Emergence of symmetries

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May 8, 2012

Abstract

The mechanism of gauge symmetry formation is discussed in the framework of multidimensional gravity. It is shown that this process is strictly connected to the entropy decrease of compact space. The existence of gauge symmetries is not postulated from the beginning. They could be absent during the inflationary stage. The conditions of this effect are discussed.

1 Introduction

The idea of multidimensional space-time allows to clarify some fundamental questions, such as the problems of modern cosmology and the Standard Model which are discussed in terms of extra-dimensional gravity [1–7]. As was shown in [8], the numerical values of the fundamental parameters depend on geometry of extra dimensions. The existence of gauge symmetries may be related to isometries of extra space[1] [8–10].

(Maximally) symmetric metrics of extra space as a starting point are among the most popular in the literature, see e.g. [9, 11, 12]. This assumption makes it possible to obtain clear and valuable results. In particular, specific symmetries of extra space manifest as gauge symmetries of effective low-energy physics. At the same time we must take into account the quantum origin of space itself due to fluctuations in the space-time foam. There is no reason to assume that the geometry or/and topology of extra space is simple [13, 14]. Moreover it seems obvious that a measure $\mathfrak{M}$ of all symmetrical spaces equals zero so that the probability of their nucleation $\mathcal{P} = 0$. Hence some period of extra space symmetrization has to exist [8, 15–18].

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[1]Hereinafter we consider compact extra spaces

In the present paper we investigate the entropic mechanism of space symmetrization after its nucleation. It is shown that the stabilization of the extra space and its symmetrization are proceeding simultaneously. This process is accompanied by a decrease in entropy for the extra space and an increase in entropy for main one.

2 Time dependence of compact space geometry

As was mentioned above some mechanism of the extra space symmetrization should exist. In this Section we consider some toy models to clarify the situation.

As a common basis, consider a Riemannian manifold

$$T \times M \times M' \tag{1}$$

with the metric

$$ds^2 = G_{AB}dX^A dX^B = dt^2 - g_{mn}(t,x)dx^m dx^n - \gamma_{ab}(t,x,y)dy^a dy^b. \tag{2}$$

Here M, M' are the manifolds with spacelike metrics $g_{mn}(t,x)$ and $\gamma_{ab}(t,x,y)$ respectively, T denotes the timelike direction. The set of coordinates of the subspaces M is denoted by x; y is the same for M'. We will refer to M and M' as a main space and a compact extra space respectively. The curvature of the manifold is assumed to be arbitrary.

Firstly, consider the $(d+1)$-dimensional compact manifold $M' \times T$ with metric

$$ds^2 = dt^2 - \gamma_{ab}(t,y)dy^a dy^b; \quad \gamma_{ab}(y,t) = \eta_{ab} + h_{ab}(t,y).$$

For the Einstein-Hilbert action

$$S = \int d^d y dt \sqrt{|\gamma|} R \tag{3}$$

and in the limit $h_{ab} \ll 1$, classical equations have the form [10]

$$\Box_{d+1} h_{ab} = 0, \tag{4}$$

where

$$\Box_{d+1} \equiv \frac{1}{\sqrt{\gamma}}\partial_0(\sqrt{\gamma}\partial_0) + \frac{1}{\sqrt{\gamma}}\partial_a(\sqrt{\gamma}\gamma^{ab}\partial_b).$$

This wave equation has no static symmetrical solutions if initial conditions are arbitrary. This would mean the absence of gauge symmetries even in the modern epoch, which is unacceptable.

The situation changes considerably if we take into account the dynamics of the main manifold M. Let it possess the Friedmann-Robertson-Walker (FRW) metric and the scale factor $a(t)$ (we assume $\dot{a}(t) > 0$). The equation of motion for the metrics of the extra space M' acquires the form

$$\Box_{d+1} h_{ab} + 3H\dot{h}_{ab} = 0, \tag{5}$$

where the Hubble parameter $H = \dot{a}/a > 0$. We also took into account the form of metrics (2) and equality

$$\frac{1}{\sqrt{g}}\partial_0\sqrt{g} = 3\frac{\dot{a}}{a} = 3H > 0 \tag{6}$$

valid for 4-dimensional FRW space. The term $3H\dot{h}_{ab}$ in (5) indicates the presence of friction and gives the asymptotic $\gamma_{ab} \to \mathrm{const}$ for $t \to +\infty$.

