On the puzzle of Bremsstrahlung as described by coaccelerated observers

I. Peña,* C. Chryssomalakos,† A. Corichi,‡ and D. Sudarsky§

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México
Ciudad Universitaria, A.P. 70-543, 04510, D.F., México

(Dated: 7th February 2008)

Abstract

We consider anew some puzzling aspects of the equivalence of the quantum field theoretical description of Bremsstrahlung from the inertial and accelerated observer's perspectives. More concretely, we focus on the seemingly paradoxical situation that arises when noting that the radiating source is in thermal equilibrium with the thermal state of the quantum field in the wedge in which it is located, and thus its presence does not change there the state of the field, while it clearly does not affect the state of the field on the opposite wedge. How then is the state of the quantum field on the future wedge changed, as it must in order to account for the changed energy momentum tensor there? This and related issues are carefully discussed.

PACS numbers: 04.62.+v, 04.70.Dy, 03.70.+k

*Electronic address: `igor@nucleares.unam.mx`
†Electronic address: `chryss@nucleares.unam.mx`
‡Electronic address: `corichi@nucleares.unam.mx`
§Electronic address: `sudarsky@nucleares.unam.mx`

I. INTRODUCTION

The topic of radiation by uniformly accelerated charges has often been the source of much puzzlement and confusion, particularly when considered in the light of the equivalence principle. Much of the confusion has been removed by the realization that for observers coaccelerating with the charge there are regions of spacetime that are inaccessible. In effect, in the classical context, it has been shown that for an electromagnetic charge in uniform acceleration, the classical radiation field, as described by accelerating observers, is zero at every point in the region that is accessible to them (known as the Rindler wedge, R, see Fig. I) [1]. This would then remove any apparent contradiction between the EP and the known Bremsstrahlung. For both the inertial and accelerated observers there is radiation but, for the accelerated ones, such radiation lies beyond the regime where the static description is valid.

In the quantum version of this situation the question is posed in terms of emission of photons rather than the evaluation of radiation fields. In fact, it was shown that the standard Bremsstrahlung when viewed from the point of view of the accelerated observers –a point of view called Rindler quantization– acquires a very particular interpretation. Actually, as will be explained in more detail below, the coincidence for the prediction of photon emission rates between the inertial and accelerated descriptions makes fundamental use of the Unruh effect [2], which states that from an accelerated frame comoving with the charge, the standard Minkowskian vacuum state corresponds to a thermal state. Moreover, it is well known that for a detector uniformly accelerating in the inertial vacuum the process for which the detector absorbs a particle from the bath (a Rindler particle) as seen by a comoving (accelerating) observer is equivalent to the emission from the detector of a Minkowski particle as seen by an inertial observer [3]. Then from the point of view of an inertial observer the accelerating charge emits particles while, from an accelerated viewpoint, the charge –which is static– will emit *and* absorb Rindler particles to and from the bath.

The restriction of this effect to wedge R has been analyzed in Ref. [4], where it is shown that the emission rate of photons with fixed transverse momentum in the inertial frame coincides with the combined rate of emission and absorption of zero-energy Rindler photons with the same transverse momentum in the accelerated frame. Thus, this result gives a clear notion of the the physical equivalence between inertial and accelerated descriptions of Bremsstrahlung. However, and as is often the case in this field, the answer to one question brings in further puzzlement, and the need to answer a further one.

The calculation mentioned above, makes fundamental use of the so called zero energy Rindler modes. The reason is that for accelerated observers the charge is static and thus it can only couple to modes of zero frequency with respect to Rindler time. In fact, due to the expression of the form $0 \times \infty$ that appears in calculations involving zero energy particles in the analysis of Ref. [4], it is required the introduction, for the purpose of regularization, of a small frequency of oscillation θ for the source, which allows one to work with finite energy modes and which at the end is taken to 0. In that work the authors considered also the question of whether or not an accelerated observer sees any difference in the thermal bath due to the emission and absorption of Rindler photons. They note that in the limit of zero energy the transition rate from an n-photon state to an $(n+1)$-photon state and the rate of the inverse process became equal, thus the accelerated charge leaves the thermal bath undisrupted. From this one can conclude that *the source is in thermal equilibrium with the quantum field.* Then, from the point of view of an accelerated observer, in R, there will be no difference between the initial state of the field (the initial thermal state) and the state generated by the interaction with the accelerating charge (note the analogy with the description of the situation in the classical context that we explained above). Similarly, the state of the field in the second Rindler wedge will remain the initial thermal state, as that region could not possibly be, in anyway, influenced by what is going on in a causally disconnected region of spacetime. We could think that this conflicts with the fact that EPR influences are allowed but this is misleading: when talking about the state in the L Rindler wedge we mean the corresponding density matrix and this could not change due to actions taken on the right wedge for otherwise there would be operators pertaining to the left wedge whose expectation values could be used to determine what has occured in the right Rindler wedge. Nevertheless, it is clear that in wedge F (see Fig. I) there would be a detectable change in the state of the field and, in particular, the expectation of the energy momentum tensor in this region should be different from what it would have been if there was no interaction with the charge. In effect, this change could be computed in an inertial quantization of the field and would correspond to the final state containing the Minkowski photons emitted by the the accelerated charge. On the other hand, the quantum description of the state of the field as seen by the accelerated observer is given by the Unruh quantization scheme [2, 5]. In this description, the restriction to the L and R wedges of both, the Minkowski vacuum state and the state resulting from the interaction with the charge is, in both cases, a thermal bath, and thus it seems neither could contain the information regarding what has changed in F. The issue is then whether this situation can be analyzed in the language appropriate to accelerated observers and how would, in that case, the information about the changed situation in F be codified?

The fact that Unruh modes can be expressed by superpositions of specific modes that extend distributionally to the whole spacetime (known as *boost modes*, see Refs. [6, 7]), opens the door to analysis of issues related to physical questions outside the double wedge, and in particular in wedge F, carried out in the language appropriate for accelerated observers. In this work, we embark on such analysis for the case of a scalar field and an accelerated scalar source, and analyze from the new perspective the change in the expectation value of the energy-momentum tensor operator when evaluated at points in wedge F.

States in the Unruh quantization can be seen as states of a composite system where each of the components is the quantum field restricted to either wedges L or R. Then, having the density matrix $\hat{\rho}$ for a state, one can describe the physics, for example, in wedge L (R) by tracing out in $\hat{\rho}$ the right (left) degrees of freedom. From this procedure we obtain a density matrix $\hat{\rho}^L$ ($\hat{\rho}^R$) describing a state in wedge L (R). It is well known that when a field observable $\hat{A}$ is localized in either L or R, then the expectation value of $\hat{A}$ is determined *completely* by $\hat{\rho}^L$ or $\hat{\rho}^R$ respectively. On the other hand, when $\hat{A}$ is localized out of the double wedge, it is clear that its expectation value would not, in general, be determined solely by information encoded in $\hat{\rho}^L$ or $\hat{\rho}^R$.

There should exist some object containing this extra information carried by the state which controls how do left and right parts *combine* which is the extra element necessary for describing completely the state in all of Minkowski spacetime. We identify this object as the *entanglement matrix*. In the case of interest we were concerned about the expectation value of the energy momentum tensor in the future wedge and it seemed natural to expect its change to be encoded in the change of the entanglement matrix. This was our initial assumption, and the work intended to see how exactly is such information encoded in this case. As we will see this expectation was mistaken and the answer in our case lies, surprisingly, in $\hat{\rho}^R$. Precisely how does this matrix codifies this information will be elucidated through the rest of the manuscript.

Regarding the foundational basis of this work we must point out that the issue of *extending* the Unruh quantization outside the double wedge is somewhat subtle. What is very well established in the literature is the restriction of global states to the double wedge –for example, the restriction of states to the wedge R is interpreted as the state seen by accelerated observers which have available only the wedge R– but, as far as we know, the extension of a state in the Unruh quantization to the whole MS does not seem to be, at this time, fully investigated. One important point concerning this question is that Unruh modes are highly *singular* at the asymptotes which are taken to coincide with the horizon of the particular construction. Thus, the initial data from which one builds up the (one particle) Hilbert space in the Unruh quantization has restrictions at these asymptotes which *can influence* the solutions of the field equation in wedges F and P (this possibility has been pointed out in Ref. [7], [6]). As we explained above, in this paper we extend the usage of the Unruh description of the field to address questions related to the physics in the wedge F. We do this in the way that we consider is the most natural and we find results that are physically consistent and which coincide with what one would obtain working in the standard Minkowski quantization of the field as we show explicitly in Appendix C.

The paper is organized as follows. In Sec. II we review the main ideas of the Unruh quantization and specify our notation. In Sec. III we explain our strategy for analyzing the encoding of the information in the final state of the field. We propose a particular decomposition of the density operator of the state and use it to write an expression for the change in the expectation value of $\hat{T}_{\mu\nu}$. Nevertheless, the particular form of the source representing an accelerating particle is needed in order to obtain an explicit expression for the final state and its density operator. In Sec. IV we work out these ideas introducing a scalar accelerating pointlike source with a particular regulator and build up the S-matrix operator. In Sect. V we make the final calculations in order to obtain the change in $\langle \hat{T}_{\mu\nu} \rangle$ and we evaluate it in wedges R and F. From these results one can see how the information of the physical change in F is encoded. In Sec. VI we analyze the case of two different sources accelerating in wedges L and R in order to get more insight of the behaviour of the density operator of the respective final state. Finally, we end with some discussions and the interpretation of the results in Sec. VII. In order to carry out these steps it is important to have explicit expressions of the spacetime behaviour of the Unruh modes, which we present in Appendix A. The calculations leading to our final results are, though straightforward, very long, in Appendix B we give an explicit derivation of the main formula in this work. In Appendix C we make the same calculations as in Sec. V for the change in wedge F but using, instead, the standard plane wave representation of the field and show that both results coincide.

To eliminate unnecessary notation we shall work in two dimensional Minkowski space, this will allow us to be clearer without losing physical insight. As a matter of fact, working with a scalar field in $2D$ with $m \neq 0$ is operationally equivalent to work with the same field in 4D and fixed traverse momentum $k_\perp^2 = k_x^2 + k_y^2$, with the identification $m^2 \to k_\perp^2 + m^2$. In all this work we shall work in units in which $c = \hbar = 1$ and signature $-+$.

II. UNRUH QUANTIZATION

All the comoving observers to a particle moving with uniform proper acceleration $a = (a_\mu a^\mu)^{1/2}$ have world lines of the form

$$t = \zeta \sinh(a\tau) \qquad z = \zeta \cosh(a\tau)\,, \tag{II.1}$$

where $0 < \zeta < \infty$ and τ is the proper time of the observer, $-\infty < \tau < \infty$. Eq. (II.1) can be used to give coordinates τ, ζ to wedge R, which is known as Rindler spacetime. In these coordinates, the Minkowski metric becomes

$$ds^2 = -\zeta^2 d\tau^2 + d\zeta^2. \tag{II.2}$$

As can be seen from Eq.(II.2), wedge R is a static, globally hyperbolic spacetime so one can use standard methods of frequency splitting [5, 8, 9] for quantizing the field in that region. As a first step we shall construct directly the one particle Hilbert space for a Klein Gordon field with mass m in wedge R, using as timelike Killing field τ^a; this is called the Fulling-Rindler quantization [10]. Consider the massive Klein Gordon (KG) field equation

$$(\Box - m^2)\phi = 0 \tag{II.3}$$

restricted to wedge R and take solutions which are *positive frequency* w.r.t. the Rindler time τ (in this case $(\partial/\partial\tau)^\mu = b^\mu$ is the time translation generator, cf. Eq. (A.1)) and which vanish asymptotically. These are superpositions of the following set of modes

$$\psi_\omega(\xi) = \frac{\sqrt{\sinh \pi\omega}}{\pi} e^{-i\omega\tau} K_{i\omega}(m\zeta), \qquad \omega > 0, \tag{II.4}$$

where $\xi = (\tau, \zeta)$, and $K_{i\omega}(x)$ is the modified Bessel function of the third kind or Macdonald function. The normalization factor is chosen such that these modes are δ normalized w.r.t. the KG product:

$$\langle \psi_\omega | \psi_{\omega'} \rangle_{KG} = \frac{i}{2} \int_{\Sigma_R} \left(\psi^*_\omega \nabla_\mu \psi_{\omega'} - \psi_{\omega'} \nabla_\mu \psi^*_\omega \right) d\Sigma^\mu = \delta(\omega - \omega') \tag{II.5}$$

where Σ_R is a Cauchy surface for Rindler spacetime. Functions $K_{i\omega}(m\zeta)$ have an essential singularity (it oscillates "infinitely") in the limit $\zeta \to 0$ [11], so the $\psi_\omega(\xi)$ are not defined at the horizon ($\tau \to \pm\infty$ and $\zeta \to 0$) when $\omega \neq 0$. Modes given by Eq. (II.4), called Fulling modes, and their conjugates $\psi^*_\omega(\xi)$ form a complete set of the space of solutions to Eq.(II.3) in Rindler spacetime. One constructs the one particle Hilbert space $\mathscr{H}_R$ for a quantization of the KG field in R by Cauchy completing the space spanned by the $\psi_\omega(\xi)$ (positive frequencies) in the inner product given by Eq. (II.5) (a rigorous construction of this space is given in [12]). The space of states of the field for this quantization is the Fock space of $\mathscr{H}_R$, $\mathscr{F}(\mathscr{H}_R)$. The field operator in this quantization takes the form

$$\hat{\phi}_R(\xi) = \int_0^\infty \left(\psi_\omega(\xi) \hat{r}_\omega + \psi^*_\omega(\xi) \hat{r}^\dagger_\omega \right) d\omega, \tag{II.6}$$

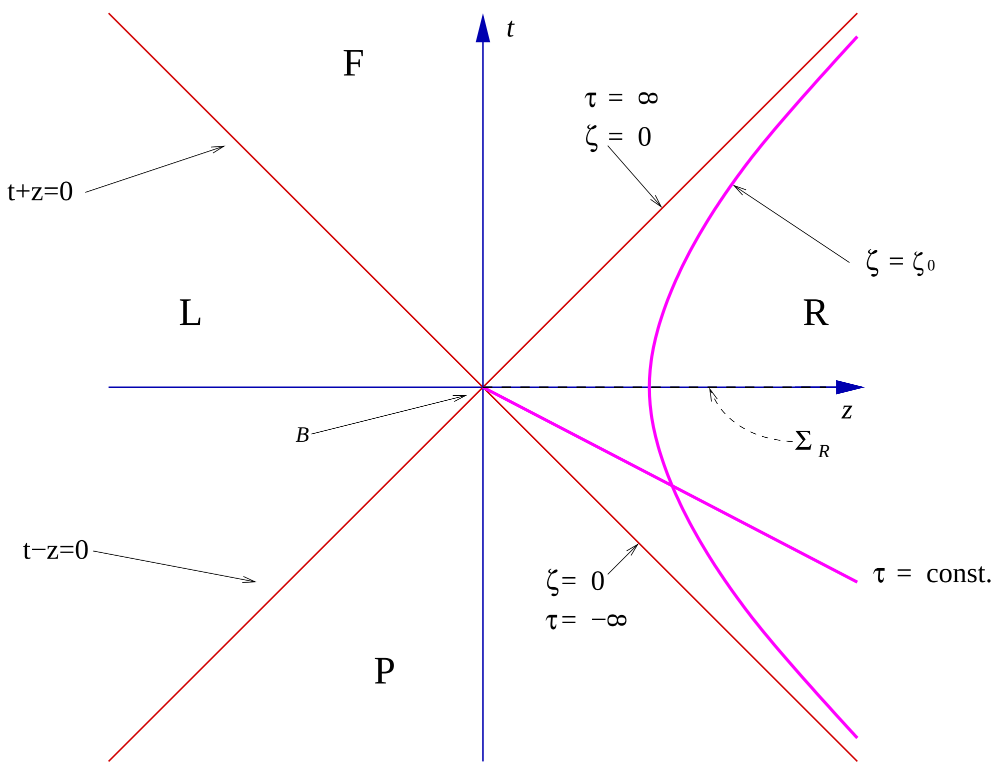

Figure 1: The geometry of Rindler space.

where the creation and annihilation operators, $\hat{r}^\dagger_\omega$ and $\hat{r}_\omega$, satisfy canonical commutation relations. Repeating the Fulling-Rindler quantization procedure in wedge L with the L-time translation generator given by $-b^\mu$ (see Eq. (A.1)) and defining the KG product on a Cauchy surface Σ_L of L, one obtains the one particle Hilbert space for the KG field in L, $\mathscr{H}_L$, and a *left* field operator analogous to Eq. (II.6).

