

# General Relativity à la string: a progress report<sup>\*†</sup>

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Preliminary results on a canonical formulation of general relativity based on an analogy with the string model of elementary particles are presented. Rather than the metric components, the basic fields of the formalism are taken to be the functions describing the embedding of four dimensional spacetime in a ten or possibly higher dimensional manifold. So far, the main drawback of the formalism is that the generator of normal deformations (“fourth constraint”) cannot be written down in closed form. The present approach is compared and contrasted with the usual one and with the canonical description of the relativistic string.

It is our intention in this note to analyze some formal analogies existing in different relativistic systems and to assess some of the current difficulties in the canonical formalism for general relativity. In particular we shall compare two different approaches to general relativity (the conventional one (I) [1, 2] and a new one based on the notion of external variables (II)) with the string model of elementary particles [3].

## I. GENERAL RELATIVITY: CONVENTIONAL FORMALISM

In the usual canonical formalism for Einstein’s theory of gravitation developed by Dirac [1] and Arnowitt, Deser and Misner (ADM) [2] the starting point is the Hilbert action

$$S = \int \left( -^{(4)}g \right)^{1/2} {}^{(4)}R d^4x \quad (1)$$

which is regarded as a functional of the metric tensor  $g_{\mu\nu}(x)$ ,  $x \in R^4$ .

It is an important feature of the Hilbert action that by adding a suitable divergence to the integrand in (1) one can switch to an alternate action density — the Dirac — ADM — lagrangian density

$$\mathcal{L} = \left( -^{(4)}g \right)^{1/2} {}^{(4)}R + \frac{\partial \mathcal{V}^\alpha}{\partial x^\alpha} \quad (2)$$

which contains no second time derivatives of the  $g_{\mu\nu}$  and furthermore contains no first time derivatives of the  $g_{0\mu}$ .

The  $g_{0\mu}$  have then vanishing conjugate momenta and enter the theory as arbitrary functions. At this stage the remaining degrees of freedom are thus those represented by the spatial metric components  $g_{ij}$  and their conjugates  $\pi^{ij}$ . The fields  $g_{ij}$ ,  $\pi^{ij}$  are however not independent, but they are restricted by the constraint equations<sup>1</sup>

$$\mathcal{H}_\perp = g^{-1/2} \left( \pi_{ij} \pi^{ij} - \frac{1}{2} (\pi_i^i)^2 \right) - g^{1/2} R \approx 0, \quad (3a)$$

$$\mathcal{H}_i = -2\pi_i^j{}_{|j} \approx 0. \quad (3b)$$

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<sup>1</sup> The “weak equality” symbol is used to emphasize that  $\mathcal{H}_\perp$  and  $\mathcal{H}_i$  have non-vanishing Poisson brackets with the canonical variables of the theory. The vertical slash denotes covariant differentiation in the spatial metric  $g_{ij}$ . Spacetime covariant derivatives are indicated by a semicolon. The letter  $R$  denotes the spatial curvature and  $g$  is the determinant of the spatial metric. To avoid confusion some spacetime quantities carry an upper left index (4) as in (1).

Geometrically speaking the  $\mathcal{H}_i$  generate arbitrary reparametrizations of the spacelike hypersurface on which the state is defined whereas  $\mathcal{H}_\perp$  generates deformations which change the location of the hypersurface in the ambient spacetime. The fact that the hypersurfaces are embedded in a common spacetime is expressed through the closure relations [4, 5]

$$[\mathcal{H}_\perp(x), \mathcal{H}_\perp(x')] = (g^{rs}(x)\mathcal{H}_s(x) + g^{rs}(x')\mathcal{H}_s(x')) \delta_{,r}(x, x'), \quad (4a)$$

$$[\mathcal{H}_r(x), \mathcal{H}_\perp(x')] = \mathcal{H}_\perp(x) \delta_{,r}(x, x'), \quad (4b)$$

$$[\mathcal{H}_r(x), \mathcal{H}_s(x')] = \mathcal{H}_r(x') \delta_{,s}(x, x') + \mathcal{H}_s(x) \delta_{,r}(x, x'). \quad (4c)$$

In this theory it is in principle possible to fix the gauges by imposing particular coordinate conditions on the surface and also by fixing the time slicing. The fixation of the spacetime coordinates amounts therefore to bring in four extra constraints besides (3). After this is done one is left with only two independent pairs of canonical variables per space point. These degrees of freedom appear in the weak field approximation as the two polarization states per wave vector  $k$  of a massless spin two graviton propagating on a flat background.

