Extra dimensions as a source of the electroweak model

S.V. Bolokhov
PFUR, 6 Miklukho-Maklaya St., Moscow 117198, Russia. E-mail: bol-rgs@yandex.ru

K.A. Bronnikov
Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow, Russia; Institute of Gravitation and Cosmology, PFUR, 6 Miklukho-Maklaya St., Moscow 117198, Russia.// E-mail: kb20@yandex.ru

S.G. Rubin
National Research Nuclear University "MEPHI" , 31 Kashirskoe Sh., Moscow 115409, Russia.// E-mail: sergeirubin@list.ru

The Higgs boson of the Standard model is described by a set of off-diagonal components of the multidimensional metric tensor, as well as the gauge fields. In the low-energy limit, the basic properties of the Higgs boson are reproduced, including the shape of the potential and interactions with the gauge fields of the electroweak part of the Standard model.

PACS numbers: 04.50.+h; 98.80.-k; 98.80.Cq; 11.30.Ly

I. INTRODUCTION

The Standard model (SM) of particle physics is a basis of modern physics. Its certain shortcomings may probably be removed without essentially changing the basic structure of the theory. However, the origin of the SM is still remaining problematic. Why does the theory possess this particular symmetry $SU(2) \otimes U(1)$ in the present case? What is the origin of the Higgs field, gauge fields and matter fields? Why precisely three generations of fermions are realized? There are no unambiguous answers to these questions. The problems that exist even in the supersymmetric version of the SM make it necessary to extend the SM in search for an acceptable version [1].

On the other hand, multidimensional gravity provides broad opportunities in explaining diverse phenomena, such as the inflaton potential and a nonzero value of the cosmological constant [2]. The models built on the basis of multidimensional gravity are intrinsically consistent because they contain a stabilization mechanism for extra dimensions [6]. Even at a phenomenological level, addition of nonminimally coupled Higgs fields to the Ricci scalar leads to nontrivial effects. As was shown in [3], one could associate the inflaton and the Higgs field in this case.

In addition, it is well known that gauge fields, including those of the SM, are introduced in a natural way on the basis of compact extra spaces (see, e.g., [4, 5]). The off-diagonal components of the metric tensor are interpreted at low energies as gauge fields belonging to the algebra of the symmetry group of the extra space. The gauge symmetry of the Lagrangian thus follows from the symmetry of extra dimensions. Thus there are clear indications that extra dimensions can be a basis for the SM. In this paper, we study the possibility of constructing the Higgs sector of the SM, interacting with gauge fields, by choosing the proper geometry of the extra factor space in the spirit of the Kaluza-Klein approach.

We will suggest a way of geometrization for not only the gauge fields but also the Higgs bosons, which transform according to the fundamental representation of the gauge group. As in the popular scheme of Higgs-gauge unification [13], we identify the Higgs field with nondiagonal components of the metric, but in a different manner. In this way we are able to obtain the standard Higgs field Lagrangian containing interaction with the gauge fields and corresponding to the structure of the boson sector of the SM in the low-energy limit. The fermion sector is not discussed.

Some previous papers devoted to multidimensional unified theories contain attempts to describe the Higgs field geometrically. In particular, in [7–9], the possible introduction of an effective Higgs field was studied in the framework of 6-, 7- and 8-dimensional models. A characteristic feature of this approach was a sufficient economy of the number of extra dimensions in the qualitative reproduction of the gauge theories of physical interactions. This economy was, however, achieved at the expense of introducing complex quantities into the multidimensional metric and employing additional degrees of freedom connected with conformal (Weyl) transformations of the original manifold.

In [10, 11], the authors considered the conditions for emergence of an irreducible scalar field multiplet due to dimensional reduction in multidimensional Kaluza-Klein models. The conditions found are of *sufficient* nature and were formulated in the framework of a very interesting but sophisticated mathematical scheme on the basis of purely group-theoretical considerations, using the method of intertwining operators and a generalization of Dynkin's diagram technique. Let us also note that the scheme used by these authors is arranged for the case of free gauge fields with a particular choice of the extra manifold topology in the form of the factor space G/H, where G is the gauge group and H its stationary subgroup.

Geometrization of the Higgs sector of the SM in the framework of the Kaluza-Klein approach was also performed in the recent paper [12]. This model used a specific geometric structure (3-brane) in the background of a nontrivial geometry of extra dimensions (squashed three-sphere) as well as an alternative mass generation mechanism for vector bosons using fluctuations of the brane.

In the present paper, we try to follow the technically simplest geometric approach in which the initial Lagrangian does not contain any dynamic variables other than the metric tensor of the multidimensional manifold. We do not invoke additional structures like branes or assumptions about some specific complicated extra space geometry. In this approach, there is no necessity to analyze any sophisticated group structure of the manifold or to introduce any non-metric degrees of freedom.

This approach allows one not only to reproduce the conventional form of the Higgs sector of the SM but also to find deflections from it. The latter can be of particular value due to the soon expected work of the LHC at its full capacity and a possible negative result in search for Higgs particles in the predicted mass range.

II. STATEMENT OF THE PROBLEM. THE METRIC STRUCTURE OF THE SPACE $\mathbb{M}^D$

A. Preliminaries

Let there be a D-dimensional Riemannian manifold $\mathbb{M}^D$ with the metric tensor of the form

$$(g_{AB}) = \left(\begin{array}{c|c|c|c} \overline{g}_{\mu\nu}(x,y) & g_{\mu a}(x,y) & & \\ \hline g_{b\nu}(x,y) & g_{ab}(y) & g_{a9}(x,z) & \\ \hline & g_{9b}(x,z) & g_{99} & \\ \hline & & & g_{mn}(z), \end{array}\right) \tag{1}$$

where only the most important (for this discussion) metric components are written. Contributions of other components to the action are not considered. Here and henceforth, the indices assume the following values:

$A, B, \ldots = 1, \ldots, D$ (the full dimension is $D = 9 + d_3$);

$\alpha, \beta, ..., \mu, \nu, ... = 1, ..., 4$ (the quantities $\overline{g}_{\mu\nu}(x,y)$ contain the metric of our 4D space $\mathbb{M}_1$ $g_{\mu\nu}(x) = g^{(1)}_{\mu\nu}(x)$, see Section IIIB);

$a, b, ... = 5, ..., 8$ (the notation $g_{ab} = g^{(2)}_{ab} = \gamma_{ab}$ will also be used; the space with this metric will be called $\mathbb{V}^4$);

$I, J, K = 5, ..., 9$ (the subspace with the metric $g_{IK} = g^{(2)}_{IK}$ will be denoted $\mathbb{M}_2$);

$m, n, ... = 10, ..., 9 + d_3$ (the subspace with the metric $g_{mn} = g^{(3)}_{mn}$ will be denoted $\mathbb{M}_3$);

i, j, k are used as group indices.