Wedges L and R, are causally disconnected and thus neither $\mathscr{F}(\mathscr{H}_L)$ nor $\mathscr{F}(\mathscr{H}_R)$ can be used to represent the field algebra in Minkowski spacetime, as the Fock space of the standard plane wave quantization does. However, one can consider a second quantum field construction in all of Minkowski spacetime with one particle Hilbert space given by

$$\mathscr{H}_U = \mathscr{H}_L \oplus \mathscr{H}_R, \tag{II.7}$$

and the space of states given by [5]

$$\mathscr{F}(\mathscr{H}_U) = \mathscr{F}(\mathscr{H}_R) \otimes \mathscr{F}(\mathscr{H}_L). \tag{II.8}$$

The quantum field construction defined by Eq. (II.7) is called *Unruh quantization.* The field operator in this quantum construction takes the form

$$\hat{\phi}(x) = \hat{\phi}_L(x) \otimes \hat{1}_R + \hat{1}_L \otimes \hat{\phi}_R(x). \tag{II.9}$$

Whenever there is no confussion, we denote by $\hat{\phi}_L(x)$ the operator $\hat{\phi}_L(x) \otimes \hat{1}_R$ and the analogous for $\hat{\phi}_R$. Clearly, these operators commute,

$$\left[\hat{\phi}_L(x), \hat{\phi}_R(x')\right] = 0, \tag{II.10}$$

reflecting the fact that the regions L and R are causally disconnected.

Constructed in this way, it may seem that in the Unruh quantization the field operator is made up of modes which have support only in the double wedge $L \cup R$ and thus that cannot describe the physics outside this region. Nevertheless, recall that the one particle Hilbert spaces $\mathscr{H}_L$ and $\mathscr{H}_R$ are made up of positive frequency modes which have initial data on either Σ_L or Σ_R. These two latter Cauchy surfaces can be seen as the restriction of some Minkowski spacetime Cauchy surface Σ to L or R respectively, then the initial data of modes in the left or right quantizations of the field define unique solutions of the KG equation in all of Minkowski spacetime. In fact, as we explain in the next subsection and in Appendix A, there exist superpositions of plane waves, called Unruh modes, which coincide with Fulling modes when restricted to either wedge L or R and that are zero when restricted to the opposite wedge [6]. The elements of $\mathscr{H}_U$ describe modes of the field defined distributionally in the whole of Minkowski spacetime. See the last paragraph of the next subsection for further discussion.

A. Boost modes and Unruh modes

The original quantization approach used by Unruh [2] does not give explicitly the functional form of Unruh modes since he works only with their restrictions to the horizons. However, for the purposes of this work we do need the functional form in all of MS of these set of modes, which are linear combinations of *boost modes.* We will introduce both sets of modes in this section following Ref. [6].

Consider the space of global classical solutions to Eq. (II.3). One can look for solutions in this space which have positive frequency w.r.t. the boost parameter τ in wedges L and R. This class of solutions, *Minkowski Bessel Modes* were originally presented and studied by Gerlach [13].

Narozhny *et.al.* [6] and Fulling and Unruh [7] call them *boost modes* (BM), name that we also prefer. Take an (unnormalized) plane wave solution to Eq. (II.3) of the form

$$P_p^{\pm}(x) = (2\pi)^{-1/2}\, e^{\mp i(\omega_p t - pz)}, \tag{II.11}$$

where $x = (t, z)$, $\omega_p = \sqrt{p^2 + m^2} > 0$ and the upper (lower) sign corresponds to positive (negative) frequency w.r.t. inertial time. Changing the variable p to the *rapidity* θ:

$$m \sinh(\theta) = p, \qquad m \cosh(\theta) = \omega_p, \qquad -\infty < \theta < \infty, \tag{II.12}$$

then one can define boost modes as the following superposition of plane waves [6]

$$\begin{aligned} B_\omega^{\pm}(x) &= \frac{1}{2^{1/2}\sqrt{2\pi}} \int_{-\infty}^{\infty} P_\theta^{\pm}(x) e^{-i\omega\theta}\, d\theta \\ &= \frac{1}{2^{3/2}\pi} \int_{-\infty}^{\infty} e^{\mp im(t\cosh\theta - z\sinh\theta)} e^{-i\omega\theta}\, d\theta, \end{aligned} \tag{II.13}$$

where $-\infty < \omega < \infty$.

Note that in general, Eq. (II.13) cannot be interpreted as the definition of a function. In particular, it is divergent at the origin, $(t = 0, z = 0)$ and thus, cannot stand as a global solution to the KG equation if it is considered as a function. However, one can avoid these problems if one considers these quantities as distributions, thus requiring the smearing with suitable test functions. Therefore, we will consider this set of modes, as well as Unruh modes (cf. Eqs. (II.18)), as distributions when constructing the quantum field. Note that this is a consistent procedure as the field $\hat{\phi}(x)$ has, by itself, a distributional character. For the purposes of this work, we will consider only test functions of compact support in M.

Formally, boost modes are orthogonal in the KG inner product (Eq. (II.5) over Σ_M),

$$\langle B_\omega^{\pm} | B_\mu^{\pm} \rangle_{KG} = \pm\delta(\omega - \mu), \tag{II.14}$$

and they are eigenfunctions of the *boost operator* [13] (inside the four wedges of MS),

$$\mathbf{B} B_\omega^{\pm} = -i\omega B_\omega^{\pm} \qquad \text{where} \qquad \mathbf{B} = t\frac{\partial}{\partial z} + z\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau}, \tag{II.15}$$

and thus, the modes $B_\omega^{\pm}$ are *positive (negative)* frequency KG solutions w.r.t. the boost parameter τ in wedges L and R whenever $\omega > 0$ ($\omega < 0$). Note that in dividing the modes into positive and negative frequency modes one is dropping out the $\omega = 0$ mode which could, in principle, be a source of trouble [6].

From now on we will only use boost modes which are positive frequency w.r.t. inertial time t, B_ω^+, and will drop the superscript: $B_\omega \equiv B_\omega^+$. One can use this set of modes as a *basis* of the positive inertial frequency solution space of eq. (II.3). Actually, one can give a quantization of the KG field unitary equivalent to the standard positive frequency plane waves one [6]:

$$\hat{\phi}(x) = \int_{-\infty}^{\infty} d\omega\, \left(B_\omega(x)\hat{b}_\omega + B_\omega^*(x)\hat{b}_\omega^\dagger \right), \tag{II.16}$$

where $[\hat{b}_\omega, \hat{b}_{\omega'}^\dagger] = \delta(\omega - \omega')$. Because the transformation is unitary these quantum field descriptions share the same vacuum

$$\hat{b}_\omega \left|0_M\right\rangle = \hat{a}_p \left|0_M\right\rangle = 0. \tag{II.17}$$

Unruh's idea [2] for giving a field quantization associated to accelerated observers consists in constructing a set of modes R_ω and L_ω from combinations of boost modes and their conjugates such that they are positive frequency w.r.t. the boost parameter (accelerated time) in wedges R and L respectively and zero on the opposite wedge. These modes are defined by

$$R_\omega(x) = \frac{1}{\sqrt{2\sinh(\pi\omega)}}\left[e^{\pi\omega/2}B_\omega(x) - e^{-\pi\omega/2}B^*_{-\omega}(x)\right], \tag{II.18a}$$

$$L_\omega(x) = \frac{1}{\sqrt{2\sinh(\pi\omega)}}\left[e^{\pi\omega/2}B_{-\omega}(x) - e^{-\pi\omega/2}B^*_{\omega}(x)\right], \tag{II.18b}$$

where $\omega > 0$. Note that, since they are defined in terms of boost modes, these definitions are global. These equations can be inverted:

$$B_\omega(x) = \frac{1}{\sqrt{2\sinh(\pi\omega)}}\big[e^{\pi\omega/2}R_\omega(x) + e^{-\pi\omega/2}L^*_\omega(x)\big], \tag{II.19a}$$

$$B_{-\omega}(x) = \frac{1}{\sqrt{2\sinh(\pi\omega)}}\big[e^{\pi\omega/2}L_\omega(x) + e^{-\pi\omega/2}R^*_\omega(x)\big]. \tag{II.19b}$$

It can be seen that, when restricted to R, Unruh modes R_ω coincide with Fulling modes, Eq. (II.4), and the analogous situation for L_ω in L [13] (see Appendix A) and, therefore, initial data of Unruh modes coincides with that of Fulling modes. It is known that solutions of the field equation can be represented by their initial data [8] and thus Cauchy completing the space spanned by the L_ω and R_ω modes one would obtain, respectively, $\mathscr{H}_R$ and $\mathscr{H}_L$.

The field operator in the Unruh quantization takes the form of Eq. (II.9) with $\hat\phi_L(x)$, $\hat\phi_R(x)$ expressed in terms of Unruh modes:

$$\hat\phi(x) = \hat\phi_L(x) + \hat\phi_R(x) = \int_0^\infty d\omega\big(R_\omega(x)\hat{1}_L\otimes\hat{r}_\omega + L_\omega(x)\hat{l}_\omega\otimes\hat{1}_R + \text{H.C.}\big), \tag{II.20}$$

where $\hat{r}_\omega$ and $\hat{l}_\omega$ are annihilation operators in $\mathscr{F}(\mathscr{H}_R)$ and $\mathscr{F}(\mathscr{H}_L)$ respectively.

We note that recently there has been some controversy about the Unruh quantization. Narozhny *et.al.* [6] claim that the *Unruh quantization is not a valid quantization scheme for all of MS* (as the boost modes quantization is, see Eq. (II.16)). To support this they argue, in particular, that the expansion of $\hat\phi$ in Unruh modes, Eq. (II.20), does not exhaust all the degrees of freedom of the quantum field in Minkowski space. This is so, they say, because when "evaluating at the origin", boost modes have a *singularity* when $\omega = 0$. The authors of Ref. [6] logic is that, consequently, in the transition from Eq. (II.16) to Eq. (II.20) —using Eqs. (II.19)— one should take the Cauchy principal value of the integral at the origin, which excludes the $\omega = 0$ mode. They assert that without this mode, the remaining set of boost modes loses the property of being complete and then lacks the possibility of spanning every (one particle) state of the field. (Ref. [6], p. 025004-12) On the other hand, Fulling and Unruh [7] have argued against this claim. They state that since the mode expansion is an integral (Lebesgue measure) and thus one mode is of *zero measure*, the omission of the $\omega = 0$ mode is quite harmless in the mode expansion of the field and the Unruh quantization is valid for expressing the restrictions to $L \cup R$ of global MS states. In this work we adhere to the position of Fulling and Unruh for the following reasons. As we have said above, boost modes should be considered as *distributions* and thus, evaluating them at one single point has no meaning. Moreover, we recall that the field itself evaluated at one point has no meaning either, the quantum field is a distributional object and only its convolution with a test function is defined.

In their reply, Fulling and Unruh forcefully argue that the Unruh quantization scheme is valid on the double wedge $L \cup R$ but, however, they indicated that they are not fully confident on the possibility of *extending* an Unruh state to all of MS ([7] p.048701-2). Let's be more precise on this issue. The initial data for the Unruh quantization consists of smooth functions of compact support on both Cauchy surfaces Σ_R and Σ_L. For definiteness, take $\Sigma_R = \{(0,z)|z > 0\}$ and $\Sigma_L = \{(0,z)|z < 0\}$. For these Cauchy surfaces, functions of compact support would be zero at the origin $(0,0)$. Nevertheless, a full MS quantization should consider the set of all C_0^∞ initial data over the Cauchy surface

$$\Sigma_M = \Sigma_L \cup \Sigma_R \cup \{0\}, \tag{II.21}$$

which includes, of course, functions which are not zero at the origin. In this respect, Fulling and Unruh [7] note that "The treatment of initial data at the origin is mathematically subtle, and data at that point may influence the solution of the field equation in regions F and P". This issue is quite relevant for our work. Although we are not giving any formal proof of the fact that the Unruh quantization *can be extended* to all of MS, we show that at least for the particular case we study, the Unruh quantization provides the same physical results as the standard flat space quantization. This we take as an indication that both quantum descriptions are generally equivalent.

