Signature of Einstein-Cartan theory

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(Dated: September 22, 2023)

We study the physical effects of torsion as predicted by the Einstein-Cartan theory in the test particle approximation and the non-relativist limit. We first present the corresponding non-relativistic Hamiltonian for a 2-spinor. Then, we solve an idealized reflection and transmission problem for a non-relativistic spin-$\frac{1}{2}$ beam travelling across a spin-polarized target. We identify deviations in the spin polarizations of the reflected and transmitted as observables capable of distinguishing Einstein-Cartan from standard general relativity. If measured, this effect would constitute compelling evidence for the presence of spacetime torsion.

arXiv:2309.11536v1 [gr-qc] 20 Sep 2023

General Relativity (GR) has passed every single experimental verification at Solar system scales [1], and it is regarded as the paradigmatic theory of gravity. Nevertheless, a plethora of modified gravity theories have been proposed with the goal, in most cases, to improve our description at cosmological scales [2]. One of the earliest modified gravity theories —dating back 100 years— goes by the name of Einstein–Cartan theory (EC) [3, 4], which was re-derived when constructing gravitational theories that are invariant under the local Poincaré group [5, 6] (for a comprehensive review see Ref. 7).

In EC, the geometrical description of gravity is given in terms of the metric and a torsion-full connection. The torsion tensor is algebraically linked with the so-called spin density tensor [see Eq. (1) below]. Consequently, torsion does not propagate and is only non-zero inside matter with spin density. Moreover, the theory reduces to GR whenever torsion vanishes. Thus, tests conducted in vacuum, as occurs effectively for the most popular GR tests, cannot empirically distinguish between GR and EC. Still, knowing the fundamental gravitational degrees of freedom, particularly when attempting to quantize gravity, is of the utmost relevance. Although one can construct alternative torsion theories (see, e.g., Ref. 8), the fact that EC reproduces the GR phenomenology in the torsion-free limit suggests that any empirically adequate theory should contain EC as the dominant contribution.

The EC action is symbolically identical to the Einstein–Hilbert action of GR, but, in the former, the Ricci curvature depends on the (potentially torsion-full) connection. We use the first order formalism of gravity [9] that allows us to incorporate spinor more naturally and where the basic geometrical variables are the frame 1-forms e^μ_a and the independent spin connection 1-forms $\omega_{a\mu\nu}$ [10]. Note that the frame forms contain the metric information while the spin connection contains the torsional degrees of freedom. The variation of the EC action with respect to $\omega_{a\mu\nu}$ yields

$$T^\rho{}_{bc}(e^a_\rho e^b_\mu e^c_\nu + e^a_\mu e^b_\nu e^c_\rho + e^a_\nu e^b_\rho e^c_\mu) = 8\pi G \Sigma^a{}_{\mu\nu}, \qquad (1)$$

where $T^\mu{}_{ab}$ is the torsion 2-form, $\Sigma^a{}_{\mu\nu} \equiv 2\delta\mathcal{L}_{\rm M}/\delta\omega_a{}^{\mu\nu}$ is the spin density, $\mathcal{L}_{\rm M}$ is the matter Lagrange function (such that, when multiplied with the volume 4-form, can be integrated to produce the matter action), and G stands for Newton's gravitational constant. On the other hand, the action variation with respect to e^μ_a generates an Einstein-like equation for a torsion-full Einstein tensor. Relevantly, upon using Eq. (1) to replace torsion in terms of $\Sigma^a{}_{\mu\nu}$, the Einstein-like equation differs from the corresponding GR equation by terms proportional to G^2 [7].