The practical implementation of the coordinate fixing is unfortunately fraught with difficulties which have prevented so far the construction of an actual canonical quantum theory of gravity. In the first place it is not a simple matter to fix the gauge freedom in such a manner as to ensure a proper parametrization of spacetime through coordinates, although some of the proposed choices look reasonable [2, 6, 7]. A second difficulty is that the reduced Hamiltonian associated to the coordinate conditions proposed so far cannot be written down in closed form and usually appears as a highly non-local expression in the canonical fields. This brings virtually to a halt the construction of the quantum theory because of the formidable problems of ordering which must be solved *ex-novo* at each order of perturbation theory in the expression for the Hamiltonian.

Yet another difficulty arises in the so-called maximal slicing ( $\pi^i_i = 0$ ), which appears to be the gauge condition most exhaustively investigated from the point of view of ensuring a proper parametrization of spacetime [7]. The difficulty in question is that ordering problems appear here already at the level of interpreting the Poisson brackets of the basic fields as commutators, because  $q$ -numbers appear nontrivially on the right hand side of the commutation relations. Such difficulties do not arise however for the ADM variables [2, 8], but unfortunately there is not much evidence that the ADM gauge defines a good system of spacetime coordinates.

The difficulties mentioned above are by no means exclusive to the gravitational field and they also appear, for example, in the string model which bears in many respects a striking analogy with Einstein's theory of gravitation. In the case of the string, because of the simple geometrical nature of the model, it is possible to circumvent the ordering problem by means of the DDF variables [9] as suggested by the interpretation of the theory in the framework of the dual models of hadrons.

In what follows we would like to examine the possibility of extending some of the useful concepts of the string model into general relativity. Although we have not been successful in this attempt we feel that the comparative discussion of the two systems is interesting by itself and leads to useful critical remarks.

## II. THE STRING MODEL

Here we consider  $n + 1$  fields  $y^A(x, t)$ ,  $x \in R$ . The functions  $y^A$  parametrize a two dimensional surface  $V_2$  embedded in an  $N + 1$  dimensional Minkowski space of metric

$$ds^2 = d\tilde{y} \cdot d\tilde{y} = \eta_{AB} dy^A dy^B = -(dy^0)^2 + \sum_1^N (dy^A)^2. \quad (5)$$

The two dimensional surface is spanned by the motion of the (one dimensional) string in the  $N + 1$  dimensional space.

The action for the system is taken to be

$$S = \int \left( -^{(2)}g \right)^{1/2} dx dt \quad (6)$$

where  $(-^{(2)}g)^{1/2} dx dt$  is the area element on  $V_2$ . The string is assumed to have a finite length and one has to impose Poincaré invariant boundary conditions at its ends in order to obtain a relativistic theory. The boundary conditions imply that the endpoints move transversally with the speed of light. The canonical formalism based on (6) leads to a

vanishing canonical Hamiltonian (due to the time reparametrization invariance of (6)) and to constraints of the form

$$\mathcal{H}_1 = \tilde{\pi} \cdot \frac{\partial \tilde{y}}{\partial x} \approx 0, \quad (7a)$$

$$\mathcal{H}_\perp = \frac{1}{2} \left| \frac{\partial \tilde{y}}{\partial x} \right|^{-1} \left( \tilde{\pi}^2 + \left( \frac{\partial \tilde{y}}{\partial x} \right)^2 \right) \approx 0. \quad (7b)$$