B. Minkowski vacuum, general states and L-R correlations

To find a basis of $\mathscr{F}(\mathscr{H}_U)$, Eq. (II.8), we need a basis for $\mathscr{F}(\mathscr{H}_R)$ and $\mathscr{F}(\mathscr{H}_L)$. For $\mathscr{F}(\mathscr{H}_R)$ we choose an orthonormal basis whose elements are R-states $|J\rangle_R$ which have a definite number of Rindler particles n_J. Let J_{ω_m} be the number of particles in this state whose frequencies are centered in the particular mode ω_m, $m = 0, 1, \ldots$. Then, the state $|J\rangle_R$ is defined by the set

$$J = \left\{J_{\omega_0}, J_{\omega_1}, \ldots, J_{\omega_m}, \ldots\right\}, \qquad \sum_{m=0}^{\infty} J_{\omega_m} = n_J\,, \tag{II.22}$$

Note that only a finite number of the J_{ω_m} are $\neq 0$. State $|J\rangle_R$ can be build up from the Rindler vacuum in the following manner

$$|J\rangle_R = |J_{\omega_0}\, J_{\omega_1} \cdots\rangle_R = N_J\, (\hat{r}^\dagger_{\omega_0})^{J_{\omega_0}} \cdots (\hat{r}^\dagger_{\omega_m})^{J_{\omega_m}} \cdots |0\rangle_R, \tag{II.23}$$

where N_J is a normalization factor such that $\langle J|J\rangle = 1$, and $\hat{r}^\dagger_{\omega_m}$ creates a Rindler particle with frequency centered at ω_m. State $|J\rangle_R$ has Rindler energy

$$E(J) = \sum_{m=0}^{\infty} \omega_m J_{\omega_m}. \tag{II.24}$$

This is the state's energy associated with the boost Killing field. Given two different elements $|J\rangle_R$, $|K\rangle_R$ of this basis we have that

$${}_R\langle J|K\rangle_R = \delta(J,K) \equiv \delta_{J_{\omega_0},K_{\omega_0}} \cdots \delta_{J_{\omega_m},K_{\omega_m}} \cdots\,, \tag{II.25}$$

where $\delta_{J_{\omega_m},K_{\omega_m}}$ are Kronecker deltas. One can obtain a basis of $\mathscr{F}(\mathscr{H}_L)$ in an analogous way. The set of all $|J\rangle_L \otimes |K\rangle_R$ where J and K are of the form of Eq. (II.22) is a basis of $\mathscr{F}(\mathscr{H}_U)$. Any state $|g\rangle \in \mathscr{F}(\mathscr{H}_U)$ can be casted in terms of this basis, and it is defined by a particular function $G(J,K)$:

$$|g\rangle = \sum_{J,K} G(J,K)\, |J\rangle_L\, |K\rangle_R. \tag{II.26}$$

Here, the sums run over the space of all possible *particle distributions* J and K, that is, they are of the form

$$\sum_J = \prod_{m=0}^{\infty} \sum_{K_{\omega_m}=0}^{\infty} . \tag{II.27}$$

Entangled states in the Unruh quantization (those which cannot be expressed in the form $|g_1\rangle_L \otimes |g_2\rangle_R$) have non trivial correlations between left and right states. The inertial Minkowski vacuum, $|0_M\rangle$, is an entangled state in the Unruh quantization:

$$|0_M\rangle = \sum_K e^{-\pi E(K)} |K\rangle_L |K\rangle_R = \sum_{J,K} e^{-\pi E(K)} \delta(J,K) |J\rangle_L |K\rangle_R. \tag{II.28}$$

Given that $|0_M\rangle$ is an entangled state, when restricted to the R wedge, it fails to be a pure state [14] and its description is in terms of a density matrix. The norm of the Minkowski vacuum is now given by

$$\langle 0_M|0_M\rangle = \prod_{m=0}^{\infty} \frac{1}{1-e^{-2\pi\omega_m}}. \tag{II.29}$$

The fact that this well behaved state in the plane wave quantization has an infinite norm in the Unruh quantization is a consequence of the non unitarily equivalence between these quantizations. Recall that different, even not unitarily equivalent, representations of the field algebra are "physically equivalent" in the sense of Fell's theorem (see Ref. [5]) and thus one expects to obtain the same physical information from the computation of expectation values in either the inertial or the Unruh scheme, provided that the Unruh quantization is a faithful representation of the field algebra[1]. We can say that our results point in this direction since, as we shall see in Sec. V and Appendix C, we obtain, in both quantization schemes, the same physical result.

III. POSING THE PROBLEM: THE MATRIX OF ENTANGLEMENT

In this work we shall be concerned with the difference between the initial vacuum state and the final state which results from the initial state and the interaction of the classical scalar source in uniform acceleration. At the same time we are interested with the restriction of these sates to the different regions and thus we shall employ the language of density matrices as indicated by the posing of the questions raised in the introduction.

The trajectory of this source is a branch of an hyperbola lying in one Rindler wedge of spacetime which we choose it to be R. The final state of such interaction can be obtained perturbatively by the application of the respective S-matrix operator to the inertial vacuum state (this operator will be constructed in detail in Sect. IV):

$$|f\rangle = \hat{S}\,|0_M\rangle\,. \tag{III.1}$$

Recall from Sec. II that the L_ω modes of the field are zero when evaluated in wedge R and thus the scalar source can only excite R modes of the field. Then, for this case the $\hat{S}$ operator takes the form

$$\hat{S} = \hat{1}_L \otimes \hat{S}_R. \tag{III.2}$$

[1] We thank Hanno Sahlmann for pointing this out.

On the other hand, expressed in the Unruh quantization scheme, state $|f\rangle$ takes the form

$$|f\rangle = \sum_{J,K} F(J,K)\,|J\rangle_L\,|K\rangle_R. \tag{III.3}$$

All the information of the state $|f\rangle$ is encoded in the function $F(J,K)$, and hence one should be able to read from the change in this function, $\Delta F = F(J,K) - F_{vac}(J,K)$, the change in the energy momentum tensor due to the effects of the source. In particular, this change should, at the same time, codify the fact that no change is produced inside the wedge R and that a dramatic change occurs in wedge F. Nevertheless, this information is more clearly encoded in the change in the density matrix of the state, $\Delta\hat{\rho} = \hat{\rho}_f - \hat{\rho}_{vac}$ since it can be splitted directly into left, right and *entangled* contributions as we will explain in the following. This is a second reason for using the language of density matrices.

We already know that, when considered as a state of the composite system $\mathscr{F}_L \otimes \mathscr{F}_R$, the inertial vacuum state is an entangled state (see [14] for an intuitive explantion of this), and thus one expects that also state $|f\rangle$ will be entangled. In this sense, since both states are pure, their density operators cannot be written in the form [15]

$$\hat{\rho}' = \hat{\rho}'^{\,L} \otimes \hat{\rho}'^{\,R}, \tag{III.4}$$

where $\hat{\rho}'^{\,L}$ and $\hat{\rho}'^{\,R}$ represent the respective partial density matrices defined by

$$\hat{\rho}^{L,R} \equiv \mathrm{Tr}_{L,R}\,\hat{\rho}. \tag{III.5}$$

Nevertheless, one can introduce a *traceless operator* $\hat{\rho}^e$ which encodes all the information of the entanglement of the state. We propose that the density operators for these states can be written as

$$\hat{\rho} = \hat{\rho}^L \otimes \hat{\rho}^R + \hat{\rho}^e. \tag{III.6}$$

In the Unruh quantization, the $\hat{\rho}^e$ operator, which we will call the *matrix of entanglement*, encodes the information of the correlation between left and right components of the state. When computing expectations of operators localized in either wedges L or R the matrix of entanglement plays no role, in fact, it can be shown that for any operator of the form $\hat{A}_L \otimes \hat{1}_R$ and a state described by Eq. (III.6) one has that

$$\mathrm{Tr}(\hat{A}_L \otimes \hat{1}_R\,\hat{\rho}) = \mathrm{Tr}(\hat{A}_L\,\hat{\rho}^L), \tag{III.7}$$

and thus $\mathrm{Tr}(\hat{A}_L \otimes \hat{1}_R\,\hat{\rho}^e) = 0$ (and the analogous if the operator has the form $\hat{1}_L \otimes \hat{A}_R$). The information encoded in $\hat{\rho}^e$ can only be retrieved when computing expectations of operators with L and R components and thus, for the case of observables (made up of field operators) this information is only present when one evaluates the expectation values in wedge F. For example, in the computations of [4] the information encoded in $\hat{\rho}^e$ does not enter in the results, which in principle are *incompatible* with the change in $\langle\hat{T}_{\mu\nu}\rangle$ in wedge F. It is therefore that we expected that the information about this change would be encoded in $\hat{\rho}^e$.

We can write the inertial vacuum $|0_M\rangle$ in the fashion of Eq. (III.6):

$$\hat{\rho}_{vac} = \hat{\rho}^L_{vac} \otimes \hat{\rho}^R_{vac} + \hat{\rho}^e_{vac}\,. \tag{III.8}$$

From Eq. (II.28) it follows that

$$\hat{\rho}_{vac} = |0_M\rangle\,\langle 0_M| = Z \sum_{JK} E_J E_K\,|K\rangle_L\,|K\rangle_R\;{}_L\langle J|\;{}_R\langle J|\,, \tag{III.9}$$

where we have defined

$$E_J \equiv e^{-\pi E(J)} \tag{III.10}$$

(see Eq. (II.24)), and we have introduced the normalization factor Z defined by

$$Z^{-1} \equiv \sum_J E_J^2 \tag{III.11}$$

in order to have $\mathrm{Tr}(\hat{\rho}_{vac}) = 1$. Taking the partial R and L traces in Eq. (III.9) we obtain respectively

$$\hat{\rho}^L_{vac} = Z \sum_J E_J^2 \, |J\rangle_{L\,L}\langle J| \, , \qquad \hat{\rho}^R_{vac} = Z \sum_J E_J^2 \, |J\rangle_{R\,R}\langle J| \, . \tag{III.12}$$

Using Eqs. (III.12) and (III.9) we can write

$$\begin{aligned} \hat{\rho}^e_{vac} &= \hat{\rho}_{vac} - \hat{\rho}^L_{vac} \otimes \hat{\rho}^R_{vac} \\ &= Z \sum_{J,K} E_J E_K \Big(\, |J\rangle_L \, |J\rangle_{R\,L}\langle K| \, {}_R\langle K| - Z E_J E_K \, |J\rangle_L \, |K\rangle_{R\,L}\langle J| \, {}_R\langle K| \, \Big). \end{aligned} \tag{III.13}$$

For the density matrix for state $|f\rangle$ we have

$$\hat{\rho}_f = \hat{\rho}^L_f \otimes \hat{\rho}^R_f + \hat{\rho}^e_f. \tag{III.14}$$

Now we turn back to our own specific case and concentrate particularly in the change in the density matrices induced by the interaction of the field with the source. Exploiting the fact that for the accelerated source the $\hat{S}$ matrix is an R operator one finds directly from Eq. (III.1) and Eq. (III.2) that

$$\hat{\rho}_f = \hat{\rho}^L_{vac} \otimes \hat{S}_R \, \hat{\rho}^R_{vac} \, \hat{S}^\dagger_R + \hat{S} \, \hat{\rho}^e_{vac} \, \hat{S}^\dagger. \tag{III.15}$$

From this last equation we can identify

$$\hat{\rho}^L_f = \hat{\rho}^L_{vac} \qquad \hat{\rho}^R_f = \hat{S}_R \, \hat{\rho}^R_{vac} \, \hat{S}^\dagger_R \tag{III.16}$$

and

$$\hat{\rho}^e_f = \hat{S} \, \hat{\rho}^e_{vac} \, \hat{S}^\dagger. \tag{III.17}$$

We want now to express the change in the state of the field induced by the interaction in terms of the change of the density matrix. Note that when restricted to L this change is $\delta\hat{\rho}^L_f = \hat{\rho}^L_f - \hat{\rho}^L_{vac} = 0$. The changes $\delta\hat{\rho}^R_f = \hat{\rho}^R_f - \hat{\rho}^R_{vac}$ and $\delta\hat{\rho}^e_f = \hat{\rho}^e_f - \hat{\rho}^e_{vac}$ deserve special attention since they depend on the operator $\hat{S}$, note that they are traceless.

The total change in the density matrix can be written as

$$\begin{aligned} \delta\hat{\rho} &= \hat{\rho}^L_{vac} \otimes \delta\hat{\rho}^R_f + \delta\hat{\rho}^L_f \otimes \hat{\rho}^R_{vac} + \delta\hat{\rho}^L_f \otimes \delta\hat{\rho}^R_f + \delta\hat{\rho}^e \\ &= \hat{\rho}^L_{vac} \otimes \delta\hat{\rho}^R_f + \delta\hat{\rho}^e, \end{aligned} \tag{III.18}$$

where $\delta\hat{\rho}^e \equiv \hat{\rho}^e_f - \hat{\rho}^e_{vac}$ and we have used $\delta\hat{\rho}^L_f = 0$. Eq. (III.18) thus reflects in a clear language what we know about the change in the quantum field. As we mentioned in Sect. I, there is theoretical evidence that when one restricts state $|f\rangle$ to wedge R one would obtain the same thermal bath as that of the inertial vacuum [4], that is, in this wedge there is no effective change in the state. A priori one may think that this means that $\delta\hat{\rho}^R_f = 0$. However, the state should have changed in order to account for the radiation emitted by the source, which should be measurable in F. Thus one might conclude that all the information of the physical change in the state is encoded in the change in the matrix of entanglement, $\delta\hat{\rho}^e$, which can only be retrieved in wedge F. To be more specific, consider an operator of the form $\hat{A} = \hat{1}_L \otimes \hat{A}_R$ on $\mathscr{F}_L \otimes \mathscr{F}_R$. From Eq. (III.18) one has that the change in its expectation is given by

$$\delta\mathrm{Tr}(\hat{A}\hat{\rho}) \equiv \mathrm{Tr}(\hat{A}\,\hat{\rho}_f) - \mathrm{Tr}(\hat{A}\,\hat{\rho}_{vac}) = \mathrm{Tr}(\hat{A}_R\,\delta\hat{\rho}_f^R). \tag{III.19}$$

Note that the operator $\hat{A}_R$ can be localized *anywhere* in Minkowski spacetime as, for example, the operator $\hat{1} \otimes \hat{\phi}_R(x)$, which is not identically zero outside wedge L. However, if the operator $\hat{A}$ has non trivial L and R components then the change is given by

$$\delta\mathrm{Tr}(\hat{A}\hat{\rho}) = \mathrm{Tr}(\hat{A}\;\hat{\rho}_{vac}^L \otimes \delta\hat{\rho}_f^R) + \mathrm{Tr}(\hat{A}\;\delta\hat{\rho}^e), \tag{III.20}$$

that is, in this case the change in the expectation may come from both $\delta\hat{\rho}_f^R$ and $\delta\hat{\rho}^e$ contributions.