Now, in the Standard Model of particle physics, the only action terms that couple to the spin connection are those associated with spinors. Thus, we take $\mathcal{L}_{\rm M}$ as the matter Lagrangian for a spin-$\frac{1}{2}$ Dirac field Ψ [11], which contains a covariant derivative of the form [12, ch. 7.10.3]

$$\nabla_a\Psi = \left(\partial_a + \frac{1}{4}\omega_{a\mu\nu}\gamma^{\mu\nu}\right)\Psi, \qquad \gamma^{\mu\nu} \equiv \frac{[\gamma^\mu,\gamma^\nu]}{2}. \qquad (2)$$

Since the other terms in this matter Lagrangian do not depend on the connection, Eq. (1) becomes

$$T^\rho{}_{\mu\nu} = 4\pi G \epsilon_{\mu\nu}{}^{\rho\sigma} J_{5\sigma}, \qquad (3)$$

where $T^\rho{}_{\mu\nu}$ are the torsion components in the e^μ_a basis and $J^\mu_5 \equiv {\rm i}\bar{\Psi}\gamma^5\gamma^\mu\Psi$ is the source's axial current. Notice that we do not impose that the source field Ψ obeys the corresponding equations of motion.

In this Letter, we compare a reflection and transmission problem for a test spinor in GR and EC when traversing a source with $J^\mu_5 \neq 0$. Concretely, we compute the spin polarization angle deflection for a non-relativistic beam of polarized neutrons. We show that EC predicts a shift in the reflected and transmitted neutron polarization that is linear in G and not present in GR. Thus, if this shift is measured, it would constitute a smoking gun for spacetime torsion. We emphasize that, even though other interactions could generate a similar spin deviation [13], EC effects could eventually be identified by their dependence on the target parameters. Furthermore, since EC introduces no new fundamental constants, our calculations contain no free parameters.

We should mention that the search for spin-dependent gravitational effects is not new (for a review, see Ref. 14).

However, most proposals to look for torsion either lie outside the realm of EC by allowing for a propagating torsion [15–21], or focus on a background torsion, ignoring the torsion source [22–27]. In addition, there are EC theory tests in situations with less experimental control, such as particle collisions [28–30]. Moreover, it has been suggested to look for torsion sourced by classical angular momentum [31], which, however, has been disputed [32, 33]. We emphasize that our analysis offers three advantages over proposed observational tests. First, it focuses on EC, which, as is argued above, is expected to provide the dominant contribution in any theory that reduces to GR. Second, the only modification to conventional physics that we assume is the introduction of a non-vanishing torsion. Third, even though the effects of torsion happen while the beam goes through the target, the detectors may be placed in a local vacuum, which offers an enormous practical advantage.

We turn to find the evolution of a test Dirac spinor ψ subject to curvature and torsion, as described in EC theory and for a given target, and obtain a non-relativistic approximation for a beam of such particles. The equation of motion for ψ reads

$$\mathrm{i}\gamma^{\mu} e^{a}_{\mu}\left(\partial_a + \Gamma_a\right) - \frac{3\pi G}{8} J^{\mu}_{5}\gamma_5\gamma_{\mu}\psi = m\psi, \tag{4}$$

where $\Gamma_a \equiv \omega^{\mathrm{GR}}_{a\mu\nu}\gamma^{\mu\nu}/4$ and $\omega^{\mathrm{GR}}_{a\mu\nu}$ is the metric connection (which is torsion independent) and J^{μ}_{5} is the axial current of the medium. In writing Eq. (4), we used Eq. (3) and the identity $\{\gamma_{\rho}, \gamma_{\mu\nu}\} = -2\mathrm{i}\epsilon_{\rho\mu\nu\sigma}\gamma_5\gamma^{\sigma}$. We also employed the test particle approximation, in which the target's spin density is an approximation for J^{μ}_{5}. GR only differs from EC in the last term on the left-hand side of Eq. (4), which is absent in GR.

