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Loop quantum cosmology and slow roll inflation

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In loop quantum cosmology the big bang is replaced by a quantum bounce which is followed by a robust phase of super-inflation. We show that this phase has an unforeseen implication: in presence of suitable inflationary potentials it funnels all dynamical trajectories to conditions which virtually guarantee a slow roll inflation with more than 68 e-foldings, without any input from the pre-big bang regime. This is in striking contrast to the situation in general relativity where it has been argued that the a priori probability of obtaining a slow roll inflation with $\mathcal{N}$ e-foldings is suppressed by a factor $e^{-3\mathcal{N}}$.

I. A PRIORI PROBABILITY OF INFLATION

The a priori likelihood of inflation in the early universe has drawn considerable attention over the last two decades (for early papers see e.g. [1, 2, 3]). With the more recent successes of inflationary models in providing a natural explanation of structure formation, it is all the more interesting to inquire if a sufficiently long, slow roll inflation requires fine tuning of initial conditions or if it occurs generically in a given theoretical paradigm (see e.g. [4, 5, 6, 7]).

A mathematically natural approach to this analysis invokes Laplace's principle of indifference [8] to calculate the *a priori* probability of a slow roll inflation. Here, one uses the canonical Liouville measure $d\mu_L$ and a flat probability distribution $P(s) = 1$ on the space of solutions of the theory under consideration [1]. Then the a priori probability is given by the *fractional* Liouville volume occupied by the sub-space of solutions in which a sufficiently long, slow roll inflation occurs. Further physical input can provide a sharper probability distribution $P(s)$ and a more reliable likelihood than the 'bare' a priori probability. However, a priori probabilities can be directly useful if they are very low or very high. In these cases, it would be an especially heavy burden on the fundamental theory to come up with the physical input that significantly alters them.

Calculating the a priori probability can be subtle because the total Liouville measure of the space of all solutions is often infinite [3]. However, sometimes it is possible to introduce physically motivated regularization schemes and show that the desired probability is insensitive to the details of the scheme. Recently, this approach was used by Gibbons and Turok [7] to argue that the probability of $\mathcal{N}$ e-folds of a slow roll, single field inflation is suppressed by a factor of $e^{-3\mathcal{N}}$ in general relativity. They concluded that, even if a fundamental theory allows inflation, an extremely sharp probability distribution $P(s)$ is needed in order to explain *why inflation actually occurred*.

The purpose of this communication is to show that the situation is reversed in loop quantum cosmology (LQC): Given suitable inflationary potentials every solution enjoys an inflationary phase and the a priori probability of obtaining at least 68 e-foldings, desired from phenomenological considerations, is extremely close to 1. Away from the Planck regime, LQC is virtually indistinguishable from general relativity. However, in the Planck regime, there are huge differences [9, 10, 11, 12, 13, 14, 15, 16, 17] and these are crucial to our analysis. In particular, the big-bang is replaced by a non-singular big bounce and initial conditions can be specified at the bounce in a fully controlled fashion. There is a robust phase of *super-inflation* immediately after the bounce [18, 19] which, surprisingly, shepherds most of the LQC solutions to phase space regions from which a long, slow roll, exponential expansion is almost inevitable. Although several phenomenological consequences of this phase have been studied (see, e.g., [20, 21, 22, 23]), its implications to slow roll inflation had not been noticed. In particular, super-inflation (as well as the bounce) was ignored in the previous analysis [24] of the a priori likelihood of inflation in LQC.