In order to understand the physical change in the state we shall evaluate the change in the expectation of the energy-momentum tensor operator $\hat{T}_{\mu\nu}$. Recall that in Minkowski spacetime $\langle\hat{T}_{\mu\nu}(x)\rangle$ is defined in the point-splitting description as a coincidence limit [5]:

$$\langle\hat{T}_{\mu\nu}(x)\rangle_f = \lim_{x'\to x} t_{\mu\nu'}F(x,x'), \tag{III.21}$$

where

$$F(x,x') = \langle f|\,\hat{\phi}(x)\hat{\phi}(x')\,|f\rangle - \langle 0_M|\,\hat{\phi}(x)\hat{\phi}(x')\,|0_M\rangle \tag{III.22}$$

and $t_{\mu\nu'}$ is the differential operator

$$t_{\mu\nu'} = \nabla_\mu\nabla_{\nu'} - \tfrac{1}{2}g_{\mu\nu}(\nabla_\sigma\nabla^{\sigma'} + m^2). \tag{III.23}$$

Thus, all one needs to compute the expectation of $\hat{T}_{\mu\nu}$ is the change in the two point function $\langle\hat{\phi}(x)\hat{\phi}(x')\rangle$. We have from Eq. (II.20) that

$$\hat{\phi}(x)\hat{\phi}(x') = \hat{\phi}_L(x)\hat{\phi}_L(x') + \hat{\phi}_L(x)\hat{\phi}_R(x') + \hat{\phi}_R(x)\hat{\phi}_L(x') + \hat{\phi}_R(x)\hat{\phi}_R(x'). \tag{III.24}$$

To simplify the notation, let us define

$$\hat{\phi} \equiv \hat{\phi}(x), \qquad \hat{\phi}' \equiv \hat{\phi}(x'). \tag{III.25}$$

From Eq. (III.24) and Eq. (III.18) we have that the total change in the expectation of the two point operator is

$$\begin{aligned}\mathrm{Tr}(\hat{\phi}\hat{\phi}'\delta\hat{\rho}) &= \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_L\;\hat{\rho}_{vac}^L \otimes \delta\hat{\rho}^R) + \mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_R\;\hat{\rho}_{vac}^L \otimes \delta\hat{\rho}^R) + \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\delta\hat{\rho}^e) + \mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_L\delta\hat{\rho}^e)\\ &= \mathrm{Tr}_L(\hat{\phi}_L\hat{\phi}'_L\hat{\rho}_{vac}^L)\mathrm{Tr}_R\,(\delta\hat{\rho}^R) + \mathrm{Tr}_R\;(\hat{\phi}_R\hat{\phi}'_R\delta\hat{\rho}^R) + \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\delta\hat{\rho}^e) + \mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_L\delta\hat{\rho}^e).\end{aligned} \tag{III.26}$$

To get Eq. (III.26) we have used that $\mathrm{Tr}(\hat{A}_{L,R}\;\hat{\rho}^e) = 0$ (see Eq. (III.7)) and that

$$\mathrm{Tr}_L(\hat{\phi}_L\;\hat{\rho}_{vac}^L) = \sum_J E_J^2\;{}_L\langle J|\,\hat{\phi}_L\,|J\rangle_L = 0, \tag{III.27}$$

since states with different number of particles are orthogonal.

Eq. (III.26) is the farthest that we can get to reduce $\mathrm{Tr}(\hat{\phi}\hat{\phi}'\delta\hat{\rho})$ using only the fact that we are considering a source with support totally contained in R and the properties of the matrix of entanglement. Note that the last two terms in the r.h.s. of Eq. (III.26) are zero when evaluating both $x, x' \in L$ or R since in these wedges the modes L_ω and R_ω cannot be different from zero simultaneously. To go further in our calculation we shall introduce the explicit form of the operator $\hat{S}$, which we do in the next section.

IV. AN ACCELERATED SCALAR SOURCE AND ITS INTERACTION

We are going to use a scalar source $j(x)$ to model a *scalar particle with uniform acceleration*. Let this scalar current $j(x)$ interact with the field with an interaction Hamiltonian density given by

$$\hat{H}_I(x) = \sqrt{-g}\, j(x)\hat{\phi}(x), \tag{IV.1}$$

where g is the determinant of the metric. The final state $|f\rangle$ of the field *after* this interaction is given by the application of the S-matrix to the inertial vaccuum state:

$$|f\rangle = \hat{S}\,|0_M\rangle \qquad \hat{S} = \hat{\mathrm{T}}\exp\left[-i\int_{\Sigma_{in}}^{\Sigma_{out}} \hat{H}_I(x)\, d^2x\right], \tag{IV.2}$$

where Σ_{in}, Σ_{out} are Cauchy hypersurfaces where the interaction begins and ends respectively, and $\hat{\mathrm{T}}$ is the time order operator.

The interaction occurs inside wedge R and thus the *in* and *out* Cauchy hypersurfaces should bound this region and, at the same time, in order to define states in the Unruh quantization scheme, they have to be Cauchy hypersurfaces of the double wedge $L \cup R$. We define Σ_{in} as the surface constructed by the union of $\{t = 0, z \leq 0\}$ and a spatial surface inside wedge R which begins in the bifurcation point of the horizons and deviates slightly from the $\zeta = 0$, $\tau = -\infty$ horizon (see Fig. I). Σ_{out} is defined analogously but its restriction to R is an spatial surface which deviates slightly from the $\zeta = 0$, $\tau = \infty$ horizon. The initial vacuum state is defined over Σ_{in} and the final state of the field $|f\rangle$ is defined over Σ_{out}. Therefore, although the interaction is present inside wedge R, the state $|f\rangle$ is defined *after* the interaction and one can evaluate expectations in this state of operators localized in wedges R and F.

Our calculation of $|f\rangle$ (and its density matrix) will be in terms of Rindler coordinates for which $\hat{\mathrm{T}}$ orders up operators with respect to the time coordinate τ. Using Eq. (IV.1) one can put the final state of the interaction in the form

$$|f\rangle = \mathrm{T}\Big(\hat{1} - i\int d^4x\,\sqrt{-g}\,j(x)\hat{\phi}(x) - \frac{1}{2}\iint d^4x\,d^4x'\,\sqrt{-g}\sqrt{-g'}j(x')j(x')\hat{\phi}(x)\hat{\phi}(x')\Big)\,|0_M\rangle + \hat{O}(q^3), \tag{IV.3}$$

where the integrations are over the same region as in Eq. (IV.2). It is useful to define the (formal) operator

$$\hat{\phi}_I \equiv \int d^2x\,\sqrt{-g}\,j(x)\hat{\phi}(x). \tag{IV.4}$$

Note that $\hat{\phi}_I$ is of order q. We are going to apply the Wick theorem to the r.h.s. of Eq. (IV.3):

$$\mathrm{T}\big(\hat{\phi}(x)\hat{\phi}(x')\big) = \mathrm{N}\big(\hat{\phi}(x)\hat{\phi}(x')\big) + \langle 0|\,\mathrm{T}\Big(\hat{\phi}(x)\hat{\phi}(x')\Big)\,|0\rangle\,, \tag{IV.5}$$

where the normal ordering, N, and the vacuum, $|0\rangle$, are with respect to the quantization scheme we are dealing with. Using Eq. (IV.5) we expand Eq. (IV.3) to get

$$|f\rangle = \hat{S}\,|0_M\rangle = (1 - \mathscr{G})\,|0_M\rangle - i\hat{\phi}_I\,|0_M\rangle - \frac{1}{2}\mathrm{N}(\hat{\phi}_I\hat{\phi}_I)\,|0_M\rangle + \mathcal{O}(q^3), \tag{IV.6}$$

where

$$\mathscr{G} = \frac{1}{2}\int_{-\infty}^{\infty} d^2x \int_{-\infty}^{\infty} d^2y\; j(x)j(y)\,\langle 0_M|\,\mathrm{T}\Big(\hat{\phi}(x)\hat{\phi}(y)\Big)\,|0_M\rangle \tag{IV.7}$$

is of second order in q.

A uniformly accelerating scalar particle follows a trajectory which in Rindler coordinates corresponds to the locus of $\zeta = \zeta_0$. From the point of view of Rindler observers, this scalar source corresponds to a static scalar *current*:

$$j'(x) = q\delta(\zeta - \zeta_0). \tag{IV.8}$$

This source transforms as a scalar, using Eq. (II.1) it can be transformed to inertial coordinates

$$j'(x) = q\frac{\delta(z - \sqrt{t^2 + a^{-2}})}{a\sqrt{t^2 + a^{-2}}}, \tag{IV.9}$$

where $\zeta_0 = 1/a$ and a is the proper acceleration of the source. In order to avoid expressions of the form $0 \times \infty$ one has to introduce some regularization factor to Eq. (IV.8). Inspired in Higuchi *et.al.* [4], who regularize an electric charge and show the consistency of their regulator (we have explained their results in Sec. I), we introduce an oscillating factor to Eq. (IV.8):

$$j(x) = q\cos(\theta\tau)\delta(\zeta - \zeta_0) \tag{IV.10}$$

and at the end of our calculations we shall take the limit $\theta \to 0$. In fact, for slow oscillations ($\theta \ll a$) the source is expected to interact with the field as if it were a constant charge q at each τ.

The current $j(x)$ has support totally contained in R, and thus one can see from Eqs. (II.20) and (IV.4) that only R-modes of the field will get excited by the accelerated source. The operator $\hat{\phi}_I$ takes the form

$$\hat{\phi}_I = \int_R d^2x\, \sqrt{-g}\, j(x)\hat{\phi}_R(x) = \int_0^\infty d\omega\, \Big(\Upsilon_\omega \hat{r}_\omega + \Upsilon^*_\omega \hat{r}^\dagger_\omega\Big), \tag{IV.11}$$

where

$$\Upsilon_\omega \equiv \int_R d^2x\, \sqrt{-g} j(x) R_\omega(x) = \Psi_\theta \delta(\omega - \theta), \qquad \Psi_\theta = q\sqrt{\sinh(\pi\theta)}\, \zeta_0 K_{i\theta}(m\zeta_0). \tag{IV.12}$$

To get Eq. (IV.12) we have used Eq. (A.11), $\omega > 0$, and we have chosen to work with $\theta > 0$ (note that the regulator is an even function of θ). Note that $K_{i\omega}(z)$ is real for real ω and z. Finally, Eq. (IV.11) takes the form

$$\hat{\phi}_I = q\sqrt{\sinh(\pi\theta)}\, \zeta_0 K_{i\theta}(m\zeta_0)\big[\hat{r}_\theta + \hat{r}^\dagger_\theta\big]. \tag{IV.13}$$

The role of the regulator we have chosen is to couple the source to an R mode of the field with frequency θ instead of the mode with frequency zero which is somehow pathological.

V. THE CHANGE OF $\langle \hat{T}_{\mu\nu} \rangle$ AT WEDGES L AND R

Now we are in a position to compute explicitly $\delta\hat{\rho}^R$ and $\delta\hat{\rho}^e$ in order to evaluate Eq. (III.26). We will work out this calculation perturbatively up to second order in q, which turns to be the first relevant contribution. From Eq. (IV.6) we have

$$\hat{S} = (1 - \mathscr{G}) - i\hat{\phi}_I + \frac{1}{2}N(\hat{\phi}_I\hat{\phi}_I) + O(q^3), \tag{V.1}$$

and from this equation we can write the density matrix for the final state as

$$\hat{\rho}_f = \hat{\rho}^{(0)}_f + \hat{\rho}^{(1)}_f + \hat{\rho}^{(2)}_f + O(q^3), \tag{V.2}$$

where $\hat{\rho}_f^{(0)}$ corresponds to the (second order) renormalized inertial vacuum density matrix given by

$$\hat{\rho}_f^{(0)} = Q\hat{\rho}_{vac} \qquad Q \equiv (1 - 2\mathrm{Re}(\mathscr{G})). \tag{V.3}$$

We shall compute the change in the density matrix with respect to the renormalized vacuum density operator:

$$\delta\hat{\rho}_{ren} = \hat{\rho}_f - \hat{\rho}_f^{(0)}. \tag{V.4}$$

Tracing out the left degrees of freedom in Eq. (V.2) we have

$$\hat{\rho}_f^R = \hat{\rho}_f^{R\,(0)} + \hat{\rho}_f^{R\,(1)} + \hat{\rho}_f^{R\,(2)} + O(q^3), \tag{V.5}$$

where

$$\hat{\rho}_f^{R\,(1)} = -i[\hat{\phi}_I, \hat{\rho}_{vac}^R], \tag{V.6}$$

$$\hat{\rho}_f^{R\,(2)} = \frac{1}{2}\big(\hat{\rho}_{vac}^R N(\hat{\phi}_I\hat{\phi}_I)^\dagger + 2\hat{\phi}_I\hat{\rho}_{vac}^R\hat{\phi}_I + N(\hat{\phi}_I\hat{\phi}_I)\hat{\rho}_{vac}^R\big). \tag{V.7}$$

Note that $\hat{\rho}_f^{R\,(1)}$ is traceless. For the entanglement matrix we have

$$\delta\hat{\rho}^e = \hat{\rho}_f^{e\,(1)} + \hat{\rho}_f^{e\,(2)} + O(q^3), \tag{V.8}$$

where $\hat{\rho}_f^{e\,(1)}$ and $\hat{\rho}_f^{e\,(2)}$ are defined as Eqs. (V.6) and (V.7) changing $R \to e$.

The first order contribution to Eq. (III.26) is

$$\mathrm{Tr}(\hat{\phi}\hat{\phi}'\delta\hat{\rho}^{(1)}) = \mathrm{Tr}_L(\hat{\phi}_L\hat{\phi}'_L\hat{\rho}_{vac}^L)\mathrm{Tr}(\hat{\rho}_f^{R\,(1)}) + \mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_R\hat{\rho}_f^{R\,(1)}) + \mathrm{Tr}((\hat{\phi}_L\hat{\phi}'_R + \hat{\phi}'_L\hat{\phi}_R)\hat{\rho}_f^{e\,(1)}). \tag{V.9}$$

The first term in the r.h.s. of Eq. (V.9) is zero since $\hat{\rho}_f^{R\,(1)}$ is traceless. From the expression for $\hat{\rho}_f^{R\,(1)}$, Eq. (V.6), it can be proved that

$$\mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_R\hat{\rho}_f^{(1)}) = \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\hat{\rho}_f^{(1)}) = \mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_L\hat{\rho}_f^{(1)}) = 0. \tag{V.10}$$

This can be understood heuristically from the fact that these traces represent a sum of brackets in the Minkowski vacuum of three field operators, which are necessarily null. From Eq. (V.10) and the definition of $\hat{\rho}_f^e$ we have

$$\begin{aligned}\mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\hat{\rho}_f^{e\,(1)}) &= \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\hat{\rho}_f^{(1)}) - \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\hat{\rho}_{vac}^L \otimes \hat{\rho}_f^{R\,(1)}) \\ &= \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\hat{\rho}_f^{(1)}) \\ &= 0.\end{aligned} \tag{V.11}$$

To get the second equality in Eq. (V.11) we have used Eq. (III.27) to conclude that

$$\mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\,\hat{\rho}_{vac}^L \otimes \hat{\rho}_f^{R\,(1)}) = \mathrm{Tr}_L\,(\hat{\phi}_L\hat{\rho}_{vac}^L)\mathrm{Tr}_R\,(\hat{\phi}'_R\hat{\rho}_f^{R\,(1)}) = 0\,. \tag{V.12}$$

Note that this last equation is valid for any order. We have then proved that $\mathrm{Tr}(\hat{\phi}\hat{\phi}'\delta\hat{\rho}^{(1)}) = 0$.

