Andreev hidden spinor coordinates: Standard Model, gravity and ^{3}He

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This paper is prepared for a special memorial issue of J. Low Temp. Phys. dedicated to memory of Alexander Andreev. The paper considers some ideas of Andreev on the fermion-boson transmutation, which look somewhat contradictory, but their consideration and development may lead to new physics on the more fundamental trans-Planckian level.

PACS numbers:

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I. INTRODUCTION. ANDREEV AND SPINOR COORDINATE

A.F. Andreev wrote several papers[1–4] devoted to the one of the most puzzling problem in modern physics – the conjecture of the superselection rule, which forbids the linear superpositions of states with even and odd numbers of fermions. This superposition is incompatible with the Lorentz invariance, since the $O(2\pi)$ transformation changes sign of the fermion wave function but does not change the sign of of the wave function of boson. This leads to the superselection rule.[5] According to Andreev, the superselection rule is not self-consistent. Instead he suggested that the spacetime must be extended. In addition to the x, y, z, t coordinates there should be the special spinor coordinate, which characterizes the internal spinor symmetry on the fundamental level.

Although the particular scenario proposed by Andreev does not work in known physics, his attempts to construct a consistent theory of the fundamental space-time, which may allow for linear superposition of bosons and fermions, deserve respect and further development. It is not excluded that on the more fundamental level, say, at the trans-Planckian energy scale, the boson-fermion transmutation becomes possible. Then the extension of the coordinate space to include the extra spinor degrees of freedom can be the right direction to probe the new physics. Anyway, the idea of the extension of the internal (spin) space, even if it does not destroy the superselection rule, leads to interesting consequences for superfluid ^{3}He. Maybe it will also lead to new physics on the more fundamental level, since ^{3}He serves as the platform for simulation of many directions in physics.[6]

Here we consider several examples of the extension of the internal spinor symmetry. This in particular includes the extension of internal Lorentz group from $SO(1,3)$ to $SO(1,4)$, which is discussed for the extension of Standard

Model,[7] see Sections IV and V. The corresponding relativistic physics, which emerges in the vicinity of the Dirac points in the spectrum of the planar phase of superfluid ^{3}He,[8,9] is discussed in Sections III and VI. The superposition of states with different number of particles suggested by Andreev and the birth of time crystals are in Section VII. The fermion-boson transmutation due to quantum anomalies and also the 1+1 quantum field theory, which is based on the Andreev proposal, are discussed in Section VIII. The extension of Standard Model to lattice theory with several Weyl and Dirac points also suggests the possibility to observe processes with creation or annihilation of single fermions (see Sec. IX).

II. PLANAR PHASE OF SUPERFLUID ^{3}HE

The spin triplet p-wave pairing in superfluid ^{3}He has the following 2×2 matrix of the gap function:

$$\hat{\Delta} = A^i_\alpha p_i \sigma^\alpha i\sigma_y \,, \tag{1}$$

where σ^α are the Pauli matrices for spin and $A_{\alpha i}$ is the 3×3 complex matrix of the order parameter, see the book[10]. In the planar phase of superfluid ^{3}He the particular representative of the order parameter is:

$$A_{\alpha i} = c_\perp \left(\delta^i_\alpha - \hat{z}_\alpha \hat{z}^i\right) \,, \tag{2}$$

where $c_\perp$ is the "speed of light" in the transverse direction. All the other degenerate states of the planar phase are obtained by spin, orbital and phase rotations of the group $G = SO(3)_S \times SO(3)_L \times U(1)$. The corresponding Bogoliubov-de Gennes (BdG) Hamiltonian for fermionic quasiparticles is

$$H(\mathbf{p}) = \begin{pmatrix} \epsilon(p) & \hat{\Delta} \\ \hat{\Delta}^\dagger & -\epsilon(p) \end{pmatrix} = \epsilon(p)\tau_3 + \tau_1(\sigma_x p_x + \sigma_y p_y) i\sigma_y \,, \tag{3}$$

where τ_i are the Pauli matrices in the particle-hole space, and $\epsilon(p)$ is particle spectrum in normal state of ^{3}He, which in the vicinity of the Fermi surface at $p = p_F$ is $\epsilon(p) = c_\parallel(p - p_F)$. Here $c_\parallel = v_F$ is the Fermi velocity of the normal Fermi liquid.