II. LOOP QUANTUM COSMOLOGY

In the LQC treatment of simple cosmological models, the big bang and big crunch singularities are naturally resolved [25]. The origin of this resolution lies in the *quantum geometry effects* that are at the heart of loop quantum gravity [26, 27, 28]. Exotic matter is not needed; indeed matter fields can satisfy all the standard energy conditions. Detailed analysis has been carried out in a variety of models: the k=0, ± 1 FRW space-times with or without a cosmological constant [10, 11, 13]; Bianchi models [14, 15] which admit anisotropies and gravitational waves; and Gowdy models [16] which admit inhomogeneities, and therefore an infinite number of degrees of freedom. The FRW models have been studied most extensively, using both analytical and numerical methods to solve the exact quantum equations of LQC [10, 11, 12, 17]. In these models, the big bang and the big-crunch are replaced by a quantum bounce, which is followed by a robust phase of super-inflation. Interestingly, full quantum dynamics, including the bounce, is well-approximated by certain effective equations. (For a recent review, see [29].)

In this paper we restrict ourselves to the phenomenologically more interesting case of the k=0 FRW model (although the method is applicable also to the k=1 case). The matter source will be a scalar field with positive kinetic energy and a suitable potential. Since the prior discussion of probabilities is based on general relativity, to facilitate comparison we use effective equations rather than the full quantum theory. Finally, we will use the natural Planck units c=$\hbar$=G=1 (rather than 8πG=1, often employed in cosmology).

In LQC, spatial geometry is encoded in the volume of a fixed, fiducial cell, rather than the scale factor a; $v = (\mathrm{const}) \times a^3$. The conjugate momentum is denoted by b. On solutions to Einstein's equations $b = \gamma H$ [17], where $H = \dot{a}/a$ is the standard Hubble parameter and γ is the Barbero-Immirzi parameter of LQC whose value, $\gamma \approx 0.24$, is fixed by the black hole entropy calculation. However, LQC modifies Einstein dynamics and on solutions to the LQC effective equations we have

$$H \;=\; \frac{1}{2\gamma\lambda}\,\sin 2\lambda b \;\approx\; \frac{0.93}{\ell_{\mathrm{Pl}}}\,\sin 2\lambda b \tag{2.1}$$

where $\lambda^2 \approx 5.2\ell_{\mathrm{Pl}}^2$ is the 'area-gap', the smallest non-zero eigenvalue of the area operator. In LQC, b ranges over $(0, \pi/\lambda)$ and general relativity is recovered in the limit $\lambda \to 0$. Quantum geometry effects modify the geometric, left side of Einstein's equations. In particular, the

Friedmann equation becomes

$$\frac{\sin^2 \lambda b}{\gamma^2 \lambda^2} \;=\; \frac{8\pi G}{3}\,\rho \;=\; \frac{8\pi G}{3}\,\big(\frac{\dot\phi^2}{2} + V(\phi)\big)\,. \tag{2.2}$$

To compare with the standard Friedmann equation $(\dot a/a)^2 = (8\pi/3)\rho$, it is often convenient to use (2.1) to write (2.2) as

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\,\rho\,\big(1 - \frac{\rho}{\rho_{\rm crit}}\big) \tag{2.3}$$

where $\rho_{\rm crit} = \sqrt{3}/32\pi^2\gamma^3 \approx 0.41\rho_{\rm Pl}$. By inspection it is clear from Eqs (2.1) - (2.3) that, away from the Planck regime —i.e., when $\lambda b \ll 1$ or, $\rho \ll \rho_{\rm crit}$— we recover classical general relativity. However, the modifications in the Planck regime are drastic. The main features of this new physics can be summarized as follows.

In general relativity, the Friedmann equation implies that if the matter density is positive, $\dot a$ cannot vanish. Thus the solution represents *either* a contracting universe *or* an expanding one. By contrast, the LQC modified Friedmann equation (2.3) implies that at $\rho = \rho_{\rm crit}$, $\dot a$ vanishes. This is a quantum bounce. To its past, the solution represents a contracting universe with $\dot a < 0$ and to its future, an expanding one with $\dot a > 0$.

