\tilde{\gamma}_{\rm E}}b}\frac{\Lambda_*^2}{M_{\rm Pl}^2}\left(\frac{\boldsymbol{P}^{(2)}}{k^4\left[1-\frac{\Lambda_*^2}{e^{\tilde{\gamma}_{\rm E}}b}\gamma_2^Q(k^2)\right]}+\frac{\boldsymbol{P}^{(0)}}{k^4\left\{(D-2)^2-\frac{\Lambda_*^2}{e^{\tilde{\gamma}_{\rm E}}b}\left[4(D-1)\gamma_0^Q(k^2)+D\gamma_2^Q(k^2)\right]\right\}}\right).\end{aligned}$$

Replacing the quantum corrections (B13) in the above expression, we get

$$\begin{aligned}\boldsymbol{\mathcal{O}}^{-1}(k)&=-\frac{4}{e^{\tilde{\gamma}_{\rm E}}b}\frac{\Lambda_*^2}{M_{\rm Pl}^2}\left(\frac{\boldsymbol{P}^{(2)}}{k^4\left[1-\frac{2\Lambda_*^2}{e^{\tilde{\gamma}_{\rm E}}b}\frac{\beta_{\rm Ric}}{M_{\rm Pl}^2}\ln\left(\frac{k^2}{\Lambda_*^2}\right)\right]}+\right.\\&\qquad\left.+\frac{\boldsymbol{P}^{(0)}}{k^4\left\{(D-2)^2-\frac{\Lambda_*^2}{e^{\tilde{\gamma}_{\rm E}}b}\left[8(D-1)\frac{\beta_R}{M_{\rm Pl}^2}\ln\left(\frac{k^2}{\Lambda_*^2\,\delta_0^2}\right)+2D\frac{\beta_{\rm Ric}}{M_{\rm Pl}^2}\ln\left(\frac{k^2}{\Lambda_*^2}\right)\right]\right\}}\right)\\&=-\frac{4}{e^{\tilde{\gamma}_{\rm E}}b}\frac{\Lambda_*^2}{M_{\rm Pl}^2k^4}\left\{\frac{\boldsymbol{P}^{(2)}}{1+\epsilon_2\ln\left(\frac{k^2}{\Lambda_*^2}\right)}+\frac{\boldsymbol{P}^{(0)}}{(D-2)^2\left[1+\epsilon_0\ln\left(\frac{k^2}{\Lambda_*^2}\right)+\frac{8(D-1)\,\Lambda_*^2\,\beta_R}{(D-2)^2e^{\tilde{\gamma}_{\rm E}}b\,M_{\rm Pl}^2}\ln\delta_0^2\right]}\right\}\\&=-\frac{4}{e^{\tilde{\gamma}_{\rm E}}b}\frac{\Lambda_*^2}{M_{\rm Pl}^2k^4}\left[\frac{\boldsymbol{P}^{(2)}}{1+Q_2}+\frac{\boldsymbol{P}^{(0)}}{(D-2)^2\,(1+Q_0+Q_b)}\right],\end{aligned} \tag{B19}$$

where we defined

$$Q_2:=\epsilon_2\ln\frac{k^2}{\Lambda_*^2}\,,\qquad Q_0:=\epsilon_0\ln\frac{k^2}{\Lambda_*^2}\,,\qquad Q_b:=\frac{8(D-1)\,\Lambda_*^2\,\beta_R}{(D-2)^2\,e^{\tilde{\gamma}_{\rm E}}b\,M_{\rm Pl}^2}\ln\delta_0^2\,, \tag{B20}$$

$$\epsilon_2:=-\frac{2\beta_{\rm Ric}\Lambda_*^2}{b\,e^{\tilde{\gamma}_{\rm E}}M_{\rm Pl}^2}\,,\qquad \epsilon_0:=-\frac{2[D\beta_{\rm Ric}+4(D-1)\beta_{\rm R}]\Lambda_*^2}{(D-2)^2\,b\,e^{\tilde{\gamma}_{\rm E}}M_{\rm Pl}^2}\,. \tag{B21}$$

In $D=4$ dimensions, Eq. (B21) reads

$$\epsilon_2=-\frac{2\beta_{\rm Ric}\Lambda_*^2}{b\,e^{\tilde{\gamma}_{\rm E}}M_{\rm Pl}^2}\,,\qquad \epsilon_0=-\frac{2(\beta_{\rm Ric}+3\beta_{\rm R})\Lambda_*^2}{b\,e^{\tilde{\gamma}_{\rm E}}M_{\rm Pl}^2}\,,\qquad \epsilon_0-\epsilon_2=-\frac{6\Lambda_*^2}{b\,e^{\tilde{\gamma}_{\rm E}}M_{\rm Pl}^2}\beta_R=3\epsilon_2\frac{\beta_R}{\beta_{\rm Ric}}\,. \tag{B22}$$

In particular, from Eq. (A11) one has

$$\beta_{\rm Ric}+3\beta_R=\frac{3}{2}\frac{\beta_3}{(4\pi)^2}\,, \tag{B23}$$

and

$$\epsilon_0=\frac{3\epsilon_2}{2}\frac{\beta_3}{\beta_2}\ll\epsilon_2\,,\qquad \epsilon_2-\epsilon_0=\epsilon_2\frac{2\beta_2-3\beta_3}{2\beta_2}\simeq\epsilon_2\,, \tag{B24}$$

where in Eq. (B24) we assumed that $\beta_3 \ll \beta_2$, as is usually the case since β_3 depends on the number of fields $N_{\rm fields}$, while β_2 does not [70, formulæ (9.1) and (9.8)]. From Eq. (A11), this implies

$$\frac{\beta_{\rm Ric}+3\beta_R}{\beta_{\rm Ric}} \ll 1\,, \tag{B25}$$

which can happen because $\beta_R < 0$.

