An analytic cosmological solution of Poincare gauge gravity with a pseudoscalar torsion

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A cosmology of Poincare gauge theory is developed, and its analytic solution is obtained. The calculation results agree with observational data and can be compared with the ΛCDM model. The cosmological constant puzzle, the coincidence and fine tuning problem are relieved naturally at the same time. The cosmological constant turns out to be the intrinsic torsion and curvature of the vacuum universe and is derived from the theory naturally rather than added artificially. The dark energy originates from geometry, includes the cosmological constant but differs from it. The analytic expression of the state equations of the dark energy and the density parameters of the matter and the geometric dark energy are derived. The full equations of linear cosmological perturbations and the solutions are obtained.

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I. Introduction

The discovery of the accelerated expansion of the universe motivates a large variety of theoretical works to explain it. In order to account for the acceleration the Einstein equation has to be modified and then two approaches are developed. One is to introduce "dark energy" in the right-hand side in the framework of general relativity (see [1] for recent reviews). The another is to modify the left-hand side of the equation, called modified gravitational theories, e.g., $f(R)$ gravity (see [2] for recent reviews). A large amount of literature in every approach has been accumulated in the past years. However, none of them offers a convincing explanation of the observed results, and most of them were introduced to explain the acceleration phenomenologically, rather than emerging naturally out of fundamental physics principles. The most popular model in the fist approach is the Λ cold dark matter (ΛCDM) model which is plagued by the cosmological constant problem and the coincidence and fine tuning problem. Meanwhile, there is not the enough evidence on the validity of this model. It is shown that the thermal and mechanical stability requirement provides an evidence against the dark energy hypothesis [3]. Adding dark energy to the content of the Universe may not be the answer to the cosmic acceleration problem. In the second approach the Einstein-Hilbert Lagrangian is usually generalized to a function f of the Ricci scalar R. However, at present there are no fully realized and empirically viable $f\ (R)$ theories that explain the observed level of cosmic acceleration. Furthermore, the $f\ (R)$ theories suffer from a long-standing controversy about which frame (Einstein or Jordan) is the physical one [4]. It should be noted that although we have strong observational evidence for accelerated cosmic expansion but no compelling evidence that the

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cause of this acceleration is really a new energy component. At the same time we do not have enough independent data yet to clarify the nature of dark energy. This provides further motivation for a deeper investigation of the very nature of dark energy and the origin of the accelerated cosmic expansion. In the framework of $f(R)$ gravity, the field equation can be written as the Einstein equation with an effective energy-momentum tensor that contains all the modifications and the energy-momentum tensor of matter fields. The contributions of the modifications of gravity can be identified with some kind of geometric dark energy. This is specially advantageous since one can define an equation of state associated with such dark energy and compare it with the ΛCDM model [5]. However, the function $f(R)$ is not known a priori, none introduces a new fundamental principle that can be used as a guiding line, it is usually constructed by trial and error. In fact, as a geometric theory a modified gravity should be formulated in a gauge theoretical framework including. A famous example is the Poincare gauge theory of Gravity [6]. Some works have been done to develop a model of geometric dark energy in the Poincare gauge theory framework [7], [8], [9]. In [7] the effect of torsion is to introduce an extra-term into matter density and pressure which gives rise to an accelerated behavior of the universe. However, the torsion contributes only a constant density, it is not possible to solve the coincidence and fine tuning problem. The torsion model in [8] contributes an oscillating aspect to the expansion rate of the universe and displays features similar to those of only the presently observed accelerating universe. In [9] the Lagrangian involves too many terms and indefinite parameters, which make the field equations complicated and difficult to solve and the role of each term obscure. In order to simplify the field equations, some restrictions on indefinite parameters have to be imposed. Under these restrictions, especially, all the higher derivatives of the scale factor are excluded from the cosmological equations. In fact, starting from a well behaved Lagrangian $\frac{1}{2}R+\alpha R^2+\beta R_{\mu\nu}R^{\mu\nu}$ in quadratic gravity [10] and string theory [11] and adding a quadratic term of torsion $\gamma T^{\mu}{}_{\nu\rho}T_{\mu}{}^{\nu\rho}$ a good toy model can be obtained [12]. In addition to the simplicity the main advantage of this Lagrangian is to permit exact or analytic solutions which have not been found in previous works. In contrast with [9] the field equations are allowed to contain higher derivatives in [12]. When the macroscopic spacetime average of the spin vanishes, the solutions of the cosmological equations are found to split into two families. Each of them is related with only one torsion function, the scalar or the pseudoscalar torsion function. It has been proved [8] that only these two scalar torsion modes are physically acceptable and no-ghosts. This model has a free-ghost dynamics. It has a well posed initial value problem without any ghost or tachyonic propagation. The first family has been investigated in detail in [12], the second family corresponding to the pseudoscalar torsion function will be studied in a totally different way in this paper. Some meaningful consequences can be inferred from the solutions obtained. The cosmological constant problem and the coincidence and fine tuning problem are solved naturally.