The functions (7) admit again the geometrical interpretation of generating tangential and normal deformations of the string. They satisfy closure relations analogous to (4), namely [8]

$$\begin{aligned} [\mathcal{H}_\perp(x), \mathcal{H}_\perp(x')] &= \left( \left| \frac{\partial \tilde{y}}{\partial x} \right|^{-2}(x) \mathcal{H}_1(x) + \left| \frac{\partial \tilde{y}}{\partial x} \right|^{-2}(x') \mathcal{H}_1(x') \right) \delta'(x, x') + \\ &+ 2 \left( \left| \frac{\partial \tilde{y}}{\partial x} \right|^{-3}(x') \mathcal{H}_\perp(x) \mathcal{H}_1(x) + \left| \frac{\partial \tilde{y}}{\partial x} \right|^{-3}(x') \mathcal{H}_1(x') \right) \delta'(x, x'), \end{aligned} \quad (8a)$$

$$[\mathcal{H}_1(x), \mathcal{H}_\perp(x')] = \mathcal{H}_\perp(x) \delta'(x, x'), \quad (8b)$$

$$[\mathcal{H}_1(x), \mathcal{H}_1(x')] = (\mathcal{H}_1(x) + \mathcal{H}_1(x')) \delta'(x, x'). \quad (8c)$$

We note that the only difference between (8) and (4) is the presence of the term quadratic in the constraints on the right hand side of (8a). This term has however weakly vanishing brackets with everything, which means that (7) still ensures that all the strings are embedded in a common two dimensional Riemannian surface.

In GGRT [3] the problem of accounting for the constraints and fixing the coordinate system on the surface spanned by the string is solved by introducing a system of null surfaces  $y^0 - y^1 = t$  in  $R^{N+1}$  which reduces the problem to dealing with  $N - 1$  independent modes per point on the string<sup>2</sup>. It is also possible to introduce a more conventional spacelike gauge [12]  $y^0 = t$ . In the latter case the Dirac brackets of the basic fields are given typically by expressions of the form

$$[\alpha_m^A, \alpha_n^B] = m \delta_{m, -n} \delta^{AB} + \sum_{M \neq 0} \frac{mn}{M} \frac{1}{(p^0)^2} \alpha_{m-M}^A \alpha_{n+M}^B \quad (9a)$$

where

$$y^A(x, t) = q^A + p^A t + i \sum_{n \neq 0} \frac{1}{n} \alpha_n^A \cos(nx) e^{-int} \quad (9b)$$

Equation (9) shows that the fields  $y^A$ ,  $\pi_A$  are related to the fundamental canonical variables of the theory by a non-elementary expression. It is in fact extremely hard to approach the quantization procedure by considering the  $y^A$  as operators and (9a) as a commutation relation because of the ordering problem. A better approach is to consider the DDF operators which appear in the integral form

$$D_n^A = \frac{1}{2} \int_{-\pi}^{2\pi} \frac{dy^A(0, t)}{dt} \exp \left( n(\tilde{k} \cdot \tilde{p})^{-1} \tilde{k} \cdot \tilde{y}(0, t) \right) dt \quad (10)$$

(here  $\tilde{k}$  is an arbitrary null vector) and which obey a simple algebra. The whole string model can be built upon a systematic exploitation of this algebra.

The underlying pseudo Euclidean structure of  $R^{N+1}$  is necessary for the use of the DDF operators in that form it follows that there are orthonormal coordinates  $x, t$  such that the equations of motion can be explicitly solved in the form

$$y^A(x, t) = f^A(t - x) + f^A(t + x), \quad (11)$$

an equation which is crucial in defining the Fourier transform used by DDF.

A solution similar to (11) is of course not available in general relativity but it is nevertheless of interest to investigate what happens if one tries to cast general relativity in a string-like form, which we pass to do now.

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<sup>2</sup> Null surfaces have been introduced to analyze the dynamics of gravity by Aragone and Gambini [10] and Kaku [11]. There is however no analog in the discussion given by those authors of an ambient flat space which is heavily relied upon in the string model.

### III. GENERAL RELATIVITY À LA STRING

By analogy with the string model we postulate here that ordinary curved spacetime  $V_4$  is embedded in some Minkowski space  $R^{N+1}$  with a sufficiently high dimensionality  $N \geq 9$  so as to be able to accommodate locally a generic four-dimensional pseudo Riemannian manifold. We thus consider the spacetime  $V_4$  as a “trajectory” swept by a three dimensional string in  $R^{N+1}$ .

