Regular accelerating Universe without dark energy

A.V. Minkevich[1,2], A.S. Garkun[1] and V.I. Kudin[3]

[1]Department of Theoretical Physics, Belarussian State University, Belarus
[2]Department of Physics and Computer Methods, Warmia and Mazury University in Olsztyn, Poland
[3]Department of Technical Physics, Belarussian National Technic University, Belarus

E-mail: minkav@bsu.by, garkun@bsu.by

Abstract. Homogeneous isotropic cosmological models with two torsion functions filled with scalar fields and usual gravitating matter are built and investigated in the framework of the Poincaré gauge theory of gravity. It is shown that by certain restrictions on indefinite parameters of gravitational Lagrangian the cosmological equations at asymptotics contain an effective cosmological constant that can explain observable acceleration of cosmological expansion. The behavior of inflationary cosmological solutions at extremely high energy densities is analyzed, regular bouncing solutions are obtained. The role of the space-time torsion provoking the acceleration of cosmological expansion is discussed.

PACS numbers: 04.50.+h; 98.80.Cq; 11.15.-q; 95.36.+x

Submitted to: *Class. Quantum Grav.*

1. Introduction

The discovery of the acceleration of cosmological expansion at present epoch is the most principal achievement of observational cosmology at last time [1]. By using Friedmann cosmological equations of General Relativity theory (GR) in order to explain accelerating cosmological expansion, the notion of dark energy (or quintessence) was introduced in cosmology. According to obtained estimations, approximately 70% of energy in our Universe is related with some hypothetical form of gravitating matter with negative pressure — "dark energy" — of unknown nature. Previously a number of investigations devoted to dark energy problem were carried out (see review [2]). According to widely known opinion, the dark energy is associated with cosmological term. If the cosmological term is related to the vacuum energy density, it is necessary to explain, why it has the value close to critical energy density at present epoch (see for example [3]).

The present paper is devoted to investigation of the "dark energy" problem in the framework of the Poincaré gauge theory of gravity (PGTG), which is a natural generalization of Einsteinian GR by applying the gauge approach to the theory of gravitational interaction (see for example [11]). In fact the generalization of GR leading to PGTG is necessary, if one supposes that the Lorentz gauge field corresponding to fundamental group in physics – the Lorentz group – exists in the nature (see [4]). According to PGTG the physical space-time possesses the structure of Riemann-Cartan continuum with curvature and torsion. The investigation of isotropic cosmology built in the framework of PGTG (see [4-6] and references herein) shows that the gravitational interaction in PGTG, unlike GR and Newton's gravitation theory, can have the repulsion character in the case of gravitating systems with positive energy densities and pressures satisfying energy dominance condition. So, the gravitational repulsion effect takes place at extreme conditions (extremely high energy densities and pressures) preventing the appearance of cosmological singularity in homogeneous isotropic models (HIM) [5]. According to generalized cosmological Friedmann equations (GCFE) for HIM deduced in the framework of PGTG, all cosmological solutions including inflationary solutions are regular in metrics, Hubble parameter, its time derivative and have bouncing character. Properties of discussed HIM in PGTG coincide practically with that of GR at sufficiently small energy densities, which are much less in comparison with limiting (maximum) energy density for such models. By including cosmological term of corresponding value to GCFE, we can obtain regular cosmological solutions with observable accelerating expansion stage. However, like GR, the problem of dark energy in such theory is not solved.

From geometrical point of view, the structure of HIM in PGTG can be more complicated in comparison with models describing by GCFE. In fact in the case of homogeneous isotropic models the torsion tensor $S^{\lambda}{}_{\mu\nu} = -S^{\lambda}{}_{\nu\mu}$ can have the following non-vanishing components [7, 8]: $S^1{}_{10} = S^2{}_{20} = S^3{}_{30} = S_1(t)$, $S_{123} = S_{231} = S_{312} = S_2(t)\dfrac{R^3 r^2}{\sqrt{1-kr^2}}\sin\theta$, where S_1 and S_2 are two torsion functions of time, spatial spherical coordinates are used. The functions S_1 and S_2 have different properties with respect to transformations of spatial inversions, namely, the function $S_2(t)$ has pseudoscalar character. The GCFE follow from gravitational equations of PGTG for HIM together with $S_2 = 0$. Obtained physical consequences of GCFE have principal character. However, it is necessary to note that gravitational equations of PGTG for HIM have also other solution with non-vanishing function S_2.

The HIM with two torsion functions filled with scalar fields and usual gravitating matter are studied below in the frame of PGTG in connection with the dark energy problem. Following to [9], in Section 2 cosmological equations for such HIM are introduced. In Section 3 the solutions asymptotics of cosmological equations is analyzed. In Section 4 the bouncing character of inflationary cosmological solutions is examined.

2. Cosmological equations for HIM with two torsion functions in PGTG

At first, let us mention some general relations of the PGTG. Gravitational field is described in the frame of PGTG by means of the orthonormalized tetrad $h^i{}_\mu$ and anholonomic Lorentz connection $A^{ik}{}_\mu$ (tetrad and holonomic indices are denoted by latin and greek respectively); corresponding field strengths are torsion $S^i{}_{\mu\nu}$ and curvature $F^{ik}{}_{\mu\nu}$ tensors defined as

$$S^i{}_{\mu\,\nu} = \partial_{[\nu}\, h^i{}_{\mu]} - h_{k[\mu} A^{ik}{}_{\nu]}\,,$$

$$F^{ik}{}_{\mu\nu} = 2\partial_{[\mu} A^{ik}{}_{\nu]} + 2A^{il}{}_{[\mu} A^k{}_{|l\,|\nu]}\,.$$

We will consider the PGTG based on the following general form of gravitational Lagrangian

$$\mathcal{L}_{\rm G} = h\left[f_0\, F + F^{\alpha\beta\mu\nu}\left(f_1\, F_{\alpha\beta\mu\nu} + f_2\, F_{\alpha\mu\beta\nu} + f_3\, F_{\mu\nu\alpha\beta}\right) + F^{\mu\nu}\left(f_4\, F_{\mu\nu} + f_5\, F_{\nu\mu}\right) + f_6\, F^2 \right.$$
$$\left. + S^{\alpha\mu\nu}\left(a_1\, S_{\alpha\mu\nu} + a_2\, S_{\nu\mu\alpha}\right) + a_3\, S^\alpha{}_{\mu\alpha} S_\beta{}^{\mu\beta}\right], \tag{1}$$

where $h = \det\left(h^i{}_\mu\right)$, $F_{\mu\nu} = F^\alpha{}_{\mu\alpha\nu}$, $F = F^\mu{}_\mu$, f_i $(i = 1, 2, \ldots, 6)$, a_k $(k = 1, 2, 3)$ are indefinite parameters, $f_0 = (16\pi G)^{-1}$, G is Newton's gravitational constant. Gravitational equations of PGTG obtained from the action integral $I = \int \left(\mathcal{L}_g + \mathcal{L}_m\right) d^4x$, where $\mathcal{L}_m$ is the Lagrangian of matter, contain the system of 16+24 equations corresponding to gravitational variables $h^i{}_\mu$ and $A^{ik}{}_\mu$.