As is customary in the literature on probabilities, let us ignore the exceptional de Sitter solutions with eternal inflation. On all other solutions b decreases monotonically from $b = \pi/\lambda$ to 0. Eqs (2.2) and (2.3) imply that $b = \pi/2\lambda$ at the bounce. Thus, each solution undergoes precisely one bounce. The Hubble parameter $H = \dot a/a$ *vanishes* at the bounce and Eq.(2.1) implies that it is bounded on the solution space; $|H| \lesssim 0.93/\ell_{\rm Pl}$. By contrast, in general relativity, H is large in the entire Planck regime and diverges at the singularity.

When the potential is bounded below, $|\dot H|$ is bounded above by $10.29/\ell_{\rm Pl}^2$. The Ricci scalar —the only non-trivial curvature scalar in these models— is bounded above by $31/\ell_{\rm Pl}^2$. Thus, physical quantities which diverge at the big bang of general relativity cannot exceed certain finite, maximum values in LQC. One can also show that if $v \neq 0$ initially, it cannot vanish in finite proper time along any solution. Thus, *the LQC solutions are everywhere regular irrespective of whether one focuses on matter density, curvature or the scale factor.*

Next, the full set of space-time equations of motion can be written in terms of $v(t), \phi(t)$. These variables are subject to the constraint (2.3) and evolve via:

$$\ddot v = \frac{24\pi v}{\rho_{\rm crit}}\,\big[(\rho - V(\phi))^2 + V(\phi)(\rho_{\rm crit} - V(\phi))\big], \qquad \ddot\phi + \frac{\dot v}{v}\,\dot\phi + V_{,\phi} = 0\,. \tag{2.4}$$

Our task is to obtain the Liouville measure on the space $\mathbb{S}$ of solutions to these equations.

For this, we first construct the phase space Γ. It consists of quadruplets $(v, b;\, \phi, p_{(\phi)})$, with $\lambda b \in [0, \pi/2]$. The Liouville measure on Γ is simply $\mathrm{d}\mu_{\rm L} = \mathrm{d}v\,\mathrm{d}b\,\mathrm{d}\phi\,\mathrm{d}p_{(\phi)}$. The LQC Friedmann equation implies that these variables must lie on a constraint surface $\bar\Gamma$ defined by

$$\frac{p_{(\phi)}^2}{2v^2} + 4\pi^2\gamma^2 V(\phi) = \frac{3\pi}{2\lambda^2}\,\sin^2\lambda b\,. \tag{2.5}$$

They evolve via

$$\dot{v} = \frac{3v}{2\gamma}\,\frac{\sin 2\lambda b}{\lambda}, \quad \dot{b} = -\frac{p_{(\phi)}^2}{\pi\gamma v^2}, \quad \dot{\phi} = \frac{p_{(\phi)}}{2\pi\gamma v}, \quad \text{and} \quad \dot{p}_{(\phi)} = -2\pi\gamma |v|\, V_{,\phi}\,. \tag{2.6}$$

As is well-known, the space of solutions $\mathbb{S}$ is naturally isomorphic to a gauge fixed surface, i.e., a 2-dimensional surface $\hat{\Gamma}$ of $\bar{\Gamma}$ which is intersected by each dynamical trajectory once and only once. Since b is monotonic in each solution, an obvious strategy is to choose for $\hat{\Gamma}$ a 2-dimensional surface $b = b_o$ (a fixed constant) within $\bar{\Gamma}$. Symplectic geometry considerations unambiguously equip $\hat{\Gamma}$ —and hence the solution space $\mathbb{S}$— with an induced Liouville measure $\mathrm{d}\hat{\mu}_{\text{Ł}}$. Since the dynamical flow preserves the Liouville measure, $\mathrm{d}\hat{\mu}_{\text{Ł}}$ on $\mathbb{S}$ is *independent of the choice of* b_o. The most natural choice in LQC is to set $b_o = \pi/2\lambda$ so that $\bar{\Gamma}$ is just the 'bounce surface'. We will make this choice because it also turns out to be convenient for calculations.