The key difference between the present formalism and the usual approach described in Section I above, is that the metric components  $g_{\mu\nu}(x)$  are no longer the basic variables but, rather, they are regarded now as derived objects constructed from the functions  $y^A(x^0, x^1, x^2, x^3)$  determining the (time dependent) embedding of  $V_3$  in  $R^{N+1}$ .

The metric tensor is thus given by

$$g_{\mu\nu}(x) = \tilde{y}_{,\mu} \cdot \tilde{y}_{,\nu} = \eta_{AB} \frac{\partial \tilde{y}^A}{\partial x^\mu} \frac{\partial \tilde{y}^B}{\partial x^\nu} \quad (12)$$

with  $\eta_{AB} = \text{diag}(-1, 1, \dots, 1)$ ,  $A, B, \dots = 0 \dots N$ .

We shall use the same action as in I, namely

$$S[y] = \int \mathcal{L} d^4 x \quad (13)$$

where  $\mathcal{L}$  is the Dirac--ADM lagrangian density appearing in (2), regarded this time as a functional of the  $y^A$  through (12). The fact that  $\mathcal{L}$  contains no time derivatives of the  $g_{0\mu}$  implies that only first time derivatives of the  $y^A$  enter into the action (13). Eq. (12) shows that  $y_{,0}^A$  can enter  $\mathcal{L}$  through  $g_{0\alpha}$  only). As solely first time derivatives of the  $y^A$  appear in the action we see that we are still dealing with a system that can be put in canonical form by standard methods. We have already paid however, a stiff price by introducing the external variables  $y^A$ , namely, we have to retain all the fields instead of being able to eliminate four of them (the  $g_0$ ) at an early stage as was done in I.

A worse feature is that requiring the action (13) to be stationary under arbitrary variations of the  $y^A$  does not reproduce the equations of motion of general relativity

$$(\text{Einstein tensor})^{\alpha\beta} = G^{\alpha\beta} = 0, \quad (14)$$

but gives rather the weaker set

$$G^{\alpha\beta} \tilde{y}_{;\alpha\beta} = 0. \quad (15)$$

Equations (15) are the analog of the string equations

$$g^{\alpha\beta} \tilde{y}_{;\alpha\beta} = 0. \quad (\text{String}) \quad (16)$$

in which case  $\alpha$  and  $\beta$  refer to the two dimensional spanned by the string.

Equations (14) do not imply  $G^{\alpha\beta} = 0$  due to the identities

$$\tilde{y}_{;\alpha\beta} \cdot \tilde{y}_{;\gamma} = 0. \quad (17)$$

which show that in the generic case only six among the  $N + 1$  equations are independent. We note in passing that the identities (17) avoid the paradoxical implication  $g^{\alpha\beta} = 0$  in (16). The difficulty of having only six independent equations in (15) instead of the full Einstein set is not unsurmountable and could be circumvented by imposing in an ad-hoc fashion the additional constraints

$$G_{\perp\alpha} = 0 \quad (18)$$

where the symbol  $\perp$  refers to the unit normal to  $V_3$  lying in  $V_4$  and  $a = \perp, 1, 2, 3$ . Examination of the canonical formalism for the external variables shows in fact that one may expect (18) not to be an entirely unreasonable addition to the equations (15).

### IV. CANONICAL FORMALISM FOR EXTERNAL VARIABLES

We start from the Dirac-ADM Lagrangian density (13), which written down in detail reads

$$\mathcal{L} = g^{1/2} N (R + K_{ab} K^{ab} - (K_a^a)^2), \quad (19)$$

where  $R$  is the curvature scalar of  $V_3$  and where the extrinsic curvature  $K_{ab}$  of  $V_3$  with respect to  $V_4$  is given by

$$K_{ab} = (2N)^{-1}(-\dot{g}_{ab} + N_{a|b} + N_{b|a}). \quad (20)$$

The symbols  $N$  and  $N_a$  stand for the lapse and shift functions

$$N = \left( -^{(4)}g^{00} \right)^{1/2}, \quad N_a = g_{0a}. \quad (21)$$

A dot denotes differentiation with respect to  $x^0$ . The Lagrangian density (19) is expressed as a functional of the  $y^A$  by means of (12), (20) and (21).

