Poor man's holography: How far can it go?

Yu Tian*

College of Physical Sciences, Graduate University of Chinese Academy of Sciences, Beijing 100049, China

Xiao-Ning Wu†

Institute of Mathematics, Academy of Mathematics and System Science, CAS, Beijing 100190, China

Hongbao Zhang‡

Crete Center for Theoretical Physics, Department of Physics, University of Crete, Heraklion 71003, Greece

Abstract

One of the most exciting things in recent theoretical physics is the suspicion that gravity may be holographic, dual to some sort of quantum field theory living on the boundary with one less dimension. Such a suspicion has been supported mainly by a variety of specific examples from string theory. This paper is intended to purport the holographic gravity from a different perspective. Namely we propose that such a holography can actually be observed within the context of Einstein's gravity, where neither is spacetime required to be asymptotically AdS nor the boundary to be located at conformal infinity. We show that our holography works remarkably well at least at the level of thermodynamics and hydrodynamics. In particular, a perfect matching between the bulk gravity and boundary system is found not only for the equilibrium variation but also for the non-equilibrium entropy production, where a method of conserved current is seen to be efficient in relating the black hole perturbation in the bulk gravity and the non-equilibrium thermodynamics on the boundary.

*Electronic address: ytian@gucas.ac.cn

†Electronic address: wuxn@amss.ac.cn

‡Electronic address: hzhang@physics.uoc.gr

I. INTRODUCTION

Traditional black hole thermodynamics dealt with infinitesimal variations between stationary black hole configurations, until Iyer and Wald generalized the discussion to arbitrary linear perturbations around a stationary black hole background by the method of conserved current [1]. This situation is just like the generalization of equilibrium thermodynamics to the non-equilibrium (but near equilibrium) case, which is nowadays standard in textbooks [2]. In the black hole context, it eats up perturbations with an increase of its horizon area, while in the thermodynamics context, transportation smoothes out non-equilibrium with production of entropy. But can these two things be directly related to each other? Imagine the thermodynamic system living on a constant r surface in a spherically symmetric black hole, which is a holographic setup slightly generalizing the well studied ones based on the AdS/CFT correspondence [3, 4]. Then one can check, under appropriate holographic dictionary, whether the increase of the black hole entropy and the production of the thermodynamic entropy matches each other, which is the main motivation of this letter.

Our starting point is the following (Euclidean) holographic principle

$$Z_{\text{bulk}}[\bar{\phi}] = \int D\psi \exp(-I_{\text{FT}}[\bar{\phi}, \psi]) \tag{1}$$

for some quantum gravity theory with partition function $Z_{\text{bulk}}[\bar{\phi}]$ on some bulk space-time region and the corresponding quantum field theory with action $I_{\text{FT}}[\bar{\phi}, \psi]$ on its boundary, which is the refined and generalized version of Witten's principle [4]. Here the partition function $Z_{\text{bulk}}[\bar{\phi}]$ is evaluated by fixing the boundary value of the bulk field ϕ to be $\bar{\phi}$, which acts as some background field on the boundary, and ψ denotes all the dynamical fields in the boundary theory, which is integrated out to produce the partition function in the right hand side of (1). To be more precise, if ϕ is the metric or form fields, then the pull back of ϕ to the boundary is fixed to be $\bar{\phi}$. Infinitesimal variation of $\bar{\phi}$ in (1) gives

$$Z_{\text{bulk}}[\bar{\phi} + \delta\bar{\phi}] = Z_{\text{bulk}}[\bar{\phi}] \left\langle \exp \int_{\text{bdry}} \delta\bar{\phi} O_\phi \sqrt{\bar{g}} d^d x \right\rangle_{\text{FT}}$$

with $\sqrt{\bar{g}} d^d x$ the standard volume element on the boundary and

$$O_\phi(x) = -\frac{1}{\sqrt{\bar{g}}} \frac{\delta I_{\text{FT}}[\bar{\phi}, \psi]}{\delta\bar{\phi}(x)} \tag{2}$$

the "dual field", which should be understood as the corresponding quantum operator in the expression of expectation value.