In Sec. II, starting from the Poincare gauge principle and the Lagrangian $\frac{1}{2}R+\alpha R^2+\beta R_{\mu\nu}R^{\mu\nu}+\gamma T^{\mu}{}_{\nu\rho}T_{\mu}{}^{\nu\rho}$, which is the sum of the Starobinsky Lagrangian [13] and Yang-Mills type terms of the local rotation and translation field strength, the field equations are derived. In order to evade any unnecessary discussion regarding frames (i.e. Einstein .vs. Jordan) the theory is treated using the original variables instead of transforming to a scalar-tensor theory in contrast to $f(R)$ theories. As an exact solution a set of cosmological equations is obtained. Although we do not introduce a cosmology constant in the action it automatically emerges in the derivation of the cosmological equations and then is endowed with intrinsic character. The dark energy is identified with the geometry of the spacetime and is a function of the density and the pressure of the matter. It includes the cosmological constant but can not be identified with it. It is nothing but the intrinsic torsion or curvature of the vacuum universe. In Sec. III, the analytic expressions of the state equation and the density parameters of the matter and the geometric dark energy are derived and used to determine the values of α , β and γ. Then a theoretical value of the cosmological constant is computed and compared with the observed datum. The cosmological constant problem and the coincidence and fine tuning problem are solved naturally. In Sec. IV an analytic integral of the cosmological equation is obtained and used to evaluate the age of the universe which can be compared with observed data. In Section V the full equations of linear cosmological perturbations and the solutions are obtained. In addition, the behavior of perturbations for the sub-horizon modes relevant to large-scale structures is discussed. It is shown that our model can be distinguished from others by considering the evolution of matter perturbations and gravitational potentials. Sec. VI is devoted to conclusions.

II. Cosmological equations

The discussion in this paper are entirely classical. We consider a Poincare gauge theory of gravity [6], [7], [8], [9], in which there are two sets of local gauge potentials, the orthonormal frame field (tetrad) $e_I{}^\mu$ and the metric-compatible connection $\Gamma^{IJ}{}_\mu$ associated with the translation and the Lorentz subgroups of the Poincare gauge group, respectively. We use the Greek alphabet (μ, ν, ρ, ... $= 0,1,2,3$) to denote (holonomic) indices related to spacetime, and the Latin alphabet ($I, J, K, ... = 0,1,2,3$) to denote algebraic (anholonomic) indices, which are raised and lowered with the Minkowski metric $\eta_{IJ} = \mathrm{diag}\,(-1,+1,+1,+1)$. The field strengths associated with the tetrad and connection are the torsion $T^\lambda{}_{\mu\nu}$ and the curvature $R_{\mu\nu}{}^{\lambda\tau}$. We use the geometrized system of units in which $8\pi G = 1$, $c = 1$, start from the action

$$S = \int d^4x\sqrt{-g}\left[\left(\frac{1}{2}R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma T^\mu{}_{\nu\rho}T_\mu{}^{\nu\rho}\right) + \mathcal{L}_m\right], \tag{1}$$

where $\mathcal{L}_m$ denotes the Lagrangian of the source matter including baryonic matter, cold dark matter and radiation, α and β are two parameters with the dimension of $[L]^2$, γ is a dimensionless parameter. The vales of α, β and γ can be determined by experiment and observational data. The terms $\frac{1}{2}R$ and $\gamma T^\mu{}_{\nu\rho}T_\mu{}^{\nu\rho}$ represent weak gravity, while αR^2 and $\beta R_{\mu\nu}R^{\mu\nu}$ represent strong gravity with the dimensionless strong gravity constant α and β according to Hehl *et al* [6].