The sets of coordinates of the subspaces $\mathbb{M}_1$, $\mathbb{M}_2$ (which includes $\mathbb{V}^4$) and $\mathbb{M}_3$ will be denoted by x, y, z, respectively, and all D coordinates jointly by the letter Z.

Let us, anticipating, describe the physical meaning of some of the metric components. The off-diagonal components $g_{a\mu}$, $a = 5, ..., 8$ are connected, according to the standard Kaluza-Klein scheme [4], with gauge fields that lie in the algebra of the symmetry group of the 4D compact factor space $\mathbb{V}^4$. In what follows it will be shown that the components g_{a9} contain scalar (in $\mathbb{M}_1$) fields to be interpreted as the Higgs field. The metric tensor g_{mn} of the factor space $\mathbb{M}_3$ is of auxiliary nature and can in principle create inflaton-like scalar fields.

The basic element in this approach is the extra space $\mathbb{V}^4$ with the metric γ_{ab}. It is supposed that its isometry group T causes the symmetry of the SM Lagrangian under gauge transformations.

One usually does not discuss the reasons for a high symmetry of an extra factor space. We will also adopt this fact without proof but refer to the article [18], where such a reason is discussed. Briefly, an extra factor space is initially non-symmetric, but due to entropy transfer to the basic space it passes into a symmetric state. The entropy transfer happens due to decay of Kaluza-Klein excitations.

Our strategy will consist in choosing the group T in such a way as to provide the correct transformation law of the effective Higgs doublet that emerges from the metric components g_{a9} and belonging to the fundamental representation[1] of the electroweak gauge group $SU(2)\times U(1)$ of the SM.

The model dynamics will be specified by the D-dimensional action

$$S = \frac{1}{2} m_{\rm D}^{D-2} \int \sqrt{{}^D g}\, d^D Z \Big[R_D + \eta R_D^2 - 2\Lambda \Big] \tag{2}$$

where R_D is the scalar curvature of the Riemannian space $\mathbb{M}^D$, η and Λ are constants (parameters of the theory), $m_{\rm D}$ is the D-dimensional Planck mass. The legality of introducing curvature-nonlinear corrections to the action is validated by taking into account quantum field effects [14, 15]. Higher-order corrections other than R^2 are not considered here because the action (2) is already sufficient for the purposes of the present paper.

B. Singling out the proto-Higgs field from the extra-space metric

Consider a class T of linear transformations of the coordinates y^a of the extra space $\mathbb{V}^4$:

$$y'^a = T^a_b y^b, \qquad a, b = 5, ..., 8. \tag{3}$$

Let us assume that T forms an s-parametric Lie group of isometries of $\mathbb{V}^4$, i.e., these transformations leave its metric form-invariant:

$$g'_{ab}(y) = g_{ab}(y).$$

The generators $t^{ja}_{\ \ b}$ $(j = 1, ..., s)$ of this group, describing infinitesimal shifts,

$$y'^a = (\delta^a_b + i\varepsilon t^a_b) y^b, \qquad \varepsilon t^a_b \equiv \sum_j \varepsilon_j t^{ja}_{\ \ b}, \tag{4}$$

create the corresponding algebra of Killing vectors on the manifold $\mathbb{V}^4$.

The matrices $t^{ja}_{\ \ b}$ are assumed to be independent of y^a. In addition, it is supposed that we work in the class of coordinate systems $\{y^a\}$ adjusted to the linear action of the group T (3). It is easy to verify that these transformations which deal with the coordinates y^a only, imply the corresponding vector transformation law for the metric components g_{a9}. For infinitesimal shifts we accordingly have

$$g'_{b9}(x,z) = \frac{\partial y^a}{\partial y'^b} g_{a9}(x,z) = (\delta^a_b - i\varepsilon t^a_b) g_{a9}(x,z); \tag{5}$$

The components $g_{a9}(x,z)$ (which may be called proto-Higgs fields) and the coordinates y^a in (3) are transformed by the same representation of the group T (although with mutually reciprocal matrices belonging to this representation). This sufficiently obvious fact will be important in what follows.

C. The choice of the symmetry group T

Let us specify the group T by requiring that (a) this group should be realized by coordinate transformations in the extra space, i.e., it should be real; (b) it should be isomorphic to the electroweak group of the SM. In other words, the group T should be built as a realification of $SU(2)\times U(1)$. This can be performed by taking as a basis the sufficiently well-known scheme of embedding the unitary groups $SU(n)$ into the orthogonal groups $SO(2n)$.

---

[1] In the context of the present paper, a fundamental (*syn.*: standard, or definitive) representation of a matrix Lie group is understood as the exact representation of minimum dimension, corresponding to the standard group-theoretic nomenclature specifying this matrix group. Thus, the fundamental representation of the $SU(n)$ group is realized by $n\times n$ matrices with determinants equal to unity (the Special Unitary group).