Now we turn to the second order contribution,

$$\mathrm{Tr}(\hat{\phi}\hat{\phi}'\delta\hat{\rho}^{(2)}) = \mathrm{Tr}_L(\hat{\phi}_L\hat{\phi}'_L\hat{\rho}_{vac}^L)\mathrm{Tr}(\hat{\rho}_f^{R\,(2)}) + \mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_R\hat{\rho}_f^{R\,(2)}) + \mathrm{Tr}((\hat{\phi}_L\hat{\phi}'_R + \hat{\phi}'_L\hat{\phi}_R)\hat{\rho}_f^{e\,(2)})\,. \tag{V.13}$$

For the mixed LR terms, we have that, similarly to Eq. (V.11)

$$\begin{aligned}\mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\,\hat{\rho}_f^{e\,(2)}) &= \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\,\hat{\rho}_f^{(2)}) - \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\,\hat{\rho}_{vac}^{L}\otimes\hat{\rho}_f^{R\,(2)}) \\ &= \mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\,\hat{\rho}_f^{(2)})\,.\end{aligned} \tag{V.14}$$

Using Eq. (V.7) it can be proven directly that (see Appendix B)

$$\mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\hat{\rho}_f^{e\,(2)}) \;=\; Z\Psi_\theta^2\mathrm{Tr}(\hat{\phi}_L\hat{\phi}'_R\hat{\rho}_{vac}), \tag{V.15}$$

$$\mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_L\hat{\rho}_f^{e\,(2)}) \;=\; Z\Psi_\theta^2\mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_L\hat{\rho}_{vac})\,. \tag{V.16}$$

Also from Eq. (V.7) (see its basis expansion in Eq. (B.10)) we have that $\mathrm{Tr}(\hat{\rho}_f^{R\,(2)}) = \Psi_\theta^2$ and thus

$$\mathrm{Tr}_L(\hat{\phi}_L\hat{\phi}'_L\hat{\rho}_{vac}^L)\mathrm{Tr}(\hat{\rho}_f^{R\,(2)}) = Z\Psi_\theta^2\mathrm{Tr}_L(\hat{\phi}_L\hat{\phi}'_L\hat{\rho}_{vac}^L)\,. \tag{V.17}$$

These terms, since are proportional to the corresponding LR and LL parts of the two point function in the vacuum, are going to be absorbed in the renormalization of the final result. We have then obtained the, in principle, unexpected result that at least for the change in the expectation of the $\hat{T}_{\mu\nu}$ operator *there is no contribution from the change in the entanglement matrix*, although as one can see from Eq. (III.17) there is actually a change in this operator. All the information of this change comes from the change in the R density matrix of the state, $\delta\hat{\rho}_f^R$. As shown explicitly in Appendix B we have that

$$\mathrm{Tr}(\hat{\phi}_R\hat{\phi}'_R\hat{\rho}_f^{R(2)}) = 4\Psi_\theta^2\,\mathrm{Im}[R_\theta(x)]\,\mathrm{Im}[R_\theta(x')] + Z\Psi_\theta^2\mathrm{Tr}\big(\hat{\rho}_{vac}^R\hat{\phi}_R(x)\hat{\phi}_R(x')\big)\,. \tag{V.18}$$

Finally, adding up Eqs. (V.15)-(V.18) we have that the change in the two point function between the inertial vacuum and the state generated by the interaction of the scalar source is

$$\mathrm{Tr}(\hat{\phi}(x)\hat{\phi}(x')\delta\hat{\rho}_{ren}) = 4\Psi_\theta^2\,\mathrm{Im}[R_\theta(x)]\,\mathrm{Im}[R_\theta(x')] + \frac{Z\Psi_\theta^2}{Q}\mathrm{Tr}\big(\hat{\phi}(x)\hat{\phi}(x')\hat{\rho}_f^{(0)}\big) + O(q^3), \tag{V.19}$$

which is valid for all $x, x' \in M$ and we have used Eq. (V.3) to express $\hat{\rho}_{vac}$ in terms of the renormalized vacuum density operator $\hat{\rho}_f^{(0)}$. However, as we have said, we still have to renormalize this change in the expectation value. We define the renormalized field operator as

$$\hat{\phi}_{ren}(x) \equiv \left(1 - \frac{\Psi_\theta^2}{Q}\right)^{1/2}\hat{\phi}(x), \tag{V.20}$$

and thus the renormalized change in the expectation of the two point function is

$$C_\theta(x,x') \equiv \mathrm{Tr}(\hat{\phi}_{ren}(x)\hat{\phi}_{ren}(x')\delta\hat{\rho}_{ren}) = 4\Psi_\theta^2\,\mathrm{Im}[R_\theta(x)]\,\mathrm{Im}[R_\theta(x')] + O(q^3)\,. \tag{V.21}$$

The second order term in this change reads

$$C_\theta^{(2)}(x,x') = 4\Psi_\theta^2\,\mathrm{Im}[R_\theta(x)]\,\mathrm{Im}[R_\theta(x')]\,. \tag{V.22}$$

We have arrived to a regular expression for the change in the expectation value of the two point function and thus now we can reconsider the source as static by taking out the regulator taking the limit $\theta \to 0$. In the following we are going to evaluate explicitly Eq. (V.22) in wedges R and F. First, note that form Eq. (IV.12) and the fact that Bessel functions $K_{i\theta}(m\zeta_0)$ are regular whenever $\zeta_0 \neq 0$ we have that

$$\lim_{\theta\to 0}\Psi_\theta = 0\,. \tag{V.23}$$

Note that Ψ_θ^2 is an overall factor in $C_\theta^{(2)}(x,x')$. When computing the expectation of $\hat{\phi}(x)\hat{\phi}(x')$ the correct procedure is to take the limit $\theta \to 0$ at the end of the calculation since the field operators *couple* to the frequency θ of particles in $|f\rangle$ which are emitted and absorbed by the source.

First we will work the case when $x, x' \in R$, for we can express these points in terms of Rindler coordinates: $x = (\tau, \zeta)$, $x' = (\tau', \zeta')$. Unruh modes R_ω, when restricted to wedge R take the form of Eq. (II.4) (see also Eq. (A.11) in the Appendix). Using the fact that functions $K_{i\omega}(z)$ are real whenever ω is real and $z > 0$, we have that

$$\mathrm{Im}\big(R_\theta(x)\big) = -\frac{\sqrt{\sinh(\pi\theta)}}{\pi}\sin(\theta\tau)K_{i\theta}(m\zeta) \qquad x \in R\,. \tag{V.24}$$

And thus, from Eqs. (V.22) and (IV.12)

$$C_\theta^{(2)}(x,x') = \frac{1}{\pi^2}q^2\sinh^2(\pi\theta)\zeta_0^2K_{i\theta}^2(m\zeta_0)\sin(\theta\tau)\sin(\theta\tau')K_{i\theta}(m\zeta)K_{i\theta}(m\zeta') \qquad x, x' \in R\,. \tag{V.25}$$

Recall that R is an open set bounded by the horizons and thus $\zeta, \zeta' \neq 0$, so the Bessel functions $K_{i\theta}(m\zeta)$, $K_{i\theta}(m\zeta')$ are regular. Then we have that the second order change in the two point function between the inertial vacuum and state $|f\rangle$ in wedge R is

$$\lim_{\theta\to 0} C_\theta^{(2)}(x,x') = 0 \qquad x, x' \in R\,. \tag{V.26}$$

This resut, is consistent with the fact that, as pointed out in Ref. [4] the source is in thermal equilibrium with the field inside wedge R. Any observer inside this wedge will not be able to notice any change in the expected value of $\hat{T}_{\mu\nu}$ due to the presence of the accelerating source. Note that the specific form of the two point operator has played a crucial role to get to Eq. (V.22).

Now we proceed to evaluate Eq. (V.22) in wedge F. Recall that in F, τ is a spatial coordinate and ζ is timelike (cf. Eq. (A.2b)). Form Eq. (A.12) we have that the mode $R_\theta(x)$ restricted to F takes the form

$$R_\theta(x) = -\frac{i}{2^{3/2}}\frac{e^{-i\theta\tau}}{\sqrt{2\sinh(\pi\theta)}}\left[e^{\theta\pi}H_{i\theta}^{(2)}(m\zeta) + e^{-\theta\pi}H_{i\theta}^{(1)}(m\zeta)\right] \qquad x \in F, \tag{V.27}$$

where coordinates (τ, ζ) are defined in Eq. (A.2b) and $H_{i\theta}^{(1),(2)}$ are the first and second Hankel functions. Using the definitions of $H_{i\theta}^{(1),(2)}$ in terms of Bessel functions, Eqs. (A.19) and (A.20), it follows immediately that

$$\mathrm{Im}\big(R_\theta(x)\big) = -\frac{1}{4\sqrt{\sinh(\pi\theta)}}\big(e^{-i\theta\tau}J_{-i\theta}(m\zeta) + e^{i\theta\tau}J_{i\theta}(m\zeta)\big) \qquad x \in F\,. \tag{V.28}$$

Now using Eqs.(IV.12) and (V.28) we have that

$$\lim_{\theta\to 0} C_\theta^{(2)}(x,x') = q^2\zeta_0^2K_0^2(m\zeta_0)J_0(m\zeta)J_0(m\zeta') \qquad \text{for all } x, x' \in F\,. \tag{V.29}$$

Compare with Eq. (V.26). This expression is the (non zero) change in the two point function in wedge F due to the interaction. It contains the information of the field radiated away from the source into F. It should be noted that Eq. (V.29) is not valid at the horizons. In Appendix C we make the computation, in an inertial frame, of the same change in the expectation of the two point function, $\mathrm{Tr}(\hat{\phi}(x)\hat{\phi}(x')\delta\hat{\rho})$, for $x, x' \in F$ and obtain exactly Eq. (V.29). Therefore, at least for the particular case we are dealing with, both quantum descriptions produce the same physical results.

VI. AN EXAMPLE OF LEFT-RIGHT INTERFERENCE: TWO SOURCES WITH OPPOSITE ACCELERATION

As we have seen in the last section, the matrix of entanglement plays no significant role in the change in the expectation of $\hat{T}_{\mu\nu}$ for the interaction of an accelerated source with the inertial vacuum. However, the presence of another (accelerating) source lying in L may affect the entanglement of the final state and thus $\hat{\rho}^e$ may play a part in the change of the expectation of $\hat{T}_{\mu\nu}$. In this section we work out these calculations.

Suppose that additionally to the scalar particle considered in the latter sections we have an extra scalar particle in uniform acceleration in wedge L with scalar charge $\tilde{q}$. The perturbation of the field due to this pair of particles is given by

$$j(x) = \begin{cases} j_L(x) & x \in L \\ j_R(x) & x \in R \end{cases}, \tag{VI.1}$$

where

$$j_L(x) = \tilde{q}\cos(\tilde{\theta}\,\tau_L)\delta(\zeta_L - \tilde{\zeta}_0) \qquad j_R(x) = q\cos(\theta\,\tau_R)\delta(\zeta_R - \zeta_0)\,. \tag{VI.2}$$

Rindler coordinates in wedge L, (τ_L, ζ_L) are given by Eq. (A.2a). To avoid confusion, all along this section we will call Rindler coordinates in R: (τ_R, ζ_R) (originally we have used (τ, ζ)). Note that in order to have independent sources we have introduced the cosine regulator with a different parameter for the source in wedge L.

Using Eq. (IV.1) for $j(x)$ given by Eq. (VI.1) we have that

$$\hat{S}' = \hat{T}\exp\big[-i[(\hat{\phi}_I^L \otimes \hat{1}_R) + (\hat{1}_L \otimes \hat{\phi}_I^R)]\big] = \hat{T}\;\exp[-i\hat{\phi}_I^L] \otimes \exp[-i\hat{\phi}_I^R] \equiv \hat{S}_L \otimes \hat{S}_R, \tag{VI.3}$$

where

$$\hat{\phi}_I^L = \Psi_{\tilde{\theta}}\big(\hat{l}_{\tilde{\theta}} + \hat{l}_{\tilde{\theta}}^\dagger\big), \qquad \hat{\phi}_I^R = \Psi_\theta\big(\hat{r}_\theta + \hat{r}_\theta^\dagger\big), \tag{VI.4}$$

and $\hat{T}$ is the time order operator. In this case we have two different time parameters, τ_L and τ_R and thus $\hat{T}$ will time order L and R operators independently (recall that $\hat{\phi}_L$ and $\hat{\phi}_R$ commute). The factor Ψ_θ is given by Eq. (IV.12) and $\Psi_{\tilde{\theta}}$ is now

$$\Psi_{\tilde{\theta}} = \tilde{q}\sqrt{\sinh(\pi\tilde{\theta})}\,\tilde{\zeta}_0 K_{i\tilde{\theta}}(m\tilde{\zeta}_0)\,. \tag{VI.5}$$

Let $|g\rangle$ be the final state of this interaction, analogously to Eq. (III.15) now we have that its density matrix takes the form

$$\hat{\rho}_g = \hat{\rho}_g^L \otimes \hat{\rho}_g^R + \hat{\rho}_g^e, \tag{VI.6}$$

where

$$\hat{\rho}_g^L = \hat{S}_L \hat{\rho}_{vac}^L \hat{S}_L^\dagger, \qquad \hat{\rho}_g^R = \hat{S}_R \hat{\rho}_{vac}^R \hat{S}_R^\dagger, \qquad \hat{\rho}_g^e = \hat{S}' \hat{\rho}_{vac}^e \hat{S}'^\dagger\,. \tag{VI.7}$$

Then we have that the change in the density operators is given by

$$\delta\hat{\rho}_g = \hat{\rho}_{vac}^L \otimes \delta\hat{\rho}_g^R + \delta\hat{\rho}_g^L \otimes \hat{\rho}_{vac}^R + \delta\hat{\rho}_g^L \otimes \delta\hat{\rho}_g^R + \delta\hat{\rho}_g^e, \tag{VI.8}$$

where $\delta\hat{\rho}_g = \hat{\rho}_g - \hat{\rho}_{vac}$ and all other differences are defined analogously.