In order to manipulate the propagator analytically, at this point we make the following approximations:

$$1+Q_2 = 1+\ln\left(\frac{k^2}{\Lambda_*^2}\right)^{\epsilon_2} \simeq \left(\frac{k^2}{\Lambda_*^2}\right)^{\epsilon_2} =: u^{2\epsilon_2}\,, \tag{B26}$$

$$1+Q_0 = 1+\ln\left(\frac{k^2}{\Lambda_*^2}\right)^{\epsilon_0} \simeq \left(\frac{k^2}{\Lambda_*^2}\right)^{\epsilon_0} = u^{2\epsilon_0}\,. \tag{B27}$$

According to Eq. (B24), $\epsilon_0 \ll \epsilon_2$, so that it is sufficient to focus on ϵ_2, the largest of these two parameters. For $\epsilon_2 = -0.0175$ (observed value; see Sec. VI D), the approximation (B26) has an accuracy of 10% or better if

$$u^{2\epsilon_2} - (1+Q_2) < 0.01 \qquad \Longrightarrow \qquad 0.02 < u < 60\,. \tag{B28}$$

In the main text, we show that our cosmological model fully lies in this range of values. The other approximation we have to take into account is the self-consistency of the one-loop perturbative computation, namely,

$$\left|\ln u^{2\epsilon_2}\right| \ll 1\,, \qquad \left|\ln u^{2\epsilon_0}\right| \ll 1\,. \tag{B29}$$

Again, for $\epsilon_2 = -0.0175$ and a quantum correction smaller than 10% of the classical part, we found

$$\left|\ln u^{2\epsilon_2}\right| < 0.1 \qquad \Longrightarrow \qquad 0.06 < u < 17\,. \tag{B30}$$

This lower bound on u is larger than the one in (B28) and we can take it as a reference. For simplicity and to be even more conservative, we approximate it to

$$u \gtrsim 0.1\,, \tag{B31}$$

corresponding to an energy scale $E \gtrsim \Lambda_*/10$, which is within the conformally invariant trans-Planckian regime (36) if we identify Λ_* with $M_{\rm Pl}$.

Coming back to the calculation of the propagator, making use of the replacements (B26) and (B27) into Eq. (B19) we get

$$\begin{aligned}\boldsymbol{\mathcal{O}}^{-1}(k) &\simeq -\frac{4}{e^{\tilde\gamma_{\rm E}}b}\frac{\Lambda_*^2}{M_{\rm Pl}^2k^4}\left\{\frac{\boldsymbol{P}^{(2)}}{\left(\frac{k}{\Lambda_*}\right)^{2\epsilon_2}}+\frac{\boldsymbol{P}^{(0)}}{(D-2)^2\left[\left(\frac{k}{\Lambda_*}\right)^{2\epsilon_0}+Q_b\right]}\right\}\\ &= -\frac{4}{e^{\tilde\gamma_{\rm E}}b}\frac{\Lambda_*^{2(1+\epsilon_2)}}{M_{\rm Pl}^2k^{4+2\epsilon_2}}\left[\boldsymbol{P}^{(2)}+\frac{\boldsymbol{P}^{(0)}}{(D-2)^2\left(\frac{k}{\Lambda_*}\right)^{2(\epsilon_0-\epsilon_2)}}\right].\end{aligned} \tag{B32}$$

Here, we assumed that $Q_b \ll 1$. Indeed, in $D=4$ dimensions

$$Q_b = \frac{6\,\Lambda_*^2\,\beta_R}{e^{\tilde\gamma_{\rm E}}b\,M_{\rm Pl}^2}\ln\delta_0^2 = (\epsilon_2-\epsilon_0)\ln\delta_0^2\,, \tag{B33}$$

which is much smaller than 1 for $\epsilon_2-\epsilon_0 \simeq \epsilon_2 \ll 1$ [see Eq. (B24)] or $\delta_0 \sim 1$. This analysis justifies the assumption $Q_b = 0$ in the main text.

4. One-loop linearized equations of motion

On the same vein, we can write the equations of motion (B14) in vacuum as

$$0 = \Box\left(e^{\bar{\rm H}}\left\{\left[P^{(2)}-(D-2)P^{(0)}\right]+\frac{\frac{2\beta_{\rm Ric}}{M_{\rm Pl}^2}\Box\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)}{\exp{\rm H}_2}P^{(2)} + \frac{\Box\left[8(D-1)\frac{\beta_R}{M_{\rm Pl}^2}\ln\left(-\frac{\Box}{\Lambda_*^2\,\delta_0^2}\right)+2D\frac{\beta_{\rm Ric}}{M_{\rm Pl}^2}\ln\left(-\frac{\Box}{\Lambda_*^2}\right)\right]}{\exp{\rm H}_0}P^{(0)}\right\}\right)_{\mu\nu}^{\ \ \rho\sigma} h_{\rho\sigma}\,,$$