Let us introduce notations for separate components of the electroweak group:

$$\omega_1(\phi) = e^{i\phi} \in U(1); \qquad \omega_2(\theta_j) = A(\theta_j) + iB(\theta_j) \in SU(2). \tag{6}$$

Here ϕ and θ_j $(j = 1, ..., 3)$ are parameters of the groups $U(1)$ and $SU(2)$, respectively; $A = \mathrm{Re}(\omega_2)$, $B = \mathrm{Im}(\omega_2)$ are real 2×2 matrices which are the real and imaginary parts of an $SU(2)$ element in the fundamental representation and therefore satisfy the conditions

$$A^T A + B^T B = 1, \qquad A^T B - B^T A = 0, \qquad \det(A + iB) = 1. \tag{7}$$

We now introduce the set of 4×4 matrices T_1, T_2:

$$T_1 = \begin{pmatrix} I\cos\phi & -I\sin\phi \\ I\sin\phi & I\cos\phi \end{pmatrix}, \qquad T_2 = \begin{pmatrix} A & -B \\ B & A \end{pmatrix}, \tag{8}$$

where I is the unit 2×2 matrix.

The following statements are easily verified directly, taking into account (7):

1. Each of the sets of matrices T_1 and T_2 forms a group by matrix product.
2. The group T_1 is orthogonal and has a unit determinant (i.e., lies in $SO(4)$) and is isomorphic to the group $U(1)$.
3. The group T_2 also lies in $SO(4)$ and is isomorphic to the group $SU(2)$ in its fundamental representation.
4. Matrices of the form T_1 and T_2 commute: $T_1T_2 = T_2T_1$.

A consequence of all these facts is the following statement: *All possible products of elements $T_1 \cdot T_2$ form a real 4-parametric group T, lying in $SO(4)$ and isomorphic to the group $SU(2)\times U(1)$.* A constructive meaning of this statement is made clear in the next section. We choose as a symmetry group of the extra 4D space $\mathbb{V}^4$ the group $T = T_1 \cdot T_2$ built above.

It is important that the initial Lagrangian is invariant under the coordinate transformations belonging to the group T because it is manifestly invariant under general coordinate transformations of the space $\mathbb{V}^4$.

It is also easy to verify that the Euclidean metric

$$\gamma_{ab} = \delta_{ab} \tag{9}$$

is symmetric under transformations of the group T. That is why it can be chosen as the metric of the basic extra space $\mathbb{V}^4$. The latter is assumed to be compact, which is easily achieved, eg., by endowing it with toroidal topology.

III. A TRANSITION TO THE DYNAMIC VARIABLES OF THE SM

Our immediate task is to connect the Higgs complex doublet $h(x) \in \mathbb{C}^2$ of the SM with the metric of the extra space under consideration. Let us address to the sector of the gauge theory responsible for the Higgs field and its interaction with the gauge fields $A_\mu(x)$. The structure of the corresponding Lagrangian is well known:

$$\mathcal{L}[h(x), A_\mu(x)] = (D^\mu h)^+(D_\mu h) - V(h). \tag{10}$$

Here, $V(h)$ denotes the potential providing a spontaneous symmetry breakdown (see Section IV) while the gauge-covariant derivative has the form

$$D_\mu h = \partial_\mu h - ig_1 A'^a_\mu \frac{\sigma^a}{2} h - \frac{i}{2} g_2 B_\mu h, \tag{11}$$

where the sets of gauge fields A^a_μ corresponding to the groups $SU(2)$ and $U(1)$ are denoted by A'^a_μ and B_μ, respectively; σ_a are the Pauli matrices, g_1 and g_2 are coupling constants..

The Higgs doublet h is transformed by the fundamental representation of the electroweak group $SU(2)\times U(1)$:

$$h' = \omega_1\omega_2 h = (A + iB)e^{i\phi} h. \tag{12}$$

Let us establish a relation between the metric coefficients

$$g_{a9} \equiv H_a \tag{13}$$

and the Higgs field h. To do so, we express the 4-component field H_a, transformed by the group T, in terms of two-component columns X and Y:

$$H \equiv \begin{pmatrix} X \\ Y \end{pmatrix}, \qquad H' = TH = T_1 T_2 H, \tag{14}$$

With (8), the explicit form of the transformation (14) in the space of the 4-columns H is

$$\begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{pmatrix} A\cos\phi - B\sin\phi & -(A\sin\phi + B\cos\phi) \\ A\sin\phi + B\cos\phi & A\cos\phi - B\sin\phi \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}, \tag{15}$$

We build the combination

$$\tilde{h} = X + iY, \qquad X, Y \in \mathbb{R}^2. \tag{16}$$

It is easily verified that the field $\tilde{h}$ is transformed in the same way as the Higgs doublet (12). Therefore at full rights we identify these two doublets, $\tilde{h} \equiv h$.

A counterpart of the transformation of complex 2-columns $h' = \omega_1\omega_2 h$ in the space of real 4-columns is the equivalent transformation $H' = TH$, which means that the representations of the groups T and $SU(2)\times U(1)$ are isomorphic.

Let us note that the correspondence between the fields H and h can be described by using the matrix

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & i & 0 \\ 0 & 1 & 0 & i \end{pmatrix} \tag{17}$$

with the property

$$PP^+ = PP_R^{-1} = 1. \tag{18}$$

The matrix (17) “projects” one representation of the Higgs field, $H = (h_1, h_2, h_3, h_4)$, onto the other,

$$h = \begin{pmatrix} h_1 + ih_3 \\ h_2 + ih_4 \end{pmatrix}$$

so that

$$h = \frac{1}{\sqrt{2}} PH. \tag{19}$$

IV. LAGRANGIAN OF THE HIGGS FIELD

So far, our main task was to extract the group structure of the Higgs field and the gauge fields from the extra space metric and to find the proper symmetry of the extra space. Let us now outline a way of obtaining the Higgs field Lagrangian.

