What tells Gravity on the shape and size of an electron [1]

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Abstract

Gravitational field of an electron, fixed by experimental values of its mass, spin, charge and magnetic moment, is given by the metric of Kerr-Newman (KN) solution. Unexpectedly, this metric contains a singular ring of the Compton radius, which should be regulated resulting in a weeak and smooth source. The consistent source takes the form of an oblate vacuum bubble, bounded by a closed string of the Compton radius. The bubble turns out to be relativistically rotating and should be filled by a coherently oscillating Higgs field in a false vacuum state. The Compton size of the bubble shrinks sharply to a "point" for relativistic particle.

Introduction. It is commonly recognized now that black holes are akin to elementary particles. The Kerr-Newman solution has gyromagnetic ratio $g = 2$ as that of the Dirac electron, and the experimentally observable parameters of electron determine its asymptotical gravitational field in accord with the Kerr-Newman solution. The spin/mass ration of the electron is extremely high, $J/m \sim 10^{22}$ (we use the units $G = c = \hbar = 1$), and the *black hole horizons disappear*, opening the naked Kerr singular ring of the Compton radius $\sim 10^{-11}$ cm. It is very far from the expected pointlike electron of quantum theory. Besides, quantum theory supposes a flat minkowskian background, and this singular region should be regularized by some procedure leading to a finite and smooth source of the KN solution with a flat metric in vicinity of the electron core. It is not a priory clear that such a source can be obtained, and the aim of this paper is to describe basic elements of the given in [1] electron model which is consistent with the external KN solution and the above mentioned quantum requirements.

Structure of the KN solution. Metric of the KN solution has the form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \qquad H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}, \tag{1}$$

where $\eta_{\mu\nu}$ is metric of auxiliary Minkowski space in the Cartesian coordinates $\mathrm{x} \equiv (\mathrm{t}, \mathrm{x}, \mathrm{y}, \mathrm{z}) \in \mathrm{M}^4$, and $k^\mu(\mathrm{x}) \in \mathrm{M}^4$ is a lightlike vector field, forming

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a twisting congruence shown in Fig.1. Coordinates r, θ and ϕ_K are Kerr's oblate spheroidal coordinates (Fig.2). The KN metric is singular at the circle $r = \cos\theta = 0$, which is *branch line of the Kerr space into two sheets* r^+ *for* $r > 0$ *and* r^- *for* $r < 0$, so that the field $k^\mu(\mathrm{x})$ and the aligned with k^μ metric and vector potential of the electromagnetic (em) field,

$$\alpha^\mu_{KN} = Re \frac{e}{r + ia\cos\theta} k^\mu, \tag{2}$$

turn out to be twosheeted, taking different values on the different sheets of the same point $\mathrm{x} \in \mathrm{M}^4$. Twosheetedness represents one of the main puzzles of the KN space-time. For electron parameters, gravitational field of the KN solution is concentrated very close to singular ring, forming a circular waveguide – analog of the closed relativistic string. It has been shown in [2, 3] that the KN solution in vicinity of the Kerr ring corresponds to the obtained by Sen solution to low-energy heterotic string theory. Meanwhile, the long-term attack on the mysterious twosheetedness (Keres, Israel, Hamity, López at all, [3]) resulted in the gravitating soliton model in the form of the consistent with KN solution rotating vacuum bubble, metric of which is regularized, approaching the flat minkowskian background in the Compton region. It fixes unambiguously the form and some details of the consistent with KN gravity electron model. Following [1] we discuss basic features of

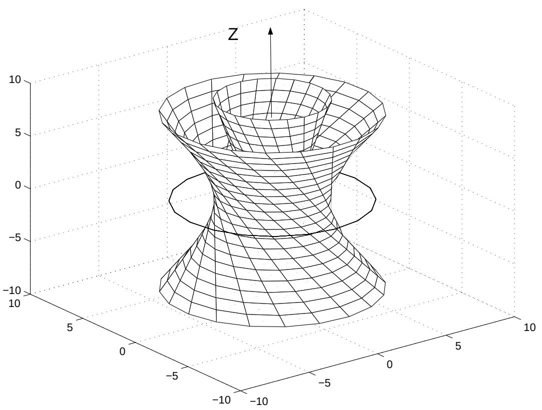

Figure 1: Congruence of the lightlike lines $k^\mu(x)$ is focused on singular ring, creating twosheeted Kerr space.