Then $\hat{\Gamma}$ is naturally coordinatized by $(\phi_{\rm B}, v_{\rm B})$, the scalar field and the volume at the bounce. Since $b = \pi/2\lambda$, the constraint (2.5) determines $p_{(\phi)}$ (or, equivalently, $\dot{\phi}$) up to sign which, without loss of generality, will be taken to be non-negative. The induced measure on $\mathbb{S}$ can be written explicitly as:

$$\mathrm{d}\hat{\mu}_{\text{Ł}} = \frac{\sqrt{3\pi}}{\lambda}\,\big[1 - F_{\rm B}\big]^{\frac{1}{2}}\,\mathrm{d}\phi_{\rm B}\,\mathrm{d}v_{\rm B} \tag{2.7}$$

where $F_{\rm B} = V(\phi_{\rm B})/\rho_{\rm crit}$ is the fraction of the total density that is in the potential energy at the bounce. The total Liouville volume of $\bar{\Gamma} \equiv \mathbb{S}$ is infinite because, although $\phi_{\rm B}$ is bounded for suitable potentials such as $m^2\phi^2$, $v_{\rm B}$ is not. However, this non-compact direction represents gauge on the space of solutions $\mathbb{S}$: If $(\phi(t), v(t))$ is a solution to (2.2) and (2.4), so is $(\phi(t), \alpha v(t))$ and this rescaling by a constant α simply corresponds to a rescaling of spatial coordinates (or of the fiducial cell) under which physics does not change. Therefore, as discussed in section IV, there is a natural prescription to calculate fractional volumes of physically relevant sub-regions of $\hat{\Gamma}$.

III. SUPER-INFLATION AND INFLATION

For our purposes it suffices to focus just on the *post bounce* part of solutions. The key question now is: What is the fractional Liouville volume in $\mathbb{S}$ occupied by solutions that exhibit a sufficiently long inflation? To answer it in detail we will use $V(\phi) = (1/2)m^2\phi^2$. Then (2.5) implies that $m\phi_{\rm B} \in [-0.90, 0.90]$ and we are led to set $m = 6 \times 10^{-7}\mathrm{M_{Pl}}$ by phenomenological considerations, [30] (recall that we have set G=1 rather than 8πG=1).

Let us first focus on the part $\mathbb{S}^+$ of solutions on which ϕ is non-negative at the bounce surface $\hat{\Gamma}$. Then, the problem can be divided into three parts using the fraction $F_{\rm B}$. In each part, one can introduce suitable approximations to analyze dynamics.

(i) $F_{\rm B} < 10^{-4}$: *Kinetic energy dominated bounce.* At the bounce the Hubble parameter H vanishes. However, there is a short phase of super-inflation lasting a fraction of a Planck second during which H increases very rapidly to its maximum value $H_{\rm max} = 0.93$. At this point $\dot{H}$ vanishes and then H starts decreasing and continues to decrease during the rest of the evolution. Since $\dot{\phi} > 0$, the inflaton climbs up the potential during super-inflation and continues to do so afterward super-inflation till it reaches a turn-around point at $\dot{\phi} = 0$.

Then it starts descending. Very soon after that, $\ddot{\phi}$ vanishes. This is the onset of slow roll inflation during which $\dot{H}/H^2$ is in the range $1.6 \times 10^{-2} - 3.3 \times 10^{-10}$. The time required to reach this onset from the bounce is in the range of $10^6 - 10^2$ $\mathrm{s_{pl}}$ where $\mathrm{s_{Pl}}$ denotes Planck seconds. The number of e-foldings during inflation is given approximately by

$$\mathcal{N} \approx 2\pi \left(1 - \frac{\phi_o^2}{\phi_{\max}^2}\right) \phi_o^2 \ln \phi_o \tag{3.1}$$

where ϕ_o is the value of the scalar field at the *onset* of inflation. ϕ_o increases monotonically with $\phi_{\rm B}$ (and is always larger than $\phi_{\rm B}$). For $\phi_{\rm B} = 0.99$, we have $\phi_o = 3.24$ and $\mathcal{N} = 68$. Thus, *for a kinetic energy dominated bounce, there is a slow roll inflation with over* 68 *e-foldings for all* $\phi_{\rm B} > 1$, i.e., $F_{\rm B} > 4.4 \times 10^{-13}$.