The structure of the gauge field contribution to the Lagrangian is studied below, therefore here we will only consider the terms that do not contain the metric components $g_{a\mu}(x)$ responsible for the gauge fields. Assuming here and further on $g_{a\mu}(x) = 0$, we obtain a space-time with the product structure $\mathbb{M}^D = \mathbb{M}_1 \otimes \mathbb{M}_2 \otimes \mathbb{M}_3$ and the block-diagonal metric

$$ds^2 = g_{AB} dZ^A dZ^B = g^{(1)}_{\mu\nu}(x) dx^\mu dx^\nu + g^{(2)}_{IK}(x,z) dy^I dy^K + g^{(3)}_{mn}(z) dz^m dz^n \tag{20}$$

Recall that we are considering a space-time with the metric (1) and the factor space dimensions $\dim \mathbb{M}_1 = 4$, $\dim \mathbb{M}_2 = 5$, $\dim \mathbb{M}_3 = d_3$. From a comparison with (1) it is seen that

$$g^{(2)}_{IK}(x,z) = \left(\begin{array}{c|c} g_{ab} & g_{a9}(x,z) \\ \hline g_{9b}(x,z) & g_{99} \end{array} \right) \tag{21}$$

Let us write down the components involved in the action (2) in terms of (21). For the scalar curvature of the whole D-dimensional space we obtain

$$
\begin{aligned}
R_D &= \overline{R}_1 + \overline{R}_{2+3} + K_H, \\
K_H &= -\nabla_\alpha \mathcal{X}^\alpha + \frac{1}{4} g^{IK,\alpha} g_{IK,\alpha} - \frac{1}{4}\mathcal{X}^\alpha \mathcal{X}_\alpha, \qquad \mathcal{X}^\alpha := g^{IK} g_{IK}{}^{,\alpha}
\end{aligned}
\tag{22}
$$

where $g_{IK} = g^{(2)}_{IK}(x,z)$, while a bar over R means that the curvature is calculated in the corresponding subspace taken separately. In particular, $\overline{R}_{2,3}$ is the curvature of the subspace $\mathbb{M}_2 \otimes \mathbb{M}_3$, equal to

$$
\begin{aligned}
\overline{R}_{2+3} &= \overline{R}_2 + \overline{R}_3 + V_H, \\
V_H &= -\nabla_n \mathcal{Y}^n + \frac{1}{4} g^{IK,n} g_{IK,n} - \frac{1}{4}\mathcal{Y}^n \mathcal{Y}_n, \qquad \mathcal{Y}^n := g^{IK} g_{IK}{}^{,n};
\end{aligned}
\tag{23}
$$

here $\overline{R}_2 = 0$ since the metric g_{IK} does not depend on y^I.

The derivatives in x^α, forming the expression K_H, contribute to the kinetic term of the Higgs field Lagrangian. Consider the first term $\int d^D x \sqrt{g_D} R_D$ in the action integral (2). Substituting (22) into it and singling out the total derivative ∂_α, we obtain

$$
-\sqrt{g_D}\nabla_\alpha \mathcal{X}^\alpha = -\sqrt{g_1 g_2 g_3}\nabla_\alpha \mathcal{X}^\alpha = -\sqrt{g_3}\partial_\alpha\Big(\sqrt{g_1\, g_2}\mathcal{X}^\alpha\Big) + \sqrt{g_1 g_3}\mathcal{X}^\alpha \partial_\alpha \sqrt{g_2},
\tag{24}
$$

where $g_1 = |\det(g_{\mu\nu})|$, $g_2 = |\det(g_{IK})|$, $g_3 = |\det(g_{mn})|$, and g_3 does not depend on x^α.

The expression V_H in (23), in which the derivatives are taken with respect to the coordinates z^n, ultimately contribute to the potential of the Higgs field. Similarly to (24), let us single out the total derivative ∂_m in the action integral:

$$
-\sqrt{g_D}\nabla_m \mathcal{Y}^m = -\sqrt{g_1}\partial_m\Big(\sqrt{g_2\, g_3}\mathcal{Y}^m\Big) + \sqrt{g_1 g_3}\mathcal{Y}^m \partial_m \sqrt{g_2},
\tag{25}
$$

where g_1 does not depend on z^m.

In what follows, we work in the slow-change approximation (as compared to the Planck scale $m_{\rm D}$) suggested in [6], according to which each derivative $\partial_\mu \equiv \partial/\partial x^\mu$ is considered as an expression containing a small parameter ε, and in the equations of motion we take into account only terms of orders not higher than $O(\varepsilon^2)$. No small parameter is assigned to the derivative $\partial/\partial z^m$. As has been shown in [6], this approximation is even applicable at Grand unification energies (under reasonable assumptions), to say nothing of the more modest weak interaction scale. Owing to the above-said, we can restrict ourselves in the kinetic term to the expression (see (13)) $H_{a,\alpha}H_a^{;\alpha}$, neglecting all further corrections, whereas in the expression for the Higgs field potential we preserve all quantities up to $O(H^4)$ (under the additional assumption $H_a H_a \ll 1$).

In the second term of the action (2), containing R_D^2, it is sufficient to take the expression for R_D in the linear approximation with respect to the quantity $H_a H_a$, now without singling out total derivatives and neglecting expressions with $H_{a,\alpha}H_a^{;\alpha}$ due to the above-said about the kinetic term. Therefore, we can write

$$
R_D^2 \approx \left(-\nabla_m \mathcal{Y}^m + \frac{1}{4} g^{IK,m} g_{IK,m}\right)^2.
\tag{26}
$$

Let us now express the relations obtained in terms of the proto-Higgs field H_a, involved, according to (13), in the expression for the metric of the factor space $\mathbb{M}_2$ in the form

$$
g^{(2)}_{IK} = g_{IK} = \begin{pmatrix}
-1 & 0 & 0 & 0 & H_1(x,z) \\
0 & -1 & 0 & 0 & H_2(x,z) \\
0 & 0 & -1 & 0 & H_3(x,z) \\
0 & 0 & 0 & -1 & H_4(x,z) \\
H_1(x,z) & H_2(x,z) & H_3(x,z) & H_4(x,z) & \epsilon
\end{pmatrix} = \begin{pmatrix} -\delta_{ab} & H_a \\ H_b & 1 \end{pmatrix},
\tag{27}
$$

where $\epsilon = \pm 1$. It is then easy to obtain

$$
\begin{aligned}
& g_2 = |g|, \qquad g = \epsilon + H_a H_a, \\
& (g^{IK}) = \begin{pmatrix} (H_a H_b)/g - \delta_{ab} & H_a/g \\ H_b/g & 1/g \end{pmatrix}, \\
& \mathcal{X}_\alpha = \frac{2}{g} H_a H_{a,\alpha}, \qquad \mathcal{Y}_m = \frac{2}{g} H_a H_{a,m}.
\end{aligned}
\tag{28}
$$