The canonical momenta are defined by

$$\tilde{\pi}(x) = \frac{\delta}{\delta \dot{\tilde{y}}(x)} \int d^3x' \mathcal{L}(x') \quad (22)$$

which gives, after some calculation<sup>3</sup>,

$$\tilde{\pi} = g^{1/2} (-2G_{\perp\perp} \tilde{n} + 2(K^{ab} - K_m^m g^{ab}) \tilde{y}_{|ab}) \quad (23)$$

Here  $\tilde{n}$  denotes the unit normal to  $V_3$  lying in  $V_4$ :

$$\tilde{n} = N^{-1} \left[ \dot{\tilde{y}} - (\dot{\tilde{y}} \cdot \tilde{y}^{|i} \tilde{y}_{,i}) \right] \quad (24)$$

and  $G_{\perp\perp}$  is the double projection of the Einstein tensor along  $\tilde{n}$ :

$$-2G_{\perp\perp} = K_{ab} K^{ab} - (K_m^m)^2 - R. \quad (25)$$

The normal (24) satisfies the normalization condition,

$$\tilde{n} \cdot \tilde{n} = -1, \quad (26)$$

and the extrinsic curvature is related to  $\tilde{n}$  by

$$K_{ab} = \tilde{n} \cdot \tilde{y}_{|ab}. \quad (27)$$

Now we note that the six vectors  $\tilde{y}_{|ab}$  are perpendicular to  $V_3$  (this is just the  $V_3$  version of the identities (17)). Also the normal  $\tilde{n}$  is perpendicular to  $V_3$ . It thus follows that the three components of  $\tilde{\pi}$  on  $V_3$  vanish. We then get the three constraints

$$\mathcal{H}_i = \tilde{\pi} \cdot \tilde{y}_{,i} \approx 0, \quad (28)$$

which are the analog of (7a) for the string. The  $\mathcal{H}_i$  defined by (28) generate reparametrizations on  $V_3$  and they satisfy consequently the closure relations (4c). It follows from (28), for example that  $y^A$  and  $\pi_A$  transform as scalars and scalar densities respectively under changes of coordinates in  $V_3$ , which was of course to be expected.

The fourth constraint (analog to (7b)) for the string is obtained in principle by solving (23) as a system of nonlinear algebraic equations for  $n^A$  as a function of  $\pi_A$  and  $y^A$  and imposing afterwards the normalization condition (26). In the case of the string this procedure yields (7b). In fact the string analog of (23) reads simply

$$\tilde{\pi} = \left| \frac{\partial \tilde{y}}{\partial x} \right| \tilde{n} \quad (\text{string}), \quad (29)$$

which upon squaring and using (26), gives (7b). The solution of (23) is however considerably harder and there seems to be no way of obtaining a simple closed form for

$$\tilde{n} = \tilde{n}(\tilde{y}, \tilde{\pi}). \quad (30)$$

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<sup>3</sup> Note added (2016): Actually the right-hand side of (23) is nothing but the Lagrangian density obtained by dropping the factor  $N$  in (19). See [14].

This problem might be circumvented to some extent with the help of the additional constraints (18) which we pass to discuss now.

As we mentioned before, even if we imagine having the solution (30), the formalism does not reproduce Einstein's theory. In fact, if we count the number of independent degrees of freedom of the theory we find:  $2(N + 1) - 4$  (first class constraints)  $- 4$  (gauge conditions)  $= 2(N - 3)$ . That is we have  $N - 3$  degrees of freedom per point, which recalling that  $N \geq 9$  is at least an excess of four over the required number of two for general relativity.