Now we compute the change $\mathrm{Tr}(\hat{\phi}\hat{\phi}'\delta\hat{\rho}_g)$, analogously to the case of the single accelerating particle the first relevant term is of second order in q and $\tilde{q}$. Using Eq. (III.24) and Eq. (VI.8) It can be seen that

$$\begin{aligned}\mathrm{Tr}(\hat{\phi}\hat{\phi}'\delta\hat{\rho}_g^{(2)}) =& \mathrm{Tr}_L\left(\hat{\phi}_L\hat{\phi}_L'\delta\hat{\rho}_g^{L\,(2)}\right) + \mathrm{Tr}_R\left(\hat{\phi}_R\hat{\phi}_R'\delta\hat{\rho}_g^{R\,(2)}\right)\\ &+ Z\Psi_\theta^2\mathrm{Tr}_L\left(\hat{\phi}_L\hat{\phi}_L'\hat{\rho}_{vac}^L\right) + Z\Psi_{\tilde{\theta}}^2\mathrm{Tr}_R\left(\hat{\phi}_R\hat{\phi}_R'\hat{\rho}_{vac}^R\right)\\ &+ \mathrm{Tr}\big((\hat{\phi}_L\hat{\phi}_R' + \hat{\phi}_R\hat{\phi}_L')\,\delta\hat{\rho}_g^{L\,(1)}\otimes\delta\hat{\rho}_g^{R\,(1)}\big)\\ &+ \mathrm{Tr}\big((\hat{\phi}_L\hat{\phi}_R' + \hat{\phi}_R\hat{\phi}_L')\delta\hat{\rho}_g^{e}\big)\,.\end{aligned} \tag{VI.9}$$

The first four terms in the r.h.s. of Eq. (VI.9) are analogous to Eq. (V.17) and and Eq. (V.18) for the single source case. In effect, we have that

$$\mathrm{Tr}(\hat{\phi}_R\hat{\phi}_R'\delta\hat{\rho}_g^{R(2)}) = 4\Psi_\theta^2\,\mathrm{Im}[R_\theta(x)]\,\mathrm{Im}[R_\theta(x')] + Z\Psi_\theta^2\mathrm{Tr}\big(\hat{\rho}_{vac}^R\hat{\phi}_R(x)\hat{\phi}_R(x')\big), \tag{VI.10}$$

$$\mathrm{Tr}(\hat{\phi}_L\hat{\phi}_L'\delta\hat{\rho}_g^{L(2)}) = 4\Psi_{\tilde{\theta}}^2\,\mathrm{Im}[L_{\tilde{\theta}}(x)]\,\mathrm{Im}[L_{\tilde{\theta}}(x')] + Z\Psi_{\tilde{\theta}}^2\mathrm{Tr}\big(\hat{\rho}_{vac}^L\hat{\phi}_L(x)\hat{\phi}_L(x')\big)\,. \tag{VI.11}$$

Similarly to Eq. (V.6), it can be seen that

$$\delta\hat{\rho}_g^{L\,(1)} = -i(\hat{\phi}_I^L\hat{\rho}_{vac}^L - \hat{\rho}_{vac}^L\hat{\phi}_I^L), \tag{VI.12}$$
$$\delta\hat{\rho}_g^{R\,(1)} = -i(\hat{\phi}_I^R\hat{\rho}_{vac}^R - \hat{\rho}_{vac}^R\hat{\phi}_I^R), \tag{VI.13}$$

and from these equations we have that

$$\mathrm{Tr}\big((\hat{\phi}_L\hat{\phi}_R' + \hat{\phi}_R\hat{\phi}_L')\,\delta\hat{\rho}_g^{L\,(1)}\otimes\delta\hat{\rho}_g^{R\,(1)}\big) = 4\Psi_{\tilde{\theta}}\Psi_\theta\big(\mathrm{Im}(L_{\tilde{\theta}}(x))\mathrm{Im}(R_\theta(x')) + \mathrm{Im}(R_{\tilde{\theta}}(x))\mathrm{Im}(L_\theta(x'))\big)\,. \tag{VI.14}$$

On the other hand, it can be shown that the second order contribution from the matrix of entanglement is

$$\mathrm{Tr}\big((\hat{\phi}_L\hat{\phi}_R' + \hat{\phi}_R\hat{\phi}_L')\delta\hat{\rho}_g^{e\,(2)}\big) = Z\big(\Psi_{\tilde{\theta}}^2 + \Psi_\theta^2\big)\;\mathrm{Tr}\big((\hat{\phi}_L\hat{\phi}_R' + \hat{\phi}_R\hat{\phi}_L')\hat{\rho}_{vac}\big), \tag{VI.15}$$

which, as in the case of a single source, also corresponds to terms that will be absorbed in the renormalization of the final change in the expectation value. Adding up all the contributions, Eq. (VI.9) takes the form

$$\begin{aligned}\mathrm{Tr}(\hat{\phi}\hat{\phi}'\delta\hat{\rho}_g^{(2)}) =& 4\big[\Psi_{\tilde{\theta}}\mathrm{Im}(L_{\tilde{\theta}}(x)) + \Psi_\theta\mathrm{Im}(R_\theta(x))\big]\big[\Psi_{\tilde{\theta}}\mathrm{Im}(L_{\tilde{\theta}}(x')) + \Psi_\theta\mathrm{Im}(R_\theta(x'))\big]\\ &+ Z\big(\Psi_{\tilde{\theta}}^2 + \Psi_\theta^2\big)\;\mathrm{Tr}\big(\hat{\phi}(x)\hat{\phi}(x')\hat{\rho}_{vac}\big)\,.\end{aligned} \tag{VI.16}$$

Thus, the renormalized change in the two point function reads, up to second order

$$\mathrm{Tr}(\hat{\phi}_{ren}\hat{\phi}_{ren}'\delta\hat{\rho}_{g\,ren}) = 4\big[\Psi_{\tilde{\theta}}\mathrm{Im}(L_{\tilde{\theta}}(x)) + \Psi_\theta\mathrm{Im}(R_\theta(x))\big]\big[\Psi_{\tilde{\theta}}\mathrm{Im}(L_{\tilde{\theta}}(x')) + \Psi_\theta\mathrm{Im}(R_\theta(x'))\big] + \ldots \tag{VI.17}$$

After taking the limits $\tilde{\theta}, \theta \to 0$ and evaluating at $x = (\tau_F, \zeta_F)$ and $x' = (\tau_F', \zeta_F')$ we obtain

$$\lim_{\tilde{\theta},\theta\to 0}\mathrm{Tr}(\hat{\phi}_{ren}(x)\hat{\phi}_{ren}(x')\delta\hat{\rho}_{g\,ren}) = \big(\tilde{q}\tilde{\zeta}_0K_0(m\tilde{\zeta}_0) + q\zeta_0K_0(m\zeta_0)\big)^2 J_0(m\zeta_F)J_0(m\zeta_F')\,. \tag{VI.18}$$

As expected, if we turn off the charge in wedge L ($\tilde{q} \to 0$) we recover our previos result, Eq. (V.29). For the case of the two accelerating sources it turns out that also all the contribution to the change in the expectation value of the two point function comes solely from the change in the partial matrices, $\delta\hat{\rho}_g^L$, $\delta\hat{\rho}_g^R$. In particular, the interference term, Eq. (VI.14), is determined by the latter pair of matrices, that is, for the case we have just analyzed all the information of the change in the expectation of $\hat{T}_{\mu\nu}$ is *only* encoded in $\delta\hat{\rho}_g^L$, $\delta\hat{\rho}_g^R$.

VII. DISCUSSION

One of the main goals of our work was to reconcile the fact that the final state of the field appears to remain an undisrupted thermal state in both the left and right Rindler wedges, with the expected change induced by the source on field observables, such as the energy momentum, in the future wedge. This issue is recasted in terms of the codification in the state of the field of the pertinent information that exhibits the change in the expectation value of $\hat{T}_{\mu\nu}$.

At the beginning it was our belief that, since there is no change in the expectation of $\hat{T}_{\mu\nu}$ in wedge R (neither in L) the physical change in wedge F cannot be induced by the particular behaviours of the state when restricted to either wedge. Hence, the information of this change information should have been encoded in some part of the state which is not represented by any of the restricted density operators $\hat{\rho}_f^L$ and $\hat{\rho}_f^R$ (or in their respective changes). It is in this sense is that we proposed the decomposition of $\hat{\rho}_f$ given by Eq. (III.6) with the particular introduction of the *matrix of entanglement* $\hat{\rho}^e$. As we explained in Sec. III this operator plays no role when computing expectations of observables localized in wedges L and R and thus was a good candidate to account for the change of $\langle\hat{T}_{\mu\nu}\rangle$ in wedge F. Nevertheless, we computed this change perturbatively and found that it has contributions only from the change in the density matrix describing the state in the wedge R (see Eqs. (V.18) and (V.21)). That is, when evaluating $\langle\hat{T}_{\mu\nu}\rangle$ in either wedges R and F, its change is determined solely by the characterization of the state in wedge R. This result contrasts with our initial expectation that the information about the change of $\langle\hat{T}_{\mu\nu}\rangle$ would be encoded in the change in the entanglement matrix, $\delta\hat{\rho}^e$.

In order to obtain this result, we had to introduce by hand a particular regularizing function, $\cos(\theta\tau)$, into the current describing a uniformly accelerating scalar source with the prescription to take the limit $\theta \to 0$ at the end of the calculation. As mentioned earlier, this regulator was inspired by a similar computation done in [4]. Despite the seemingly artificial choice of this regulator, we have shown that when making the regulator independent calculation of the change in the same expectation $\langle\hat{\phi}(x)\hat{\phi}(x')\rangle$, using a plane wave quantization scheme, one obtains the same results as in the Unruh scheme with such regulator (see Appendix C). One can give an heuristic explanation to the fact that this regulator is physically correct as follows. In principle, if one is precise, one would like to describe the radiation due to a real physical particle, which should be described as a quantum object itself. However, this description has a serious drawback regarding our wish to describe the source as a *uniformly accelerating* particle: A quantum particle does not move in a definite trajectory and thus assigning to it a particular acceleration is impossible. On the other hand, the nature of the quantum field is distributional and therefore the correct quantum description of the interacting source should be in terms of test functions of compact support (note that Eq. (IV.10) lacks this property) and therefore, the source would correspond to an extended object. To such object we can not naturally ascribe a *uniform* acceleration: if the object is to maintain “its shape” along its trajectory then different parts should have different proper accelerations. However, we know from [4] that a treatment using classical point-like sources (with definite proper acceleration) together with a certain type of regulator produce physically correct results (in particular, results that are fully consistent with the Equivalence Principle).

The regulator used in [4] consisted in the introduction of an artificial oscillation with frequency θ in the strength of the source, to identify therefore expressions of the form $0 \times \infty$ occurring in the calculation and to proceed to carry all calculations to the end before taking the limit $\theta \to 0$. We are thus assuming that the introduction of such regulator, along with the prescription to take the limit $\theta \to 0$ at the end of the calculations reflects in an effective way the description of a quantum source in uniform acceleration interacting with the field. (Nevertheless, the robustness of the result would be ensured if one confirms that the same physical behaviour is to be obtained for a wider class of regularizing functions.) It is however worthy to emphasize that in the inertial calculation of Appendix C there was no need to introduce such a regulator.

From these considerations it follows that our calculations make sense only if the limit $\theta \to 0$ is taken at the end of the computation of the expectation values. Actually, one may be tempted to take this limit directly into the density matrix $\hat{\rho}_f$ of the state. Ignoring for a moment details regarding the precise notion of limit of an operator, one can see that due to the overall factor Ψ_θ, which goes to zero as $\theta \to 0$, every single term in the expansion of $\hat{\rho}_f$ is also zero except for those terms proportional to $\hat{\rho}_{vac}$. Thus, one would conclude that there has been no change in the state of the field in R due to the presence of the source. As this was the only potential contribution to the change in the expectation of $\hat{T}_{\mu\nu}$, we would be led to the erroneous conclusion that there is no change in this quantity.

From the calculations in Sect. V we concluded that the information of the change in $\langle \hat{T}_{\mu\nu} \rangle$ is encoded in $\delta\hat{\rho}^R$. We now want to consider *how* is it encoded. The answer to this question relies on a subtle interplay of the field operator $\hat{\phi}(x)$ and the density matrix in the present formalism. Let us focus on the details of our specific calculation: The overall factor Ψ_θ^2 in the second order contribution to $\hat{\rho}_f^R$ comes from the fact that the source is located in wedge R. In this wedge Unruh modes $R_\theta(x) \to 0$ as $\theta \to 0$ (whenever x is not at the horizon). When computing $\mathrm{Tr}(\hat{\phi}_R \hat{\phi}'_R \hat{\rho}_f^{R\,(2)})$ the field operators are sensitive to the frequency θ. In fact, they only excite modes with $\omega = \theta$ as could be expected on Rindler energy conservation grounds. The particular form of $\hat{\rho}_f^R$ determines the structure of the contribution given by Eq. (V.22) which in turn, due to the different behaviors of the Unruh modes in wedges R and F, is zero in the former and not zero in the latter.

We interpret these results by saying that the "0" Rindler energy modes, which were of such concern in regard to the definition of the theory (see discussion at the end of Sec. II A), are in effect, essential in order to obtain in the accelerated frame description identical results as in the inertial one. Physically we could think that these modes are excited by the slightest quantum fluctuations of a realistic quantum particle and that their excitation would be directly felt in the future wedge. These results seem to be in accordance with the spirit of those obtained in [16] where it is argued that the zero energy modes seemed to be undetectable (with an appropriate definition of detectability) when confining the detection to the right wedge. Finally, it is our belief that this work has helped in clarifying the questions raised at the beginning. Furthermore, there is a technically analogous situation which indicates that a stationary particle just outside of the horizon of a stationary black hole could be "emitting" towards its interior. This work shows a clear path to studying the changes in the energy momentum tensor in that situation.

Acknowledgments

The authors would like to thank very helpful discussions with George Matsas and Hanno Sahlmann. This work was in part supported by grants DGAPA-UNAM IN108103-3 and CONACYT 43914-F and 36581-E. I.P. would like to thank the CONACYT Graduate scholarship and SNI-assistant grant.