Any homogeneous isotropic gravitating system in PGTG is characterized in general case by three functions of time: the scale factor of Robertson-Walker metrics R and two torsion functions S_1 and S_2. Below the spherical coordinate system is used and the tetrad is taken in diagonal form. Then the curvature tensor has the following non-vanishing tetrad components denoted by means of the sign ˆ :

$$F^{\hat 0\hat 1}{}_{\hat 0\hat 1} = F^{\hat 0\hat 2}{}_{\hat 0\hat 2} = F^{\hat 0\hat 3}{}_{\hat 0\hat 3} \equiv A_1, \qquad F^{\hat 1\hat 2}{}_{\hat 1\hat 2} = F^{\hat 1\hat 3}{}_{\hat 1\hat 3} = F^{\hat 2\hat 3}{}_{\hat 2\hat 3} \equiv A_2,$$
$$F^{\hat 0\hat 1}{}_{\hat 2\hat 3} = F^{\hat 0\hat 2}{}_{\hat 3\hat 1} = F^{\hat 0\hat 3}{}_{\hat 1\hat 2} \equiv A_3, \qquad F^{\hat 3\hat 2}{}_{\hat 0\hat 1} = F^{\hat 1\hat 3}{}_{\hat 0\hat 2} = F^{\hat 2\hat 1}{}_{\hat 0\hat 3} \equiv A_4$$

with

$$\begin{aligned} A_1 &= \dot H + H^2 - 2HS_1 - 2\dot S_1, \\ A_2 &= \frac{k}{R^2} + \left(H - 2S_1\right)^2 - S_2^2, \\ A_3 &= 2\left(H - 2S_1\right) S_2, \\ A_4 &= \dot S_2 + HS_2, \end{aligned} \tag{2}$$

where $H = \dot R/R$ is the Hubble parameter and a dot denotes the differentiation with respect to time.

The Bianchi identities in this case are reduced to two following relations:

$$\begin{aligned} \dot A_2 + 2H\left(A_2 - A_1\right) + 4S_1 A_1 + 2S_2 A_4 &= 0, \\ \dot A_3 + 2H\left(A_3 - A_4\right) + 4S_1 A_4 - 2S_2 A_1 &= 0. \end{aligned} \tag{3}$$

The system of gravitational equations for HIM corresponding to gravitational Lagrangian (1) has the following form

$$a\,(H-S_1)\,S_1-2bS_2^2-2f_0A_2+4f\left(A_1^2-A_2^2\right)+2q_2\left(A_3^2-A_4^2\right)=-\frac{\rho}{3}, \tag{4}$$

$$a\left(\dot{S}_1+2HS_1-S_1^2\right)-2bS_2^2-2f_0\,(2A_1+A_2)-4f\left(A_1^2-A_2^2\right)-2q_2\left(A_3^2-A_4^2\right)=p, \tag{5}$$

$$f\left[\dot{A}_1+2H\,(A_1-A_2)+4S_1A_2\right]+q_2S_2A_3-q_1S_2A_4+\left(f_0+\frac{a}{8}\right)S_1=0, \tag{6}$$

$$q_2\left[\dot{A}_4+2H\,(A_4-A_3)+4S_1A_3\right]-4f\,S_2A_2-2q_1S_2A_1-(f_0-b)\,S_2=0, \tag{7}$$

where

$$a=2a_1+a_2+3a_3, \qquad b=a_2-a_1,$$
$$f=f_1+\frac{f_2}{2}+f_3+f_4+f_5+3f_6\,,$$
$$q_1=f_2-2f_3+f_4+f_5+6f_6, \qquad q_2=2f_1-f_2,$$

ρ is the energy density, p is the pressure and the average of spin distribution of gravitating matter is supposed to be equal to zero. Equations (4)–(5) lead to generalization of Friedmann cosmological equations of GR, which does not contain high derivatives for the scale factor R, if $a=0$ (see below). Moreover, equations (6)–(7) take more symmetric form, if $2f=q_1+q_2$. Then by using the Bianchi identities (3), the system of gravitational equations for HIM take the following form:

$$-2b\,S_2^2-2f_0A_2+4f\left(A_1^2-A_2^2\right)+2q_2\left(A_3^2-A_4^2\right)=-\frac{1}{3}\rho, \tag{8}$$

$$-2b\,S_2^2-2f_0\,(2A_1+A_2)-4f\left(A_1^2-A_2^2\right)-2q_2\left(A_3^2-A_4^2\right)=p, \tag{9}$$

$$f\left[\left(\dot{A}_1+\dot{A}_2\right)+4S_1\,(A_1+A_2)\right]+q_2\,S_2\,(A_3+A_4)+f_0S_1=0, \tag{10}$$

$$q_2\left[\left(\dot{A}_3+\dot{A}_4\right)+4S_1\,(A_3+A_4)\right]-4f\,S_2\,(A_1+A_2)-(f_0-b)\,S_2=0. \tag{11}$$

The system of equations (8)–(11) together with definition of curvature functions (2) is the base of our investigation of HIM below. Note also that the conservation law for spinless matter has the usual form:

$$\dot{\rho}+3H\,(\rho+p)=0. \tag{12}$$

In order to investigate inflationary cosmological models we will consider below HIM filled with non-interacting scalar field ϕ minimally coupled with gravitation and gravitating matter with equation of state in the form $p_m=p_m(\rho_m)$ (values of gravitating matter are denoted by means of index "m"). Then the energy density ρ and the pressure p take the form

$$\rho=\frac{1}{2}\dot{\phi}^2+V+\rho_m \quad (\rho>0), \quad p=\frac{1}{2}\dot{\phi}^2-V+p_m, \tag{13}$$

where $V = V(\phi)$ is a scalar field potential. By using the scalar field equation in homogeneous isotropic space

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \tag{14}$$

we obtain from (12)–(13) the conservation law for gravitating matter

$$\dot{\rho}_m + 3H\left(\rho_{\rm m} + p_{\rm m}\right) = 0. \tag{15}$$

From (8)–(9) follows that

$$A_1 + A_2 = \frac{1}{12f_0}\left(\rho - 3p\right) - \frac{b}{f_0}S_2^2. \tag{16}$$