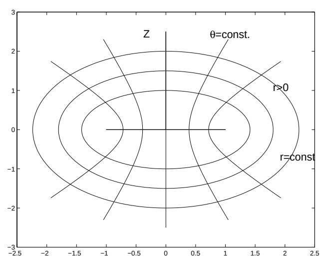

Figure 2: Oblate coordinate system (r, θ) covers the Kerr space twice, for $r > 0$ and $r < 0$. Truncation of the sheet $r < 0$ creates the source at $r = 0$.

the regular KN electron model as a gravitating soliton. The most wonderful fact is emergence of the quantum condition for spin of the KN soliton, as a consequence of the pure classical relations completed by the condition on periodicity of the Wilson loop (analog of the Bohm-Aharonov effect). *As a new result, we obtain that the bubble electron, which has the Compton radius at rest, is to be quickly shrinking in size by a relativistic boost.*

Relativistically rotating disk as source of the Kerr geometry. Kerr's coordinates r, θ, ϕ_K are related to Cartesian ones as follows

$$x + iy = (r + ia)\exp\{i\phi_K\}\sin\theta, \quad z = r\cos\theta, \quad \rho = r - t. \tag{3}$$

The null vector field k^μ is determined by the differential form

$$k_\mu dx^\mu = dt + \frac{z}{r}dz + \frac{r(xdx + ydy)}{r^2 + a^2} - \frac{a(xdy - ydx)}{r^2 + a^2}, \tag{4}$$

where the function $Y(\mathrm{x})$ is determined by the Kerr theorem. In Kerr's coordinates it is $Y(\mathrm{x}) = \mathrm{e}^{\mathrm{i}\phi_\mathrm{K}}\tan\theta/2$. In equatorial plane ($z = \cos\theta = 0$), the Kerr congruence is focused on the Kerr ring, approaching the ring tangentially

$$k|_{r=\cos\theta=0} = dt - (xdy - ydx)/a = dt - ad\phi. \tag{5}$$

Therefore, the Kerr ring is lightlike, similar to the DLCQ circle of M-theory.

Truncation of the 'negative' Kerr sheet (H.Keres, 1967; W.Israel, 1968) created a disklike source, $r = 0$, spanned by the Kerr ring. V.Hamity (1976) showed that the disklike source represents a rigid and relativistically rotating membrane, with the lightlike boundary.

The regular bubble model was proposed by López, which suggested to truncate the negative sheet along the ellipsoidal shell at $r = r_e > 0$, which covers the singular ring. There appears a rotating bubble source with a flat interior. One sees that the external KN metric 1 is matched with the flat interior by the condition $H = 0$, which fixes the boundary of the bubble at

$$r = r_e = e^2/(2m). \tag{6}$$

It follows from (3) that *the bubble takes the form of a highly oblate ellipsoid of the Compton radius $a \approx \hbar/m$ and thickness $2r_e = e^2/m$.*[2]

Gravitating soliton. In [1], the bubble shell model was extended to a smooth field model of the domain wall bubble interpolating between the external KN background and a false vacuum state inside the bubble (for details see [1]). The Kerr singular ring is suppressed by *the supersymmetric*

[2]The exact value of the disk radius is $r_b = \sqrt{a^2 + r_e^2} \approx a + \frac{1}{2}\delta$, where $\frac{\delta}{a} \approx 3 \cdot 10^{-46}$.

vacuum state with a flat metric inside the bubble, the Kerr closed string is formed by the em field concentrating at the edge rim of the bubble.

Regularization of the em field. Regularization is performed by the Higgs mechanism of broken symmetry. One of the complex chiral fields, say Φ is considered as a Higgs field, which takes the non-zero vev $\Phi = \Phi_0 \exp(i\chi)$ inside the bubble and pushes out the em field. The typical Lagrangian yields the equation

$$\chi,_{\mu} + e\alpha_{\mu} = 0. \tag{7}$$

The cut-off r_e determines maximum of the KN vector potential

$\alpha^{str} = \alpha^{max}_{KN} = e/r_e = 2m/e$, which is concentrated in the form of a closed string at the edge border of the disklike bubble. In agreement with (5), the longitudinal component of α^{str} forms a closed Wilson loop along the boundary of the disk, which yields $\oint e\alpha^{str}_{\mu} dl^{\mu} = 4\pi ma$. Using Kerr's relation $J = ma$, we obtain for the loop integral of (7) over the disk boundary

$$\oint \chi,_{\mu} dl^{\mu} = 2\pi n = -4\pi J, \tag{8}$$

which indicates *quantization of the spin-projection,* $|J| = n/2, \quad n = 1, 2...$

Similarly, the time component of (7) yields $\dot{\chi} = \omega = 2m$, resulting in *oscillations of the Higgs field with the frequency* $2m$, what is typical for the "oscillon" type of the solitonic models.

Finally, let us consider one more important consequence of the Kerr relation $J = ma$, which states that the electron should have at rest the Compton radius $a_0 = \hbar/2m_0$, corresponding to its rest mass m_0. One can rewrite it in the form $J = m_{rel} a_{rel}$, where $m_{rel} = m_0/\sqrt{1 - (v/c)^2}$ and $a_{rel} = a_0\sqrt{1 - (v/c)^2}$, and reveal that the radius a_{rel} shrinks sharply by the boost. Therefore, the Compton rest-size radius a_0 can only be exhibited by the very low-energy scattering.

References

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