where $\bar{\mathrm{H}}$ is taken at the zero order in the perturbation and, for $\Lambda_{\rm hd} \lesssim E \ll \Lambda_*$, we can make the replacements (B10) and (B11):

$$0 = \Box\left(e^{\bar{\mathrm{H}}}\left\{\left[P^{(2)} - (D-2)P^{(0)}\right] + \frac{\frac{2\beta_{\rm Ric}}{M_{\rm Pl}^2}\Box\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)}{e^{\tilde{\gamma}_{\rm E}}\left(-b\frac{\Box}{\Lambda_*^2}\right)}P^{(2)}\right.\right.$$
$$\left.\left. + \frac{\Box\left[8(D-1)\frac{\beta_R}{M_{\rm Pl}^2}\,\ln\left(-\frac{\Box}{\Lambda_*^2\,\delta_0^2}\right) + 2D\frac{\beta_{\rm Ric}}{M_{\rm Pl}^2}\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)\right]}{e^{\tilde{\gamma}_{\rm E}}b(D-2)\frac{\Box}{\Lambda_*^2}}P^{(0)}\right\}\right)_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}$$
$$= \Box\left(e^{\bar{\mathrm{H}}}\left\{\left[P^{(2)} - (D-2)P^{(0)}\right] - \frac{2\beta_{\rm Ric}\Lambda_*^2\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)}{b\,e^{\tilde{\gamma}_{\rm E}}M_{\rm Pl}^2}P^{(2)}\right.\right.$$
$$\left.\left. + \Lambda_*^2\frac{8(D-1)\beta_R\ln\left(-\frac{\Box}{\Lambda_*^2\,\delta_0^2}\right) + 2D\beta_{\rm Ric}\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)}{b\,e^{\tilde{\gamma}_{\rm E}}(D-2)M_{\rm Pl}^2}P^{(0)}\right\}\right)_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}\,.$$

Using again the completeness relations for the projectors (B3), we can factorize the operator $P^{(2)} - (D-2)P^{(0)}$ and get

$$0 = \Box\left\{e^{\bar{\mathrm{H}}}\left[P^{(2)} - (D-2)P^{(0)}\right]\left[\mathbb{1} - \frac{2\beta_{\rm Ric}\Lambda_*^2\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)}{b\,e^{\tilde{\gamma}_{\rm E}}M_{\rm Pl}^2}P^{(2)}\right.\right.$$
$$\left.\left. - \Lambda_*^2\frac{8(D-1)\beta_R\ln\left(-\frac{\Box}{\Lambda_*^2\,\delta_0^2}\right) + 2D\beta_{\rm Ric}\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)}{b\,e^{\tilde{\gamma}_{\rm E}}(D-2)^2M_{\rm Pl}^2}P^{(0)}\right]\right\}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}\,,$$

where $\mathbb{1}_{\mu\nu,\rho\sigma} := (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})/2$ is the identity tensor [76]. Finally, for $Q_b \ll 1$ we write the linearized quantum equations of motion as

$$\begin{aligned}0 &= \Box\left\{e^{\bar{\mathrm{H}}}\left[P^{(2)} - (D-2)P^{(0)}\right]\left[\mathbb{1} + \epsilon_2\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)\,P^{(2)} + \epsilon_0\,\ln\left(-\frac{\Box}{\Lambda_*^2}\right)\,P^{(0)}\right]\right\}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}\\ &= \Box\left\{e^{\bar{\mathrm{H}}}\left[P^{(2)} - (D-2)P^{(0)}\right]\left[\mathbb{1} + Q_2\,P^{(2)} + Q_0\,P^{(0)}\right]\right\}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma}\,.\end{aligned} \tag{B34}$$

In the presence of matter, the equations of motion are (87). Expanding them at the linear level around the Minkowski background [indeed, the metric $\eta_{\mu\nu}$ is an exact solutions of the equations of motion (87) also including the quantum corrections E^Q], the contribution of the nonlocal form factor is of zero order in the fluctuations. Hence, it is diagonal and we can write the modified Einstein's equations (87) as

$$\left[e^{\mathrm{H}^{(0)}}\right]_{\mu\nu}^{\sigma\tau} G_{\sigma\tau}^{(1)} + \frac{2E_{\mu\nu}^{Q(1)}}{M_{\rm Pl}^2} = \frac{1}{M_{\rm Pl}^2}\left[e^{\mathrm{H}^{(0)}}\right]_{\mu\nu}^{\sigma\tau} T_{\sigma\tau}^{(1)}\,, \tag{B35}$$

where $^{(0)}$ and $^{(1)}$ stand for, respectively, the zero-order and the first-order expansion in the metric or matter perturbations. There is no $\mathrm{H}^{(1)}$ contribution, as shown in [27]. The left-hand side of Eq. (B35) at the linear level is given by Eq. (B14). Hence, the equations of motion (B35) at the linear level read