By using (16) and the formula following from definition of curvature functions A_3 and A_4

$$A_3^2 - A_4^2 = 4A_2\,S_2^2 - 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2 - \left(\dot{S}_2 + HS_2\right)^2,$$

we find from gravitational equations (8)–(9) the following expressions for A_1 and A_2:

$$A_1 = -\frac{1}{12f_0 Z}\left[\rho + 3p - \frac{\alpha}{2}\left(\rho - 3p - 12bS_2^2\right)^2\right] \\ -\frac{\alpha\varepsilon}{Z}\left(\rho - 3p - 12bS_2^2\right)S_2^2 + \frac{3\alpha\varepsilon f_0}{Z}\left[\left(HS_2 + \dot{S}_2\right)^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right], \tag{17}$$

$$A_2 = \frac{1}{6f_0 Z}\left[\rho - 6bS_2^2 + \frac{\alpha}{4}\left(\rho - 3p - 12bS_2^2\right)^2\right] \\ -\frac{3\alpha\varepsilon f_0}{Z}\left[\left(HS_2 + \dot{S}_2\right)^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right], \tag{18}$$

where $Z \equiv 1 + \alpha\left(\rho - 3p - 12\left(b + \varepsilon f_0\right)S_2^2\right) = 1 + \alpha\left(4V - \dot{\phi}^2 + \rho_{\rm m} - 3p_{\rm m} - 12\left(b + \varepsilon f_0\right)S_2^2\right)$, $\alpha \equiv f/(3f_0^2)$, $\varepsilon \equiv q_2/f$ (hence, $q_2 = 3\alpha\,\varepsilon f_0^2$). By using (13)–(16) and the following relation obtained from definition of A_3 and A_4

$$A_3 + A_4 = \dot{S}_2 + 3HS_2 - 4S_1S_2, \tag{19}$$

we find for the torsion function S_1 from (10) the following expression:

$$S_1 = -\frac{3\alpha}{4Z}\left[\frac{\partial V}{\partial \phi}\dot{\phi} \;+ H\left(Y + 2\dot{\phi}^2\right) - 4\left(2b - \varepsilon f_0\right)S_2\,\dot{S}_2\right], \tag{20}$$

where

$$Y \equiv \left(\rho_{\rm m} + p_{\rm m}\right)\left(3\frac{dp_{\rm m}}{d\rho_{\rm m}} - 1\right) + 12\varepsilon f_0 S_2^2.$$

Then by using formulas (16) and (19) we find from (11) the following second order differential equation for the torsion function S_2:

$$\varepsilon\left[\ddot{S}_2 + 3H\dot{S}_2 + 3\dot{H}S_2 - 4\left(\dot{S}_1 - 3HS_1 + 4S_1^2\right)S_2\right] \\ -\frac{1}{3f_0}\left(\rho - 3p - 12bS_2^2\right)S_2 - \frac{(f_0 - b)}{f}S_2 = 0\,. \tag{21}$$

The obtained expressions (17)–(18) for curvature functions A_2 and A_1 together with their definition (2) give the generalization of cosmological Friedmann equations for HIM:

$$\frac{k}{R^2}+(H-2S_1)^2=\frac{1}{6f_0Z}\left[\rho+6\left(f_0Z-b\right)S_2^2+\frac{\alpha}{4}\left(\rho-3p-12bS_2^2\right)^2\right]$$
$$-\frac{3\alpha\varepsilon f_0}{Z}\left[\left(HS_2+\dot{S}_2\right)^2+4\left(\frac{k}{R^2}-S_2^2\right)S_2^2\right], \quad (22)$$

$$\dot{H}+H^2-2HS_1-2\dot{S}_1=-\frac{1}{12f_0Z}\left[\rho+3p-\frac{\alpha}{2}\left(\rho-3p-12bS_2^2\right)^2\right]$$
$$-\frac{\alpha\varepsilon}{Z}\left(\rho-3p-12bS_2^2\right)S_2^2+\frac{3\alpha\varepsilon f_0}{Z}\left[\left(HS_2+\dot{S}_2\right)^2+4\left(\frac{k}{R^2}-S_2^2\right)S_2^2\right]. \quad (23)$$

These equations contain the torsion function S_1 determined by (20) and the torsion function S_2, satisfying the equation (21). Obtained equations contain three indefinite parameters: indefinite parameter α determining the scale of extremely high energy densities [4], parameter b with dimension of parameter f_0 and the parameter ε without dimension. We have to analyze all these equations in order to investigate HIM with pseudoscalar torsion function in the frame of PGTG.

3. Asymptotics of cosmological solutions for HIM with pseudoscalar torsion function

The structure of obtained equations (20)–(23) describing HIM with two torsion functions is essentially more complicated in comparison with the case of HIM with vanishing function S_2. Note that if $S_2 = 0$ the equation (21) vanishes and the cosmological equations (22)–(23) are transformed to GCFE containing the only indefinite parameter α [4,5].

Now we will analyze the following question: by what restrictions on indefinite parameters the cosmological solutions for HIM with pseudoscalar torsion function have the asymptotics in agreement with actual observations. By taking into account that various parameters of HIM have to be small at asymptotics, when values of energy density are sufficiently small, we see from (21), that if $|\varepsilon| \ll 1$, the pseudoscalar torsion function has at asymptotics the following value:

$$S_2^2=\frac{f_0(f_0-b)}{4fb}+\frac{\rho-3p}{12b}. \quad (24)$$

Then we have at asymptotics: $Z \to (b/f_0)$, $S_1 \to 0$ and the cosmological equations (22)–(23) at asymptotics take the form of cosmological Friedmann equations with cosmological constant:

$$\frac{k}{R^2}+H^2=\frac{1}{6b}\left[\rho+\frac{3\left(f_0-b\right)^2}{4f}\right], \quad (25)$$

$$\dot{H} + H^2 = -\frac{1}{12b}\left[\rho + 3p - \frac{3\left(f_0 - b\right)^2}{2f}\right]. \tag{26}$$

From equations (25)–(26) we see, that parameter b has to be very close to f_0, but smaller than f_0. The value of b leading to observable acceleration of cosmological expansion depends on the scale of extremely high energy density defined by α^{-1}. If we take into account that the value of $\frac{3}{4}(f_0 - b)^2/f = \frac{1}{4}\alpha^{-1}(1 - b/f_0)^2$ is equal approximately to $0.7\rho_{\rm cr}$ (the critical energy density is $\rho_{\rm cr} = 6f_0H_0^2$, where H_0 is the value of the Hubble parameter at present epoch), then we obtain that $b = [1 - (2.8\rho_{cr}\alpha)^{1/2}]f_0$. If we suppose that the scale of extremely high energy densities is larger than the energy density for quark-gluon matter, but less than the Planckian energy density, then we obtain the corresponding estimation for b, which is very close to f_0. Obtained restrictions on indefinite parameters will be used below for investigation of inflationary cosmological solutions.

4. Regular inflationary cosmological solutions with two torsion functions

To obtain cosmological solution by integrating cosmological equations we have to use the equation of state of gravitating matter, which is different at different stages of cosmological evolution. So, at asymptotics one uses usually equation of state for dust matter ($\rho = 0$). At the same time, in order to obtain cosmological solution for inflationary HIM, we will use at the beginning of cosmological expansion the expressions (13) for energy density and pressure by including scalar field as one component of gravitating matter. Like GR, the inflationary stage appears, if the value of scalar fields at the beginning of cosmological expansion is sufficiently large ($\phi > 1\,M_{\rm p}$, $M_{\rm p}$ is Planckian mass) [10].

