Excited DeSitter brane worlds localized by a kink.

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We reconsider, in five-dimensional space-time, the issue of thick brane localized in the extra dimension by a kink formed by a scalar field. The localization is achieved by a sine-Gordon potential. Apart from a fundamental brane (discovered by Koley and Kar [1]) where the scalar field is a monotonic function of the extra dimension), we show that a series of new solutions exist as well, labelled by the number of zeros of the scalar field. These solutions are regular, localized on the brane and mirror symmetric with respect to the extra dimension. They form a tower of "excited branes". The study of some perturbations of the solutions reveals that the new solutions are not stable. Finally, fermions are coupled to the scalar field by means of a Yukawa potential and their localization in the background of the new solutions is examined. It turns out that the excited branes can localize left and right chiral fermion either on the brane and/or in the bulk but close to the brane.

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I. INTRODUCTION

One of the main problem in the theoretical description of the Universe is the cosmological constant problem [2]. One of the most promising attemp to solve it is based on the "brane-world" models [3]; the basic idea is that our Universe is represented by a 3+1-dimensional subspace (a three-brane) embedded into an higher dimensional space-time (the bulk). In this context, the first challenge is to reconciliate the extra dimensions with the fact that low energy physics is very well described by a conventional 3+1-dimensional space-time in which numerous laws (e.g. Newton's law) are tested with a very high degree of accuracy. It is nevertheless challenging to emphasize the Einstein equations in higher dimensional space-times and to study and classify the possibly relevant solutions. One way to attack the various problems consists in trying to localize the matter fields and gravity on the brane in the case where the extra dimensions are non compact [4, 5]; another way would be to have solutions where the geometry of the extra-dimensions naturally comes out to be a compact manifold [4].

Here we will follow the ideas developped in several papers [6–8] and consider branes which are supported by appropriate topological deffects living in the extra-dimensions. In these papers, the brane is supposed to be Minskowski space-time (no vacuum energy) or de Sitter space-time with a cosmological constant (or brane tension) leading to an inflating brane. No bulk cosmological constant is assumed. In contrast, in [5] the authors take advantage of a cosmological constant in the bulk : $\Lambda_{4+n} \neq 0$ and manage to enforce the localisation of gravity by a fine tuning of Λ_{4+n} in function of the other constants characterizing the topological defects.

Assuming only one extra dimension, a lot of work has been done about thick brane scenarios [9–12] where five-dimensional gravity is coupled to one or more scalars and an appropriate potential leading to localized solutions in the extra dimension. The relation of gravity localisation with different types of potential for one scalar field was analyzed namely in [13] and in [14–16]; here the relevant potential derive from a superpotential allowing for explicit solutions to the field equations. The Sine-Gordon soliton potential was used in [1]. In a series of papers, two scalar fields were used to construct a so called Bloch brane [17–19] which is also based on a superpotential.

Here, we will reconsider the existence of branes within the sine-Gordon model. One feature of the brane solution found in [1] is that it exists only for a particular relation between the five dimensional Newton constant and the bulk cosmological constant Λ. In this paper, we reinvestigate the model of [1] and study the field equations numerically for generic value of the constant Λ. Our numerical result strongly suggest that the Koley-Kar brane-world is the first solution of a tower of solutions characterized by the number of nodes of the scalar field. All these solutions are mirror symmetric with respect to the four dimensional brane and exist for specific values of the cosmological constant (like a spectral value while the five dimensional Newton constant is fixed. For the other values of Λ, the warp factor become singular on one side of the bulk space. The excited solutions have important consequences on the localization of masless fermion on the brane. In particular, both chiral component of the fermion can be localized on or close to the brane while it is know that only the left component is localizable in the background of the Koley-Kar fundamental solution. We also checked that the tower-like solutions of the type studied here are also present with other type of potential admitting kink in the flat space, e.g. the so called $\lambda\phi^4$ potential. The model and the equations are presented in Sect. II. In particular we consider the case where the four dimensional brane is an Einstein space with a independant cosmological constant. The various type of solutions are discussed in Sect. III and Sect. IV is devoted to the localization of the fermions. Finalize, we analyze the stability property of the first few elements of the tower in Sect. V and give some conclusions in Sect. VI.