Appendix A: UNRUH MODES AND REPRESENTATIONS OF BOOST MODES

One can generalize Rindler coordinates to all Rindler wedges assigning always the respective coordinate τ to the parameter associated to the generator of boosts about the origin in the z direction:

$$b^\mu = a \left[z \left(\frac{\partial}{\partial t} \right)^\mu + t \left(\frac{\partial}{\partial z} \right)^\mu \right] . \tag{A.1}$$

In wedge L one should do an exception in order to to guarantee that the future direction coincides with that of inertial time, in this case, τ^μ is $-b^\mu$. In wedges F and P the coordinate τ is spacelike. We have [17]:

$$z = -\,\zeta\cosh(a\tau) \qquad t = \zeta\sinh(a\tau) \qquad (t,z)\in L, \tag{A.2a}$$
$$z = \zeta\sinh(a\tau) \qquad t = \zeta\cosh(a\tau) \qquad (t,z)\in F, \tag{A.2b}$$
$$z = -\,\zeta\sinh(a\tau) \qquad t = -\zeta\cosh(a\tau) \qquad (t,z)\in P, \tag{A.2c}$$

where $\zeta > 0$ in each region.

Boost modes, Eq. (II.13), should be thought of as distributions and thus one cannot evaluate them in one particular point. However, what we can do is to apply this distributions to test functions which have support defined on a certain open region. The integral which defines the boost modes can be expressed in terms of the accelerated coordinates (τ,ζ) given by Eqs. (II.1) and (A.2). As Unruh modes are defined by boost modes, from this operation we can express Unruh modes in accelerated coordinates too.

For example, let f be a test function in M with $\mathrm{supp}(f)\subset R$, the evaluation of B_ω at function f is given by

$$B_\omega[f] = \frac{1}{2^{3/2}\pi}\int_{-\infty}^{\infty} d\theta\, e^{-i\omega\theta}\int_R d^2x\, e^{im\,(z\sinh(\theta)-t\cosh(\theta))} f(t,z)\,. \tag{A.3}$$

Since the space-time integration is in wedge R we can express the d^2x integral in terms of Rindler coordinates (c.f. Eq.(II.1)):

$$B_\omega[f] = \frac{1}{2^{3/2}\pi}\int_{-\infty}^{\infty} d\theta\, e^{-i\omega\theta}\int_R d^2x\, e^{im\zeta\sinh(\theta-\tau)}\, f(\tau,\zeta)\,. \tag{A.4}$$

Changing the integration order and using the following relation for the Bessel functions [18]

$$K_\nu(x) = \frac{1}{2}e^{\frac{i}{2}\nu\pi}\int_{-\infty}^{\infty} d\alpha\, e^{\nu\alpha}e^{-ix\sinh(\alpha)}, \tag{A.5}$$

it can be seen that

$$B_\omega[f] = \frac{1}{\pi\sqrt{2}}e^{\frac{\omega\pi}{2}}\int_R d^2x\, e^{-i\omega\tau}K_{i\omega}(m\zeta) f(\tau,\zeta)\,. \tag{A.6}$$

By analogous arguments it can be seen that in other wedges boost modes take the following form [13] (coordinates in the following eqs. are respective to each wedge defined by Eqs. (II.1) and (A.2)).

$$B_\omega|_R(\tau,\zeta) \;=\; \frac{1}{\pi\sqrt{2}}e^{\frac{\omega\pi}{2}}e^{-i\omega\tau}K_{i\omega}(m\zeta), \tag{A.7}$$
$$B_\omega|_L(\tau,\zeta) \;=\; \frac{1}{\pi\sqrt{2}}e^{\frac{-\omega\pi}{2}}e^{i\omega\tau}K_{i\omega}(m\zeta), \tag{A.8}$$
$$B_\omega|_F(\tau,\zeta) \;=\; -\frac{i}{2^{3/2}}e^{\frac{\omega\pi}{2}}e^{-i\omega\tau}H^{(2)}_{i\omega}(m\zeta), \tag{A.9}$$
$$B_\omega|_P(\tau,\zeta) \;=\; \frac{i}{2^{3/2}}e^{\frac{-\omega\pi}{2}}e^{-i\omega\tau}H^{(1)}_{i\omega}(m\zeta), \tag{A.10}$$

where $H_\nu^{(1),(2)}$ are Hankel functions. Note that in wedge L, $B_\omega(\tau,\zeta)$ has *negative frequency* w.r.t. τ because we have chosen the time translation generator in L to be $\tau^\mu = -b^\mu$, where b^μ is the boost generator. Using Eqs. (A.7-A.10) in Eqs. (II.18) one obtais directly the representations of Unruh modes in each wedge. Modes R_ω are given by:

$$
\begin{aligned}
R_\omega|_R(\tau,\zeta) &= \frac{1}{\pi}\sqrt{\sinh\pi\omega}e^{-i\omega\tau}K_{i\omega}(m\zeta), && \text{(A.11)}\\
R_\omega|_F(\tau,\zeta) &= -\frac{i}{2^{3/2}}\frac{e^{-i\omega\tau}}{\sqrt{2\sinh(\pi\omega)}}\left[e^{\omega\pi}H^{(2)}_{i\omega}(m\zeta)+e^{-\omega\pi}H^{(1)}_{i\omega}(m\zeta)\right], && \text{(A.12)}\\
R_\omega|_L(\tau,\zeta) &= 0, && \text{(A.13)}\\
R_\omega|_P(\tau,\zeta) &= \frac{i}{2^{3/2}}\frac{e^{-i\omega\tau}}{\sqrt{2\sinh\pi\omega}}\left[H^{(1)}_{i\omega}(m\zeta)+H^{(2)}_{i\omega}(m\zeta)\right]. && \text{(A.14)}
\end{aligned}
$$

And L_ω modes are

$$
\begin{aligned}
L_\omega|_R(\tau,\zeta) &= 0, && \text{(A.15)}\\
L_\omega|_F(\tau,\zeta) &= -\frac{i}{2^{3/2}}\frac{e^{i\omega\tau}}{\sqrt{2\sinh(\pi\omega)}}\left[e^{\omega\pi}H^{(2)}_{i\omega}(m\zeta)+e^{-\omega\pi}H^{(1)}_{i\omega}(m\zeta)\right], && \text{(A.16)}\\
L_\omega|_L(\tau,\zeta) &= \frac{1}{\pi}\sqrt{\sinh\pi\omega}e^{-i\omega\tau}K_{i\omega}(m\zeta), && \text{(A.17)}\\
L_\omega|_P(\tau,\zeta) &= \frac{i}{2^{3/2}}\frac{e^{i\omega\tau}}{\sqrt{2\sinh\pi\omega}}\left[H^{(1)}_{i\omega}(m\zeta)+H^{(2)}_{i\omega}(m\zeta)\right]. && \text{(A.18)}
\end{aligned}
$$

Here we put some useful relations for the Hankel functions [18] from which one can simplify the expressions for Unruh modes

$$H^{(1)}_\nu(z)=\frac{1}{i\sin(\nu\pi)}\left[J_{-\nu}(z)-e^{-i\nu\pi}J_\nu(z)\right], \tag{A.19}$$

$$H^{(2)}_\nu(z)=\frac{1}{i\sin(\nu\pi)}\left[e^{i\nu\pi}J_\nu(z)-J_{-\nu}(z)\right]. \tag{A.20}$$

These latter equations are used to obtain Eq. (V.28).

Appendix B: SECOND ORDER CALCULATIONS

Along this work we do several second order calculations of expectation values of $\hat\phi(x)\hat\phi(x')$. In this Appendix we want to put the details of the calculus of $\mathrm{Tr}\big(\hat\rho^R_f\hat\phi_R(x)\hat\phi_R(x')\big)$ which leads to Eq. (V.18); all the other calculi are analogous to this one.

The state $|f\rangle$ takes the form of Eq. (II.26) in the Unruh quantization, in effect, from Eq. (IV.6) one can express $|f\rangle$ as (from now on we will omit the $\otimes$ in the Unruh states)

$$|f\rangle=\sum_{J,K}F(J,K)\,|J\rangle_L\,|K\rangle_R,\qquad F(J,K)=F_0(J,K)+F_1(J,K)+F_2(J,K)+\mathcal{O}(q^3), \tag{B.1}$$

where the first term is $F_0(J,K)=\mathcal{Q}F_{vac}(J,K)$ ($\mathcal{Q}$ is defined above Eq. (IV.7)), and $F_{vac}(J,K)$ is the left-right superposition function defining the inertial vacuum state $|0_M\rangle$ (cf. Eq. (II.28)):

$$F_{vac}(J,K)=Z^{1/2}e^{-\pi E(K)}\delta(J,K), \tag{B.2}$$

where Z is the normalization factor defined in Eq. (III.11). In order to express the other terms in Eq. (B.1), it is useful to define the normalization factor $N^\alpha_\theta(K)$ by

$$N^\alpha_\theta(K)=\begin{cases}\sqrt{K_\theta+1} & \alpha=+\\ \sqrt{K_\theta} & \alpha=-\end{cases}, \tag{B.3}$$

where K_θ is the particle component of state $|K\rangle_{L,R}$ in the mode with frequency centered on $\omega = \theta$. Using Eq. (IV.13) in Eq. (IV.6) we find

$$\begin{aligned}
F_0(J,K) &= \mathcal{Q} Z^{1/2} e^{-\pi E(K)} \delta(J,K), && \text{(B.4)}\\
F_1(J,K) &= -i Z^{1/2} \Psi_\theta\, e^{-\pi E(J)} \Big(N_\theta^+(J) \delta(K, J+1_\theta) + N_\theta^-(J) \delta(K, J-1_\theta) \Big), && \text{(B.5)}\\
F_2(J,K) &= -\frac{1}{2} Z^{1/2} \Psi_\theta^2\, e^{-\pi E(J)} \Big(N_\theta^-(J) N_\theta^-(J-1_\theta)\, \delta(K, J-2_\theta) \\
&\quad + 2 N_\theta^-(J) N_\theta^+(J-1_\theta)\, \delta(K,J) + N_\theta^+(J) N_\theta^+(J+1_\theta)\, \delta(K, J+2_\theta) \Big) . && \text{(B.6)}
\end{aligned}$$

Eq. (B.4) is actually Eq. (II.28) with the extra $\mathcal{Q}$ factor; $E(J)$ is defined by Eq. (II.24). The δ functions are defined by Eq. (II.25). The $J - 1_\theta$ which appears in the last term in the r.h.s. of Eq. (B.5) corresponds to a normalized state $|J - 1_\theta\rangle_R$ defined by

$$\hat{r}_\theta\, |J\rangle_R = N_\theta^-(J)\, |J - 1_\theta\rangle_R \qquad \text{(B.7)}$$

with particle content $J - 1_\theta = \{J_{\omega_0}, \ldots, J_\theta - 1, \ldots\}$. The other terms are defined analogously.

The density matrix of state $|f\rangle$ reads

$$\hat{\rho}_f = |f\rangle \langle f| = \sum_{J,K,J',K'} F(J,K) F(J',K')^* \;\; |J\rangle_L \, |K\rangle_R \; {}_L\!\langle J'| \, {}_R\!\langle K'| \; . \qquad \text{(B.8)}$$

Let's write it in the following manner

$$\hat{\rho}_f = \hat{\rho}_f^{(0)} + \hat{\rho}_f^{(1)} + \hat{\rho}_f^{(2)} + \mathcal{O}(q^3) \, . \qquad \text{(B.9)}$$

Taking the trace over the left degrees of freedom to this expression we obtain that

$$\hat{\rho}_f^{R(2)} = Z \Psi_\theta^2 \Big[-\frac{1}{2} (e^{-2\pi\theta} - 1)^2 \hat{\rho}_a - \frac{1}{2} (e^{2\pi\theta} - 1)^2 \hat{\rho}_b + \hat{\rho}_c \Big], \qquad \text{(B.10)}$$

where

$$\begin{aligned}
\hat{\rho}_a &= \sum_K e^{-2\pi E(K)} \sqrt{K_\theta + 2} \sqrt{K_\theta + 1} \;\; |K\rangle_{R\,R}\langle K + 2_\theta| \, , && \text{(B.11)}\\
\hat{\rho}_b &= \sum_K e^{-2\pi E(K)} \sqrt{K_\theta} \sqrt{K_\theta - 1} \;\; |K\rangle_{R\,R}\langle K - 2_\theta| \, , && \text{(B.12)}\\
\hat{\rho}_c &= \sum_K e^{-2\pi E(K)} \Big(-2K_\theta + e^{2\pi\theta} K_\theta + e^{-2\pi\theta} (K_\theta + 1) \Big) \; |K\rangle_{R\,R}\langle K| \; . && \text{(B.13)}
\end{aligned}$$

Writing the field operator as

$$\hat{\phi}_R(x) = \sum_{\alpha = +,-} \sum_{m=0}^{\infty} R_{\omega_m}^\alpha(x) \hat{r}_{\omega_m}^\alpha \qquad \text{(B.14)}$$

where

$$R_{\omega_m}^+(x) = R_{\omega_m}^*(x), \qquad R_{\omega_m}^-(x) = R_{\omega_m}(x), \qquad \hat{r}_{\omega_m}^+ = \hat{r}_{\omega_m}^\dagger, \qquad \hat{r}_{\omega_m}^- = \hat{r}_{\omega_m}, \qquad \text{(B.15)}$$

we have

$$\begin{aligned}
\mathrm{Tr}\big(\hat{\rho}_a \hat{\phi}_R(x) \hat{\phi}_R(x') \big) = \sum_K \sum_{\alpha,\alpha'} \sum_{m,n=0}^{\infty} & e^{-2\pi E(K)} \sqrt{K_\theta + 2} \sqrt{K_\theta + 1}\, N_{\omega_n}^{\alpha'}(K) N_{\omega_m}^{\alpha}(K + 1_{\omega_n}^{\alpha'}) \times \\
& \times R_{\omega_m}^\alpha(x) R_{\omega_n}^{\alpha'}(x')_R \Big\langle K + 2_\theta | K + 1_{\omega_m}^\alpha + 1_{\omega_n}^{\alpha'} \Big\rangle_R , \qquad \text{(B.16)}
\end{aligned}$$

where $\big|K + 1^{\pm}_{\omega_m}\big\rangle_R \equiv |K \pm 1_{\omega_m}\rangle_R$. The expectation value appearing in the r.h.s. of Eq. (B.16) gives

$$_R\Big\langle K + 2_\theta | K + 1^{\alpha}_{\omega_m} + 1^{\alpha'}_{\omega_n} \Big\rangle_R = \delta_{\alpha,+}\ \delta_{\alpha',+}\ \delta_{\omega_m,\theta}\ \delta_{\omega_n,\theta}\,. \tag{B.17}$$