For us it is important that the planar phase has the operator of discrete symmetry $C = \sigma_3$, which commutes with the BdG Hamiltonian:[11]

$$CH = HC \,. \tag{4}$$

C is the combined symmetry: the combination of spin rotation by angle π about z-axis and rotation of the phase of the BdG wave function by $-\pi/2$. In terms of the order parameter $A_{\alpha i}$ in Eq.(2) it is the spin rotation by π followed by change of the phase of the order parameter by π.

In the other representation of the planar phase:

$$\tilde{H} = U^\dagger H U \;,\; U = \begin{pmatrix} 1 & 0 \\ 0 & i\sigma \end{pmatrix} , \tag{5}$$

the Hamiltonian and the corresponding symmetry operator are:

$$\tilde{H} = \epsilon\tau_3 + \tau_1(\sigma_x p_x + \sigma_y p_y) \;,\; \tilde{C} = \tau_3\sigma_z \;,\; \tilde{C}\tilde{H} = \tilde{H}\tilde{C} \,. \tag{6}$$

III. SPINOR COORDINATE IN PLANAR PHASE

The reason why we considered the symmetry C of the Hamiltonian is that it demonstrates an example when the discrete symmetry can be extended to continuous $U(1)$ symmetry:

$$U_C(\alpha) = \exp\left(-i\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2} + iC\sin\frac{\alpha}{2}\right) \;,\; C = U_C(\pi) \,. \tag{7}$$

The symmetry $C = U_C(\pi)$ is the combined symmetry: the combination of spin rotation by angle π and rotations of the phase by $-\pi/2$ (the latter corresponds to rotation by π of the phase of the order parameter). In the representation of Eq.(5) the extension of discrete $\tilde{C}$ to continuous $U_{\tilde{C}}(\alpha)$.

While the rotation $O(2\pi)$ of spins changes sign, the combined rotation does not, $U_C(2\pi) = O(2\pi)U(\pi) = 1$. Of course, this does not solve the main problem, because such compensation of the sign by extra spinor degrees of freedom is not fundamental, and takes place only in a special model. However, it shows that if something similar takes place on the trans-Planckian level, this would destroy the superselection conjecture. This demonstrates that the introduction of the additional degree in the internal space suggested by Andreev can be productive.

IV. SPINOR COORDINATE FOR STANDARD MODEL EXTENSION

It is interesting that the similar extension of discrete symmetry to the continuous symmetry has been suggested for Dirac spinors in Standard Model.[7] Let us consider the parity transformation. It is the combined symmetry $P = P_c P_s$, which contains the coordinate transformation $P_c\Psi(\mathbf{r},t) = \Psi(-\mathbf{r},t)$ and internal symmetry of spinor $P_s = \gamma_0$, where γ_a is the Dirac matrix. Now similar to Eq.(7) we have the transformation of the discrete internal parity P_s of spinors to the combined continuous symmetry:

$$U_{P_s}(\alpha) = \exp\left(-i\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2} + iP_s\sin\frac{\alpha}{2}\right) \ , \ P_s = U_{P_s}(\pi)\,. \tag{8}$$

So, instead of the Lorentzian spin group $SO(1,3)$ with 6 generator we have one extra generator for internal degrees of freedom. As is shown in Ref.[7] such extension can be naturally included into the $SO(1,4)$ group. As a result one has unusual situation, when the coordinate space is (3+1)-dimensional and obeys the the $SO(1,3)$ group with 6 generators, while the internal spin space is (4+1)-dimensional and obeys the the $SO(1,4)$ symmetry group with 10 generators. Instead of one extra degree of freedom we have $10 - 6 = 4$ extra degrees of freedom.