II. THE MODEL AND THE EQUATIONS

Along with Koley and Kar [1] (and more recently [20]), we consider the action

$$S = \int d^5x\sqrt{-g}[\frac{1}{2\kappa_5^2}(R-2\Lambda) - \frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi - V(\phi)] \tag{1}$$

where $\kappa_5^2 = 8\pi G_5$ with G_5 the 5-dimensional Newton constant and Λ is the 5-dimensional cosmological constant. The real scalar field ϕ interact through the potential $V(\phi)$. In this note we have considered both the sine-Gordon potential and the quartic double potential

$$V(\phi) = p(1+\cos(\frac{2\phi}{q})) \quad \text{or} \quad V(\phi) = p(\frac{\phi^2}{q}-1)^2 \tag{2}$$

where p, q are real constants. These potentials lead qualitatively to the same results but we will discuss essentially the case of the sine-Gordon potential. We are interested in solutions of the field equations associated to the action (1) with a metric of the form

$$ds^2 = e^{-2A(y)}(ds_4^2) + dy^2 \tag{3}$$

where the extra coordinate is denoted y and the function $\exp(-2A(y))$ is the warp factor (note that the function $A(y)$ in our notation differs by a sign from the one of [20]). The scalar field is assumed to depend on y only : $\phi = f(y)$. The 4-dimensional space whose metric is denoted ds_4^2 and is assumed to be an Einstein space, i.e. such that

$$R^{(4)}_{\mu\nu} = \frac{H^2}{4} g^{(4)}_{\mu\nu} \tag{4}$$

where H^2 is a constant which can be negative. A real (resp. negative, nul) H^2 corresponds to deSitter (resp. Anti de Sitter, Minkowski) 4-dimensional space-time but the metric $g^{(4)}$ can actually describe *any* Einstein space.

The relevant components of the 5-dimensional Ricci tensor are then

$$R_0^0 = R_1^2 = R_2^2 = R_3^3 = 3e^{2A}H^2 + A'' - 4(A')^2 \quad , \quad R_4^4 = 4A'' - 4(A')^2 \tag{5}$$

from now on, the prime denotes the derivative with respect to y. The field equations read

$$A'' = \frac{\kappa}{3}(f')^2 + H^2 \exp 2A \quad , \quad \kappa \equiv \kappa_5^2 \tag{6}$$

$$A'^2 = \frac{\kappa}{12}(f')^2 - \frac{1}{6}\kappa V(f) - \frac{\Lambda}{6} + H^2 \exp 2A \tag{7}$$

$$f'' = 4A'f' + \frac{\partial V}{\partial f} \tag{8}$$

These equations depend in principle on five constants : the parameters of the potential p, q the gravity parameter κ and the bulk and the brane cosmological constants, respectively Λ, H. The parameter q defines the scale of the scalar field and can be set to one. Independently, p can be set to a fixed value by a suitable rescaling of y, Λ and of κ. In the following we adopt the normalisation $q = 1$, $p = 1/4$. Using (6) and (8) to determine $A(y), f(y)$, the cosmological constant Λ then appears as a "constant of motion" through Eq.(7). As a consequence, only κ, H need to be specified to construct a solution.

III. ANALYSIS OF THE SOLUTIONS

A. Case $H = 0$

The equations (6),(8) have to be solved with appropriate boundary conditions. Without loosing generality, the coordinate y can be translated in such a way that $\phi(0) = 0$; we want the sine-Gordon field to approach a vacuum for $y \to \infty$, so we require $\phi(\infty) = \pi/2$. In addition, the condition $A(0) = 0$ can be set by a suitable normaliation of t , the brane is then located at $y = 0$. This leaves the parameter $A'(0) \equiv C$ to be chosen in order to specify a boundary value problem.

The hypothesis of a symmetric space time under the reflection $y \to -y$ is fulfilled if $C = 0$. However, assuming for a while space-time limited to the subspace with $y \geq 0$, the equations can be integrated for $y \in [0, \infty]$ with arbitrary values of C. The boundary conditions are then

$$A(0) = 0 \quad , \quad A'(0) = C \quad , \quad f(0) = 0 \quad , \quad f(\infty) = \frac{\pi}{2} \tag{9}$$