(ii) $10^{-4} < F_{\rm B} < 0.5$: *The intermediate case.* The LQC departures from general relativity are now increasingly significant. The super-inflation era is similar to case (i). However, now $\phi_{\rm B}$ is higher and $\dot{\phi}_{\rm B}$ lower while, as before, H is very high at the end of super-inflation. Therefore, the coefficient of friction, H/m^2, is large and one arrives at the slow roll conditions within 10-100 $s_{\rm Pl}$ after the bounce. Consequently, now the change $(\phi_o - \phi_{\rm B})$ is negligible, a key feature not shared by regime (i). At the onset of slow roll inflation, the Hubble parameter is now given to an excellent approximation by

$$H_o \approx \Big[\, \frac{8\pi}{3}\, \rho_{\rm crit}\, F_{\rm B}(1 - F_{\rm B}) \,\Big]^{1/2} \;\approx\; 1.9 \,\big[\, F_{\rm B}(1 - F_{\rm B}) \,\big]^{1/2} \tag{3.2}$$

and decreases *very* slowly with $\dot{H}/H^2 < 3.5 \times 10^{-10}$. Thus, the Hubble parameter is essentially frozen to the value (3.2). This value is very high, in the range $1.9 \times 10^{-2}\ \mathrm{s_{Pl}^{-1}}$ to $9.3 \times 10^{-1}\ \mathrm{s_{Pl}^{-1}}$. The Hubble freezing is an LQC phenomenon: It relies on the fact that H acquires its largest value $H_{\max} = 0.93\,\mathrm{s_{Pl}^{-1}}$ at the end of super-inflation (and, in the case under consideration, $\dot{\phi}_{\rm B}$ is not large enough to decrease H more than two orders of magnitude). Eq. (3.1) implies that throughout this range of $F_{\rm B}$ there are more than 68 e-foldings.

(iii) $0.5 < F_{\rm B} < 1$: *Potential energy dominated bounce.* Now the LQC effects dominate. Again, because $\dot{\phi} > 0$, the inflaton climbs up the potential but now the turn around $(\dot{\phi} = 0)$ occurs *during super-inflation!* The change $(\phi_o - \phi_{\rm B})$ is even more negligible because the kinetic energy at the bounce is lower than that in case (ii). The Hubble parameter again freezes at the onset of inflation to the value given in (3.2). The slow roll conditions are easily met as $\dot{H}/H^2$ is less than 1×10^{-11} when $\ddot{\phi} = 0$ (or soon thereafter). A difference from the slow roll inflation of (i) and (ii) above is that H continues to grow during the slow roll because we are in the super-inflation phase. There are many more than 68 e-foldings already in the deep Planck regime where the matter density is greater than half the critical density.

Finally, let us consider the part $\mathbb{S}^-$ of the solution space on which $\phi_B < 0$. The main difference now is that the inflaton starts rolling down the potential immediately after the bounce. As before, in case (ii) the Hubble freezing occurs soon after the end of super-inflation and in (iii) during super-inflation. The value of H_o is again given by (3.2). In case (i), differences can arise from $\mathbb{S}^+$ because now the kinetic energy is very large at the bounce point so the inflaton can transit from a negative to a positive value before the onset of inflation. But after the onset, the situation is the same as in (ii). In this case, there are more than 68 e-foldings if $F_{\rm B} > 1.4 \times 10^{-11}$ or $\phi_{\rm B} \notin [-5.7, 0]$.