In order to investigate inflationary cosmological solutions at extremely high energy densities, by using (13)–(15) and (20) we transform cosmological equations (22)–(23) and equation (21) for S_2-function to the following form

$$\begin{aligned} H^2 &\left\{\left[Z + \frac{3}{2}\alpha\left(Y + 2\dot{\phi}\right)\right]^2 + 3\alpha\varepsilon f_0 S_2^2 Z\right\} \\ &+6\alpha H\left\{\left[Z + \frac{3}{2}\alpha\left(Y + 2\dot{\phi}^2\right)\right] \times \left[\frac{\partial V}{\partial\phi}\dot{\phi} - 2\left(2b - \varepsilon f_0\right)S_2\dot{S}_2\right] + \varepsilon f_0 S_2\dot{S}_2 Z\right\} \\ &+9\alpha^2\left[\frac{\partial V}{\partial\phi}\dot{\phi} - 2\left(2b - \varepsilon f_0\right)S_2\dot{S}_2\right]^2 + 3\alpha\varepsilon f_0\left[\dot{S}_2^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right]Z \\ &-\frac{1}{6f_0}\left[\rho_{\rm m} + \frac{1}{2}\dot{\phi} + V - 6bS_2^2 + \frac{1}{4}\alpha\left(\rho_{\rm m} - 3p_{\rm m} + 4V - \dot{\phi}^2 - 12bS_2^2\right)^2\right]Z \\ &+\left(\frac{k}{R^2} - S_2^2\right)Z^2 = 0, \end{aligned} \tag{27}$$

$$
\begin{aligned}
&\dot{H}\left[1+\frac{3\alpha}{2Z}\left(Y+2\dot{\phi}^2\right)\right]+H^2\left\{1+\frac{3\alpha}{2Z}\left(Y+2\dot{\phi}^2\right)-\frac{9\alpha^2}{2Z^2}\left(Y+2\dot{\phi}^2\right)\left(Y+2\dot{\phi}^2-12\varepsilon f_0 S_2^2\right)\right.\\
&\qquad\left.-\frac{9\alpha}{2Z}\left[3\frac{d^2p_{\rm m}}{d\rho_{\rm m}^2}\left(\rho_{\rm m}+p_{\rm m}\right)^2+\left(3\frac{dp_{\rm m}}{d\rho_{\rm m}}-1\right)\left(1+\frac{dp_{\rm m}}{d\rho_{\rm m}}\right)\left(\rho_{\rm m}+p_{\rm m}\right)+4\dot{\phi}^2\right]\right\}\\
&\qquad-\frac{3\alpha}{Z}H\left\{\left[4\frac{\partial V}{\partial\phi}\dot{\phi}+2\left(2b-7\varepsilon f_0\right)S_2\dot{S}_2\right]+\frac{3\alpha}{Z}\left[\left(\frac{\partial V}{\partial\phi}\dot{\phi}-2\left(2b-\varepsilon f_0\right)S_2\dot{S}_2\right)\right.\right.\\
&\qquad\left.\left.\times\left(Y+2\dot{\phi}^2-12\varepsilon f_0 S_2\dot{S}_2\right)+\left(Y+2\dot{\phi}^2\right)\times\left(\frac{\partial V}{\partial\phi}\dot{\phi}-4\left(b+\varepsilon f_0\right)S_2\dot{S}_2\right)\right]\right\}\\
&\qquad+\frac{3\alpha}{Z}\left\{\frac{\partial^2 V}{\partial\phi^2}\dot{\phi}^2-\left(\frac{\partial V}{\partial\phi}\right)^2\right.\\
&\qquad-\frac{6\alpha}{Z}\left(\frac{\partial V}{\partial\phi}\dot{\phi}-2\left(2b-\varepsilon f_0\right)S_2\dot{S}_2\right)\times\left(\frac{\partial V}{\partial\phi}\dot{\phi}-4\left(b+\varepsilon f_0\right)S_2\dot{S}_2\right)\\
&\qquad\left.-2\left(2b-\varepsilon f_0\right)\left(\dot{S}_2^2+S_2\ddot{S}_2\right)\right\}\\
&\qquad=-\frac{1}{12f_0Z}\left[\rho_{\rm m}+3p_{\rm m}-2\left(V-\dot{\phi}^2\right)-\frac{1}{2}\alpha\left(\rho_{\rm m}-3p_{\rm m}+4V-\dot{\phi}^2-12bS_2^2\right)^2\right]\\
&\qquad-\frac{\alpha\varepsilon}{Z}\left(\rho_{\rm m}-3p_{\rm m}+4V-\dot{\phi}^2-12bS_2^2\right)S_2^2\\
&\qquad+3\frac{\alpha\varepsilon f_0}{Z}\left[\left(HS_2^2+\dot{S}^2\right)^2+4\left(\frac{k}{R^2}-S_2^2\right)S_2^2\right],
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\ddot{S}_2\left[1-\frac{12\alpha}{Z}\left(2b-\varepsilon f_0\right)S_2^2\right]+3\dot{H}S_2\left[1+\frac{\alpha}{Z}\left(Y+2\dot{\phi}^2\right)\right]\\
&\qquad-9\frac{\alpha}{Z}H^2S_2\left[Y+6\dot{\phi}^2+3\frac{d^2p_{\rm m}}{d\rho_{\rm m}^2}\left(\rho_{\rm m}+p_{\rm m}\right)^2\right.\\
&\qquad+\left(3\frac{dp_{\rm m}}{d\rho_{\rm m}}-1\right)\left(1+\frac{dp_{\rm m}}{d\rho_{\rm m}}\right)\left(\rho_{\rm m}+p_{\rm m}\right)+\frac{\alpha}{Z}\left(Y+2\dot{\phi}^2\right)\\
&\qquad\left.\times\left(Y+2\dot{\phi}^2-12\varepsilon f_0 S_2^2\right)\right]+3HS_2\left\{1-4\frac{\alpha}{Z}\left(4\frac{\partial V}{\partial\phi}\dot{\phi}-3\left(2b+\varepsilon f_0\right)S_2\dot{S}_2\right)\right.\\
&\qquad-6\frac{\alpha^2}{Z^2}\left[\left(\frac{\partial V}{\partial\phi}\dot{\phi}-2\left(2b-\varepsilon f_0\right)S_2\dot{S}_2\right)\left(Y+2\dot{\phi}^2-12\varepsilon f_0 S_2\dot{S}_2\right)\right.\\
&\qquad\left.\left.+\left(Y+2\dot{\phi}^2\right)\left(\frac{\partial V}{\partial\phi}\dot{\phi}-4\left(b+\varepsilon f_0\right)S_2\dot{S}_2\right)\right]\right\}\\
&\qquad-9\frac{\alpha^2}{Z^2}S_2\left[H\left(Y+2\dot{\phi}^2\right)+2\left(\frac{\partial V}{\partial\phi}\dot{\phi}-2\left(2b-\varepsilon f_0\right)S_2\dot{S}_2\right)\right]^2\\
&\qquad-6\frac{\alpha}{Z}S_2\left[\left(\frac{\partial V}{\partial\phi}\right)^2-\frac{\partial^2 V}{\partial\phi^2}\dot{\phi}^2+2\left(2b-\varepsilon f_0\right)\dot{S}_2^2\right.
\end{aligned}
$$

$$+\frac{6\alpha}{Z}\left(\frac{\partial V}{\partial\phi}\dot{\phi}-2\left(2b-\varepsilon f_0\right)S_2\dot{S}_2\right)\left(\frac{\partial V}{\partial\phi}\dot{\phi}-4\left(b+\varepsilon f_0\right)S_2\dot{S}_2\right)\Bigg]$$

$$-\frac{1}{\varepsilon}\left[\frac{1}{3f_0}\left(\rho_{\rm m}-3p_{\rm m}+4V-\dot{\phi}^2-12bS_2^2\right)+\frac{f_0-b}{f}\right]S_2=0. \tag{29}$$