Henceforth, we work in the weak-field regime for gravity, keeping only linear terms in G. The metric is linearized around the Minkowski metric η_{ab} as $g_{ab} = \eta_{ab} + Gh_{ab}$, where h_{ab} is found by solving the equations of motion. The GR spin connection is $\mathcal{O}(G)$:

$$\omega^{\mathrm{GR}}_{a\mu\nu} \approx -Ge^{\rho}_{a}(\partial_{\mu}h_{\nu\rho} - \partial_{\nu}h_{\mu\rho}). \tag{5}$$

We consider a static target that, for all practical purposes, can be considered infinite in the two orthogonal directions, y and z, and uniform between the $x = 0$ and $x = a > 0$ planes. Notice that we choose Minkowski coordinates to be compatible with the target's symmetries so that the metric perturbations are diagonal and time-independent. According to Eq. (5), the only non-zero components of $\omega^{\mathrm{GR}}_{a\mu\nu}$ are $\omega^{\mathrm{GR}}_{00i} \approx G\partial_i h_{00}$ and $\omega^{\mathrm{GR}}_{ijk} \approx G\partial_k h_{ij}$ (notice that $h_{ij} = 0$ if $i \neq j$). This implies that Γ_a is proportional to the 4×4 matrices

$$\gamma^{0i} = \begin{bmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{bmatrix} \quad \text{and} \quad \gamma^{ij} = -\mathrm{i}\epsilon^{ij}{}_{k}\begin{bmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{bmatrix}, \tag{6}$$

where ϵ_{ijk} is the completely antisymmetric tensor with $\epsilon_{123} = 1$.

We further assume that the beam of spin-$\frac{1}{2}$ particles is non-relativistic. To obtain the corresponding non-relativistic equation, we use the well-known Foldy-Wouthuysen [34] procedure, namely, a series of unitary spinor transformations that block-diagonalizes the Hamiltonian $\tilde{H}$ order by order in p_i/m, where p_i are the components of momentum. When we multiply Eq. (4) on the left by γ^0, we get an equation of the form $\mathrm{i}\partial_t\psi = \tilde{H}\psi$ [35] for

$$\tilde{H} = \gamma^0 m + \mathcal{E} + \mathcal{O}, \tag{7}$$

where

$$\mathcal{E} = \frac{3\pi G}{8} J^{i}_{5}\gamma^0\gamma_5\gamma_i \tag{8}$$

and

$$\mathcal{O} = -\mathrm{i}\gamma^0\Gamma_0 - \mathrm{i}\gamma^0\gamma^i\partial_i - \mathrm{i}\gamma^0\gamma^i\Gamma_i + \frac{3\pi G}{8}J^0_5\gamma_5. \tag{9}$$

Here, $\mathcal{E}$ and $\mathcal{O}$ denote the even and odd terms, respectively [36]. According to Ref. 34, to the lowest order in p_i/m, the block diagonal Hamiltonian $\tilde{H}'$ has the form

$$\tilde{H}' \approx \gamma^0 m + \mathcal{E} + \frac{\gamma^0}{2m}\mathcal{O}^2, \tag{10}$$

and the particle (antiparticle) 2×2 Hamiltonian corresponds to the upper (lower) block. We can then write a Shrödinger-like equation acting on a Pauli (i.e., two-component) spinor.

Observe that the terms containing Γ_a are odd, which enter Eq. (7) either squared or in an anticommutator with another odd operator. By inspection, we can conclude that all contributions containing Γ_a enter $\tilde{H}$ as a multiple of the identity matrix, which commutes with all spin operators. Therefore, to linear order in G, the interactions mediated by the metric cannot mix the spinorial components of the beam. However, this is not the case for torsion interactions. Since we are primarily interested in distinguishing EC from GR by measuring changes in spin, we omit the metric interactions from here on to estimate the effect's size.

The relevant non-relativistic Hamiltonian for a particle travelling in the $+x$ direction up to the first order in p_i/m and G becomes

$$H = \frac{p^2}{2m} + \frac{3\pi G}{8}\left[\frac{\sigma_x J^0_5 p}{2m} + \frac{\sigma_x p (J^0_5 \bullet)}{2m} - \sum_{j=1}^{3}(J_5)^j\sigma_j\right], \tag{11}$$

where $p \equiv p_x = -i\partial_x$. In Eq. (11), the symbol $\bullet$ emphasizes that p also acts on the vector on which H operates. It comes as no surprise that J^0_5 and J^i_5 play different roles since, in this limit, there are preferred notions of space and time.