When gravity is considered, then such extension produces the gravity in terms of the rectangular vielbein.[8,9]

V. RECTANGULAR VIELBEIN: DIMENSION OF SPIN SPACE IS LARGER THAN DIMENSION OF COORDINATE SPACE

For the internal $SO(1,4)$ group of the spin space we need five anticommuting matrices Γ^a with $a = (0,1,2,3,4)$. They can be constructed from the Dirac γ-matrices:

$$\Gamma^0 = \gamma_0 = \tau_1 \ , \ a = 0 \tag{9}$$

$$\Gamma^a = \gamma_a = i\tau_2\sigma^a \ , \ a = 1,2,3\,, \tag{10}$$

$$\Gamma^4 = -i\gamma_5 = i\tau_3 \ , \ a = 4\,. \tag{11}$$

Extra 4 generators of the $SO(1,4)$ symmetry group are $\sigma^a\tau_1$, where $a = 1,2,3$, and $i\tau_2$. One can check, that each of three operators $\sigma^a\tau_1$ plays the role of internal parity P_s in Eq.(8), i.e. they represent different extensions of discrete parity to continuous symmetries.[7]

When gravity is considered, then instead of the quadratic 4×4 gravitational tetrads, we have the 4×5 rectangular gravitational vielbein. This is because the coordinate space is the conventional (3+1)-dimensional spacetime, while the spin space has dimension (4+1). The corresponding inverse Green's function of Dirac fermions is expressed in terms of rectangular vielbein e^μ_a in the following way:

$$G^{-1}(M) = e^\mu_a\Gamma^a p_\mu + M\,, \tag{12}$$

where the spin and coordinate indices of the vielbein e^μ_a are $a = (0,1,2,3,4)$ and $\mu = (0,1,2,3)$.

Although the viellbein is rectangular the coordinate spacetime is described by the 4×4 metric. This can be seen from the spectrum of Dirac fermions, which can be obtained from the product of two matrix Green's function with opposite signs of the mass terms:

$$\begin{aligned} G^{-1}(-M)G^{-1}(M) &= (e^\mu_a\Gamma^a p_\mu + M)(e^\nu_b\Gamma^b p_\nu - M) = \\ &= -(\eta^{ab}e^\mu_a e^\nu_b p_\mu p_\nu + M^2)\,, \end{aligned} \tag{13}$$

where

$$\eta^{ab} = (-1,1,1,1,1)\,. \tag{14}$$

This shows that the poles of the Green's function describe the spectrum of massive Dirac particles

$$g^{\mu\nu}p_\mu p_\nu + M^2 = 0\,, \tag{15}$$

in the (3+1)-dimensional spacetime with the metric $g^{\mu\nu}$:

$$g^{\mu\nu} = \eta^{ab}e^\mu_a e^\nu_b\,. \tag{16}$$

The extra dimensions of spin space are hidden inside the metric. That is why, if only the metric of spacetime is known one has no complete information on the spin degrees of freedom in our quantum vacuum. On the level of the metric one cannot resolve between different dimensions of the internal space and its transformation properties are not known. This justifies the concern by Andreev, that the symmetry arguments in support of the superselection rule may not work.

VI. RECTANGULAR VIELBEIN IN PLANAR PHASE AND ITS HIDDEN GRAVITATIONAL GLOBAL MONOPOLE

The relativistic physics also takes place in the planar phase of superfluid ^{3}He. It emerges in the vicinity of the two Dirac points in the quasiparticle spectrum at $\mathbf{p} = \pm p_F\hat{\mathbf{z}}$. If to the planar phase ordfer parameter the s-wave component is added, it produces the mass M for these Dirac fermions.[9] It appears that this relativistic physics also experiences the extension of spin space, although for the reason different from that in Sec. III. This provides vielbein with mixed dimensions (4+1 dimension of internal space and 3+1 dimension of spacetime coordinates).[8,9]