In the classical (saddle point) approximation, the bulk partition function is given by

$$Z_{\text{bulk}}[\bar{\phi}] = \exp(-I_{\text{bulk}}[\bar{\phi}])$$

with $I_{\text{bulk}}[\bar{\phi}]$ the on-shell action (Hamilton's principal functional). So the above holographic principle leads to

$$-\frac{\delta I_{\text{bulk}}[\bar{\phi}]}{\delta \bar{\phi}(x)} = \sqrt{\bar{g}}\, \langle O_\phi(x) \rangle_{\text{FT}}\,, \tag{3}$$

where the left hand side is just the canonical momentum conjugate to ϕ by virtue of the Hamilton-Jacobi equation regarding the boundary as the "time" slice. Now turn to the Minkowskian signature. The discussion in this case is similar to the above, but subtleties arise when one further considers correlation functions [5], which does not concern us in the present letter. For the bulk to be (asymptotic) AdS space-time and the boundary to be its conformal boundary, it is well known that the dual field theory is a (local) CFT. But in more general cases, e.g. asymptotically flat bulk and/or boundary at finite distance [6, 7], the field theory may be both non-conformal and non-local [8], which is not easy to study. However, macroscopic aspects of the general cases (called the general bulk/boundary correspondence) can still be understood. In the macroscopic point of view, the boundary system is described by thermodynamics and hydrodynamics, where we identify the expectation value in (3) with the macroscopic (classical) mechanical quantity $O_\phi(x)$.

Two examples are of special interest. One is the case of ϕ to be the metric $g_{\mu\nu}$, where $\bar{\phi}$ is just the induced metric $\bar{g}_{ab}$ on the boundary. Then the Minkowskian version of (3) tells us that the stress-energy tensor of the boundary system is given by the Brown-York tensor (see (10) for the explicit form)

$$t_{ab}(x) = \frac{2}{\sqrt{-\bar{g}}} \frac{\delta I_{\text{bulk}}[\bar{g}]}{\delta \bar{g}_{ab}(x)}, \tag{4}$$

where the bulk action is taken to be the standard Einstein-Hilbert action plus the Gibbons-Hawking term. The other is the case of ϕ to be the electromagnetic potential A_μ. Similarly, the dictionary is that the electric current of the boundary system is given by

$$j^a(x) = \frac{1}{\sqrt{-\bar{g}}} \frac{\delta I_{\text{bulk}}[\bar{A}]}{\delta \bar{A}_a(x)} = -n_\mu F^{\mu a}, \tag{5}$$

where the bulk action is just the Maxwell one in addition to the gravitational part and n^μ the outward normal vector to the boundary. Now we explore the macroscopic aspects of the general bulk/boundary correspondence in both equilibrium and non-equilibrium.

II. THE CORRESPONDENCE IN EQUILIBRIUM: THERMODYNAMICS

First of all, we must check the consistency of basic thermodynamic relations in the correspondence. Consider the $(d+1)$-dimensional Reissner-Nödstrom (RN) black hole

$$ds^2_{d+1} = \frac{dr^2}{f(r)} - f(r)dt^2 + r^2 d\Omega^2_{d-1},$$
$$f(r) = \varepsilon + \frac{r^2}{l^2} - \frac{2m}{r^{d-2}} + \frac{Q^2}{r^{2d-4}}, \qquad d\Omega^2_{d-1} = \hat{g}^{(\varepsilon)}_{ij}(x)dx^i dx^j,$$
$$A = \sqrt{\frac{d-1}{8\pi(d-2)G}}\frac{Q}{r^{d-2}}dt \tag{6}$$

with negative cosmological constant[1] in the Einstein-Maxwell theory as our bulk space-time (in equilibrium). Here m is the mass parameter, Q the charge parameter of the black hole, and $\hat{g}^{(\varepsilon)}_{ij}(x)$ the metric on the "unit" sphere, plane or hyperbola for ε equal to 1, 0 or -1 respectively, where in the planar or hyperbolic case some standard compactification is assumed. The boundary is the hypersurface $r = r_c$ outside the outer horizon, with an induced metric

$$ds^2_d = -f_c dt^2 + r_c^2 d\Omega^2_{d-1}, \qquad f_c := f(r_c). \tag{7}$$

Due to static nature (with time-like Killing vector ∂_t) of both the bulk space-time and the boundary, and maximum symmetry on a time slice of the boundary, the boundary system is obviously in equilibrium. From the identification (1) of the Euclidean partition function, an argument of conical singularity leads to the conclusion that the entropy and temperature of the boundary system are equal to the Bekestein-Hawking entropy