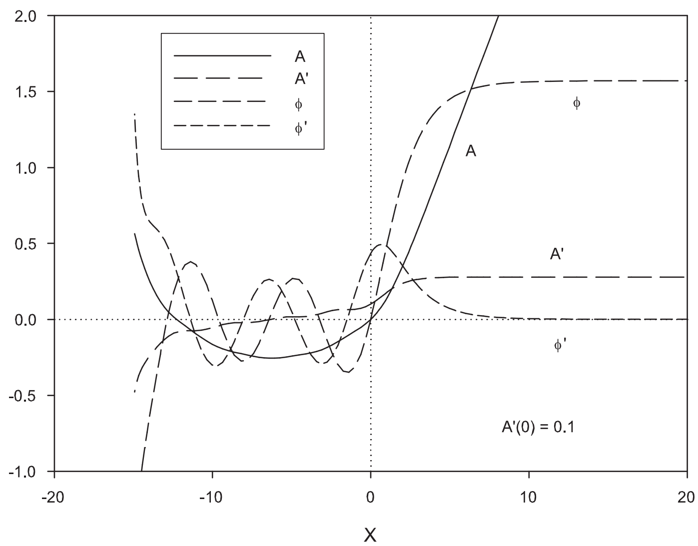


FIG. 1. The functions A, A', f, f' for $\kappa = 1$, $p = 0.25$ and $C = 0.1$

For $C = 0$, the explicit solution of Koley, Kar is recovered

$$A(y) = \frac{\kappa}{3} \ln \cosh(ky) \quad , \quad f(y) = 2 \arctan(\exp ky) - \frac{\pi}{2} \tag{10}$$

with

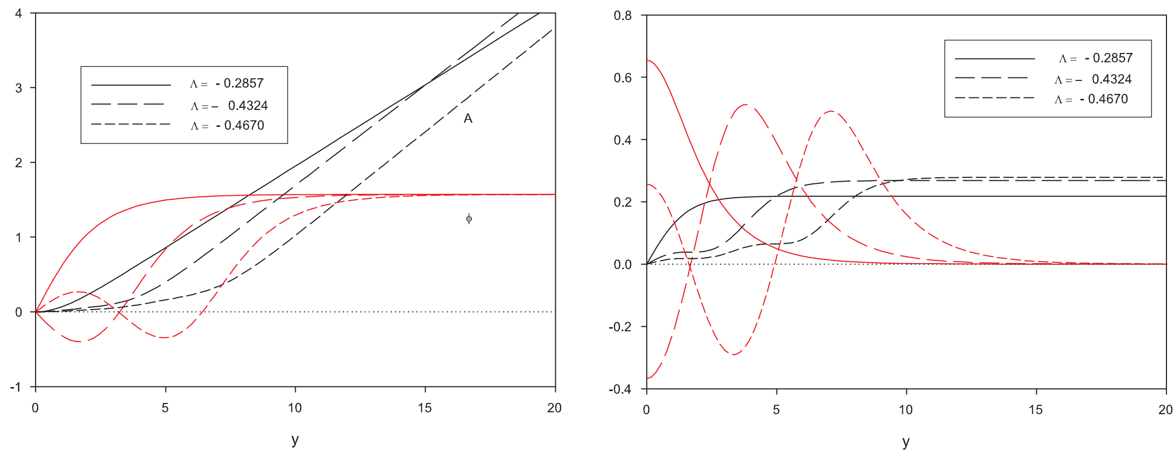
$$k^2 = \frac{12p}{4\kappa + 3} \quad , \quad \Lambda = -\frac{8p\kappa^2}{4\kappa + 3} \tag{11}$$

In particular, the bulk cosmological constant is fixed by the five dimensional coupling constant and the parameter p.

For $C > 0$, the system of equation above has, to our knowledge, no explicit solution. We therefore solve the equation by a numerical method. We used a collocation method for boundary-value ordinary differential equations, equipped with an adaptive mesh selection procedure [22]. Our solutions were constructed with a relative error of order 10^{-8}. The solution corresponding to $\kappa_5 = 1$ and $C = 0.1$ is presented on Fig. 1. Continuing the numerical solution on the negative part of the y-axis, the numerical results show that, for generic values of C, the solution develops oscillations and then becomes divergent when y is small enough. Accordingly, the solution cannot be continued of the full axis $y \in [-\infty, \infty]$. The analysis reveals, however, that for fine tuned values of the parameter C, the solution has $f(y_c) = A'(y_c) = 0$ for some y_c. For these values of C and y_c, the solution can be continued into a regular, mirror symmetric solution (after a translation by y_c) on the full axis y.