Hence in Eqs. (24) and (25)

$$\partial_\alpha \sqrt{g_2} = \mathcal{X}_\alpha/2 = \frac{1}{g}\, H_a H_{a,\alpha}, \qquad \partial_m \sqrt{g_2} = \mathcal{Y}_m/2 = \frac{1}{g}\, H_a H_{a,m}. \tag{29}$$

As a result, omitting total derivatives, we obtain

$$\sqrt{g_D}\; R_D = \sqrt{g_D}\; (\overline{R}_1 + \overline{R}_3) + \sqrt{g_1 g_3}\; \frac{\operatorname{sign} g}{2\sqrt{|g|}} \Big(H_{a,\alpha} H_a^{;\alpha} + H_{a,n} H_a^{;n} \Big). \tag{30}$$

The terms with H_a form a standard expression of the form $\frac{1}{2}(\partial H)^2 - V(H)$, where $(\partial H)^2$ is obtained from expressions with the derivatives $\partial/\partial x^\alpha$, and $V(H)$ from expressions with the derivatives $\partial/\partial z^m$.

The expression for R_D gives the correct sign (plus) of the kinetic term of the Higgs field if $g > 0$, therefore we choose $\epsilon = 1$, whence

$$\frac{\sqrt{g_1 g_3}}{2\sqrt{g}} H_{a,\alpha} H_a^{;\alpha} = \frac{1}{2}\sqrt{g_1 g_3} H_{a,\alpha} H_a^{;\alpha} \Big(1 - \frac{1}{2} H_b H_b + \ldots \Big). \tag{31}$$

The quantity R_D^2 reads

$$R_D^2 \approx \Big(\overline{R}_3 - \nabla_m \mathcal{Y}^m + \frac{1}{4} g^{IK,m} g_{IK,m} \Big)^2 = \Big(\overline{R}_3 - 2 H_a \nabla_m H_a^{;m} - \frac{3}{2} H_{a,m} H_a^{;m} \Big)^2. \tag{32}$$

Lastly, in the third term of the action (2) we simply expand $\sqrt{g_2}$ in power series with respect to $H_a H_a$,

$$\sqrt{g_2} \approx 1 + \frac{1}{2} H_a H_a - \frac{1}{8} (H_a H_a)^2. \tag{33}$$

Substituting the resulting relations to the action (2), we have

$$\begin{aligned} S_{\rm eff} \;=\; & \frac{1}{2} m_{\rm D}^{D-2} \int_{\mathbb{M}^D} d^4x\; d^5y\; d^{d_3}z \sqrt{g_1 g_3} \Big[\overline{R}_1 + \frac{1}{2} H_{a,\alpha} H_a^{;\alpha} \\ & + \overline{R}_3 \Big(1 + \frac{1}{2} H_a H_a - \frac{1}{8} (H_a H_a)^2 \Big) + \eta \Big(\overline{R}_3 - 2 H_a \nabla_m H_a^{;m} - \frac{3}{2} H_{a,m} H_a^{;m} \Big)^2 \\ & + \frac{1}{2} H_{a,n} H_a^{;n} \Big(1 - \frac{1}{2} H_a H_a \Big) - 2\Lambda \Big(1 + \frac{1}{2} H_a H_a - \frac{1}{8} (H_a H_a)^2 \Big) \Big] \end{aligned} \tag{34}$$

It is now easy to pass over from the proto-Higgs field H_a, $a = 5,6,7,8$, to the Higgs field h_i, $i = 1,2$. It follows from the expression (19) that $H_a H_a = h_i^* h_i$ and so on. Therefore the expression (34) in terms of the Higgs field has the form

$$S_{\rm eff} \;=\; \frac{1}{2} m_{\rm D}^{D-2} \int_{\mathbb{M}^D} d^4x\; d^5y\; d^{d_3}z \sqrt{g_1 g_3} \Big[\overline{R}_1 + \frac{1}{2} h^*_{i,\alpha} h_i^{;\alpha} - V(h) \Big], \tag{35}$$

$$\begin{aligned} V(h) \;=\; & \overline{R}_3 \Big(1 + \frac{1}{2} h_i^* h_i - \frac{1}{8} (h_i^* h_i)^2 \Big) + \eta \Big(R_3 - h_i^* \nabla_m h_i^{;m} h_i \nabla_m h_i^{**,m} - \frac{3}{2} h^*_{i,m} h_i^{;m} \Big)^2 \\ & + \frac{1}{2} h^*_{i,n} h_i^{;n} \Big(1 - \frac{1}{2} h_i^* h_i \Big) - 2\Lambda \Big(1 + \frac{1}{2} h_i^* h_i - \frac{1}{8} (h_i^* h_i)^2 \Big). \end{aligned} \tag{36}$$

Up to now, the geometry of the factor space $\mathbb{M}_3$ remained arbitrary. Let us now, for simplicity, choose $\mathbb{M}_3$ to be one-dimensional (so that the full dimension is $D = 10$) and compact, i.e., a ring of a certain radius $b_3 = b$. Thus $z \in [0, 2\pi)$ and $g_{zz} = \zeta b^2$, $\zeta = \pm 1$. We will also assume that $\mathbb{M}_2$ is compact and has the form of a torus of certain radius b_2 along all five directions.