The variational principle yields the field equations for the tetrad $e_I{}^\mu$ and the connection $\Gamma^{IJ}{}_\mu$ [12]:

$$R_{\nu\mu} - \frac{1}{2}g_{\nu\mu}R = T_{\nu\mu} + T_{(g)\nu\mu}, \tag{2}$$

and

$$T^\nu{}_{\tau\nu}\delta^\mu_\lambda - T^\nu{}_{\lambda\nu}\delta^\mu_\tau + T^\mu{}_{\nu\tau}\delta^\nu_\lambda = e^I{}_\lambda e^J{}_\tau\left(s_{IJ}{}^\mu + s_{(g)IJ}{}^\mu\right), \tag{3}$$

where $T_{\nu\mu} := e_{I\mu}\partial\left(\sqrt{-g}\mathcal{L}_m\right)/\partial e_I{}^\nu$ and $s_{IJ}{}^\mu := \partial\left(\sqrt{-g}\mathcal{L}_m\right)/\partial\Gamma^{IJ}{}_\mu$ are energy-momentum tensor and spin tensor of the source matter, respectively, while

$$\begin{aligned} T_{(g)\nu\mu} &= -\alpha\left(4R_{\nu\mu} - g_{\nu\mu}R\right)R - \beta\left(2R^\rho{}_\nu R_{\rho\mu} + 2R^{\rho\sigma}R_{\nu\rho\mu\sigma} - g_{\nu\mu}R_{\rho\sigma}R^{\rho\sigma}\right) \\ &\quad +\gamma\left(4\partial_\tau e^{I\lambda}\left(e_{I\nu}T_{\mu\lambda}{}^\tau - e_{I\lambda}T_{\mu\nu}{}^\tau\right) + 4\partial_\tau T_{\mu\nu}{}^\tau + g_{\nu\mu}T^\lambda{}_{\rho\sigma}T_\lambda{}^{\rho\sigma} - 4T^\lambda{}_{\nu\tau}T_{\lambda\mu}{}^\tau\right), \end{aligned} \tag{4}$$

and

$$\begin{aligned} s_{(g)IJ}{}^{\mu} &= -4\alpha\left(e_{[I}{}^{\nu}e_{J]}{}^{\tau}\Gamma^{\mu}{}_{\nu\tau}R + e_{[J}{}^{\mu}e_{I]}{}^{\nu}\left(\Gamma^{\lambda}{}_{\lambda\nu}R - \partial_{\nu}R\right) + e_{[I}{}^{\nu}e_{J]}{}^{\mu}Re^{K}{}_{\tau}\partial_{\nu}e_{K}{}^{\tau}\right) \\ &\quad -4\beta e_{J}{}^{\lambda}\left(e_{I}{}^{[\mu}\partial_{\nu}R_{\lambda}{}^{\nu]} + e_{I}{}^{[\nu}R_{\lambda}{}^{\mu]}e^{K}{}_{\tau}\partial_{\nu}e_{K}{}^{\tau} + e_{I}{}^{\tau}\Gamma^{[\nu}{}_{\nu\tau}R_{\lambda}{}^{\mu]} + e_{I}{}^{[\nu}R_{\tau}{}^{\mu]}\Gamma^{\tau}{}_{\nu\lambda}\right) \\ &\quad -4\gamma e_{I\nu}e_{J}{}^{\tau}T^{\nu\mu}{}_{\tau}, \end{aligned} \tag{5}$$

are the energy-momentum and the spin of this kind of "geometric dark energy" corresponding to the terms $\alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma T^{\mu}{}_{\nu\rho}T_{\mu}{}^{\nu\rho}$ in (1). Note that the energy-momentum tensor $T_{\nu\mu}$ of type (0, 2) should not be confused with the torsion tensor $T^{\lambda}{}_{\mu\nu}$ of type (1, 2). If $\alpha = \beta = \gamma = 0$, these equations become the field equations of Einstein-Cartan-Sciama-Kibble theory. Furthermore, for $T^{\lambda}{}_{\mu\nu} = 0$ we come back to General Relativity.