$$S = \frac{\Omega^{(\varepsilon)}_{d-1} r_h^{d-1}}{4G} \tag{8}$$

and local (red shifted) Hawking temperature [6]

$$T = \frac{T_H}{\sqrt{f_c}} = \frac{f'(r_h)}{4\pi\sqrt{f_c}} \tag{9}$$

of the bulk black hole. Here $\Omega^{(\varepsilon)}_{d-1}$ is the volume of the "unit" sphere, plane or hyperbola, and r_h the radius of the outer horizon satisfying $f(r_h) = 0$. Due to the bulk/boundary dictionary (4), the stress-energy tensor of the boundary system is given by the Brown-York tensor

$$t_{ab} = \frac{1}{8\pi G}(Kg_{ab} - K_{ab} - Cg_{ab}), \qquad K := K_{ab}g^{ab} \tag{10}$$

[1] The case with positive or vanishing cosmological constant can also be included by formally allowing $l^2 < 0$ or $l^2 \to \infty$ respectively, where ε can only be positive [16].

on the boundary with K_{ab} its extrinsic curvature and C some constant, which can be easily shown to have a form of ideal fluid

$$t_{ab} = \epsilon u_a u_b + p(u_a u_b + g_{ab})$$

with the velocity $u_a = (-\sqrt{f_c}, 0, \cdots, 0)$, the energy density

$$\epsilon = -\frac{d-1}{8\pi G}\frac{\sqrt{f_c}}{r_c} + C, \tag{11}$$

and the pressure

$$p = \frac{d-2}{8\pi G}\frac{\sqrt{f_c}}{r_c} + \frac{1}{16\pi G}\frac{f'_c}{\sqrt{f_c}} - C. \tag{12}$$

As well, the electric current (5) of the boundary system is

$$j^a = -n_\mu F^{\mu a}(r_c) = (-\sqrt{\frac{(d-1)(d-2)}{8\pi G f_c}}\frac{Q}{r_c^{d-1}}, 0, \cdots, 0). \tag{13}$$

Since the volume of the boundary system is

$$V = \Omega_{d-1}^{(\varepsilon)} r_c^{d-1}, \tag{14}$$

the energy density (11) gives the total energy

$$E = \Omega_{d-1}^{(\varepsilon)}(-\frac{d-1}{8\pi G}\sqrt{f_c} r_c^{d-2} + C r_c^{d-1}), \tag{15}$$

while the electric current (13) gives the total charge

$$\Omega_{d-1}^{(\varepsilon)}\sqrt{\frac{(d-1)(d-2)}{8\pi G}}Q$$

that coincides with the physical charge of the black hole. The proportion coefficient here is not essential, so we will take Q as the total charge in the following discussion.

From the expressions (8,9,12,14) above, recasting E in (15) as a function of (S, V, Q) (eliminating m by the condition $f(r_h) = 0$), one can check that the standard thermodynamic relations

$$\frac{\partial E}{\partial S} = T, \qquad \frac{\partial E}{\partial V} = -p \tag{16}$$

hold. Furthermore, one can obtain

$$\mu = \frac{\partial E}{\partial Q} = -\frac{d-1}{8\pi G}\frac{\Omega_{d-1}^{(\varepsilon)} Q}{\sqrt{f_c}}(\frac{1}{r_c^{d-2}} - \frac{1}{r_h^{d-2}}), \tag{17}$$

which is proportional to the difference of electric potential between the horizon and the boundary, and is the appropriate generalization of the familiar chemical potential in AdS/CFT ($r_c \to \infty$). Thus, we see that the first law

$$dE + pdV = TdS + \mu dQ \tag{18}$$

of thermodynamics holds for the boundary system.