Equation (27) can be written as

$$AH^2+2BH+C=0, \tag{30}$$

where

$$A=\left[Z+\frac{3}{2}\alpha\left(Y+2\dot{\phi}^2\right)\right]^2+3\alpha\varepsilon f_0S_2^2Z,$$

$$B=3\alpha\left\{\left[Z+\frac{3}{2}\alpha\left(Y+2\dot{\phi}^2\right)\right]\left[\frac{\partial V}{\partial\phi}\dot{\phi}-2\left(2b-2\varepsilon f_0\right)S_2\dot{S}_2\right]+\varepsilon f_0S_2\dot{S}_2Z\right\},$$

$$C=9\alpha^2\left[\frac{\partial V}{\partial\phi}\dot{\phi}-2\left(2b-\varepsilon f_0\right)S_2\dot{S}_2\right]^2+3\alpha\varepsilon f_0\left[\dot{S}_2^2+4\left(\frac{k}{R^2}-S_2^2\right)S_2^2\right]Z$$
$$-\frac{1}{6f_0}\left[\rho_{\rm m}+\frac{1}{2}\dot{\phi}^2+V-6bS_2^2+\frac{1}{4}\alpha\left(\rho_{\rm m}-3p_{\rm m}+4V-\dot{\phi}^2-12bS_2^2\right)^2\right]Z$$
$$+\left(\frac{k}{R^2}-S_2^2\right)Z^2.$$

From (30) we obtain two $H_\pm$-solutions for the Hubble parameter

$$H_\pm=\frac{-B\pm\sqrt{D}}{A}, \tag{31}$$

where $D=B^2-4AC$.

At asymptotics H_--solutions and H_+-solutions describe the stages of cosmological compression and expansion respectively [4]. The transition from H_--solution to H_+-solution takes place when $D=0$.

Now we will analyze extremum surfaces in space of independent variables ϕ, $\dot{\phi}$, S_2, $\dot{S}_2$, $\rho_{\rm m}$, in the points of which the Hubble parameter vanishes $H=0$. Extremum surfaces depend on indefinite parameters α, ε, b and in the case of open and closed models also on the scale factor R. By denoting values of variables on extremum surfaces by means of index 0, we obtain from (30) the following equation for such surfaces

$$\frac{1}{6f_0}\left[\rho_{\rm m0}+\frac{1}{2}\dot{\phi}_0^2+V_0-6bS_{20}^2+\frac{1}{4}\alpha\left(\rho_{\rm m0}-3p_{\rm m0}+4V_0-\dot{\phi}_0^2-12bS_{20}^2\right)^2\right]Z_0$$
$$-9\alpha^2\left[\left(\frac{\partial V}{\partial\phi}\right)_0\dot{\phi}_0-2\left(2b-\varepsilon f_0\right)S_{20}\dot{S}_{20}\right]^2$$
$$-3\alpha\varepsilon f_0\left[\dot{S}_{20}^2+4\left(\frac{k}{R_0^2}-S_{20}^2\right)S_{20}^2\right]Z_0-\left(\frac{k}{R_0^2}-S_{20}^2\right)Z_0^2=0, \tag{32}$$

where

$$Z_0 = 1 + \alpha\left[\rho_{\rm m0} - 3p_{\rm m0} + 4V_0 - \dot{\phi}_0^2 - 12\left(b+\varepsilon f_0\right)S_{20}^2\right].$$

The derivative of the Hubble parameter on extremum surfaces obtained from (28)–(29) is determined as

$$\begin{aligned}
&\dot{H}_0 Z_0^2\left\{1+\alpha\left[\rho_{\rm m0} - 3p_{\rm m0} + \frac{3}{2}\left(3\left(\frac{dp_{\rm m}}{d\rho_{\rm m}}\right)_0 - 1\right)\left(\rho_{\rm m0}+p_{\rm m0}\right) + 4V_0 + 2\dot{\phi}_0^2\right]\right\}\\
&\quad = \left[1+\alpha\left(\rho_{\rm m0} - 3p_{\rm m0} + 4V_0 - \dot{\phi}_0^2 - 36bS_{20}^2\right)\right]\times\left\{\frac{Z_0}{2f_0}\left[\frac{1}{2}\left(\rho_{\rm m0}-p_{\rm m0}\right)+V_0\right.\right.\\
&\quad \left. -4bS_{20}^2 + \frac{1}{4}\alpha\left(\rho_{\rm m0} - 3p_{\rm m0} + 4V_0 - \dot{\phi}_0^2 - 12bS_{20}^2\right)^2\right]\\
&\quad +3\alpha Z_0\left[\left(\frac{\partial V}{\partial\phi}\right)_0^2 - \left(\frac{\partial^2 V}{\partial\phi^2}\right)_0\dot{\phi}_0^2 + \left(4b-3\varepsilon f_0\right)\dot{S}_{20}^2 - 4\varepsilon f_0\left(\frac{k}{R_0^2} - S_{20}^2\right)S_{20}^2\right.\\
&\quad \left. -\frac{1}{3}\varepsilon\left(\rho_{\rm m0} - 3p_{\rm m0} + 4V_0 - \dot{\phi}_0^2 - 12bS_{20}^2\right)S_{20}^2\right] - 2\left(\frac{k}{R_0^2} - S_{20}^2\right)Z_0^2\\
&\quad \left. -108\alpha^2\varepsilon f_0 S_{20}\dot{S}_{20}\left[\left(\frac{\partial V}{\partial\phi}\right)_0\dot{\phi}_0 - 2\left(2b-\varepsilon f_0\right)S_{20}\dot{S}_{20}\right]\right\}\\
&\quad +6\alpha\left(2b-\varepsilon f_0\right)S_{20}^2\left\{72\alpha^2\left[\left(\frac{\partial V}{\partial\phi}\right)_0\dot{\phi}_0 - 2\left(2b-\varepsilon f_0\right)S_{20}\dot{S}_{20}\right]\right.\\
&\quad \times\left[\left(\frac{\partial V}{\partial\phi}\right)_0\dot{\phi}_0 - \left(4b+\varepsilon f_0\right)S_{20}\dot{S}_{20}\right] + 6\alpha Z_0\left[\left(\frac{\partial V}{\partial\phi}\right)_0^2 - \left(\frac{\partial^2 V}{\partial\phi^2}\right)_0\dot{\phi}_0^2\right.\\
&\quad \left. +2\left(2b-\varepsilon f_0\right)\dot{S}_{20}^2\right]\\
&\quad \left. +\frac{1}{\varepsilon}Z_0^2\left[\frac{1}{3f_0}\left(\rho_{\rm m0} - 3p_{\rm m0} + 4V_0 - \dot{\phi}_0^2 - 12bS_{20}^2\right) + \frac{f_0-b}{f}\right]\right\}. \qquad (33)
\end{aligned}$$