$$\Box\left(\cancel{e^{\mathrm{H}^{(0)}}}\left\{\left[P^{(2)} - (D-2)P^{(0)}\right] + \frac{\Box\gamma_2^Q}{e^{\mathrm{H}_2^{(0)}}}P^{(2)} + \frac{\Box\left[4(D-1)\gamma_0^Q + D\gamma_2^Q\right]}{e^{\mathrm{H}_0^{(0)}}}P^{(0)}\right\}\right)_{\mu\nu}^{\sigma\tau} h_{\sigma\tau}$$
$$= \frac{1}{M_{\rm Pl}^2}\cancel{\left[e^{\mathrm{H}^{(0)}}\right]_{\mu\nu}^{\sigma\tau}} T_{\sigma\tau}^{(1)},$$

and, factoring out the exponential form factor $\exp\mathrm{H}^{(0)}$ on both sides (which can be done since $\exp\bar{\mathrm{H}}$ is entire),

$$\frac{T_{\mu\nu}^{(1)}}{M_{\rm Pl}^2} = \Box\left\{\left[P^{(2)} - (D-2)P^{(0)}\right] + e^{-\mathrm{H}_2^{(0)}}\Box\gamma_2^Q\,P^{(2)} + e^{-\mathrm{H}_0^{(0)}}\Box\left[4(D-1)\gamma_0^Q + D\gamma_2^Q\right]P^{(0)}\right]_{\mu\nu}^{\sigma\tau} h_{\sigma\tau}\,, \tag{B36}$$

where one should not confuse the labels $^{(0)}$ and $^{(2)}$ of the projectors with the linearization subscripts. Finally, using the result (B34) and a compact vectorial notation, we can replace the quantum form factors in Eq. (B36) to end up with

$$\frac{\boldsymbol{T}^{(1)}}{M_{\text{Pl}}^2} = \Box \left[\boldsymbol{P}^{(2)} - (D-2)\boldsymbol{P}^{(0)}\right] \left[\mathbf{1} + Q_2 \boldsymbol{P}^{(2)} + Q_0 \boldsymbol{P}^{(0)}\right] \boldsymbol{h}\,. \tag{B37}$$

As stated at the end of Sec. IV E, the linearized quantum equations of motion (B37) reduce to the linearized Einstein's equations in the classical limit $\hbar \to 0$, i.e., for $Q_2 = Q_0 = 0$.

Appendix C: Geodesic completeness for massless particles

For the sake of simplicity, in this section we change notation redefining $\hat{g}^*_{\mu\nu} \to \hat{g}_{\mu\nu}$ and $\phi^* \to \phi$. Therefore, for the pair $(\hat{g}_{\mu\nu}, \phi) = (\hat{g}^{\text{FLRW}}_{\mu\nu}, M_{\text{Pl}}/\sqrt{2})$ Eq. (39) becomes

$$\hat{g}_{\mu\nu} = S(x)\, g^{\text{FLRW}}_{\mu\nu}\,, \quad \phi = \sqrt{M_{\text{Pl}}}\sqrt{2}\, S^{-\frac{1}{2}}(x)\,, \tag{C1}$$

where $g^{\text{FLRW}}_{\mu\nu}$ is the FLRW metric given in Eq. (40).

1. Geodesics for massless particles

The action for photons or general massless particles, parametrized by a parameter p, is

$$S_\gamma = \int dp\, \mathcal{L}_\gamma = \int dp\, e^{-1}(p) \phi^2 \hat{g}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu\,, \qquad \dot{} := \frac{d}{dp}\,, \tag{C2}$$

where $e(p)$ is an auxiliary field. The action (C2) is invariant under general coordinate transformations $x'^\mu = f^\mu(x^\nu)$, the Weyl conformal rescaling (10), and reparametrizations $p' = f(p)$ of the worldline, since $e(p)$ transforms as $e'(p') = e(p)(dp'/dp)^{-1}$.

The variation with respect to e gives

$$\frac{\delta S_\gamma}{\delta e} = -\int dp\, \frac{\delta e}{e^2} \phi^2\, \hat{g}_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu = 0 \quad \Longrightarrow \quad d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = 0\,, \tag{C3}$$

consistently with the fact that massless particles travel along the light-cone.

Varying with respect to x^μ, one gets the geodesic equation in the presence of the dilaton field. In the gauge $e(p) = \text{const}$,

$$\begin{aligned} \frac{D^2[\boldsymbol{g} = \phi^2 \hat{\boldsymbol{g}}]\, x^\sigma}{dp^2} &:= \frac{d^2 x^\sigma}{dp^2} + \Gamma^\sigma_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu \\ &= \frac{D^2[\hat{\boldsymbol{g}}]\, x^\sigma}{dp^2} + 2\frac{\partial_\mu \phi}{\phi} \dot{x}^\mu \dot{x}^\sigma - \frac{\partial^\sigma \phi}{\phi} \dot{x}_\mu \dot{x}^\mu = 0\,, \end{aligned} \tag{C4}$$

where

$$\Gamma^\sigma_{\mu\nu} := \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu}\right) \tag{C5}$$

is the Levi-Civita connection for the metric $\boldsymbol{g}$. When we contract Eq. (C4) with the velocity $\dot{x}_\sigma$ and we use $d\hat{s}^2 = 0$ obtained in Eq. (C3), we get the on-shell condition

$$\dot{x}_\sigma\, \frac{D^2[\hat{\boldsymbol{g}}]\, x^\sigma}{dp^2} + 2\dot{x}_\sigma\, \frac{\partial_\mu \phi}{\phi} \dot{x}^\mu \dot{x}^\sigma - \dot{x}_\sigma\, \frac{\partial^\sigma \phi}{\phi} \dot{x}^\mu \dot{x}_\mu = 0 \quad \Longrightarrow \quad \dot{x}_\sigma\, \frac{D^2[\hat{\boldsymbol{g}}]\, x^\sigma}{dp^2} = 0\,. \tag{C6}$$