We believe that solutions with an arbitrary number of oscillations in the core exist, forming an infinite tower labelled by N, the number of nodes of $f(y)$ on $y \in [0, \infty]$ (corresponding to a total on $2N - 1$ nodes). We constructed the first three elements of this tower; the profiles are presented on Fig. 2 (A, f on the left part, A', f' on the right part). Several parameters characterizing the brane solutions, namely the bulk cosmological constant and the value $|f'(0)|$ are represented as functions of κ_5 on Fig. 2 for the first three solutions. The curves corresponding to $N = 1$ is known analytically; the A function is such that $A'(\infty) = \sqrt{-\Lambda/6}$.

The pattern of solutions of the sine-Gordon equation (i.e. in the probe limit $\kappa_5 = 0$) suggests that the new solutions, found for $\kappa > 0$, are nothing else than the first elements of a tower of gravitating solutions labelled by the number of nodes of the scalar function. We now present arguments supporting this statement.



(a) A, f (b) A', f'

FIG. 2. The metric function A and scalar function f for the solutions with one, two and three nodes(left), the corresponding derivatives (right)

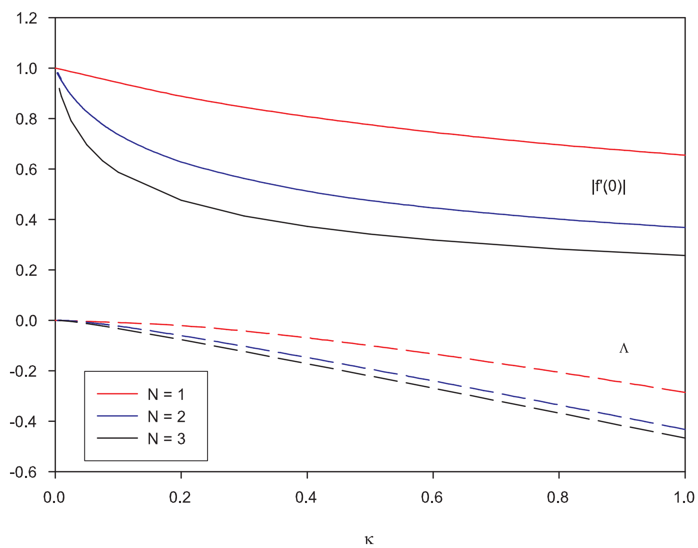


FIG. 3. The parameters $|f'(0)|$ and Λ as functions of κ_5 for $N = 1, 2, 3$

With our conventions ($q = 1, p = 1/4$ and assuming the kink to be centered at the origin: $f(0) = 0$), the sine-Gordon kink has $f'(0) = 1$, $f(\infty) = \pi/2$. For the following, let us denote a "half-kink" the part of this solution between $y = 0$ and $y = \infty$. Integrating the sine-Gordon equation with $f(0) = 0, f'(0) = s$ with $s < 1$ leads to oscillating solutions; these are periodic functions whose zeros (which we note $y = y_n$) are such that $f'(y_n) = \pm s$. Accordingly, configurations can be constructed by "gluing" a half-kink (suitably translated) at one of the zeros of a periodic solution corresponding to a value of s arbitrarily close to $s = 1$. Such configurations are off course *not* differentiable at $y = y_n$ but the coupling to gravity somehow regularizes them and leads to the family of regular solutions discussed above. Our numerical results strongly confirms this interpretation.

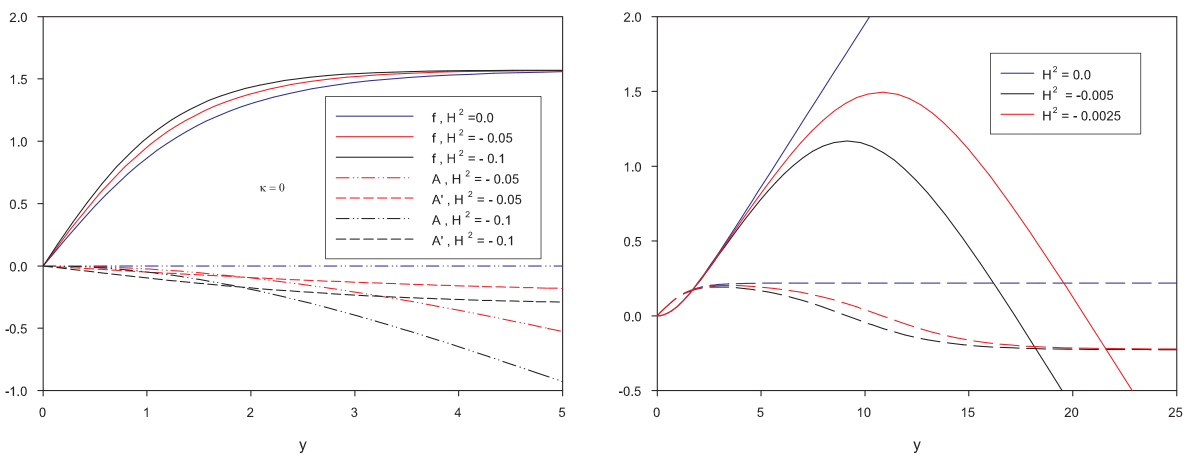
Replacing the sine-Gordon potential by the double well quartic potential given in (2) leads to the same pattern. The occurence of the "quasi-solution" configurations in approaching the probe limit makes, however, the numerical analysis rather involved for $\kappa_5 << 1$ for the two potentials.