The corresponding Dirac Γ-matrices have the following form:

$$\Gamma^0 = i\tau_2 \ , \ \Gamma^a = \tau_3\sigma^a \ (a = 1,2,3) \ , \ \Gamma^4 = \tau_1 \, . \tag{17}$$

The BdG Hamiltonian near the Dirac points at $\mathbf{p} = \pm p_F\hat{\mathbf{l}}$, where $\hat{\mathbf{l}}$ is the unit vector

$$\tilde{H} = \sum_a \Gamma^a e^i_a (p_i - qA_i) \, . \tag{18}$$

Here $\mathbf{A} = p_F\hat{\mathbf{l}}$ is the vector potential of effective gauge field acting on the massless Dirac fermions; $q = \pm 1$ is the corresponding electric charge. Since it is the Hamiltonian, the index a in Γ^a matrices has the values $a = 1,2,3,4$. The matrix e^i_a are the components of the spatial vielbein with $a = 1,2,3,4$ and $i = 1,2,3$: are

$$e^i_a = c_\perp(\delta^i_a - \hat{l}_a\hat{l}^i) \ \text{ for } a = 1,2,3 \ , \ e^i_4 = c_\parallel \hat{l}^i \, . \tag{19}$$

We ignore here the mixed components e^0_a and e^i_0.

The rectangular 4×3 vielbein give rise to the conventional 3×3 spatial metric:

$$g^{ik} = \sum_{a,b} \delta^{ab} e^i_a e^k_b \ , \ a,b = 1,2,3,4 \ , \ i,k = 1,2,3 \, , \tag{20}$$

with

$$g^{ik} = c_\parallel^2 \hat{l}^i\hat{l}^k + c_\perp^2(\delta^{ik} - \hat{l}^i\hat{l}^k) \, , \tag{21}$$

and again the nontrivial structure of tetrad is hidden.

The interesting consequence of the extension of the spin degrees of freedom is the unusual structure of the topological object – the monopole in the vielbein field. The simplest example of this gravitational monopole is the hedgehog with $\hat{\mathbf{l}}(\mathbf{r}) = \hat{\mathbf{r}}$. In the isotropic space, where $c_\parallel^2 = c_\perp^2 \equiv c^2$, the metric in Eq.(21) is flat, $g^{ik} = c^2\delta^{ik}$. The topological monopole structure of the vielbein is hidden. This is very different from the other global monopoles, which generate the solid angle deficit in the spacetime outside the monopole.

VII. SUPERFLUIDS VS TIME CRYSTALS

Most of the Andreev's arguments were based on the consideration of the coherent superposition of states with different conserved charges (angular momentum, particle number, etc.). The coherent superposition of such states is time dependent. That is why, according to Andreev 1996 paper[12], the spontaneous breaking of the global $U(1)$ symmetry accompanying the Bose condensation corresponds to thermodynamically equilibrium ground states with non-integral average particle number. The latter results in the time dependent order parameter:

$$\left\langle \hat{a}^+ \right\rangle = \mathcal{N}^{1/2} \exp\left(i\frac{\mu}{\hbar}t\right) , \tag{22}$$

where $\hat{a}^+$ is the operator of creation of bosons, μ is their chemical potential, and $\mathcal{N}$ is the number of particles in Bose condensate. Eq.(22) suggests that there are oscillations with frequency $2\pi\hbar/\mu$, and thus this looks as spontaneous breaking of the symmetry under the time translation.

The problem is that such oscillations are produced by the system in its ground state, which is rather strange. However, Wilczek in 2012 introduced the notion of the time crystals,[13] which in particular included the Bose condensates (see e.g. the papers "Superfluidity and space-time translation symmetry breaking"[14] and "Space- and time-crystallization effects in multicomponent superfluids"[15]). Although such realizations of time crystals were criticized,[16–18] now time crystals became the hot topic.