In the case of HIM without pseudoscalar torsion function S_2 the equation (32) and the formula (33) simplify and take the form obtained in ref. [4]. As it was noted in [4,6], in this case the most part of extremum surfaces play the role of bounce surfaces ($\dot{H}_0 > 0$) for scalar field potentials applying in theory of chaotic inflation. The different situation is in considering case of extremum surfaces (32). By given values of parameters α and ε‡ the bounce ($\dot{H}_0 > 0$) takes place only in limited domain of extremum surfaces (32) with negligibly small values of S_{20}. In the case $S_{20} = 0$ the equation of extremum surface (32) and the expression (33) of derivative $\dot{H}_0$ are simplified and take the following form

$$\frac{1}{6f_0}\left[\rho_{\rm m0} + \frac{1}{2}\dot{\phi}_0^2 + V_0 + \frac{1}{4}\alpha\left(\rho_{\rm m0} - 3p_{\rm m0} + 4V_0 - \dot{\phi}_0^2\right)^2\right]Z_0$$

‡ According to the conclusion obtained in Section 3 the parameter b is very close to f_0. As result we put below for numerical calculations $b = f_0$.

$$-9\alpha^2\left(\frac{\partial V}{\partial\phi}\right)_0^2\dot{\phi}_0^2-\frac{k}{R_0^2}Z_0^2=3\alpha\varepsilon f_0\dot{S}_{20}^2Z_0, \tag{34}$$

$$\begin{aligned}\dot{H}_0=&\left\{\frac{1}{2f_0}\left[\frac{1}{2}\left(\rho_{\rm m0}-p_{\rm m0}\right)+V_0+\frac{1}{4}\alpha\left(\rho_{\rm m0}-3p_{\rm m0}+4V_0-\dot{\phi}_0^2\right)^2\right]\right.\\&\left.+3\alpha\left[\left(\left(\frac{\partial V}{\partial\phi}\right)_0^2-\left(\frac{\partial^2 V}{\partial\phi^2}\right)_0\dot{\phi}_0^2+\left(4b-3\varepsilon f_0\right)\dot{S}_{20}^2\right]-\frac{2k}{R_0^2}Z_0\right\}\\&\times\left\{1+\alpha\left[\rho_{\rm m0}-3p_{\rm m0}+\frac{3}{2}\left(3\left(\frac{dp_{\rm m}}{d\rho_{\rm m}}\right)_0-1\right)\left(\rho_{\rm m0}+p_{\rm m0}\right)+4V_0+2\dot{\phi}_0^2\right]\right\}^{-1}.\end{aligned} \tag{35}$$

We see from (35) that the presence of $\dot{S}_{20}$ in this expression does not prevent from the bounce realization. Moreover, if we put $\phi=0$ and $k=0$, from (34) follows that $\varepsilon>0$.

As an example of inflationary cosmological solutions we will consider below flat HIM filled with ultrarelativistic matter $p_{\rm m}=\frac{1}{3}\rho_{\rm m}$ and scalar field with quadratic potential $V=\frac{1}{2}m^2\phi^2$. For numerical calculations we will use $m=10^{-6}M_{\rm P}$ and $\alpha^{-1}=1.2\times10^{-13}M_{\rm P}^4$. To perform the numerical integration of equations (14)–(15), (28)–(29) it is convenient to transform all variables and parameters entering these equations to dimensionless units marked by tilde

$$\begin{aligned}&t\to\tilde{t}=t/\sqrt{f_0\alpha}, && R\to\tilde{R}=R/\sqrt{f_0\alpha},\\&\rho\to\tilde{\rho}=\alpha\,\rho, && p\to\tilde{p}=\alpha\,p,\\&\phi\to\tilde{\phi}=\phi/\sqrt{f_0}, && m\to\tilde{m}=m\sqrt{f_0\alpha},\\&H\to\tilde{H}=H\sqrt{f_0\alpha}, && S_{1,2}\to\tilde{S}_{1,2}=S_{1,2}\sqrt{f_0\alpha}.\end{aligned} \tag{36}$$

The explicit form of equations (14)–(15), (28)–(29) after this transformation is similar to original form except the fact that parameters α and f_0 are cancelled in obtained equations. Particular numerical solution was found under the following value of indefinite parameter $\varepsilon=10^{-4}$. Initial conditions for $\tilde{H}$, $\tilde{\phi}$, $\tilde{S}_2$, $\tilde{S}_2'$, $\tilde{\rho}_{\rm m}$ were taken at a bounce as follows

$$\tilde{H}_0=0,\quad \tilde{\phi}_0=25,\quad \tilde{S}_{20}=0,\quad \tilde{S}_{20}'=0.001,\quad \tilde{\rho_{\rm m0}}=0.4,$$

where the prime denotes the differentiation with respect to $\tilde{t}$. Initial condition for $\tilde{\phi}_0'$ was taken to satisfy (34). Obtained solution is given in figure 1 – figure 4 and includes four stages: the compression stage (figure 1), the transition stage from compression to expansion (figure 2), the inflationary stage (figure 3) and the postinflationary stage (figure 4). The distinguishing features of obtained solution are its completely regular character. Note, that during inflationary stage number of e-folds for the scale factor is equal approximately to 76.

Similar to GR, the transition to radiation dominated stage can be realized by transformation of oscillating scalar fields (see figure 4) into particles [11]. Details of such transition in considered theory require further investigation. In particular, the presence of the oscillations of the Hubble parameter H (figure 4) can lead to some distinguishing features