III. THE CORRESPONDENCE IN NON-EQUILIBRIUM: ENTROPY PRODUCTION

If the boundary system is perturbed by some sort of external sources, various transport processes occur intending to bring the system back to equilibrium, which causes entropy production. From the bulk point of view, the ingoing boundary condition at the future horizon implies that the (material or gravitational) perturbations at the boundary should propagate to the black hole and be absorbed, which causes increase of the area of the black hole horizon. Based on the equilibrium configuration we have discussed above, there are three kinds of transport processes that we can consider, i.e. heat conduction, viscosity of fluid and charge conduction. The heat conduction is energy transportation, caused by temperature inhomogeneity. The viscosity of fluid is momentum transportation, caused by velocity inhomogeneity (shear, more precisely). The charge conduction is caused by external electric field or inhomogeneity of chemical potential. A non-relativistic framework (but allowing a curved space) is enough for small perturbations around our equilibrium configuration, with the total entropy production rate [2]

$$\Sigma = \mathbf{j}_q \cdot \nabla \frac{1}{T} - \frac{1}{T}\Pi : \nabla \mathbf{u} + \frac{1}{T}\mathbf{j}\cdot\mathbf{E} = j_q^i \nabla_i \frac{1}{T} - \frac{1}{T}\Pi^{ij}\sigma_{ij} + \frac{1}{T} j^i E_i, \tag{19}$$

where $\mathbf{j}_q$ is the heat current, Π^{ij} the dissipative part of the stress-energy tensor, $\sigma_{ij} = \nabla_{(i}u_{j)}$ the shear (assuming $\nabla \cdot \mathbf{u} = 0$), $\mathbf{j}$ the electric current, $\mathbf{E}$ the electric field, and we have assumed a homogeneous chemical potential. In fact, the physical laws of transportation tell us that the transport current $(j_q^i, \Pi^{ij}, j^i, \cdots)$ is proportional (in the linear regime) to the driving force $(\nabla_i \frac{1}{T}, -\frac{1}{T}\sigma_{ij}, \frac{1}{T}E_i, \cdots)$, while the entropy production rate is just their product. In the holographic context, this proportion factor (matrix), i.e. the transport coefficients, is determined by imposing the ingoing boundary condition at the horizon and then solving the bulk equations of motion (see e.g. [9, 10] for the traditional AdS/CFT case and [6, 11, 12] for the "finite cutoff" case). However, we do not need the precise values of them here.

It is well known in AdS/CFT that the temperature inhomogeneity and shear can both be realized by gravitational perturbations, at least for some special configurations (see e.g. [9, 10] and [13] respectively). Now we generalize the analyses to arbitrary (but small) temperature perturbation and shear field on the boundary hypersurface $r = r_c$. The temperature perturbation can be introduced by the metric perturbation

$$ds_d^2 \to ds_d^2 + 2h_{ti}dtdx^i,$$

generalizing the discussion in [9, 10], as

$$\nabla_i \frac{1}{T} = \frac{1}{f_c T}\partial_t h_{ti} \tag{20}$$

with ∇_a the background covariant derivative on the boundary. Then, we insist on the frame $u^a = (1/\sqrt{f_c}, 0, \cdots, 0)$, while turn on the shear by an off-diagonal space metric perturbation

$$ds_d^2 \to ds_d^2 + h_{ij}dx^i dx^j, \qquad i \neq j.$$

In this case the shear reads

$$\sigma_{ij} = \partial_{(i}u_{j)} - \gamma^a_{ij}u_a = \frac{1}{2\sqrt{f_c}}(\partial_i g_{tj} + \partial_j g_{ti}) - \frac{\gamma_{tij}}{\sqrt{f_c}} = \frac{1}{2\sqrt{f_c}}\partial_t h_{ij}. \tag{21}$$

The heat current j^i_q is just the energy current $-\sqrt{f_c}t^{ti}$, while Π^{ij} is just the traceless part of t^{ij}. So from (19) we have the total entropy production rate

$$\Sigma = -\frac{1}{2\sqrt{f_c}T}t^{ab}\partial_t h_{ab} - \frac{1}{\sqrt{f_c}T}n_\mu F^{\mu i}(r_c)F_{ti}(r_c), \tag{22}$$

where the contribution from the charge conduction is simply realized by electromagnetic perturbations under the holographic dictionary (5).