Our setting consists of a non-relativistic monochromatic beam of spin-$\frac{1}{2}$ neutrons [37] directed towards a

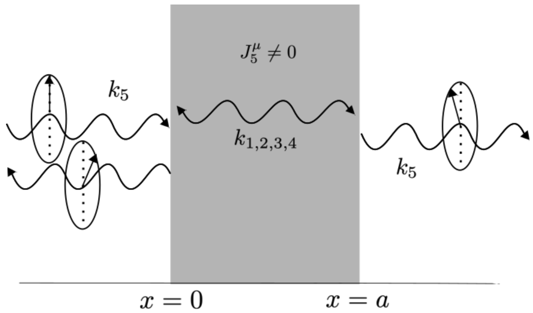

Figure 1. Problem set-up and auxiliary constructions. Our boundary conditions consist of a right-moving incident beam, whose polarization (schematically represented in the wheels) is aligned with the z axis, and a purely right-moving transmitted beam from a slab occupying the $0 \leq x \leq a$.

spin-polarized target, whose geometry is described below and with $J_5^\mu \neq 0$ within (see Fig. 1). Slowly moving neutrons are natural candidates for the beam particles as they avoid Coulomb-type interactions and have been used for decades as probes of fundamental physics [38]. They can also be handled and measured with exquisite sensitivities (see, e.g., Refs. 39–43; Ref. 44 is a review with possible future applications), and experiments where the spin polarization of a neutron beam that goes through a polarized media have already been performed [45].

The most elementary model for the target capable of distinguishing EC from GR consists of an axial current whose components are constants. Such a current is incompatible with the free Dirac equation for a massive Ψ field. Still, since any realistic target demands interactions between its constituents, we shall not impose the Dirac equations on it. Concretely, we consider the axial current

$$J_5^\mu(x) = \begin{cases} (j_5^0, j_5^x, j_5^y, j_5^z), & 0 \leq x \leq a, \\ 0, & \text{otherwise}, \end{cases} \tag{12}$$

with j_5^μ real constants satisfying $(j_5^0)^2 \gg (j_5^x)^2 + (j_5^y)^2 + (j_5^z)^2$ to ensure that the axial current is a non-relativistic timelike vector field.

To model other unavoidable short-ranged spin-independent interactions between the target and the neutron beam, we add to the Hamiltonian (11) a potential $V_0\theta(x)\theta(a-x)\mathbf{1}$ for a constant $V_0 > 0$ and where θ is the conventional Heaviside step function. We write the stationary Schrödinger equation, $H\psi = E\psi$, as a first-order linear system for the Pauli spinor $\psi = \binom{f}{g}$ and its associated momenta. On the spinor basis in which the Pauli matrix σ_3 is diagonal, the linear system of equations takes the form

$$\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix} f \\ g \\ p_f \\ p_g \end{bmatrix} = A(x)\begin{bmatrix} f \\ g \\ p_f \\ p_g \end{bmatrix}, \tag{13}$$

where

$$A(x) \equiv \begin{bmatrix} 0 & 0 & \mathrm{i} & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ 2m\mathrm{i}\left(E - V_0 + \frac{3\pi G}{8}j_5^z\right) & \frac{3\pi G}{8}[2m(j_5^y + \mathrm{i}j_5^x) - j_5^0(\delta(x) - \delta(x-a))] & 0 & -\frac{3\pi}{4}\mathrm{i}Gj_5^0 \\ \frac{3\pi G}{8}[2m(\mathrm{i}j_5^x - j_5^y) - J^0(\delta(x) - \delta(x-a))] & 2m\mathrm{i}\left(E - V_0 - \frac{3\pi G}{8}j_5^z\right) & -\frac{3\pi}{4}\mathrm{i}Gj_5^0 & 0 \end{bmatrix}.$$

Observe that the Dirac delta arises from Eq. (11) once p acts on J_5^0, which is in turn described by Eq. (12).