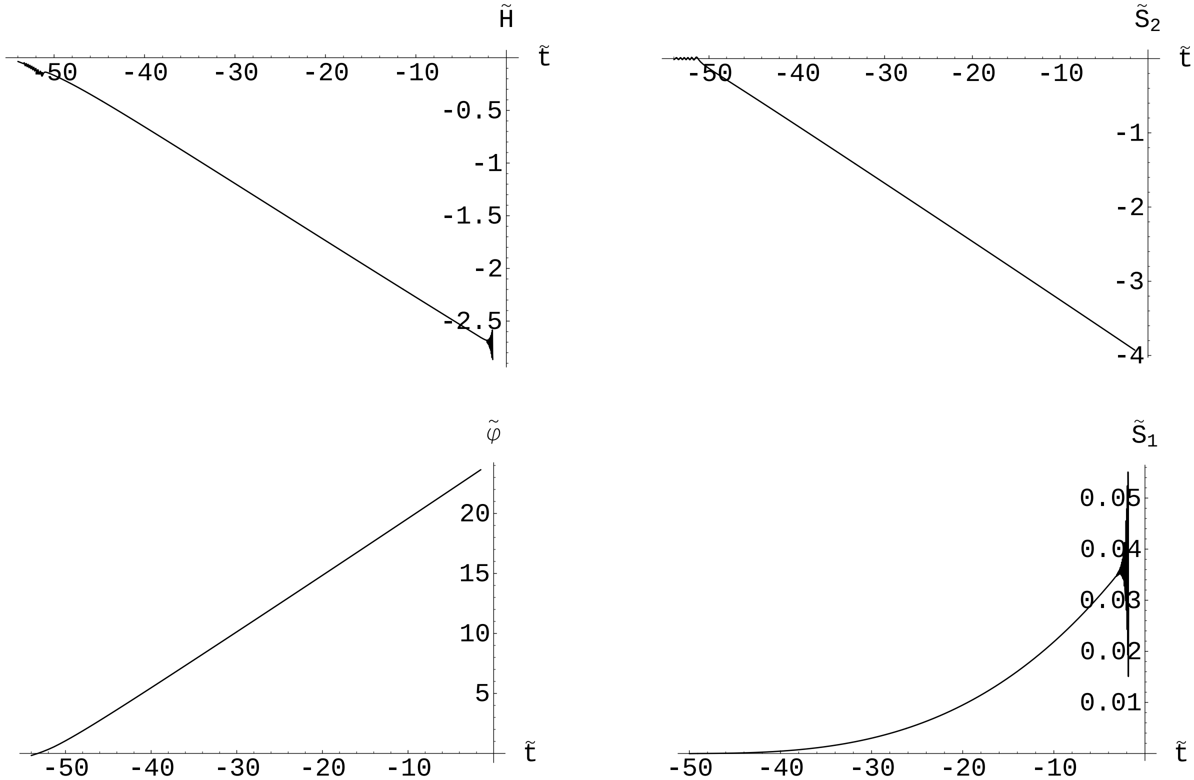

Figure 1. Compression stage.

of the inflationary scenario in the considered theory in comparison with the inflationary scenario in GR.

After transition to matter dominated stage the further evolution of the Universe in this theory is the same as in the frame of standard cosmological scenario. The transition to the accelerating expansion takes place, when the value of effective cosmological constant is greater than the matter energy density.

5. Conclusion

As it was shown, in the framework of PGTG the gravitational interaction in the case of usual gravitating systems can have the repulsion character not only at extreme conditions [4,5], but also at sufficiently small energy densities. The pseudoscalar torsion function in HIM provokes the appearance of effective cosmological constant at asymptotics of cosmological solutions that can lead to observable accelerating cosmological expansion. Quantitative agreement of the obtained result with observations depends on corresponding restrictions on indefinite parameters α, b and ε from Section 3. Numerical solution for inflationary cosmological model

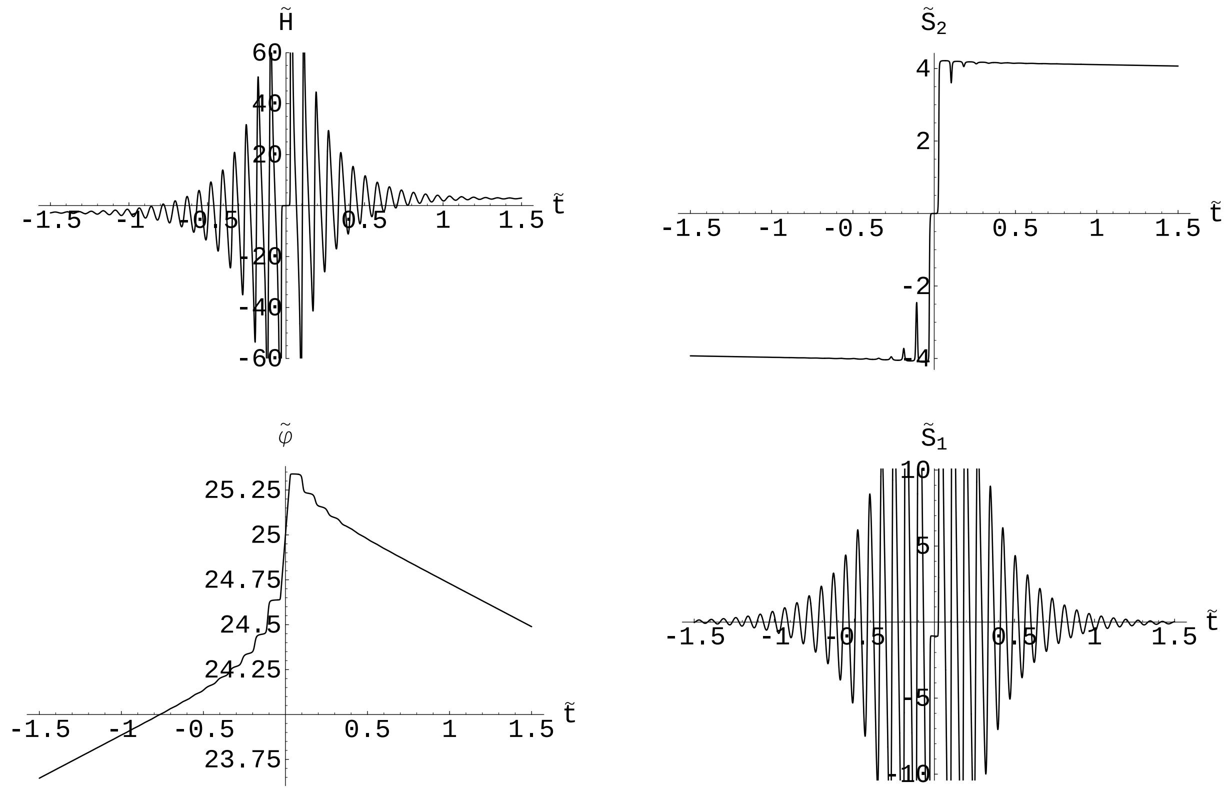

Figure 2. Transition stage.

presented at Figures 1–4 conserves its qualitative behaviour by relatively small variations of indefinite parameters and initial conditions.

The effect of acceleration of cosmological expansion in PGTG has the geometrical nature and is connected with geometrical structure of physical space-time. Hence, from the point of view of considered theory hypothetical form of gravitating matter — dark energy — is fiction.

References

[1] Riess A G et al. 1998 *Astron. J.* **116** 1009–38 (*Preprint* astro-ph/9805201); Perlmutter S.J. et al. 1999 *Astroph. J.* **517** 565–86 (*Preprint* astro-ph/9812133); Knop R A et al. 2003 *Astroph. J.* **598**, 102–37 (*Preprint* astro-ph/0309368)

[2] Sahni V and Starobinsky A 2006 *Int. J. Mod. Phys.* **D15** 2105–32 (*Preprint* astro-ph/0610026)

[3] Padmanabhan T 2006 *AIP Conf.Proc.* **861** 179–96 (*Preprint* astro-ph/0603114)

[4] Minkevich A V 2006 *Gravitation&Cosmology* **12** 11–21 (*Preprint* gr-qc/0506140)

[5] Minkevich A V 2007 *Acta Physica Polonika B* **38** 61–72 (*Preprint* gr-qc/0512123)

[6] Minkevich A V and Garkun A S 2006 *Class. Quantum Grav.* **23** 4237–47 (*Preprint* gr-qc/0512130)