From this one can read that there is only a contribution to Eq. (B.16) when the operator $\hat\phi_R(x)\hat\phi_R(x')$ creates two particles in the mode $\omega = \theta$ in the *state* defined by $\hat\rho_a$. Using Eq. (B.17) in Eq. (B.16) we have that

$$\mathrm{Tr}\big(\hat\rho_a \hat\phi_R(x)\hat\phi_R(x')\big) = R^*_\theta(x) R^*_\theta(x') \sum_K e^{-2\pi E(K)} (K_\theta + 2)(K_\theta + 1)\,. \tag{B.18}$$

To evaluate the sum in the r.h.s. of Eq. (B.18) note that from Eq. (II.27) we have that

$$\begin{aligned}\sum_K e^{-2\pi E(K)} f(K_\theta) &= \sum_{K_\theta=0}^{\infty} e^{-2\pi\theta K_\theta} f(K_\theta) \times \prod_{\substack{m=0\\ \omega_m\neq\theta}}^{\infty} \sum_{K_{\omega_m}=0}^{\infty} e^{-2\pi\omega_m K_{\omega_m}} \\ &= \langle 0_M | 0_M\rangle\ (1 - e^{-2\pi\theta}) \sum_{K_\theta=0}^{\infty} e^{-2\pi\theta K_\theta} f(K_\theta),\end{aligned} \tag{B.19}$$

where $f(K_\theta)$ is any function of K_θ and $\langle 0_M|0_M\rangle$ is given by Eq. (II.29). For latter use it is convenient to define

$$G_\theta[f(K_\theta)] \equiv \sum_{K_\theta=0}^{\infty} e^{-2\pi\theta K_\theta} f(K_\theta)\,. \tag{B.20}$$

Using Eq. (B.19) then Eq. (B.18) becomes

$$\mathrm{Tr}\big(\hat\rho_a \hat\phi_R(x)\hat\phi_R(x')\big) = \langle 0_M|0_M\rangle \,\frac{2}{(1 - e^{-2\pi\theta})^2} R^*_\theta(x) R^*_\theta(x'), \tag{B.21}$$

where we have used that

$$G_\theta[(K_\theta + 2)(K_\theta + 1)] = \frac{2}{(1 - e^{-2\pi\theta})^3}\,. \tag{B.22}$$

Analogously we have

$$\mathrm{Tr}\big(\hat\rho_b \hat\phi_R(x)\hat\phi_R(x')\big) = \langle 0_M|0_M\rangle \,\frac{2}{(e^{2\pi\theta} - 1)^2} R_\theta(x) R_\theta(x')\,. \tag{B.23}$$

Before computing $\mathrm{Tr}\big(\hat\rho_c \hat\phi(x)\hat\phi(x')\big)$ let's define

$$H_{\omega_m}[K](x,x') \equiv K_{\omega_m} R^*_{\omega_m}(x) R_{\omega_m}(x') + (K_{\omega_m} + 1) R_{\omega_m}(x) R^*_{\omega_m}(x')\,. \tag{B.24}$$

It can be verified that

$$\mathrm{Tr}\big(\hat\rho^R_{vac} \hat\phi_R(x)\hat\phi_R(x')\big) = \sum_K e^{-2\pi E(K)} \sum_{m=0}^{\infty} H_{\omega_m}[K](x,x')\,. \tag{B.25}$$

Now we compute

$$
\begin{aligned}
&\mathrm{Tr}\Big(\sum_K e^{-2\pi E(K)}\left[e^{2\pi\theta}K_\theta + e^{-2\pi\theta}(K_\theta+1)\right]|K\rangle_{RR}\langle K|\,\hat{\phi}_R(x)\hat{\phi}_R(x')\Big) = \\
&= \sum_K e^{-2\pi E(K)}\Big[\big(e^{2\pi\theta}K_\theta^2 + e^{-2\pi\theta}K_\theta(K_\theta+1)\big)R_\theta^*(x)R_\theta(x') \\
&\qquad + \big(e^{2\pi\theta}K_\theta(K_\theta+1) + e^{-2\pi\theta}(K_\theta+1)^2\big)R_\theta(x)R_\theta^*(x')\Big] \\
&\qquad\qquad + \sum_K e^{-2\pi E(K)}\big(e^{2\pi\theta}K_\theta + e^{-2\pi\theta}(K_\theta+1)\big)\sum_{\substack{m=0\\ \omega_m\neq\theta}}^{\infty} H_{\omega_m}[K](x,x')\,.
\end{aligned}
\tag{B.26}
$$

From the definition of $G_\theta[f(K_\theta)]$, Eq. (B.20), it can be shown that

$$
\begin{aligned}
e^{2\pi\theta}G_\theta[K_\theta^2] + e^{-2\pi\theta}G_\theta[K_\theta(K_\theta+1)] &= G_\theta[K_\theta(2K_\theta+1)] + G_\theta[1], && \text{(B.27)}\\
e^{2\pi\theta}G_\theta[K_\theta(K_\theta+1)] + e^{-2\pi\theta}G_\theta[(K_\theta+1)^2] &= G_\theta[(2K_\theta+1)(K_\theta+1)] + G_\theta[1], && \text{(B.28)}\\
e^{2\pi\theta}G_\theta[K_\theta] + e^{-2\pi\theta}G_\theta[(K_\theta+1)] &= G_\theta[(2K_\theta+1)]\,. && \text{(B.29)}
\end{aligned}
$$

Now we use Eq. (B.19) to simplify the r.h.s. of Eq. (B.26). From Eqs.(B.27)-(B.29) we have that

$$
\begin{aligned}
&\mathrm{Tr}\Big(\sum_K e^{-2\pi E(K)}\left[e^{2\pi\theta}K_\theta + e^{-2\pi\theta}(K_\theta+1)\right]|K\rangle_{RR}\langle K|\,\hat{\phi}_R(x)\hat{\phi}_R(x')\Big) = \\
&= \langle 0_M|0_M\rangle\left(R_\theta^*(x)R_\theta(x') + R_\theta(x)R_\theta^*(x')\right) + \sum_K e^{-2\pi E(K)}\big(2K_\theta+1)\big)\sum_{m=0}^{\infty} H_{\omega_m}[K](x,x')\,.
\end{aligned}
\tag{B.30}
$$

Then, from Eq. (B.30) we have at once that (see Eq. (B.13))

$$
\mathrm{Tr}\big(\hat{\rho}_c\hat{\phi}_R(x)\hat{\phi}_R(x')\big) = \langle 0_M|0_M\rangle\left(R_\theta^*(x)R_\theta(x') + R_\theta(x)R_\theta^*(x')\right) + \mathrm{Tr}\big(\hat{\rho}_{vac}^R\hat{\phi}_R(x)\hat{\phi}_R(x')\big), \tag{B.31}
$$

where we have used Eq. (B.25). Finally, from Eqs. (B.21), (B.23), (B.31) and (B.10) we have

$$
\begin{aligned}
\mathrm{Tr}\big(\hat{\rho}_f^{R(2)}\hat{\phi}_R(x)\hat{\phi}_R(x')\big) =& \Psi_\theta^2\big(-R_\theta(x)R_\theta(x') - R_\theta^*(x)R_\theta^*(x') + R_\theta^*(x)R_\theta(x') + R_\theta(x)R_\theta^*(x')\big)\\
&+ Z\Psi_\theta^2\mathrm{Tr}\big(\hat{\rho}_{vac}^R\hat{\phi}_R(x)\hat{\phi}_R(x')\big)\,.
\end{aligned}
\tag{B.32}
$$

and Eq. (V.18) follows directly.

Appendix C: INERTIAL CALCULATION

In this Appendix we want to show that one obtains exactly Eq. (V.29) when computing the change in $\langle\hat{\phi}(x)\hat{\phi}(x')\rangle$ in the inertial scheme. For this case we are going to use the scalar source given by Eq. (IV.9) written in the form

$$
j(x) = q\zeta_0\frac{\delta(z-\sqrt{t^2+\zeta_0^2})}{\sqrt{t^2+\zeta_0^2}}, \tag{C.1}
$$

where $\zeta_0 = 1/a$ and a is the acceleration of the source. Using the inertial field operator

$$
\hat{\phi}(x) = \int_{-\infty}^{\infty} dp\,\big(\psi_p(x)\hat{a}_p + \psi_p^*(x)\hat{a}_p^\dagger\big), \tag{C.2}
$$

where $\psi_p(x)$ represents a plane wave with frequency $\omega_p = +\sqrt{p^2+m^2}$,

$$\psi_p(x) = \frac{1}{\sqrt{2\pi}\sqrt{2\omega_p}} e^{-i\omega_p t + ipz}, \tag{C.3}$$

in Eqs. (IV.1) and (IV.2), it can be seen that the second order renormalized change in $\langle\hat{\phi}(x)\hat{\phi}(x')\rangle$ between states $|f\rangle$ and $|0_M\rangle$ is given by

$$C_{in}^{(2)}(x,x') = 4\mathrm{Im}[Q(x)]\mathrm{Im}[Q(x')], \tag{C.4}$$

where

$$Q(x) = \int_M d^2x' \int_{-\infty}^{\infty} dp\, j(x)\psi_p^*(x')\psi_p(x)\,. \tag{C.5}$$

Recall that for the quantized scalar field we are considering the positive frequency function is given by

$$i\Delta^{(+)}(x,x') \equiv \langle 0_M|\,\hat{\phi}(x)\hat{\phi}(x')\,|0_M\rangle = \int_{-\infty}^{\infty} dp\,\psi_p^*(x')\psi_p(x) \tag{C.6}$$

and from [6] we have the following result

$$\Delta^{(+)}(x,0) = \frac{1}{4} \times \begin{cases} H_0^{(2)}(m\sqrt{t^2-z^2}) & t > |z| \\ \frac{2i}{\pi}K_0(m\sqrt{z^2-t^2}) & |t| < |z| \\ -H_0^{(1)}(m\sqrt{t^2-z^2}) & t < -|z| \end{cases}. \tag{C.7}$$

From now on we shall suppose that $x \in F$. Using the fact that $\Delta^{(+)}(x,x') = \Delta^{(+)}(x-x',0)$ and Eq. (C.7) it can be seen that

$$\mathrm{Im}[Q(x)] = \frac{q\zeta_0}{4}\int_{-\infty}^{t^-} \frac{dt'}{\sqrt{t'^2+\zeta_0^2}} J_0(m\sqrt{-2\sigma(t')}), \tag{C.8}$$

where

$$\sigma(t') = \frac{1}{2}\left(-(t-t')^2 + \left(z - \sqrt{t'^2+\zeta_0^2}\right)^2\right) \tag{C.9}$$

and $\sigma(t^-) = 0$. To get Eq. (C.8) we have used that $H_\nu^{(2)}(y) = J_\nu(y) - iY_\nu(y)$ and the fact that $J_0(y)$, $Y_0(y)$ are real for $y \geq 0$. Making the changes of variables $t' = \zeta_0\sinh(\tau/\zeta_0)$ and $u = \sqrt{-2\sigma(\tau)}$ we obtain

$$\mathrm{Im}[Q(x)] = \frac{q\zeta_0}{2}\int_0^{\infty} \frac{uJ_0(mu)}{\sqrt{4\zeta_0^2\zeta^2 + (u^2+\zeta_0^2-\zeta^2)^2}}\,du = \frac{q\zeta_0}{2}K_0(m\zeta_0)J_0(m\zeta), \tag{C.10}$$

were we have used $x = (t,z)$ and $t = \zeta\cosh(\tau_p/\zeta_0)$, $z = \zeta\sinh(\tau_p/\zeta_0)$ (see Eq. (A.2b)). The derivation that leads to the second equality in Eq. (C.10) is analogous to that which leads to Eq. §13.54(1) of [11]. Thus we have proved that

$$C_{in}^{(2)}(x,x') = q^2\zeta_0^2 K_0(m\zeta_0)^2 J_0(m\zeta)J_0(m\zeta'), \tag{C.11}$$

which coincides functionally with Eq. (V.29). In this computation one has to apply Wick theorem with the notion of time and normal ordering associated to the inertial time parameter t. However, the effect of this choice of time does not show up in the physical change of the two point function but only in renormalization terms. Note that, in contrast to the accelerated frame calculation, we did not need to introduce any regulator into the current (neither any cutoff as in the inertial frame calculation in [16]).

---

[1] D. G. Boulware, Ann. Phys. (N.Y.) **124**, 169 (1980).
[2] W. G. Unruh, Phys. Rev. **D14**, 870 (1976).
[3] W. G. Unruh and R. M. Wald, Phys. Rev. **D29**, 1047 (1984).
[4] A. Higuchi, G. E. A. Matsas, and D. Sudarsky, Phys. Rev. **D45**, R3308 (1992).
[5] R. M. Wald, *Quantum field theory in curved space-time and black hole thermodynamics* (University Press, Chicago, 1985).
[6] N. B. Narozhny, A. M. Fedotov, B. M. Karnakov, V. D. Mur, and V. A. Belinskii, Phys. Rev. **D65**, 025004 (2002).
[7] S. A. Fulling and W. G. Unruh, Phys. Rev. **D70**, 048701 (2004).
[8] B. S. Kay, Commun. Math. Phys. **62**, 55 (1978).
[9] A. Ashtekar and A. Magnon, Proc. Roy. Soc. Lond. **A346**, 375 (1975).
[10] S. A. Fulling, Phys. Rev. **D7**, 2850 (1973).
[11] G. N. A. Watson, *A Treatise on the Theory of Bessel Functions* (The University Press, Cambridge, UK, 1922).
[12] B. S. Kay, Commun. Math. Phys. **100**, 57 (1985).
[13] U. H. Gerlach, Phys. Rev. **D38**, 514 (1988).
[14] P. Martinetti and C. Rovelli, Class. Quant. Grav. **20**, 4919 (2003).
[15] M. Horodecki, P. Horodecki, and R. Horodecki, *in: Quantum Information: An Introduction to Basic Theoretical Concepts and Experiments* (Springer-Verlag, 2001), Springer Tracts in Modern Physics, 173.
[16] A. Higuchi, G. E. A. Matsas, and D. Sudarsky, Phys. Rev. **D46**, 3450 (1992).
[17] D. G. Boulware, Phys. Rev. **D11**, 1404 (1975).
[18] A. Erdélyi, *Higher trascendental functions*, vol. II (Robert E. Krieger, USA, 1953).