Our central task is to check whether (22) matches the black hole side. For clarity, we first assume that the equilibrium background is uncharged, i.e. $Q = 0$. Since in this case the gravitational perturbation and electromagnetic perturbation are decoupled from each other, it turns out that the first two kinds of transport processes and the charge conduction are decoupled, which allows us to discuss them separately. As a warm-up, we first consider electromagnetic perturbations, which clearly illustrates our approach. For convenience, we take the gauge $n^\mu A_\mu = 0$, so the entropy production rate (22) on the boundary is obviously of order $\mathcal{O}(A_a^2)$. On the bulk side, the physical picture is that the electromagnetic wave caused by the boundary perturbation propagates to the black hole, which will be absorbed and render the horizon area to increase. Here we use a conserved current to relate the horizon and the boundary. Since $\xi = \partial_t$ is Killing and the stress-energy tensor $T_{\mu\nu}$ of the wave satisfies $D_\mu T^{\mu\nu} = 0$ with respect to the background covariant derivative D_μ, we have the conservation law

$$D_\mu(T^\mu_\nu \xi^\nu) = 0 \tag{23}$$

of the current $T^\mu_\nu \xi^\nu = T^\mu_t$. Suppose the non-equilibrium region has compact support on the boundary, which naturally gives rise to the corresponding compact support for both perturbed bulk and perturbed horizon. Then integrating the above equation over the perturbed bulk and using Gauss law, we end up with

$$\int_H T^\mu_t \lambda_\mu = \int_{\text{bdry}} T^\mu_t n_\mu. \tag{24}$$

where H is the perturbed horizon and λ^μ is the affinely parameterized null generators of H. So we have The standard technique of Raychaudhuri equation implies [14]

$$\int_H T^\mu_t \lambda_\mu = T_H \delta S. \tag{25}$$

The integrand in the right hand side of (24) is just

$$n_\mu T^\mu_t(r_c) = -n_\mu F^{\mu i}(r_c) F_{ti}(r_c), \tag{26}$$

which together with (25) and (22) without the gravitational part gives

$$\delta S = \frac{1}{T_H} \int_{\text{bdry}} n_\mu T^\mu_t(r_c) = \int_{\text{bdry}} \Sigma, \tag{27}$$

where we have used the relation $T_H = \sqrt{f_c}T$. Thus we conclude that the entropy increase on the bulk side and the entropy production on the boundary side match exactly.

Next, we consider gravitational perturbations, realizing the heat conduction and viscosity of fluid. We introduce the gravitational perturbation with gauge

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}, \qquad n^\mu h_{\mu\nu} = 0 \tag{28}$$

in the bulk,[2] while in addition all diagonal elements of $h_{\mu\nu}$ vanish on the boundary. The

[2] Rigorously speaking, the Schwarzschild coordinates are not perfect for our discussion that involves in-falling processes. But our conclusion can be shown to hold in the Eddinton coordinates.

above metric implies the extrinsic curvature

$$K_{ab} = \frac{1}{2}\mathcal{L}_n g_{ab} = \frac{1}{2}n^\mu \partial_\mu g_{ab} \to \frac{1}{2}n^\mu \partial_\mu (g_{ab} + h_{ab}) \tag{29}$$

for any hypersurface of constant r, up to $\mathcal{O}(h_{ab})$ even after perturbation (28). On the boundary side, since the background Brown-York tensor $t^{(0)}_{ab}$ has no off-diagonal elements, the entropy production rate (22) is of order $\mathcal{O}(h^2_{ab})$. To leading order, the entropy production rate (22) without the electromagnetic part can be worked out as

$$\Sigma = -\frac{1}{16\pi G\sqrt{f_c}T}(-K^{(0)}h_{ab}\partial_t h^{ab} - \frac{1}{2}n^\mu \partial_\mu h_{ab}\partial_t h^{ab} + 2K^{(0)c}_b h_{ca}\partial_t h^{ab}), \tag{30}$$

where we have used the fact that the trace of h_{ab} vanishes on the boundary. Here the indices are lowered (or raised) with the background metric g_{ab}. From the theory of gravitational waves (see, e.g. [15]), we know that the effective stress-energy tensor $T_{\mu\nu}$ of the wave is just $-\frac{1}{8\pi G}$ times $G^{(2)}_{\mu\nu}$, the second order contribution of $h_{\mu\nu}$ to the Einstein tensor, which satisfies $D_\mu T^{\mu\nu} = 0$ to order $\mathcal{O}(h^2_{ab})$. Then similar strategy as in the electromagnetic case follows. Especially, to evaluate the right hand side of (24), we should know the explicit form of $G^{(2)\mu}_t n_\mu$. Some lengthy but straightforward calculation [16] gives