It is clear that the system in its ground state cannot have physically observable oscillations. If the ocsillations can be detected using some stationary device, this would mean that the detector is excited. The energy of the system is transferred to the detector, which means that the system is not in the ground state. So, if the Bose condensate is in its stable ground state, such as superfluid ^{4}He with $\mu < 0$ at $T = 0$, the oscillations of the order parameter in Eq.(22) are not observable.

There are several ways of how the oscillations can be observed, but in all cases the system is either perturbed, or the number of particles is not strongly conserved. The latter takes place if we consider the possibility of proton decay suggested by the Grand Unification Theories (GUTs), which in particular leads to the non-conservation of the helium atoms. Measuring the intensity of the decay of the helium atoms in superfluid ^{4}He, one can observe oscillations between the states with different number of atoms in the condensate.

Another source of the decay of the superfluid ^{4}He is the expansion of the Universe. The probability of evaporation of atoms from this Bose liquid is:[19]

$$w \propto \exp\left(-\frac{\pi|\mu|}{\hbar H}\right) . \tag{23}$$

where H is the Hubble parameter. This looks as if the quantum vacuum in expanding Universe has the effective temperature $T = H/\pi$, which leads to the activation process of evaporation of atoms from the liquid to the vacuum.

So, in principle, the oscillations are observable, but they become unobservable in the limit of the infinite decay time, when the system can be considered in its ground state. That is why the notion of time crystals requires consideration of the interplay between different time scales. The typical example is provided by the spontaneous coherent precession of spin, which can be considered in terms of the Bose condensate of magnons.[20,21] The corresponding $U(1)$ symmetry is the symmetry under spin rotations around the direction of magnetic field, and the corresponding $U(1)$ charge is the spin projection on the direction of magnetic field or, which is the same, the number of magnons.

Both the coherent spin precession and the Bose condensates in superfluids experience the off-diagonal long-range order. This is seen if one compares the operator of creation of particle $\hat{a}_0^+$ with the operator $\hat{S}^+$ of creation of spin projection on axis z, whose expectation value is:

$$\left\langle \hat{S}^+ \right\rangle = \mathcal{S}_x + i\mathcal{S}_y = S_\perp e^{i\omega t} \,, \tag{24}$$

where ω is precession frequency.

We can use the Holstein-Primakoff transformation, which expresses the spin operators in terms of the operators of creation and annihilation of magnons:

$$\hat{a}_0 \sqrt{1 - \frac{\hbar a_0^\dagger a_0}{2\mathcal{S}}} = \frac{\hat{\mathcal{S}}^+}{\sqrt{2\mathcal{S}\hbar}} \,, \tag{25}$$

$$\sqrt{1 - \frac{\hbar a_0^\dagger a_0}{2\mathcal{S}}}\, \hat{a}_0^\dagger = \frac{\hat{\mathcal{S}}^-}{\sqrt{2\mathcal{S}\hbar}} \,, \tag{26}$$

$$\hat{\mathcal{N}} = \hat{a}_0^\dagger \hat{a}_0 = \frac{\mathcal{S} - \hat{\mathcal{S}}_z}{\hbar} = \frac{\mathcal{S}(1-\cos\beta)}{\hbar} \,. \tag{27}$$

where β is the tipping angle of precession. This gives the Eq.(22) for the vacuum expectation value of the operator of boson annihilation:

$$\langle \hat{a}_0 \rangle = \mathcal{N}^{1/2} e^{i\omega t} = \sqrt{\frac{2\mathcal{S}}{\hbar}}\, \sin\frac{\beta}{2}\, e^{i\omega t} \,. \tag{28}$$

This shows that the precession frequency plays the role of the chemical potential of magnons, $\hbar\omega = \mu$.

This time dependence is observable: the spin precession is seen in the NMR experiments due to spin-orbit interaction. On the other hand, the spin-orbit interaction violates the $U(1)$ symmetry of spin rotations, which leads to the non-conservation of the number $\mathcal{N}$ of magnons in the magnon Bose condensate. If the spin-orbit interaction is neglected, the number of magnons is conserved, and the coherent spin precession represents the time crystal – it is the ground state of the system at fixed number $\mathcal{N}$ of magnons. But without the spin-orbit coupling the precession is not observable, the detector would not detect the time crystal.