So the dynamics of the main space M could cause the stabilization of the extra space M'. Note that friction usually means entropy increasing in any system.

As a more complex example consider a gravity with higher order derivatives and the action in the form

$$S = \int d^{D+1}z\sqrt{G}f(R), \tag{7}$$

where $z = (t, x, y)$ and $G = |\det g \cdot \det \gamma|$. The metric of extra space (2) is chosen in the form $\gamma_{ab} = e^{\beta(t)}\gamma_{ab}^{(1)}(y)$. We also use inequality

$$R_M \ll R_{M'} \tag{8}$$

for the Ricci scalar of the main space R_M and the Ricci scalar of the extra space $R_{M'}$.

After some tedious calculation, see [7, 18] we obtain the following effective lagrangian

$$L = R_M + \frac{1}{2}K(\phi)(\partial\phi)^2 - V(\phi) \tag{9}$$

where we have introduced an effective scalar field

$$\phi \equiv R_{M'} = kd(d-1)e^{-2\beta(t)}. \tag{10}$$

Recall that we have $k = \pm 1$ for positive and negative curvature in extra dimensions, respectively, so that ϕ has different signs in these cases by definition. The forms of the functions $V(\phi)$ and $K(\phi)$ were obtained in [7, 18]. Their explicit form is irrelevant in this case.

If ϕ oscillates near a minimum ϕ_m of the potential V, equation of motion is rather simple

$$\ddot{\phi} + 3H\dot{\phi} + V'_\phi/K(\phi_m) = 0. \tag{11}$$

Due to the presence of friction, the Ricci scalar of extra space tends to a constant,

$$\phi = R_{M'} \to \phi_m \tag{12}$$

and the extra space M' acquires maximally symmetrical form.

The Ricci scalar $R_M \sim 1/a(t)^2 \to 0$ for the FRW metric of the main space. It means that inequality (8) holds true at large times.

As in the previous case, see (5), it is the dynamics of the main space that is responsible for the friction in the extra space and its stabilization. This indicates the presence of entropy flow from the extra space M' to the main one M [19, 20].

3 Entropy and symmetry formation

In the previous section we saw that stabilization of an extra space occurs simultaneously with an influx of entropy into the main space which acts as a thermostat. At the same time a symmetrization of extra space takes place. This means that an entropy decrease leads to an extension of the symmetry group of the compact extra space. Let us see to what extent this observation is general. More definitely, we prove the following

Statement

Let M be a smooth manifold, G_1 and G_2 are two given metrics on it. If the number of Killing vectors of metric G_1 is less then the number of Killing vectors of metric G_2 then the entropy of G_1 is greater than the entropy of G_2.

We will use the well known definition of the Boltzmann entropy S. It links entropy to a number of microstates Ω, $S = k_B \ln \Omega$. Other definitions are discussed in [21, 22] for example.

Let us consider a compact smooth manifold M. We suppose that every metric G on M defines a microstate. More definitely, two metrics G_1 and G_2 on M define the same microstate if and only if they are equal in each of the points $P \in M$.

The definition of a macrostate is as follows. Let v be an arbitrary smooth vector field defined globally on the smooth manifold M. Any shift along the integral path of vector field v corresponds to a diffeomorphism M on itself. We define a macrostate as a set of metrics G that are connected by shifts. As an example, a 2-dim torus with a bulge, being shifted, still represents the same macrostate. Another macrostate is determined by the addition of another bulge. So this definition seems reasonable.

The statistical weight of a given macrostate is the number of microstates. The latter is a continuum set for any classical system. The concept of microstates is correctly defined at a quantum level where the set of energy levels is known. However, the quantization of geometry is a yet unsolved problem. That is why any discussion of a metric on scale less than the Planck scale L_P is pointless. Thus shifts less than Planck scale should not be taken into account when counting statistical weight (see discussion in [23]). Therefore a number of shifts along various integral paths is assumed to be finite.