We see therefore that even if we bring  $N$  down to its minimum value of 9, as we shall do tentatively from now on, we need four additional first class constraints besides the  $\mathcal{H}_\mu$ . It is quite reasonable to take these new constraints to be (18). In fact the  $G_{\perp\mu}$  are constructed from the  $g_{ab}$  and  $K_{ab}$  only, which means that they can in principle be expressed, via (30), as functions of the canonical variables  $\tilde{y}, \tilde{\pi}$ .

Now, if we are going to impose the constraints (18), we need to solve (23) for  $\tilde{n}$  only when  $G_{\perp\perp} = 0$ . This will result in changing the constraints by linear combinations of themselves and will therefore not change the dynamics of the system.

When  $G_{\perp\perp} = 0$  (23) can be written as

$$\pi^A \approx W_B^A n^B, \quad (31)$$

with

$$W_B^A = 2g^{1/2}(g^{ad}g^{bc} - g^{ab}g^{cd})y_{|ab}^A y_{|cd}^B. \quad (32)$$

The matrix  $W$  defined by (32) regarded as a mapping of  $R^{10}$  onto  $R^{10}$  does not have an inverse because it maps to zero the three vectors  $\tilde{y}_{,i} (i = 1, 2, 3)$ . However when restricted to the sub-space orthogonal to the  $\tilde{y}_{,i}$ ,  $W$  will have an inverse in the generic case. Let us denote that inverse by  $M$ . The matrix  $M$  is therefore defined as giving that solution of (31),

$$n^B = M_A^B \pi^A \quad (33)$$

which satisfies

$$\tilde{n} \cdot \tilde{y}_{,i} = 0. \quad (34)$$

It follows from (32) that  $M$  is constructed from the  $y^A$  and their derivatives and that  $M_{AB} = \eta_{AC} M_B^C$  is symmetric. The eight constraints of the theory can then be expressed in terms of  $M$  as follows

$$-2G_{\perp\perp} = K_{ab}K^{ab} - (K_m^m)^2 - R \approx \frac{1}{2}g^{-1/2}M_{AB}\pi^A\pi^B - R \approx 0, \quad (35a)$$

$$-G_{\perp i} = (K_i^k K_m^m \delta_i^k)_{|k} \approx (M_{AB}\pi^B)_{,i} y^{A|m}{}_{|m} - (M_{AB}\pi^B)_{,m} y^{A|i}{}_{|i} \approx 0, \quad (35b)$$

$$\mathcal{H}_\perp = g^{1/2}(\tilde{n}^2 + 1) \approx g^{1/2}((M^2)_{AB}\pi^A\pi^B + 1) \approx 0, \quad (35c)$$

$$\mathcal{H}_i = \tilde{\pi} \cdot \tilde{y}_{,i} \approx 0. \quad (35d)$$

The essential problem at this point is to prove that the eight constraints (35) are first class. It does not seem possible to do this without knowing more about the form of  $M_{AB}$ . We plan to investigate this matter in the future.

If the constraints (35) are indeed first class their compatibility is ensured and the theory is consistent. Furthermore we are then sure that we are dealing exactly with Einstein's equations because the only way in which  $G_{\perp\mu}$  can vanish on every three dimensional space like hypersurface of  $V_4$  is that all ten equations  $G_{\alpha\beta} = 0$  hold. On the other hand, if the system (35) is not first class we would be merely selecting by means of (35a), (35b) special coordinates on  $V_4$  (i.e., fixing the gauge) instead of reducing the number of physical degrees of freedom of the theory. The formalism would not reproduce Einstein's theory in that case.

### Final Remarks

The theory as we have presented it here is not complete but we feel it deserves further investigation. It is quite possible that the actual value of  $N$  is not relevant in a final, as yet hypothetical, complete form. We must keep in mind in this connection that the possibility of embedding a four dimensional manifold in  $R^{10}$  holds only in a very local sense and that non-trivial problems are already encountered in trying to embed globally a smooth two dimensional manifold [13] in  $R^3$ . However the existence of the variables  $y^A$  gives us more freedom to construct field variables which do not exist in the conventional theory and which could possibly lead to a canonical formulation of general relativity different from the conventional one. In this sense it could be interesting to try to find the analog of the DDF operators for the string model. Unfortunately we have not been able as yet to obtain any definite result along this direction.

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