Let us specify $h_i(x,z)$, implying that the z dependence should be periodic with the period 2π. We choose the simplest form

$$h_i(x,z) = \chi_i(x)(a \sin z + c), \qquad a, c = \text{const}. \tag{37}$$

Substituting (37) into the action integral (35) for the fields $h_a(x,z)$, we integrate it over the extra dimension z and bring it to the form

$$A_\chi = \int d^4x \sqrt{g_1(x)}\, L_\chi, \qquad L_\chi = \chi^{*,\alpha} \chi_{,\alpha} - V(\chi^* \chi). \tag{38}$$

In doing so, we take into account that the first term in (35) should take the form of the Einstein-Hilbert action $\frac{1}{2}m_4^2\int\sqrt{g_1}R_1$, where $m_4^2 = 1/(8\pi G)$ is the Planck mass squared, so that

$$m_4^2 = m_{10}^8(2\pi)^6 b_2^5 b. \tag{39}$$

As a result, for the kinetic term in the main approximation in the sense described above (recall that only this approximation is considered) we obtain

$$\frac{1}{8}m_4^2(a^2+2c^2)\chi_i^{*,\alpha}\chi_{i,\alpha}. \tag{40}$$

Comparing it with (38), we obtain the normalization condition

$$\frac{1}{8}m_4^2(a^2+2c^2) = 1. \tag{41}$$

Note that the quantities a, b, c have the dimensionality of length.

For the potential $V(\chi)$, summing the contributions from all three terms in (2) and taking into account (41), we obtain the following expression:

$$\begin{aligned} V(\chi) &= m_4^2\Lambda + \frac{m_4^2}{4}\left[\Lambda(a^2+2c^2) - \frac{\zeta a^2}{2b^2}\right]\chi_i^*\chi_i \\ &+ \frac{m_4^2}{64}\left[-4\Lambda(3a^4+12a^2c^2+4c^4) + \zeta\frac{a^2}{b^2}(3a^2+4c^2) - \eta\frac{a^2}{b^4}(51a^2+32ac+128c^2)\right](\chi_i^*\chi_i)^2. \end{aligned} \tag{42}$$

Let us compare this expression with the standard Higgs potential [19]

$$V = \frac{1}{2}\lambda^2\left[(\chi_i^*\chi_i)^2 - \frac{1}{2}v^2\right]^2, \tag{43}$$

where the vacuum mean value is $v \approx 246$ GeV while the quantity $\lambda \le 1$ can be expressed in terms of the Higgs boson mass $m_\chi = \lambda v$. The latter is expected to be in the range $\sim 100 \div 300$ GeV.

Equalizing the potentials (42) and (43) term by term, we obtain:

$$\Lambda = \frac{\lambda^2 v^4}{8m_4^2}, \tag{44}$$

$$\frac{a^2}{4b^2} = \lambda^2 v^2\left(1 + \frac{v^2}{2m_4^2}\right), \tag{45}$$

$$-\eta v^2\left(204\frac{v^2}{m_4^2} + \frac{13c^2+16ac}{b^2}\right) = 1. \tag{46}$$

In (45) it has been taken into account that according to (44), $\Lambda > 0$, therefore to have a correct sign of the coefficient by $\chi_i^*\chi_i$ it is necessary to put $\zeta = 1$, i.e., the direction z is timelike. In obtaining (46) we have used the relations (41) and (44); also, since $v^2/m_4^2 \sim 10^{-32}$, the corresponding additions to quantities of the order of unity are neglected; it is also remembered that λ is of the order of 1.

Furthermore, as is clear from (41), the quantities a and c are not greater than the Planck length by order of magnitude, and from (45) it is evident that $a^2/b^2 \sim 10^{-32}$, so that the scale of the tenth dimension b should be much larger than the Planck one (but still within the empirical constraint $b \lesssim 10^{-17}$ cm corresponding to the TeV energy scale [16]). Moreover, returning to the dimensionless quantities H_a and h_i, one can see that according to (37) they are of the order $\lesssim 10^{-16}$ at $|\chi_i|$ close to their vacuum value $v/\sqrt{2}$. So the assumption $H_aH_a = h_i^*h_i \ll 1$, used in (34) and (35), is well justified.

The Higgs field mass $m_\chi = \lambda v$, according to (41) and (45), satisfies the relation

$$b^2 m_\chi^2 = 2 - c^2 m_4^2/2. \tag{47}$$

By choosing the parameter c it can be made a quantity of the order of the vacuum value v, much smaller than $1/b$, which has the order of (at least) a few TeV. Precisely this is required in the SM [19].

Lastly, from (46) it follows that the parameter η in this model should be rather a large negative quantity, $-\eta \sim 10^{30}v^{-2} \sim 10^{-2}$ cm^2. However, being applied to the gravitational field in our space $\mathbb{M}_1$, the correction ηR^2 cannot be significant at curvatures $R < 1$ cm^{-2}, i.e., at curvature radii larger than 1 cm.

We can conclude that the parameters of the Higgs field potential (42), obtained here due to a specific choice of the extra-dimensional metric components, agree with those involved in the SM.

V. INTERACTION BETWEEN THE HIGGS FIELD AND THE GAUGE FIELDS OF THE SM

We have singled out the components of the metric tensor interpreted as the Higgs bosons. Meanwhile, the way of singling out gauge fields from the extra-dimensional metric is well known. Namely, the metric (1) is represented in a standard form (see, e.g., [17]), where the following components of the total metric (1) will be of interest for us:

$$\overline{g}_{\mu\nu}(x,y) = g_{\mu\nu}(x) + g_{ab}k_i^a(y)A_\mu^i(x)k_j^b(y)A_\nu^j(x), \qquad i,j = 1,2,3,4 \tag{48}$$

$$g_{\mu a}(x,y) = g_{ab}k_i^a(y)A_\mu^i. \tag{49}$$

Here, k_i^a is a Killing vector of the subspace $\mathbb{V}^4$ with the metric g_{ab}, $A_\mu^i(x)$ are gauge fields in the algebra of the symmetry group T of $\mathbb{V}^4$.