B. Case $H^2 < 0$

Let us pose $H^2 = -K^2$. In this case, we could not find explicit solutions for generic values of κ. For $\kappa_5 = 0$, the equation for A can be solved, leading to

$$A(y) = \frac{1}{2}\log(1 - \tanh^2(Hy)) \quad \longrightarrow \exp(-2A) = \cosh^2(Hy) \quad . \tag{12}$$

This function is such that $A = -K^2y^2/2$ for $y << 1$ and $A = -|K|y$ for $y >> 1$. Accordingly, the warp factor $\exp(-2A)$ increases for large y and the brane cannot be localized. Solving the equations numerically for $\kappa_5 > 0$, reveals that, asymptotically, the warp factor keeps diverging for large y. Several plots of such solutions are presented in Fig. 4.



(a) A, f (b) A, f

FIG. 4. Left: The metric function A and scalar function f, f' for $\kappa = 0$ and three values of H^2. Right: the metric function A (solid) and the derivative (dashed) for $\kappa = 1$ and three values of H^2.

C. Case $H^2 > 0$

In this case also, we could not find explicit solutions for generic values of κ. For $\kappa = 0$, the equation for A can be solved, leading to

$$A(y) = \frac{1}{2}\log(1 + \tan^2(Hy)) \quad \longrightarrow \exp(-2A) = \cos^2(Hy) \tag{13}$$

It is such that $A(y) = H^2y^2/2 + O(y^3)$ for $y << 1$. The warp factor vanishes $y = \pi/(2H)$, while the components of the Riemann tensor remain finite. Solving (numerically) the equation for $f(y)$ in this background reveals that the scalar field does not stabilize to a constant for $y \to \pi/(2H)$.

Investigating the equations numerically for $\kappa > 0$ reveals that the peculiar behaviour of the $\kappa = 0$ limit (in particular the vanishing of the warp factor at a finite value extra dimension coordinate y) persists.

IV. LOCALIZATION OF THE FERMIONS ON THE BRANE

The coupling of the Dirac action to the brane is usually realized by means of the fermionic action

$$S_F = \int d^5x\sqrt{-g}(\overline{\psi}\Gamma^M D_M\psi - m\overline{\psi}\psi - \eta\overline{\psi}F(\phi)\psi \ . \tag{14}$$

The parameter η represents the coupling of the fermion to the scalar field. The function $F(\phi)$ is usually chosen as $F(\phi) = \phi$ leading the a conventional Yukawa potential. (the alternative choice $F(\phi) = \sin(\phi)$ is also used [1, 20]). The matrices Γ^M are the Dirac matrices in curved space. To solve the underlying Dirac equations, one usually adopts a coordinate z defined according to $dz = exp(A(y))dy$. This transforms the line element into a conformally flat metric

$$ds^2 = e^{-2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \tag{15}$$

With this coordinate, the Dirac matrices are given by $\Gamma^M = (e^A\gamma^\mu, e^A\gamma^5)$ where γ^μ, γ^5 are the usual Dirac matrices in flat space. The fermion function is usually decomposed according to

$$\psi(x, z) = \sum_n \psi_{L,n}\alpha_{Ln}(z) + \sum_n \psi_{R,n}\alpha_{Rn}(z) \tag{16}$$

with the chiral spinor $\psi_{L,n} = -\gamma^5\psi_{L,n}$ and $\psi_{R,n} = \gamma^5\psi_{R,n}$ and the sum n run over the spectrum of the 4-dimensional solutions.