Therefore, $D^2[\hat{\boldsymbol{g}}]\, x^\sigma/dp^2$ must be proportional to the velocity,

$$\frac{D^2[\hat{\boldsymbol{g}}]\, x^\sigma}{dp^2} = f\, \dot{x}^\sigma\,, \qquad f = \text{const}\,, \tag{C7}$$

which is null on the light-cone. Under a reparametrization $q = q(p)$ of the worldline, Eq. (C7) turns into

$$\frac{d^2x^\sigma}{dq^2} + \hat{\Gamma}^\sigma_{\mu\nu}\frac{dx^\mu}{dq}\frac{dx^\nu}{dq} = \frac{dx^\sigma}{dp}\left(\frac{dp}{dq}\right)^2\left(f\frac{dq}{dp} - \frac{d^2q}{dp^2}\right). \tag{C8}$$

Choosing the p-dependence of q such as to make the right-hand side of Eq. (C8) vanish, we end up with the geodesic equation in affine parametrization. Hence, we can redefine $q \to \lambda$ and, finally, get the affinely parametrized geodesic equation for photons in the metric $\hat{g}_{\mu\nu}$,

$$\frac{D^2[\hat{\boldsymbol{g}}]\, x^\sigma}{d\lambda^2} = 0\,. \tag{C9}$$

This result can be obtained also noting that the geodesic equation is invariant under a Weyl transformations, where λ is the conformally transformed affine parameter [47, Example 12.2].

We can now investigate the conservations laws based on the symmetries of the metric. Let us consider the scalar

$$\xi \coloneqq \hat{g}_{\mu\nu}v^\mu\frac{dx^\nu}{d\lambda} = \hat{g}_{\mu\nu}v^\mu\dot{x}^\nu\,, \tag{C10}$$

where v^μ is a generic vector and, from now on, $\dot{} = d/d\lambda$. Taking the derivative of Eq. (C10) with respect to λ and using Eqs. (C5) and (C9), we get

$$\frac{d\xi}{d\lambda} = \frac{1}{2}v^\mu\partial_\mu\hat{g}_{\rho\nu}\dot{x}^\rho\dot{x}^\nu + \hat{g}_{\mu\nu}\partial_\rho v^\mu\dot{x}^\nu\dot{x}^\rho = \frac{1}{2}[\mathcal{L}_v\hat{g}]_{\rho\nu}\,\dot{x}^\rho\dot{x}^\nu\,, \tag{C11}$$

where $\mathcal{L}_v\hat{\boldsymbol{g}}$ is the Lie derivative of $\hat{g}_{\mu\nu}$ by a vector field v^μ. Thus, if v^μ is a Killing vector field, namely $\mathcal{L}_v\hat{\boldsymbol{g}} = 0$, then the scalar (C10) is affinely constant:

$$\frac{d\xi}{d\lambda} = \frac{d}{d\lambda}\left(\hat{g}_{\mu\nu}v^\mu\dot{x}^\nu\right) = 0\,. \tag{C12}$$

2. Geodesic completeness in conformal gravity

After the rescaling (C1), the exact solution is Eq. (41), Minkowski spacetime in presence of the dilaton:

$$d\hat{s}^2 = -d\tau^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2\,, \qquad \phi = \sqrt{M_{\rm Pl}}\sqrt{2}\,a(\tau)\,. \tag{C13}$$

However, it is crucial to note that the dilaton does not appear in the geodesic equations (C9) and in the conservation equation (C12), which is only a function of $\hat{g}_{\mu\nu}$.

It is well known that Minkowski spacetime has ten Killing vectors, four of which are

$$v_0^\mu = (1,\,0,\,0,\,0)^\mu\,, \qquad v_1^\mu = (0,\,1,\,0,\,0)^\mu\,, \qquad v_2^\mu = (0,\,0,\,1,\,0)^\mu\,, \qquad v_3^\mu = (0,\,0,\,0,\,1)^\mu\,. \tag{C14}$$

When replaced in Eq. (C12), these vectors give the following conserved quantities,

$$\xi_0 \coloneqq \hat{g}_{\mu\nu}\,v_0^\mu\dot{x}^\nu = \hat{g}_{00}\,\dot{\tau} = -\dot{\tau}\,, \tag{C15a}$$

$$\xi_1 \coloneqq \hat{g}_{\mu\nu}\,v_1^\mu\dot{x}^\nu = \hat{g}_{11}\,\dot{x}^1 = \dot{x}^1\,, \tag{C15b}$$

$$\xi_2 \coloneqq \hat{g}_{\mu\nu}\,v_2^\mu\dot{x}^\nu = \hat{g}_{22}\,\dot{x}^2 = \dot{x}^2\,, \tag{C15c}$$

$$\xi_3 \coloneqq \hat{g}_{\mu\nu}\,v_3^\mu\dot{x}^\nu = \hat{g}_{33}\,\dot{x}^3 = \dot{x}^3\,, \tag{C15d}$$

which are the geodesic equations for massless particles in Minkowski spacetime. Since Eqs. (C15) are well defined everywhere for any value of the affine parameter λ, then this spacetime is geodesically complete. The condition (C3), i.e., $d\hat{s}^2 = 0$, only imposes the following consistency relation between the conserved quantities $\xi_0, \xi_1, \xi_2, \xi_3$ in Eq. (C15):

$$d\hat{s}^2 = -\xi_0^2 + \xi_1^2 + \xi_2^2 + \xi_3^2 = 0\,, \tag{C16}$$

which simply states that massless particles travel on the light-cone.