$$n_\mu G^{(2)\mu}_t(r_c) = \frac{1}{2\sqrt{f_c}}(-\frac{1}{2}n^\mu \partial_\mu h_{ab}\nabla_t h^{ab} + 2K^{(0)c}_a h_{cb}\nabla_t h^{ab} - K^{(0)a}_t h^{bc}\nabla_a h_{bc} + \nabla_a J^a), \tag{31}$$

where $\nabla_t = \partial_t$ in our case and we do not need the explicit form of the order $\mathcal{O}(h^2_{ab})$ current J^a. Substituting the above equation into (24) and noting the "momentum constraints"

$$\nabla_a t^{(0)ab} = \frac{1}{8\pi G}\nabla_a(K^{(0)}g^{ab} - K^{(0)ab}) = 0 \tag{32}$$

when comparing with (30), we again obtain

$$\delta S = \int_{\text{bdry}} \Sigma.$$

To sum up, for the uncharged background, we see perfect matching between the entropy production from the above three kinds of transport processes on the boundary and the entropy increase of the black hole in the bulk.

The charged case ($Q \neq 0$) is a little more complicated, which corresponds to a non-vanishing chemical potential (17) on the boundary side. From the transportation point of view, in this case the heat conduction and charge conduction are coupled to each other, which has a nice dual description on the bulk side [9]. So we must treat the electromagnetic and gravitational perturbations altogether here. Fortunately, it turns out that we still have a conserved current of the form

$$J^{\mu}_{\rm EM} + J^{\mu}_{\rm G} - \frac{1}{2}q^{\mu\nu}G^{(0)}_{\mu\nu}\delta^{\mu}_{t} - q^{\mu\nu}G^{(0)}_{\nu t} + \frac{1}{2}q^{\nu}_{\nu}G^{(0)\mu}_{t}, \tag{33}$$

where $J^{\mu}_{\rm EM}$ and $J^{\mu}_{\rm G}$ are the electromagnetic and gravitational conserved current, respectively, $q_{\mu\nu}$ the induced second order metric perturbation, and $G^{(0)}_{\mu\nu}$ the background Einstein tensor [16]. However, under the gauge $n^{\mu}q_{\mu\nu} = 0$ and using the concrete configuration (6), the last three terms in (33) do not contribute in (24). Eventually, it can be shown that $J^{\mu}_{\rm EM} + J^{\mu}_{\rm G}$ correctly gives the bulk entropy increase on one hand and the boundary entropy production on the other.

IV. DISCUSSION

We have shown the bulk/boundary correspondence at least at the level of thermodynamics and hydrodynamics, where in particular, a perfect matching between the bulk gravity and boundary system is exactly derived for entropy production on both sides by resorting to the conserved current. Compared to AdS/CFT correspondence, our bulk/boundary correspondence is more general in the following sense. First, we do not require the bulk space-time to be asymptotically AdS. Second, our boundary is not required to be located at conformal infinity. Actually our discussion can be applied to any bulk space-time with metric

$$ds^2_{d+1} = \frac{dr^2}{f(r)} - f(r)dt^2 + a(r)d\Omega^2_{d-1},$$

i.e. static with spatial (excluding r) homogeneity and isotropy, satisfying Einstein's equation. The Rindler case recently introduced in the gravity/fluid correspondence [7] belongs to this class, in which our approach can be checked to work. On the other hand, our boundary system, by construction, is not necessarily conformal, which implies that the entropy can also been produced by the bulk viscosity on the boundary. Using the same approach, one can actually show that the bulk/boundary correspondence also exists for such an entropy production [16].

We conclude with various issues worthy of further investigation. For one thing, we have worked only to second order perturbation so far. It is interesting to see if the whole procedure can be made to any higher order. For another, we have worked merely within the context of

Einstein's gravity with Maxwell field. It is worthwhile to see if our correspondence can also be valid for the cases with other matter fields and even higher derivative gravity theories. Finally, it is interesting to explore the relation between the conserved current in [1] and ours. Moreover, the meaning of the conserved current (33) in the charged case seems rather unclear, as well as the corresponding conserved quantity. We hope to address these issues elsewhere.

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