After some standard manipulations (see e.g. in [1],[20]), the effective potential determining the z-dependent prefactors $\alpha_{Ln}(z)$, $\alpha_{Ln}(z)$ for the chiral components of the Dirac spinors are given by

$$V_L(y) = e^{-2A}(\eta^2\phi^2 - \eta\frac{\partial\phi}{\partial y} + \eta\phi\frac{\partial A}{\partial y})|_{y=y(z)} \tag{17}$$

$$V_R(y) = e^{-2A}(\eta^2\phi^2 + \eta\frac{\partial\phi}{\partial y} - \eta\phi\frac{\partial A}{\partial y})|_{y=y(z)} \tag{18}$$

It is known that the fundamental Koley-Kar brane leads to a vulcano shaped potential V_L which can localize the fermion on the brane while V_R is bell shaped, admitting no bound states. Accordingly, the right handed fermions cannot be localized on the brane.

Owing on the new excited branes constructed in the previous section, it is natural to study the shape of V_L and V_R corresponding to the excited solution. This is illustrated by Fig. 5. It turns out that the shape of the potential corresponding to the exited brane is quite different from the one of the fundamental brane and presents some interesting features. On Fig.5 the potentials $V_{L,R}$ corresponding to $\kappa = 1$, $\eta = 1$ and the N=1 and N=2 solutions are superposed. For the first excited brane, it turns out that V_R presents a negative valley on the brane (i.e. for $z = 0$) and at least one bound state should exist there. In contrast, the potential V_L presents a local maximum on the brane and a local minimum at some finite value $y = \pm y_m$. It means that left handed fermions could be bounded in this region; the corresponding fundamental wave function should be localized symmetrically in the two valleys. Decreasing the Yukawa coupling constant κ, it turns out that the potentials V_L and V_R both present a local minimum on the brane; accordingly both species of fermions could be localized.

Taking the next excitation, reveals that the corresponding effective potentials develop more and more local minima in the bulk, as shown on Fig.6.

V. STABILITY

A. Setup

We consider the following ansatz for the metric and matter field perturbations:

$$ds^2 = e^{-2(A+\epsilon A_1(t,y))}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-\epsilon B_1(t,y)}dy^2 + \epsilon C_1(t,y)dtdy \ , \ f(t,y) = f_0 + \epsilon f_1(t,y), \tag{19}$$

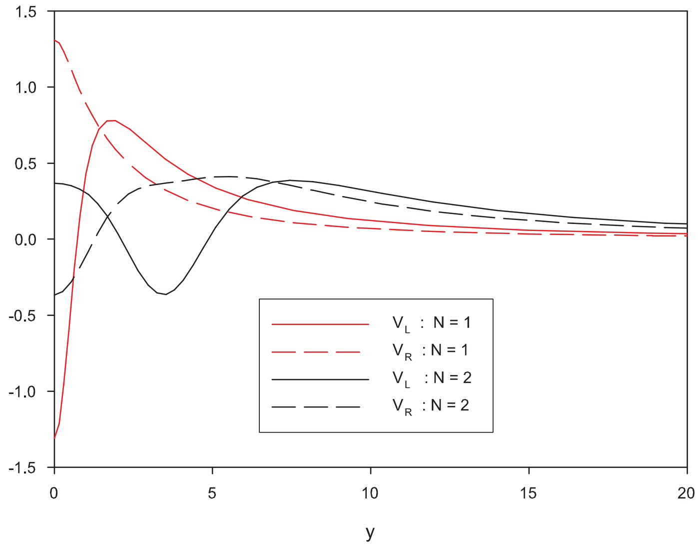
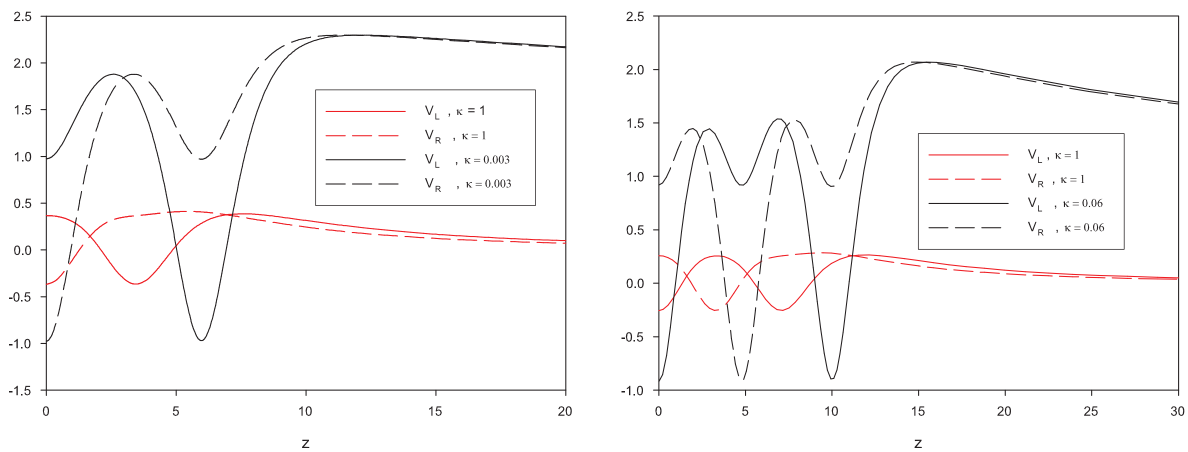