3. Geodesic incompleteness in general relativity

In contrast with the case of conformal gravity, the FLRW spacetime in Einstein gravity only possesses three out of the four Killing vectors (C15):

$$\xi_1 = a^2(\tau)\,\dot{x}^1\,, \qquad \xi_2 = a^2(\tau)\,\dot{x}^2\,, \qquad \xi_3 = a^2(\tau)\,\dot{x}^3\,. \tag{C17}$$

If we now replace the above conserved quantities in the FLRW line element before the rescaling, which is zero for light,

$$ds^2_{\rm FLRW} = a^2(\tau)\left[-d\tau^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2\right] = 0, \tag{C18}$$

we get the geodesic equation for $a(\tau) \neq 0$:

$$\dot{\tau}^2 = \frac{\xi_1^2 + \xi_2^2 + \xi_3^2}{a^4(\tau)}\,. \tag{C19}$$

For the sake of simplicity, we assume the spatial section to be flat and that it is well defined for $\tau \neq 0$. We can restrict our attention to $\tau > 0$ and integrate Eq. (C19) for the case of a radiation-dominated universe, where $a(\tau) = \tau/\tau_0$ and τ_0 is conformal time today. The result for $\dot{\tau} > 0$, $\tau > 0$, and $\lambda > 0$ is

$$\int d\tau\, \frac{\tau^2}{\tau_0^2} = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}\int d\lambda \quad \Longrightarrow \quad \lambda = \frac{1}{3\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}}\frac{\tau^3}{\tau_0^2} \quad \Longrightarrow \quad \tau = \left(3\tau_0^2\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}\right)^{\frac{1}{3}}\lambda^{\frac{1}{3}}\,. \tag{C20}$$

On the other hand, for $\dot{\tau} < 0$, $\tau > 0$, and $\lambda < 0$, we get

$$-\int d\tau\, \frac{\tau^2}{\tau_0^2} = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}\int d\lambda \quad \Longrightarrow \quad \lambda = -\frac{1}{3\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}}\frac{\tau^3}{\tau_0^2} \quad \Longrightarrow \quad \tau = \left(3\tau_0^2\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}\right)^{\frac{1}{3}}(-\lambda)^{\frac{1}{3}}\,. \tag{C21}$$

Since in Eq. (C20) the time coordinate $\tau(\lambda)$ of the massless particle evolves from the big-bang singularity for a finite amount of the affine parameter λ, then spacetime is not geodesically complete. In other words, we do not have any information about the past evolution for $\lambda \leqslant 0$. The two solutions (C20) and (C21) cannot be patched together in order to get a new solution defined for all λ, since the function $\tau(\lambda)$ is nonanalytic in $\lambda = 0$.

Appendix D: Spatial correlation function and power spectrum

The spatial correlation function for a generic field $\Psi(\boldsymbol{x})$ is defined by

$$\begin{aligned}
\langle\Psi(\boldsymbol{x})\Psi(\boldsymbol{y})\rangle &= \langle\Psi(\boldsymbol{x})\Psi(\boldsymbol{x}-\boldsymbol{r})\rangle \\
&=: \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\,P_\Psi(k)e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \\
&= \int_0^{2\pi} d\phi \int_0^{+\infty}\frac{dk\,k^2}{(2\pi)^3}\,P_\Psi(k)\int_{-1}^{1} du\,e^{ikru} \\
&= \int_0^{+\infty}\frac{dk\,k^2}{(2\pi)^2}\,P_\Psi(k)\int_{-1}^{1} du\,e^{ikru} \\
&= \int_0^{+\infty}\frac{dk\,k^2}{(2\pi)^2}\,P_\Psi(k)\frac{2\sin(kr)}{kr} \\
&= \int_0^{+\infty}\frac{dk\,k^2}{2\pi^2}\,P_\Psi(k)\frac{\sin(kr)}{kr}\,,
\end{aligned} \tag{D1}$$

where $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y}$, $r = |\boldsymbol{r}|$, and $k = |\boldsymbol{k}|$. It is common to define the dimensionless power spectrum as

$$\Delta^2_\Psi(k) := \frac{k^3}{2\pi^2}\,P_\Psi(k)\,, \tag{D2}$$

so that Eq. (D1) turns into

$$\langle\Psi(\boldsymbol{x})\Psi(\boldsymbol{y})\rangle = \int_0^{+\infty}\frac{dk}{k}\,\Delta^2_\Psi(k)\,\frac{\sin(kr)}{kr}\,. \tag{D3}$$