The gauge field Lagrangian is obtained in the conventional way from a decomposition of the multidimensional scalar curvature, starting from the ansatz (48) characteristic of the Kaluza-Klein theories. As a result, from the original multidimensional action (2), after integration over the compact submanifolds, one singles out the effective action of the form

$$S_{\text{eff}} = \text{const}\int_{\mathbb{M}_3} d^{d_3}z \int_{\mathbb{M}_1} d^4x\sqrt{|g_{\mu\nu}|}\left(\frac{R_4}{2\varkappa} - \frac{1}{4}\text{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu})\right), \tag{50}$$

which includes the 4D scalar curvature R_4 and the Lagrangian of free Yang-Mills fields [4]. We will no more discuss this well-known part of the Lagrangian. Our purpose will be a consideration of its part containing the Higgs field (see the previous section) and its interaction with the gauge fields.

We will need the structure constants f_{jkl} involved in the equation for the Killing vectors $k_j^a(y)$ since the same structure constants determine the algebra of the four gauge fields $A_\mu^i(x)$. To find the form of the structure constants, let us note that one of the possible representations of the Killing vectors is determined by the matrices t_{ab}^j from (4). Indeed, a displacement along the vectors

$$k_j^a(y) \equiv i t_b^{ja} y^b$$

does not change the metric because t_b^{ja} are generators of the symmetry group. Evidently, the nonzero structure constants f_{jkl} are the structure constants of the group T_2, coinciding with the structure constants ε_{jkl} of the $SU(2)$ group due to their isomorphism (see Eq. (54) below).

To construct a gauge-invariant derivative $D_\mu H$ corresponding to the group $T = T_1 \cdot T_2$ we need its generators. The generator $\hat{t}_0$ of the group T_1 is found directly:

$$\hat{t}_0 = \frac{\partial}{\partial\phi}T_1\Big|_{\phi=0} = \begin{pmatrix} O & -I \\ I & O \end{pmatrix}. \tag{51}$$

The generators $\hat{t}_j$ $(j = 1,2,3)$ of the group T_2 are easily constructed from the generators $\tau_j = -i\sigma_j/2$ of the group $SU(2)$ satisfying the commutation relations $[\tau_j, \tau_k] = \epsilon_{jkl}\tau_l$, where σ_j are the Pauli matrices. Since $\tau_j \equiv \partial\omega_2/\partial\theta_j\big|_{\theta=0}$, taking into account (6) and (8), we obtain

$$\hat{t}_j = \frac{\partial}{\partial\theta_j}T_2\Big|_{\theta=0} = \begin{pmatrix} \text{Re}(\tau_j) & -\text{Im}(\tau_j) \\ \text{Im}(\tau_j) & \text{Re}(\tau_j) \end{pmatrix}. \tag{52}$$

Since τ_2 is real while τ_1 and τ_3 are pure imaginary, we can write

$$\hat{t}_1 = \begin{pmatrix} O & i\tau_1 \\ -i\tau_1 & O \end{pmatrix}, \qquad \hat{t}_2 = \begin{pmatrix} \tau_2 & O \\ O & \tau_2 \end{pmatrix}, \qquad \hat{t}_3 = \begin{pmatrix} O & i\tau_3 \\ -i\tau_3 & O \end{pmatrix}. \tag{53}$$

It is easy to verify that the corresponding commutators coincide with the commutators of the $SU(2)$ group:

$$[\hat{t}_j, \hat{t}_k] = \epsilon_{jkl}\hat{t}_l, \tag{54}$$

which corresponds to isomorphism between representations of the groups T_2 and $SU(2)$.

The structure of the term $L_{\text{int}}(A,H)$, containing interaction between the field H and the gauge fields A_μ, can be obtained from general considerations. Since the initial Lagrangian is invariant under general coordinate transformations and therefore under those belonging to the group T, the fields A and H must enter into the Lagrangian in a certain gauge-invariant combination. The latter is well-known and has the form

$$\begin{aligned} L_{\rm int}(A,H) &= \mathcal{N} g^{\mu\nu}(D_\mu H)^+(D_\nu H), \\ (D_\mu H)_a &= \left(\delta_{ab}\partial_\mu + A^i_\mu(x)\hat{t}_{i,ab}\right) H_b, \end{aligned} \tag{55}$$

where $\mathcal{N}$ is a certain constant factor determined from the expression for the kinetic term of the Higgs field. The coupling constants and the charge factors are normalized to unity. It is convenient to split the set of gauge fields according to the factor groups $SU(2)$ and $U(1)$:

$$A^j_\mu(x)\hat{t}_{j,ab} \equiv \sum_{m=1,2,3} A^m_\mu(x)\hat{t}_{m,ab} + B_\mu(x)\hat{t}_{0,ab}.$$

To pass over to the full set of dynamic variables of the SM, it is necessary to express the Lagrangian $L_{\rm int}(A,H)$ in terms of the complex scalar doublet h. An explicit transition from the H-representation to the h-representation is given by the relations

$$X = \frac{1}{2}(h^* + h), \qquad Y = \frac{i}{2}(h^* - h), \tag{56}$$

easily obtainable from (16) along with Eqs. (53) for the generators. The term $L_{\rm int}(A,H)$ then becomes a certain function $\tilde{L}_{\rm int}(A,h)$ of the fields h and their derivatives:

$$L_{\rm int}(A^j_\mu(x), H_a(x)) = \tilde{L}_{\rm int}(A^j_\mu(x), h_i(x)). \tag{57}$$