Let us compare statistical weights of two metrics G_1 and G_2 with the same number of shifts at manifold M. Let G_1 have no Killing vectors and G_2 possesses a global Killing field. Shifts along Killing vector of G_2 lead to the same microstate by definition. So the statistical weight of G_1 is greater than the statistical weight of G_2. A similar argument is correct in the general case as well when the number of Killing vectors of metrics G_1 is less then the number of Killing vectors of metrics G_2. This statement is also valid for the entropy which is the nondecreasing function of the statistical weight. Therefore

$$S_1 > S_2. \tag{13}$$

The statement is proved.

Taking into account the results of the previous sections we can conclude that the entropy of the extra space decreases with time while its isometry group becomes larger. The existence of gauge symmetries turns out to be the consequence of entropy increasing in the whole space. Entropy of compact extra space is decreasing however and the main space acts as a thermostat.

4 Consequences

According to condition (12) a Ricci scalar of internal metric of extra space is a constant with good accuracy whenever condition (8) holds. Meantime the latter can be violated at the inflationary stage due to metric fluctuations of the main space.

Let us estimate the characteristic size L_{extra} of extra space, insensitive to this fluctuations. During the inflationary stage we have [14]

$$R_3 = 12H^2 = \frac{32\pi}{M_{\text{Pl}}^2} V(\phi) = 16\pi \left(\frac{m}{M_{\text{Pl}}}\right)^2 \phi^2.$$

The last equality holds for a quadratic inflaton potential $V(\phi) = \frac{1}{2}m^2\phi^2$. Metric fluctuations of the main space have the form

$$\delta R_3 = 32\pi \left(\frac{m}{M_{\text{Pl}}}\right)^2 \phi\, \delta\phi \sim \frac{m}{M_{\text{Pl}}} m^2.$$

Here we take into account the approximate equality $\phi \sim M_{\text{Pl}}$ during the inflationary stage. Field fluctuations $|\delta\phi| = H/(2\pi)$ are connected to the scale factor

$$H = \sqrt{\frac{8\pi}{3}\frac{V(\phi)}{M_{\text{Pl}}^2}} = \sqrt{\frac{4\pi}{3}}\frac{m}{M_{\text{Pl}}}\phi$$

in the usual manner. For the size of extra space not to be disturbed, it should satisfy the inequality

$$L_{\text{extra}} \sim \frac{1}{\sqrt{R_{M'}}} < \frac{1}{\sqrt{\delta R_3}} \sim \frac{1}{m}\sqrt{\frac{M_{\text{Pl}}}{m}} \sim 10^{-24}\ \text{cm}. \tag{14}$$

Otherwise metric fluctuations of the main space would influence the geometry of the extra space and any symmetries would be absent during inflation.

Let us suppose that a relaxation time t_{rel} of dynamical processes in the extra space is less than the period of inflation $t_{\text{infl}} \approx 10^{-37}$, one can obtain the estimation

$$L_{\text{extra}} \sim t_{\text{rel}} < t_{\text{infl}} \sim 10^{-27}\text{cm}.$$

LHC collider could find extra space provided its size is larger than $\sim 10^{-18}$ cm. Evidently if the LHC succeeds in finding an extra space it would mean the absence of gauge symmetries at the inflationary stage. They arise during the stage of reheating or later.

5 Discussion

It is known that the idea of extra space leads to a set of observational effects. One of the most important is the connection between the gauge symmetries of the low-energy theory with the symmetries of the extra space. We elaborated the mechanism for the gauge symmetries formation related to the entropy flow from the extra space to the main one.

Due to the entropy increasing in the whole space, a compact subspace undergoes the process of symmetrization during some time after its quantum nucleation. Relaxation time of symmetry restoration depends on many aspects and could overcome the period of inflation.

One could reasonably suppose that the entropy of the extra space decreases until a widest symmetry is restored. On the other side we need specific isometries to explain observable symmetries of low energy physics, $SU(2) \times U(1)$ for example. Could they be represented by a most widest symmetry mentioned above? The answer is not evident. From this point of view the result of the paper [24] is rather promising: every compact Lie group can be realized as the full isometry group of a compact, connected, smooth Riemannian manifold.