Appendix E: Two-point correlation function of the energy-momentum tensor

In this section, we provide all the details about the computation of the two-point correlation function of the perturbed energy-momentum tensor $T^{(1)}_{\mu\nu}$. For the sake of simplicity, we omit the indices, which are hidden in the boldface notation. Hence, from the linearized equations of motion (B37) we can get the two-point function

$$\begin{aligned}
&\frac{1}{M_{\rm Pl}^4}\langle \boldsymbol{T}^{(1)}(x)\,\boldsymbol{T}^{(1)}(y)\rangle \\
&\quad = \Box_x \left[\boldsymbol{P}^{(2)}_x - (D-2)\boldsymbol{P}^{(0)}_x\right]\left[\boldsymbol{1} + Q_2\boldsymbol{P}^{(2)}_x + Q_0\boldsymbol{P}^{(0)}_x\right]\Box_y \left[\boldsymbol{P}^{(2)}_y - (D-2)\boldsymbol{P}^{(0)}_y\right]\left[\boldsymbol{1} + Q_2\boldsymbol{P}^{(2)}_y + Q_0\boldsymbol{P}^{(0)}_y\right] \\
&\qquad \times \langle \boldsymbol{h}(x)\boldsymbol{h}(y)\rangle \\
&\quad = \int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\boldsymbol{\mathcal{O}}^{-1}(k)\,k^4 \\
&\qquad \times \left[\boldsymbol{P}^{(2)} - (D-2)\boldsymbol{P}^{(0)}\right]\left[\boldsymbol{1} + Q_2\boldsymbol{P}^{(2)} + Q_0\boldsymbol{P}^{(0)}\right]\left[\boldsymbol{P}^{(2)} - (D-2)\boldsymbol{P}^{(0)}\right]\left[\boldsymbol{1} + Q_2\boldsymbol{P}^{(2)} + Q_0\boldsymbol{P}^{(0)}\right] \\
&\quad \overset{\text{(B19)}}{=} \int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\,k^4 \left[\boldsymbol{P}^{(2)} - (D-2)\boldsymbol{P}^{(0)}\right]\left[\boldsymbol{1} + Q_2\boldsymbol{P}^{(2)} + Q_0\boldsymbol{P}^{(0)}\right] \\
&\qquad \times \left[\boldsymbol{P}^{(2)} - (D-2)\boldsymbol{P}^{(0)}\right]\left[\boldsymbol{1} + Q_2\boldsymbol{P}^{(2)} + Q_0\boldsymbol{P}^{(0)}\right]\left(-\frac{4}{b\,e^{\tilde\gamma_{\rm E}}}\frac{\Lambda_*^2}{M_{\rm Pl}^2k^4}\right)\left[\frac{\boldsymbol{P}^{(2)}}{1+Q_2} + \frac{\boldsymbol{P}^{(0)}}{(D-2)^2\,(1+Q_0+Q_b)}\right]. \qquad \text{(E1)}
\end{aligned}$$

Making iterated use of the completeness properties (B15) in the product of the first and third brackets with the last bracket, we can further simplify Eq. (E1),

$$\begin{aligned}
&\langle \boldsymbol{T}^{(1)}(x)\,\boldsymbol{T}^{(1)}(y)\rangle \\
&\quad = -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\left[\boldsymbol{1} + Q_2\boldsymbol{P}^{(2)} + Q_0\boldsymbol{P}^{(0)}\right]\left[\boldsymbol{1} + Q_2\boldsymbol{P}^{(2)} + Q_0\boldsymbol{P}^{(0)}\right]\left[\frac{\boldsymbol{P}^{(2)}}{1+Q_2} + \frac{\boldsymbol{P}^{(0)}}{1+Q_0}\right] \\
&\quad = -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\left[\boldsymbol{1} + (2Q_2+Q_2^2)\boldsymbol{P}^{(2)}_k + (2Q_0+Q_0^2)\boldsymbol{P}^{(0)}_k\right]\left[\frac{\boldsymbol{P}^{(2)}_k}{1+Q_2} + \frac{\boldsymbol{P}^{(0)}_k}{1+Q_0}\right] \\
&\quad = -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\left[\frac{(1+Q_2)^2\boldsymbol{P}^{(2)}}{1+Q_2} + \frac{(1+Q_0)^2\boldsymbol{P}^{(0)}}{1+Q_0}\right] \\
&\quad = -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\left[(1+Q_2)\boldsymbol{P}^{(2)} + (1+Q_0)\boldsymbol{P}^{(0)}\right]. \qquad \text{(E2)}
\end{aligned}$$

Let us now display the indices and also introduce the explicit form of the projectors (B3) in the two-point function (E2):