[7] Minkevich A V 1980 *Phys. Lett.A* **80** 232–34

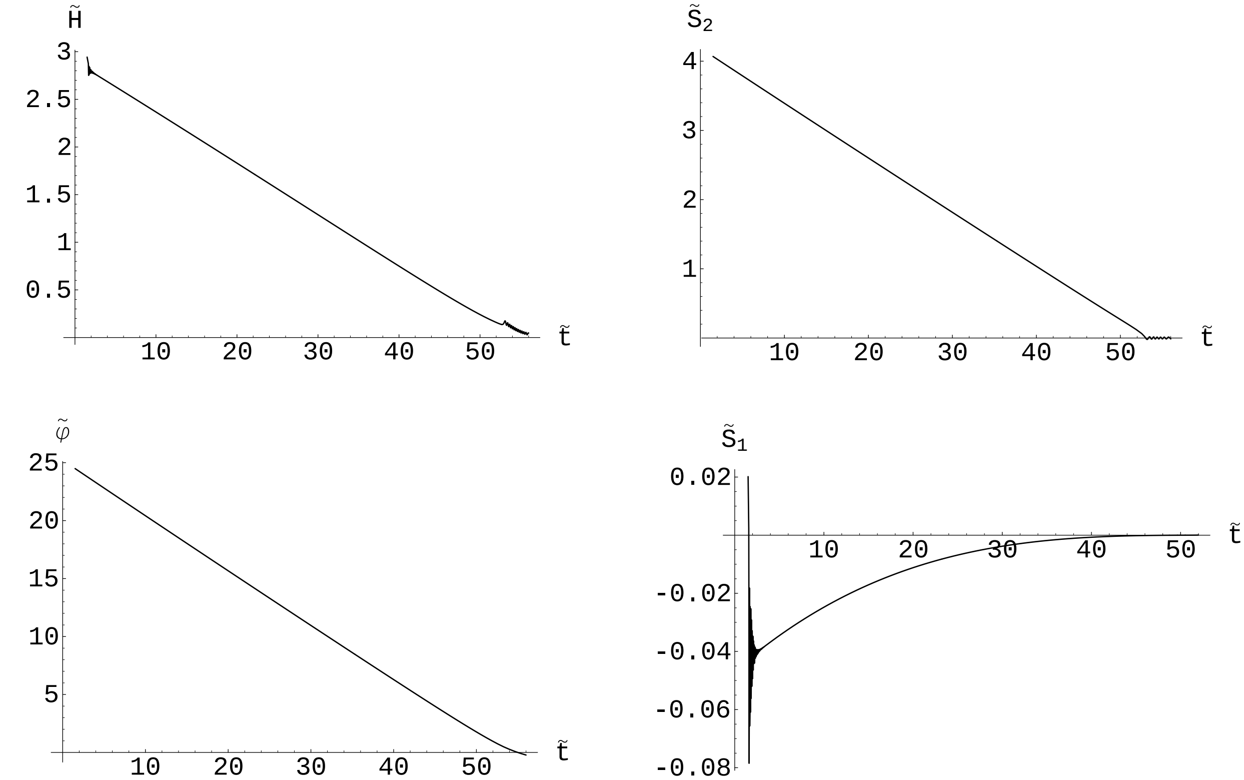

Figure 3. Inflationary stage.

[8] Kudin V I, Minkevich A V and Fedorov F I 1981 *Vestsi Akad. Nauk. BSSR. Ser. fiz.-mat.nauk* No 4 59–67

[9] Minkevich A V Garkun A S and Kudin V I 2006 *Proc.of 5th Intern. Conf. Boyai-Gauss-Lobachevsky: Methods of Non-Euclidian Geometry in Modern Physics* (Minsk: Inst. of Physics NAN Belarus) p 150–57 (*Preprint* gr-qc/0612116)

[10] Linde A 1990 *Physics of Elementary Particles and Inflationary Cosmology* (Switzerland, Chur: Harwood) p 270

[11] Blagojević M 2002 *Gravitation and Gauge Symmetries* (IOP Publishing: Bristol)

[12] Mukhanov V 2005 *Physical Foundations of Cosmology* (Cambrige University Press: New York)

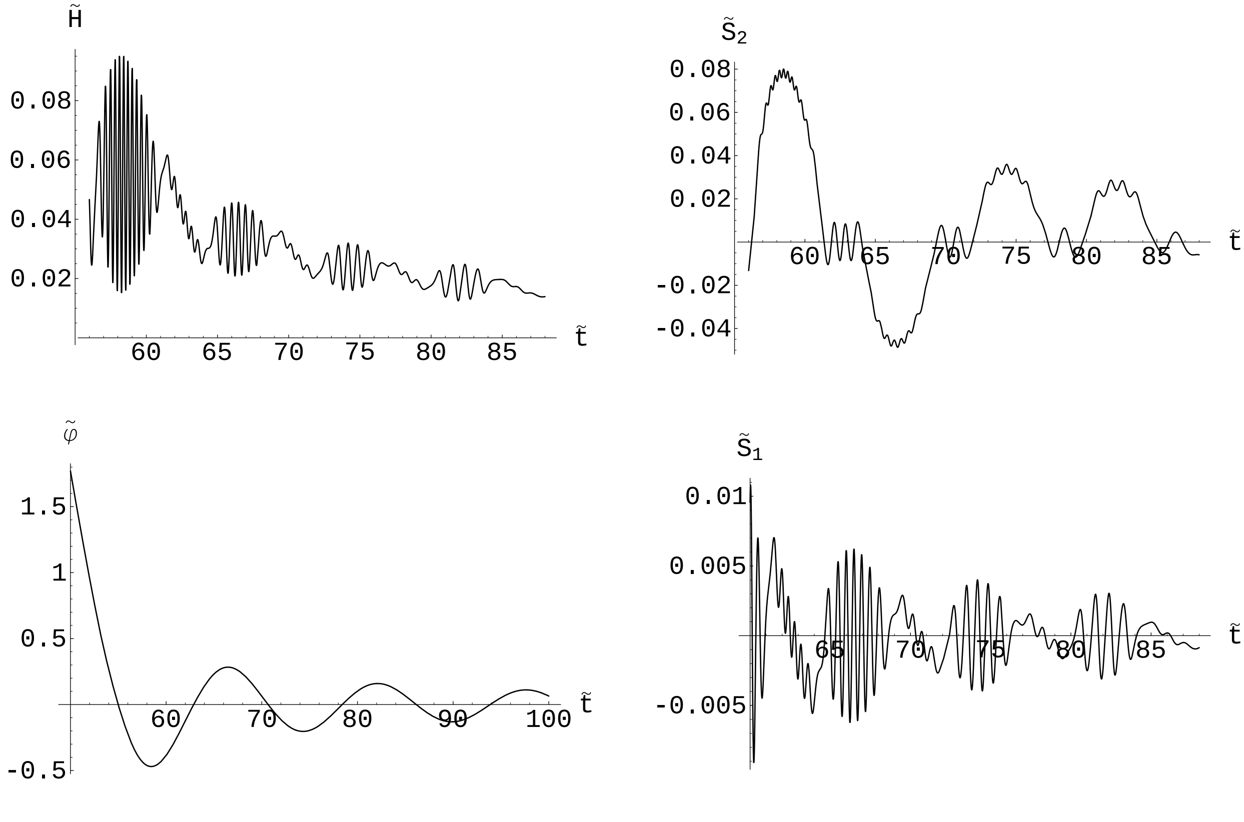

Figure 4. Postinflationary stage.