We find the solution in each spatial sector and then impose the remaining boundary conditions. For $x < 0$, we take the solution representing a right-moving 2-spinor polarized in the $+z$ direction and a left-moving 2-spinor with arbitrary polarization,

$$\begin{aligned} f(x) &= \mathrm{e}^{\mathrm{i}k_5 x} + b\mathrm{e}^{-\mathrm{i}k_5 x}, & x < 0, \\ g(x) &= c\mathrm{e}^{-\mathrm{i}k_5 x}, & x < 0, \end{aligned} \tag{14}$$

where $k_5 \equiv \sqrt{2mE}$ and b and c are constants to be determined. The overall normalization is immaterial, and we exploit it to fix the coefficient in the right-propagating contribution of f. Similarly, in $x > a$, we only consider right-moving waves:

$$\begin{aligned} f(x) &= h\mathrm{e}^{\mathrm{i}k_5 x}, & x > a, \\ g(x) &= r\mathrm{e}^{\mathrm{i}k_5 x}, & x > a. \end{aligned} \tag{15}$$

One can readily verify that Eqs. (14) and (15) solve Eq. (13).

Inside the target ($0 \leq x \leq a$), the calculation is slightly more involved. The matrix A admits four eigenvalues, which we denote by $\mathrm{i}k_n$, $n = 1, 2, 3, 4$. Their corresponding eigenvectors are designated by $\binom{\psi}{-\mathrm{i}\partial_x\psi}$ with

$$\psi = \sum_{n=1}^{4} d_n Y_n \mathrm{e}^{\mathrm{i}k_n x}, \tag{16}$$

for certain amplitudes d_n. The vectors Y_n can be taken (up to normalization) to be

$$Y_1 = Y_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \text{and} \quad Y_3 = Y_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{17}$$

It is convenient to express the eigenvalues in terms of k_5 and the parameter $\kappa \equiv \sqrt{2m(V_0 - E)}$. In what follows, we assume κ to be real and positive ($V_0 > E$), but it could also be purely imaginary, representing the situation with no damping due to non-gravitational interactions. To the lowest order in G, these eigenvalues are

$$\begin{aligned} k_1 &\approx -\mathrm{i}\kappa + \frac{3\pi G}{8}\left(j_5^0 - \mathrm{i}\frac{m(j_5^x + j_5^y + j_5^z)}{2\kappa}\right), && \text{(18a)} \\ k_2 &\approx \mathrm{i}\kappa + \frac{3\pi G}{8}\left(j_5^0 + \mathrm{i}\frac{m(j_5^x + j_5^y + j_5^z)}{2\kappa}\right), && \text{(18b)} \\ k_3 &\approx -\mathrm{i}\kappa - \frac{3\pi G}{8}\left(j_5^0 - \mathrm{i}\frac{m(j_5^x + j_5^y + j_5^z)}{2\kappa}\right), && \text{(18c)} \\ k_4 &\approx \mathrm{i}\kappa - \frac{3\pi G}{8}\left(j_5^0 + \mathrm{i}\frac{m(j_5^x + j_5^y + j_5^z)}{2\kappa}\right). && \text{(18d)} \end{aligned}$$

To obtain the amplitudes, $b, c, \{d_n\}_{n=1,2,3,4}, h, r,$ we impose the continuity of ψ across $x = 0$ and $x = a$, leading to four algebraic equations. To attain the remaining four, we integrate Eq. (13) from $-\epsilon$ to $+\epsilon$ and again from $a - \epsilon$ to $a + \epsilon$ and take the limit $\epsilon \to 0^+$. The resulting equations are aggregated to form the following system:

$$\begin{bmatrix} 0 & 0 & k_1 e^{\mathrm{i}k_1 a} & k_2 e^{\mathrm{i}k_2 a} & -k_3 e^{\mathrm{i}k_3 a} & -k_4 e^{\mathrm{i}k_4 a} & k_5 e^{\mathrm{i}k_5 a} & -\frac{3\pi}{8}Gj_5^0 e^{\mathrm{i}k_5 a} \\ 0 & 0 & -k_1 e^{\mathrm{i}k_1 a} & -k_2 e^{\mathrm{i}k_2 a} & -k_3 e^{\mathrm{i}k_3 a} & -k_4 e^{\mathrm{i}k_4 a} & -\frac{3\pi}{8}Gj_5^0 e^{\mathrm{i}k_5 a} & k_5 e^{\mathrm{i}k_5 a} \\ \frac{3\pi}{8}Gj_5^0 & k_5 & k_1 & k_2 & k_3 & k_4 & 0 & 0 \\ k_5 & \frac{3\pi}{8}Gj_5^0 & -k_1 & -k_2 & k_3 & k_4 & 0 & 0 \\ 0 & 0 & e^{\mathrm{i}k_1 a} & e^{\mathrm{i}k_2 a} & e^{\mathrm{i}k_3 a} & e^{\mathrm{i}k_4 a} & 0 & -e^{\mathrm{i}k_5 a} \\ 0 & 0 & -e^{\mathrm{i}k_1 a} & -e^{\mathrm{i}k_2 a} & e^{\mathrm{i}k_3 a} & e^{\mathrm{i}k_4 a} & -e^{\mathrm{i}k_5 a} & 0 \\ -1 & 0 & -1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} b \\ c \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ h \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\frac{3\pi}{8}Gj_5^0 \\ k_5 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}. \tag{19}$$

Because the incoming beam has only spin-up components, any non-zero c or r, which represent spin-down components in the reflected and transmitted beams, respectively, attest to a spin-dependent effect, which, in our case, is due to torsion.

In possession of the solutions to Eq. (19), we can extract observable consequences of the scattering process. For example, by splitting the spinor at $x \to -\infty$ as $\psi = \psi_i + \psi_r$, where the first term contains the incoming right-propagating wave and the second the reflected left-propagating part of ψ, we can obtain the reflection coefficient from the ratio between the reflected and incident beams. Importantly, the reflection and transmission coefficient are sensitive to Γ_a and contain order $\mathcal{O}(G)$ GR effects. These effects should be encoded in a Hamiltonian term proportional to the identity matrix and are non-vanishing outside the target; thus, they cannot be absorbed into V_0. Nevertheless, our idealized setting can offer an order-of-magnitude estimate for the angles ϕ_R and ϕ_T between the expectation value of the spin polarization of the incident and the reflected and transmitted beams, which standard GR predicts to be zero to order $\mathcal{O}(G)$. We have

$$\cos\phi_R = \frac{\psi_r^\dagger \vec{\sigma}\psi_r \cdot \psi_i^\dagger \vec{\sigma}\psi_i}{|\psi_r^\dagger \vec{\sigma}\psi_r|\,|\psi_i^\dagger \vec{\sigma}\psi_i|} = \frac{|b|^2 - |c|^2}{|b|^2 + |c|^2}, \tag{20}$$

and

$$\cos\phi_T = \frac{|h|^2 - |r|^2}{|h|^2 + |r|^2}. \tag{21}$$

Even though Eqs. (20) and (21) are ratios of quadratic in polynomials in the expansion parameter G, the coefficients c and h do not contain G-independent terms. This observation allows us to carry out Eq. (18) and the solution to Eqs. (19) only to linear order in G. The resulting expressions are lengthy, but we can consider the limit when the thickness a of the target is much larger than the real and imaginary parts of all four wavelengths inside or outside the material. For the case where $\kappa \in \mathbb{R}$, this results in

$$\phi_R \approx \frac{3\pi G}{4}\frac{k_5 m|j_5^x + j_5^y + j_5^z|}{\kappa(\kappa^2 + k_5^2)}, \tag{22}$$

$$\phi_T \approx \frac{3\pi G m a}{8\kappa} \times \sqrt{\vec{j_5}^{\,2} + 2(j_5^x j_5^y + j_5^y j_5^z + j_5^x j_5^z) + \frac{4(j_5^0)^2\kappa^2}{m^2}}. \tag{23}$$

The case where κ is imaginary produces comparable effects.