$F_B = V(\phi_{\rm B})/\rho_{\rm crit}$	Sign$[\phi_{\rm B}]$	t	ϕ	$\dot\phi$	H	$\dot H$
0	+/-	$1.6 * 10^6$	2.3	$-9.7 * 10^{-8}$	$2.8 * 10^{-6}$	$-1.2 * 10^{-13}$
$4.4 * 10^{-13}$	+	$1.2 * 10^6$	3.2	$-9.7 * 10^{-8}$	$4.0 * 10^{-6}$	$-1.2 * 10^{-13}$
	-	$2.1 * 10^6$	1.3	$9.6 * 10^{-8}$	$1.6 * 10^{-6}$	$-1.2 * 10^{-13}$
$1 * 10^{-4}$	+	$7.6 * 10^2$	$1.5 * 10^4$	$-9.8 * 10^{-8}$	$1.9 * 10^{-2}$	$-1.2 * 10^{-13}$
	-	$6.6 * 10^2$	$-1.5 * 10^4$	$9.8 * 10^{-8}$	$1.9 * 10^{-2}$	$-1.2 * 10^{-13}$
0.5	+	$1.6 * 10^1$	$1.1 * 10^6$	$-1.4 * 10^{-7}$	$9.3 * 10^{-1}$	$1.4 * 10^{-19}$
	-	$1.5 * 10^1$	$-1.1 * 10^6$	$1.4 * 10^{-7}$	$9.3 * 10^{-1}$	$-1.4 * 10^{-19}$
0.8	+	$2.0 * 10^1$	$1.3 * 10^6$	$-2.2 * 10^{-7}$	$7.4 * 10^{-1}$	$3.6 * 10^{-13}$
	-	$1.8 * 10^1$	$-1.3 * 10^6$	$2.2 * 10^{-7}$	$7.4 * 10^{-1}$	$3.6 * 10^{-13}$

TABLE I: Values of the proper time, the Hubble parameter, the scalar field and their time derivatives at onset of slow roll (where $\ddot\phi = 0$). $F_{\rm B} = V(\phi_{\rm B})/\rho_{\rm crit}$ is the ratio of the potential energy density to the total energy density at the bounce. If the value $\phi_{\rm B}$ of the scalar field is positive, the inflaton rises up the potential after the bounce while if $\phi_{\rm B}$ is negative it descends down the potential (because $\dot\phi_{\rm B}$ is assumed to be positive). For $\phi_{\rm B} > 0$, there are 68 e-foldings if $F_{\rm B} = 4.4 \times 10^{-13}$. The bounce is taken to occur at $t = 0$.

These general features of LQC dynamics emerge from analytical calculations based on approximations that are tailored to the three cases considered above. They were confirmed by detailed numerical simulations performed in MATLAB using a Runge Kutta (4,5) algorithm (ode45) to solve the set of coupled ODEs. Both relative and absolute tolerances were set at 3×10^{-14} and the preservation of the Hamiltonian constraint (2.5) to this order was verified on each solution. To ensure numerical accuracy, the natural logarithm of volume was treated as fundamental in the simulations. As noted above, the Immirzi parameter was set at 0.24 and inflaton mass 6×10^{-7} (in units c=$\hbar$=G=1). A large number of simulations were performed. Table 1 summarizes a few illustrative results.

IV. MEASURE AND PROBABILITIES

As explained in section II, the space $\mathbb{S}$ of solutions can be coordinatized by pairs $(\phi_{\rm B}, v_{\rm B})$. However, physics does not change under $(\phi_{\rm B}, v_{\rm B}) \to (\phi_{\rm B}, \alpha v_{\rm B})$, where α is a constant. In particular, the number of slow-roll e-foldings is insensitive to this rescaling of $v_{\rm B}$. Therefore, physically relevant regions $\mathcal{R}$ in $\mathbb{S}$ are those that contain complete gauge orbits: $\mathcal{R} = I \times \mathbb{R}^+$ where I is a closed interval in $[-\phi_{\rm max},\, \phi_{\rm max}]$ and $\mathbb{R}^+$ denotes the $v_{\rm B}$ axis. To calculate fractional volumes $P_{\mathcal{R}}$ of such regions it is natural to factor out by the 'volume of the gauge orbits'. This suggests an obvious strategy, commonly used in the physics literature:

$$P_{\mathcal{R}} \;=\; \lim_{v_0 \to 0} \frac{\text{Liouville Volume of } [I \times I_{v_0}]}{\text{Liouville Volume of } [I_{\rm total} \times I_{v_0}]} \;=\; \frac{\int_I {\rm d}\phi_{\rm B}\, [1 - F_{\rm B}]^{\frac{1}{2}}}{\int_{-\phi{\rm max}}^{\phi{\rm max}} {\rm d}\phi_{\rm B}\, [1 - F_{\rm B}]^{\frac{1}{2}}} \tag{4.1}$$

where $I_{v_0} = [v_0, 1/v_0]$ (with $v_0 > 0$). This physical idea can be mathematically justified by the 'group averaging technique' [31] to obtain a physical measure on $\mathbb{S}$ by averaging ${\rm d}\hat\mu_{\rm L}$ over the orbit of the 'gauge group.'

Let us now apply this strategy to calculate the probability that, prior to re-heating, there are at least 68 e-foldings of slow roll inflation or super-inflation in LQC. Since $F_{\rm B}$ ranges over $[0,1]$ and there are requisite number of e-foldings if $F_{\rm B} > 1.4 \times 10^{-11}$, it follows from (4.1) that the required probability is greater than 0.99999. Moreover, numerical simulations show that even when $F_{\rm B} \le 1.4 \times 10^{-11}$ there are at least 6.1 e-foldings. Thus the probability of obtaining at least 6.1 e-foldings is 1. By contrast, in general relativity this probability is suppressed by a factor of $e^{-18.3} \approx 1.1 \times 10^{-8}$ [7]. The minimum number of e-foldings is sensitive to the mass m of the inflaton we used. By contrast, the probability of getting at least 68 e-foldings is robust: it grows slightly if m is decreased and remains greater than 0.99 even if m increased by *two* orders of magnitude.

Thus, the situation in LQC is dramatically different from that in general relativity. Note that we used the same potential that is generally employed in the detailed calculations of probabilities [5, 7]. Furthermore, as in [7] (and unlike in [5]) we used the Liouville measure which is preserved by dynamics. The procedure we used to handle the fact that the total Liouville volume is infinite is physically and mathematically well motivated and it also constituted the basis of the regularization scheme used in [7]. Yet there is a striking contrast in the final outcome.

This can be traced back to the salient differences between LQC and general relativity at the Planck scale. Since LQC has its basis in LQG, a candidate fundamental theory of quantum gravity, it has, in particular, precise predictions in the Planck regime of the simple cosmological models that are used in the probability considerations. Consequently, we do not have to worry about setting judicious initial conditions at the singular big-bang. The bounce is regular and we considered *all possible* initial conditions there. The LQC dynamics are such that for $F_{\rm B} > 10^{-4}$ the robust super-inflation phase either suffices to yield a large number of e-foldings (case (iii)) or funnels the dynamical trajectories to the phase space region from which a sufficiently long slow roll inflation is almost inevitable (case (ii)). In fact, the second of these features persists so long as $F_{\rm B} \ge 1.4 \times 10^{-11}$ (case (i)) although the funneling mechanism is somewhat more involved. Thus, in LQC a long slow roll inflation may not result *only if* $F_{\rm B} < 1.4 \times 10^{-11}$. Since by definition $F \in [0,1]$ for all initial conditions, (4.1) implies that the probability of a sufficiently long slow roll inflation is very close to 1. Note however that this is a prediction of LQC *only* in presence of suitable potentials; if there is no potential at all, there is still a period of accelerated expansion due to super inflation but it does not yield a sufficient number of e-foldings.

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