The authors are grateful to V. Berezin, S. Bolokhov, V. Dokuchaev and Yu. Eroshenko for fruitful discussions and valuable comments. This work was supported in part by the Russian Foundation for Basic Research (project no. 09-02-00677-a).

References

[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali. The hierarchy problem and new dimensions at a millimeter. *Phys. Lett.* **B**, 429:263–272, June 1998.

[2] K. R. Dienes, E. Dudas, and T. Gherghetta. Grand unification at intermediate mass scales through extra dimensions. *Nucl. Phys. B*, 537:47–108, January 1999.

[3] L. Randall and R. Sundrum. Large Mass Hierarchy from a Small Extra Dimension. *Phys. Rev. Lett.*, 83:3370–3373, October 1999.

[4] N. Arkani-Hamed et al. Neutrino masses from large extra dimensions. *Phys. Rev.* **D**, 65(2):024032, January 2002.

[5] C. Csáki et al. Fermions on an interval: Quark and lepton masses without a Higgs. *Phys. Rev.* **D**, 70(1):015012, July 2004.

[6] G. Cacciapaglia et al. Curing the ills of Higgsless models: The S parameter and unitarity. *Phys. Rev.* **D**, 71(3):035015, February 2005.

[7] K. A. Bronnikov, R. V. Konoplich, and S. G. Rubin. The diversity of universes created by pure gravity. *Classical Quant. Grav.*, 24:1261–1277, March 2007.

[8] S. G. Rubin. On the origin of gauge symmetries and fundamental constants. *Sov. Phys. JETP*, 109:961–967, December 2009.

[9] F. Cianfrani and G. Montani. Low-Energy Sector of Eight-Dimensional General Relativity:. Electroweak Model and Neutrino Mass. *International Journal of Modern Physics D*, 17:785–798, 2008.

[10] M. Blagojevic. *Gravitation and Gauge Symmetries*. Institute of Physics Publishing, Bristol, 2002.

[11] A. Mazumdar and J. Rocher. Particle physics models of inflation and curvaton scenarios. *Phys. Rep.*, 497:85–215, January 2011.

[12] S. V. Bolokhov, K. A. Bronnikov, and S. G. Rubin. Extra dimensions as a source of the electroweak model. *Phys. Rev.* **D**, 84(4):044015, August 2011.

[13] A. Linde. Axions in inflationary cosmology. *Phys. Lett.* **B**, 259:38–47, April 1991.

[14] A. D. Linde. *Particle Physics and Inflationary Cosmology*. Harwood Academic Publishers, Switzerland, 1990.

[15] G. D. Starkman, D. Stojkovic, and M. Trodden. Large extra dimensions and cosmological problems. *Phys. Rev.* **D**, 63(10):103511, May 2001.

[16] S. M. Carroll et al. Classical stabilization of homogeneous extra dimensions. *Phys. Rev.* **D**, 66(2):024036, July 2002.

[17] U. Günther, P. Moniz, and A. Zhuk. Multidimensional Cosmology and Asymptotical AdS. *Astrophys. Space Sci.*, 283:679–684, 2003.

[18] K. A. Bronnikov and S. G. Rubin. Self-stabilization of extra dimensions. *Phys. Rev.* **D**, 73(12):124019, June 2006.

[19] E. Alvarez and M. B. Gavela. Entropy from Extra Dimensions. *Phys. Rev. Lett.*, 51:931–934, September 1983.

[20] E. W. Kolb, D. Lindley, and D. Seckel. More dimensions–Less entropy. *Phys. Rev.* **D**, 30:1205–1213, September 1984.

[21] T. A. Brun and J. B. Hartle. Entropy of classical histories. *Phys. Rev.* **E**, 59:6370–6380, June 1999.

[22] B. Hasselblatt and A. B. Katok. *Introduction to the Modern Theory of Dynamical Systems*. Cambridge University Press, Cambridge, 1996.

[23] A. Corichi, J. Díaz-Polo, and E. Fernández-Borja. Quantum geometry and microscopic black hole entropy. *Classical Quant. Grav.*, 24:243–251, January 2007.

[24] R. Saerens and W. R. Zame. The isometry groups of manifolds and the automorphism groups of domains. *Trans. Amer. Math. Soc.*, 301:413–429, April 1987.