This is the essence of the physical time crystals, which is applicable to all other coherent systems generated by the quasi-conservation of the $U(1)$ charge Q. Each system is characterized by its own relaxation time τ_Q, which is the decay time of the corresponding charge. The coherent state is time dependent, i.e. it violates the time translation symmetry. But it is not the ground state of the system, since the ground state of the decaying system has zero charge, $Q = 0$. In the strict limit $\tau_Q \to \infty$ the coherent state becomes the ground state at fixed charge Q. But in this limit the oscillations become un-observable: no breaking of time translation symmetry is seen in this limit.

So, in general the time crystals in systems with quasi-conserved charge Q do exist and they are observable. For example, due to the long relaxation time τ_Q in ^{3}He-B, we observe the AC Josephson effect between two time crystals.[22] This demonstrates that the spontaneous breaking of time translation symmetry suggested by Andreev does take place.

Andreev also applied this symmetry breaking notion to fermions, and not only to the pair condensate, which looks naturals, but also to superposition between the states with odd and even numbers of fermions.[23] This remains controversial, but it is not excluded that something like that can occur due to the quantum effects related to chiral anomaly.

VIII. CHIRAL ANOMALY AND NON-CONSERVATION OF FERMION NUMBER

It is well known that in gauge theories with nontrivial topology the violation of fermion number is possible. In the instanton process of change of the topological number of vacuum the electron number changes. The change of the fermion number, as well as the change of the baryon and lepton numbers, are determined by the topological charge. This is in the basis of the Kuzmin–Rubakov–Shaposhnikov scenario of the anomalous electroweak baryogenesis.[24] In Standard Model this leads to nonconservation by an even number of fermions.

The same takes place in condensed matter systems. For example, in superfluid ^{3}He-A the gravitational anomaly leads to creation of even number of quasiparticles. The gravitational instanton process of creation of single hopfion is accompanied by creation of 6 chiral fermions:[25]

$$\partial_\mu n^\mu_H = 6\, \partial_\mu J^\mu_5 \,. \tag{29}$$

Here J^μ_5 is the chiral current of fermionic quasiparticles in ^{3}He-A , and n^μ is the hopfion current, with hopfion density

$$n^0_H = \frac{m^2}{4\pi^2} (\mathbf{v}_s \cdot (\nabla \times \mathbf{v}_s)) \;,\; N_H = \int d^3r\, n^0_H \,, \tag{30}$$

where $\mathbf{v}_s$ is superfluid velocity and m is the mass of ^{3}He atom. This is the gravitational analog of the Kuzmin–Rubakov–Shaposhnikov scenario of the anomalous electroweak baryogenesis.[24]

Some gauge theory models allow for the topological processes that lead to the creation of just one fermion. These theories contradict different principles of quantum field theory, including the spin-statistics theorem. And usually there are the arguments, why such models cannot be realized.

However, following Andreev arguments, Bezrukov, Burnier and Shaposhnikov[26] found that at least in the 1+1 dimensions it is possible to create single fermion (see also Ref.[27]). It is created in the instanton process, at which the topological charge of the quantum vacuum changes by one. This shows that it is in principle possible that Nature may have some kind of hidden channel that allows the fermion-boson transformation. And extra spin dimension in the trans-Planckian physics may be the possible route.