FIG. 5. The effecive potentials V_L and V_R for $\kappa = 1$, $\eta = 1$ for the fundamental (red) and first exited (black) brane



(a) V_{LR} (b) V_{LR}

FIG. 6. The effective potentials for the first (left) and second (right) excited branes for $\eta = 1$ and two values of the : κ parameter: $\kappa = 1, 0.003$ for the left and $\kappa = 1, 0.006$ for the right graph.

where ϵ is a small bookkeeping parameter and A, f_0 are solutions to the background equations given in the previous section (note that, for convenience, the function f of the previous section is here renamed f_0). The form (19) is the most general ansatz compatible with the symmetries of the background spacetime. The stability issue of thick branes in five dimensions has been adressed in [21] and we basically follow the same approach.

Assuming, as usual, the fields to depend on time through a factor $e^{i\omega t}$, the nontrivial gravity perturbation equations reduce to

$$
\begin{aligned}
&3A'\left(8A_1'-B_1'\right)+B_1\left(12A'^2-\kappa f_0'^2\right)+2\left(-3A_1''+\kappa f_0'f_1'+\kappa f_1V'(f_0)\right)=0,\\
&-24A'A_1'-12B_1A'^2+3i\omega e^{2A}C_1A'+6\omega^2e^{2A}A_1+\kappa B_1f_0'^2+2\kappa f'f_1'-2\kappa f_1V'(f_0)=0,\\
&3B_1A'+6A_1'-2\kappa f_1f_0'=0.
\end{aligned} \tag{20}
$$

The perturbed matter equation leads to

$$
-2\left(-4A'f_1'+B_1V'(f_0)+f_1''\right)+f_0'\left(-i\omega e^{2A}C_1+8A_1'-B_1'\right)+2f_1\left(V''(f)-\omega^2e^{2A}\right)=0 \tag{21}
$$

The gravitational equations are not independent : solving the second and the third for B_1, C_1 and reconstructing C_1', B_1' leads to an identity for the first equation. Knowing the expressions for B_1, C_1, B_1', C_1' and further defining

$$
F := f_1 - \frac{A_1 f_0'}{A'}, \tag{22}
$$

leads to the following master equation for the perturbation $F(y)$:

$$
F'' - 4A'F' + \left(\frac{4\kappa f_0'V'(f_0)}{3A'(y)} - \frac{2\kappa^2 f_0'^4}{9A'^2} + \omega^2 e^{2A} + \frac{8}{3}\kappa f_0'^2 - V''(f_0)\right)F = 0. \tag{23}
$$

Using the following change of function and change of variable:

$$
F \to e^{\frac{3}{2}A}F, \quad dy = e^{-A}dr, \tag{24}
$$

the equation (23) can be set in the form of a Schrodinger equation :

$$
\begin{aligned}
&-\frac{d^2}{dr^2}\tilde{F} + V_p\tilde{F} = \omega^2\tilde{F},\\
&V_p = \frac{2\kappa^2\tilde{f}_0'^4}{9\tilde{A}'^2} - \frac{4\kappa e^{-2\tilde{A}}\tilde{f}_0'V'(\tilde{f})}{3\tilde{A}'} + \frac{15}{4}\tilde{A}'^2 - \frac{19}{6}\kappa\tilde{f}_0'^2 + e^{-2\tilde{A}}V''(\tilde{f}_0),
\end{aligned} \tag{25}
$$

where we defined $\tilde{f}_0(r) = f_0(y(r)),\ \tilde{A}(r) = A(y(r)),\ \tilde{F}(r) = F(y(r))$.