Notably, there exist spin-polarized targets insensitive to magnetic effects [46, 47], for which the achieved number of polarized particles is 10^{23} in a cylindrical device whose diameter and height are roughly 5cm; resulting in a density of spin-polarized particles of $2.5 \cdot 10^{26}\mathrm{m}^{-3}$. Of course, if one foregoes the insensitivity to external magnetic fields, this number could be much improved. To obtain an order-of-magnitude estimation, we consider the target to be polarized along the $+y$ direction and a realistic [38, sec. 2.3.1.] speed for the ultra-cold neutrons of $5\mathrm{ms}^{-1}$. Estimating $V_0 \approx 1.5E$, Eq. (22) yields $\phi_R \approx 5.5 \cdot 10^{-47}$ radians, and Eq. (23) for $j_5^0 = 10 j_5^y$, which ensures that the target is non-relativistic, produces $\phi_T \approx (3.3 \cdot 10^{-36})(a/\mathrm{m})$. The transmitted angle is manifestly proportional to a, implying that this effect is cumulative: the angle grows with the target's thickness (it can also be enlarged by manipulating the beam to go through the target several times). In addition, the angle of polarization of the neutrons can be measured with very high sensitivity at a rate of 10^{-7} radians per meter [45, 48, 49].

Remarkably, the deflection of the transmitted angles can be significantly amplified when the non-gravitational barrier V_0 approaches the beam energy E. To obtain the corresponding expression, we take the limit $\kappa \to 0^+$ before setting $a \gg k_j^{-1}$ for $j = 1, 2, 3, 4, 5$. The results are

$$\phi_R \approx \frac{\pi G}{2} \frac{ma|j_5^x + j_5^y + j_5^z|}{k_5} \sim 3 \cdot 10^{-36} \left(\frac{a}{\mathrm{m}}\right), \quad (24)$$

$$\phi_T \approx \frac{\pi G}{8} |j_5^x + j_5^y + j_5^z| m a^2 \sim 6 \cdot 10^{-26} \left(\frac{a}{\mathrm{m}}\right)^2, \quad (25)$$

where we used the values of the previous paragraphs to get numerical estimates.

Admittedly, achieving this limit may be technically complicated, although measurements of ϕ_R may be used to adjust κ near zero. Nevertheless, due to the dramatic increase in magnitude and the a^2 scaling behaviour contrasted with a^1 far from this limit, one can hope that an analysis of a realistic target alongside an increase in sensitivity in the angle detection and higher values J_5^μ could settle the debate between GR and EC shortly. After all, this proposal shares most of the features that allowed an improvement of several orders of magnitude in bounding departures of local Lorentz invariance attained in a few decades, reaching Planck scale sensitivity (a comprehensive list of limits on local Lorentz invariance is found in Ref. 50). In both cases, the effect is cumulative, is amplified at a certain energy, and involves high-precision experimental techniques. A realistic experiment will likely require more detailed modelling of the target. Yet, the type of observables proposed here are capable of experimentally distinguishing GR and EC, which would have far-reaching implications in astrophysics and theoretical physics.

We acknowledge an exchange with W. G. Snow and the financial support from CONACyT through the grant FORDECYT-PRONACES 140630. BAC receives funding from a UNAM-DGAPA fellowship.

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[10] We work in 4 spacetime dimensions and in units where $c = 1 = \hbar$. Indices for tangent space are denoted by Greek letters, and spacetime indices are denoted with Latin letters from the beginning of the alphabet. Repeated indices imply contraction, and Greek indices are lowered (raised) using $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ ($\eta^{\mu\nu}$, the inverse matrix of $\eta_{\mu\nu}$). Latin indices i, j, k are used for spatial Minkowskian coordinates and, when no confusion arises, are also used for the corresponding tangent space elements.
[11] Gamma matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$, and $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$. The Dirac adjoint is $\bar{\Psi} = \Psi^\dagger \gamma^0$. We adopt the Dirac representation for the gamma matrices, and σ^i denotes the usual Pauli matrices.
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