IX. ANDREEV-WEYL FERMIONS

Here we discuss another possible extension of relativistic quantum field theory, which allows the creation of single fermion. Let us consider two Weyl points separated in momentum space:

$$H(\mathbf{p}) = e^i_\alpha \sigma^\alpha (p_i - p^0_i) + f^i_\alpha \sigma^\alpha (p_i + p^0_i) \,. \tag{31}$$

This Hamiltonian dscribes two different "worlds". One is the world of the Weyl particles concentrated near the Weyl point at $\mathbf{p} = \mathbf{p}^0$, while the other particles live near the Weyl point at $\mathbf{p} = -\mathbf{p}^0$. If the distance $2p^0$ between the Weyl points is on the order of Planck scale, these worlds practically do not communicate with each other. Each world has its own gravity with its own tetrad fields, e^i_α and f^i_α, and its own metric. In each world the scattering of particle practically does not change its momentum, i.e. $\mathbf{p} = \mathbf{p}^0 + \mathbf{k} \to \mathbf{p} = \mathbf{p}^0 - \mathbf{k}$. This is the analog of the Andreev reflection, at which the momentum practically does not change but velocity changes sign. That is why they we can call such particles as Andreev-Weyl fermions.

Nevertheless there exists the true reflection, at which the momentum changes sign, $\mathbf{p} \to -\mathbf{p}$. The probability of such processes is extremely small, since it is not easy to create the perturbation of the Planck energy scale, which can lead to reflection, such as the mirror of Planck length width. Nevertheless, such reflection is possible, and then in each world the observer will detect the creation or annihilation of single fermion. In this case, for the world near the first Weyl points, the role of the hidden spin variable is played by the tetrad f^i_α, which is hidden in the second world.

The models of the type of Eq.(31) are known for lattice fermions.[28,29] The natural arrangement of the eight left and eight right Weyl fermions in one generation of Standard Model is when the Weyl points are on the vertices of the cube in the four-dimensional momentum-frequency space.[30,31] The analogous three-dimensional cube of the Weyl points in d-wave superconductors and some other possible configurations of Weyl and Dirac points are discussed in Ref.[32], which is devoted to memory of Lev Petrovich Gor'kov.

X. DISCUSSION

While the specific scenario suggested by Andreev is not realized, his suggestion of extra dimensions for spin space and the corresponding non-conservation of fermion number may occur in different realizations of the trans-Planckian physics. We considered this on several examples: (1) extension of the discrete parity to the continuous symmetry group in the planar phase of superfluid ^{3}He; (2) extension of the internal spin symmetry group $SO(1,3)$ to $SO(1,4)$ in the relativistic system; (3) the similar extension in the relativistic physics emerging in the vicinity of the Dirac points in the planar phase; (4) extension of Standard Model, which allows the baryogenesis; (5) the 1+1 quantum field theory, which allows the fermion-boson transmutation; (6) extension of Standard Model to lattice theory.

In case (1) the extra spin dimension compensates the change of sign under 2π rotation. This demonstrates, that the Andreev type scenario, which destroys the superselection rule, may in principle take place on the more fundamental level of trans-Planckian physics. In this case we must look for the different solution of the fermion-boson problem.

In cases (2) and (3) the extra spin dimensions lead to new effects. They give rise to gravity based on rectangular vielbein rather than on traditional tetrads of Einstein-Cartan gravity. It is interesting that these extra dimensions are hidden, if only the metric is probed in experiments. Example is the "invisible" gravitational global monopole in the planar phase with isotropic "speed of light". It is the singular topological structure in the gravitational vielbein field, which however does not disturb the gravitational metric.

In case (4) the baryogenesis leads to decay of atoms in superfluid ^{4}He, which in turn produces the time crystal behavior of superfluids advocated by Andreev.

The case (5) is based on the consideration of the Andreev's arguments. It is shown that in the 1+1 theory the creation of single fermion is possible, which opens the route to search for such possibility to avoid the superselection rule using extra dimensions.

In case (6) the extra degrees of freedom correspond to the worlds near far distant Weyl points in momentum space. Within each world the observer may detect the events of creation of single fermion.

So, the Andreev idea of the extension of the internal spin space, even in case if it does not destroy the superselection rule, may lead to new physics on the more fundamental level.

Acknowledgements. This work has been supported by Academy of Finland (grant 332964).

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