$$\begin{aligned}
\langle T^{(1)}_{\mu\nu}(x)\,T^{(1)}_{\rho\sigma}(y)\rangle &= -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\left[(1+Q_2)P^{(2)}_{\mu\nu\rho\sigma} + (1+Q_0)P^{(0)}_{\mu\nu\rho\sigma}\right] \\
&\overset{\text{(B3)}}{=} -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\left\{(1+Q_2)\left[\frac{1}{2}\left(\Theta_{\mu\rho}\Theta_{\nu\sigma} + \Theta_{\mu\sigma}\Theta_{\nu\rho}\right) - \frac{1}{D-1}\Theta_{\mu\nu}\Theta_{\rho\sigma}\right]\right. \\
&\qquad \left. +(1+Q_0)\frac{1}{D-1}\Theta_{\mu\nu}\,\Theta_{\rho\sigma}\right\} \\
&= -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int \frac{d^Dk}{(2\pi)^D}\, e^{ik\cdot(x-y)}\left((1+Q_2)\left\{\frac{1}{2}\left[\left(\eta_{\mu\rho} - \frac{k_\mu k_\rho}{k^2}\right)\left(\eta_{\nu\sigma} - \frac{k_\nu k_\sigma}{k^2}\right)\right.\right.\right. \\
&\qquad \left.\left. + \left(\eta_{\mu\sigma} - \frac{k_\mu k_\sigma}{k^2}\right)\left(\eta_{\nu\rho} - \frac{k_\nu k_\rho}{k^2}\right)\right] - \frac{1}{D-1}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\left(\eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}\right)\right\} \\
&\qquad \left. +(1+Q_0)\frac{1}{D-1}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\left(\eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}\right)\right). \qquad \text{(E3)}
\end{aligned}$$

Expanding the products in the brackets:

$$\langle T^{(1)}_{\mu\nu}(x)\,T^{(1)}_{\rho\sigma}(y)\rangle = -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int\frac{d^Dk}{(2\pi)^D}\,e^{ik\cdot(x-y)}\left\{(1+Q_2)\left[\frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}-\eta_{\mu\rho}\frac{k_\nu k_\sigma}{k^2}-\frac{k_\mu k_\rho}{k^2}\eta_{\nu\sigma}+\frac{k_\mu k_\rho k_\nu k_\sigma}{k^4}\right.\right.\right.$$
$$\left.+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\sigma}\frac{k_\nu k_\rho}{k^2}-\frac{k_\mu k_\sigma}{k^2}\eta_{\nu\rho}+\frac{k_\mu k_\sigma k_\nu k_\rho}{k^4}\right)$$
$$\left.-\frac{1}{D-1}\left(\eta_{\mu\nu}\eta_{\rho\sigma}-\eta_{\mu\nu}\frac{k_\rho k_\sigma}{k^2}-\frac{k_\mu k_\nu}{k^2}\eta_{\rho\sigma}+\frac{k_\mu k_\nu k_\rho k_\sigma}{k^4}\right)\right]$$
$$\left.+(1+Q_0)\frac{1}{D-1}\left(\eta_{\mu\nu}\eta_{\rho\sigma}-\eta_{\mu\nu}\frac{k_\rho k_\sigma}{k^2}-\frac{k_\mu k_\nu}{k^2}\eta_{\rho\sigma}+\frac{k_\mu k_\nu k_\rho k_\sigma}{k^4}\right)\right\}. \tag{E4}$$

According to Lorentz invariance, the following identities hold:

$$\int d^Dk\,f(k)\,k_\mu k_\nu = \frac{1}{D}\,\eta_{\mu\nu}\int d^Dk\,f(k)\,k^2\,, \tag{E5}$$
$$\int d^Dk\,f(k)\,k_\mu k_\nu k_\rho k_\sigma = \frac{1}{D(D+2)}\,(\eta_{\mu\nu}\eta_{\rho\sigma}+\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})\int d^Dk\,f(k)\,k^4\,, \tag{E6}$$

which can be easily verified contracting the left-hand sides with the Minkowski metric. Applying these identities to Eq. (E4), we obtain

$$\langle T^{(1)}_{\mu\nu}(x)\,T^{(1)}_{\rho\sigma}(y)\rangle = -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int\frac{d^Dk}{(2\pi)^D}\,e^{ik\cdot(x-y)}\left\{(1+Q_2)\frac{(D-2)(D+1)}{(D+2)(D-1)}\left[\frac{1}{2}\,(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})-\frac{1}{D}\eta_{\mu\nu}\eta_{\rho\sigma}\right]\right.$$
$$\left.+(1+Q_0)\frac{1}{D(D+2)(D-1)}\left[(D^2-3)\eta_{\mu\nu}\eta_{\rho\sigma}+\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}\right]\right\}. \tag{E7}$$

Therefore, the two-point correlation function of the perturbation of the energy density in $D=4$ dimensions is

$$\begin{aligned}\langle\delta\rho(x)\delta\rho(y)\rangle &= \langle T^{(1)}_{00}(x)T^{(1)}_{00}(y)\rangle\\ &= -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int\frac{d^Dk}{(2\pi)^D}\,e^{ik\cdot(x-y)}\,\frac{D+1}{D(D+2)}\,[(D-2)(1+Q_2)+(1+Q_0)]\\ &\overset{D=4}{=} -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int\frac{d^4k}{(2\pi)^4}\,e^{ik\cdot(x-y)}\,\frac{5}{24}\,[2(1+Q_2)+(1+Q_0)]\,.\end{aligned} \tag{E8}$$

Replacing Eqs. (B26) and (B27) into Eq. (E8), we find

$$\langle\delta\rho(x)\delta\rho(y)\rangle = -\frac{4M_{\rm Pl}^2\Lambda_*^2}{b\,e^{\tilde\gamma_{\rm E}}}\int\frac{d^4k}{(2\pi)^4}\,e^{ik\cdot(x-y)}\,\frac{5}{24}\left[2\left(\frac{k}{\Lambda_*}\right)^{2\epsilon_2}+\left(\frac{k}{\Lambda_*}\right)^{2\epsilon_0}\right]. \tag{E9}$$

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