Note that the zero mode associated with the translational invariance of the background fields leads to a trivial F. Indeed the values $A_1 = A',\ f_1 = f_0'$ imply $F = 0$.

B. Fundamental solution

For the range of the parameters considered, the fundamental solution is stable. Indeed, the potential is divergent close to the origin:

$$
\tilde{A} \approx A_{00},\ \tilde{f} \approx f_{01}r \ \Rightarrow V_p \approx \frac{2}{r}, \tag{26}
$$

and the asymptotic behaviour is

$$
\tilde{A} \approx \log r,\ \tilde{f} \approx \frac{\pi}{2} \ \Rightarrow V_p \approx \frac{4V''\left(\frac{\pi}{2}\right)+15}{4r^2}. \tag{27}
$$

The profile of the corresponding potential is presented on Fig. 7 by the solid black line. The last term in the potential V_p plays the role of the mass term, so it should be positive, leading to a positive fall-off of the eigenfunction. As a consequence, the potential is positive definite and there cannot be imaginary modes.

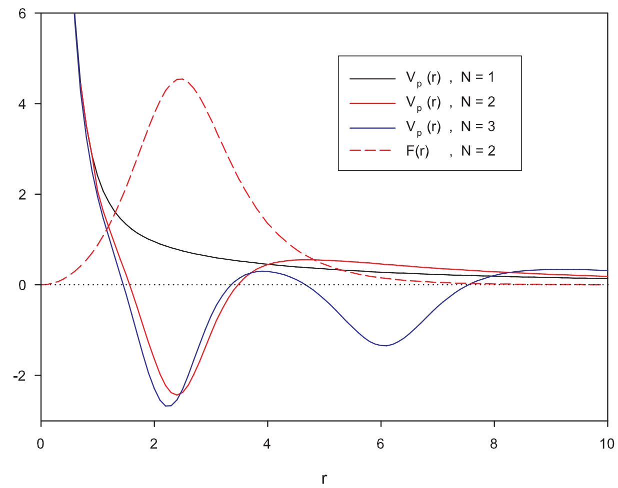


FIG. 7. Solid: potential of the perturbation master function for the solutions with one, two, three nodes. Dashed: the first normalisable eigenmode of the two node solution.

C. Excited solutions

The behaviour of the potential close to the origin (26(and at infinity (27) are the same as in the case of the fundamental solution, but the potential becomes negative in the intermediate region. The potential corresponding to the $N = 2$ and $N = 3$ solutions are represented on Fig. 7 by the red and blue lines respectively. The occurence of valleys where the potential is negative leads to bound states which possibly reveals the existence of instable modes if $\omega^2 < 0$. Indeed, we find an unstable mode for the generic solutions with one mode. For $\kappa = 1.0$ we find $\omega^2 \approx -0.695$. The profile of this eigen-mode (the normalisation is arbitrary) is supplemented on figure 7 by the dashed, red line. We further checked that the next bound state is indeed stable; this suggests that the brane characterized by N nodes of the scalar field has indeed $N - 1$ unstable modes. This speculation was, however, not tested further.

VI. CONCLUSIONS

The purpose of this note is to exhibit new families of braneworld associated to five-dimensional gravity supplemented by a scalar sector admitting kink solutions. The family of solutions is organized like a tower of excited states of a Schrodinger equation (the bulk cosmological constant playing a role of spectral parameter, taking discrete values) although the underlying equations are highly non-linear. The types of braneworlds obtained in the paper have, to our knowledge, not been constructed yet and present some potentially interesting features for the localization of both species of chiral fermions.

Often, when non linear equations admit various branches of solutions, the higher excitation are less stable than the main solution. It seems that this feature is obeyed in the present case : using a particular channel of the perturbed equations we checked that a typical solution with no nodes is stable and that the corresponding solution with one node is not. However, it could be expected that these new types of solutions is relevant in the mechanism of brane formations. Assuming brane formation in a dynamical process, possible unstable higher energy might be reached. The latter should then decay to the fundamental low energy configuration. These possibilities need further investigations.

ACKNOWLEDGMENTS

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