Let us note that the above-mentioned invariance of the Lagrangian $L(A,H)$ under transformations from the group T implies $SU(2)\times U(1)$ invariance of the Lagrangian $\tilde{L}(A,h)$. Indeed, suppose that, in the space of fields H, a T-transformation $H' = TH$ has been performed; in the space of fields h it corresponds to an $SU(2)\times U(1)$ transformation $h' = \omega_1\omega_2 h$. The fields of the pair (H', h') are connected with each other by the same transformation P from (19) as the fields of the pair (H,h). Therefore $L(A',H') = \tilde{L}(A',h')$ similarly to (57). On the other hand, due to the T-invariance, $L(A,H) = L(A',H')$. Unifying these equalities into a single chain, we obtain $\tilde{L}(A',h') = L(A',H') = L(A,H) = \tilde{L}(A,h)$, whence it follows that

$$\tilde{L}(A,h) = \text{inv}\ [SU(2)\times U(1)]. \tag{58}$$

The form of the expression bilinear with respect to the Higgs fields and invariant with respect to the group $SU(2)\times U(1)$, is well known:

$$\tilde{L}_{\rm int}(A,h) = \mathcal{N} g^{\mu\nu}(D_\mu h)^+(D_\nu h), \tag{59}$$

where $\mathcal{N}$ is a constant factor and the gauge-invariant derivative of the field h has the form

$$D_\mu h \equiv (\partial_\mu + A^m_\mu \tau_m + B_\mu I)h.$$

Further, by analogy with the previous part, we substitute the ansatz (37) into Eq. (59) and, integrating over all extra dimensions, we finally obtain

$$\tilde{L}_{\rm int}(A,\chi) = \mathcal{N}' g^{\mu\nu}(D_\mu \chi)^+(D_\nu \chi). \tag{60}$$

The normalization factor $\mathcal{N}' = 1$, as follows from comparison with the factor at the kinetic term in (38).

The expression (60) contains the standard form of interaction between the gauge fields and the Higgs field belonging to the fundamental representation of the gauge group and corresponding to the boson sector structure of the SM.

VI. DISCUSSION

We have shown in this paper that the boson sector of the unified weak and electromagnetic interactions can be reproduced in the framework of multidimensional gravity. In doing so, it has been sufficient to introduce a 5D compact extra space possessing a certain symmetry. It has turn out to be unnecessary to introduce such extended structures as strings or branes. No additional fields apart from the metric tensor have been used. Thus we have been able to indicate a possible purely geometric origin of the Higgs field.

One of the attractive features of the idea of extra dimensions in the Kaluza-Klein spirit is a natural way of introducing gauge symmetries: the off-diagonal components of the metric play the part of gauge fields. We have shown that the Higgs field can also be introduced in the framework of this paradigm: namely, we have found such components of the extra-dimensional metric that can be interpreted as the Higgs field with a correct transformation law under the action of the group $SU(2) \otimes U(1)$ and a standard coupling to the gauge fields. The potential of the Higgs field also has its standard form.

There are a number of ways to extend the SM in order to get rid of its shortcomings (see, e.g., [20]). Our results also coincide with the SM in the first approximation only. Thus, the Higgs field potential is quite a complicated function, coinciding with the well-known expression only at low energies. Apart from the Higgs bosons, there are a number of other scalar and vector fields, which is inherent to an approach employing extra dimensions. Distinctions from the SM, existing in this model, can be used for a correct interpretation of new results expected at the LHC.

Acknowledgments

This work was supported in part by the Russian Foundation for Basic Research (project no. 09-02-00677-a) and by the Federal Program "Research and Pedagogical Specialists of Innovative Russia for 2009-2013". SB and KB were also partly supported by NPK MU grant at PFUR.

---

[1] M. Carena, E. Ponton, and J. Zurita, Phys. Rev. D **82**, 055025 (2010); arXiv: 1005.4887.

[2] K.A. Bronnikov, S.G. Rubin, and I.V. Svadkovsky, Phys. Rev. D **81**, 084010 (2010); arXiv: 0912.4862. Grav. Cosmol. **15**, 32 (2009).

[3] F.L. Bezrukov, A. Magnin, and M. Shaposhnikov, Phys. Lett. B **675**, 88-92 (2009);, arXiv: 0812.4950.

[4] M. Blagojevic, *Gravitation and Gauge Symmetries* (Institute of Physics, Bristol, 2002).

[5] F. Cianfrani and G. Montani, Nuovo Cim. **122B**, 213-218 (2007); gr-qc/0701079

[6] K.A. Bronnikov and S.G. Rubin, Phys. Rev. D 73, 124019 (2006); gr-qc/0510107; K.A. Bronnikov, R.V. Konoplich and S.G. Rubin, Class. Quantum Grav. **24**, 1261 (2007); gr-qc/0610003.

[7] Yu.S. Vladimirov, *The Dimension of Physical Space-Time and Unification of Interactions* (Moscow Univ. Press, Moscow, 1987).

[8] V.A. Klimenkov and Yu.S. Vladimirov, Grav. Cosmol. **10**, 77 (2004).

[9] Yu.S. Vladimirov and S.V. Bolokhov, Gen. Rel. Grav. **37**, 2227 (2005).

[10] I.P. Volobuyev and Yu.A. Kubyshin, Teor. Mat. Fiz. **68**, 225 (1986).

[11] I.P. Volobuyev, Yu.A. Kubyshin, and J.M. Mourao, Teor. Mat. Fiz. **78**, 267 (1989).

[12] A.J. Niemi and S. Slizovskiy, Embedding Weinberg-Salam in Kaluza-Klein, arXiv: 1004.0212.

[13] N. Haba, Sh. Matsumoto, N. Okada, and T. Yamashita, Prog. Theor. Phys. **120**, 77 (2008); arXiv: 0802.3431. Yu. Hosotani, Pyungwon Ko, and M. Tanaka, Phys. Lett. B **680**, 179 (2009); arXiv: 0908.0212.

[14] A.A. Grib, S.G. Mamayev and V.M. Mostepanenko, *Vacuum Quantum Effects in Strong Fields* (Friedmann Laboratory Publ., St. Petersburg, 1994).

[15] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space.* (Cambridge-London-New York-Sydney, Cambridge Univ. Press, 1982).

[16] I. Antoniadis, Phys. Lett. B **246**, 377 (1990)

[17] P.S. Wesson and J.M. Overduin, gr-qc/9805018.

[18] S.G. Rubin, Sov. Phys. JETP **109**, 961 (2009).

[19] L.B. Okun, *Leptons and Quarks* (Elsevier, 1985).

[20] R. Foot, A. Kobakhidze, and R.R. Volkas, Phys. Lett. B **655**, 156 (2007); arXiv: 0704.1165.