For the spatially flat Friedmann-Robertson-Walker (FRW) metric

$$g_{\mu\nu} = \mathrm{diag}\left(-1, a\left(t\right)^2, a\left(t\right)^2, a\left(t\right)^2\right), \tag{6}$$

the non-vanishing torsion components with holonomic indices are given by two functions, the scalar torsion h and the pseudoscalar torsion f [8],[14]:

$$T_{ij0} = a^2 h\delta_{ij}, T_{ijk} = 2a^3 f\epsilon_{ijk}, \quad i, j, k, ... = 1, 2, 3. \tag{7}$$

The equations (2) and (3) yields the cosmological equations

$$H^2 = \frac{1}{3}\left(\rho + \rho_g\right), \tag{8}$$

$$2\dot{H} + 3H^2 = -\left(p + p_g\right), \tag{9}$$

$$\begin{aligned} &(\beta + 6\alpha)\left(\ddot{H} + \ddot{h}\right) + 6\left(\beta + 4\alpha\right)\left(H + h\right)\dot{H} + \left(5\beta + 18\alpha\right)\left(H + h\right)\dot{h} - 4\left(\beta + 3\alpha\right)f\dot{f} \\ &+3\left(\beta + 4\alpha\right)hH^2 + \left(5\beta + 18\alpha\right)h^2 H + 2\left(\beta + 3\alpha\right)h^3 - 2\left(\beta + 3\alpha\right)hf^2 + \frac{1}{4}h + \frac{1}{2}s_{01}{}^{1} = 0, \end{aligned} \tag{10}$$

and

$$\begin{aligned} &f\{2\left(\beta + 6\alpha\right)\left(\dot{H} + \dot{h}\right) + 6\left(\beta + 4\alpha\right)H^2 + 2\left(5\beta + 18\alpha\right)Hh \\ &+\left(\beta + 3\alpha\right)\left(4h^2 - 4f^2\right) - 4\gamma + \frac{1}{2}\} - \frac{1}{2}s_{12}{}^{3} = 0, \end{aligned} \tag{11}$$

where $H = \dot{a}\left(t\right)/a\left(t\right)$ is the Hubble parameter, $\dot{H} = dH/dt$, while

$$\begin{aligned} \rho_g &= -6Hh - 3h^2 + 3f^2 \\ &\quad +12\left(3\alpha + \beta\right)\left(\dot{H} + \dot{h} - Hh - h^2 + f^2\right)\left(\dot{H} + \dot{h} + 2H^2 + 3Hh + h^2 - f^2\right) \\ &\quad -6\gamma\left(h^2 + 4f^2\right), \end{aligned} \tag{12}$$

and

$$\begin{aligned} p_g &= 4Hh + h^2 - f^2 \\ &\quad +4\left(3\alpha + \beta\right)\left(\dot{H} + \dot{h} - Hh - h^2 + f^2\right)\left(\dot{H} + \dot{h} + 2H^2 + 3Hh + h^2 - f^2\right) \\ &\quad -2\gamma\left(2\dot{h} + 8Hh + h^2 + 4f^2\right), \qquad i = 1, 2, 3, \end{aligned} \tag{13}$$

are the density and the pressure of the geometric dark energy. (12) and (13) indicate that the geometric dark energy is just the gravitational field itself described by h, H and f. The source matter is a fluid characterized by the energy density $\rho = T_{00}$, the pressure $p = T_{ij}$ $(i = j)$ and the spin $s_{IJ}{}^{\mu}$. (8) and (9) lead to

$$\frac{\ddot{a}}{a} = -\frac{1}{6}\left(\rho + \rho_g + 3p + 3p_g\right). \tag{14}$$

It is easy to see that when $\alpha = \beta = \gamma = 0$ and $h = f = 0$, (8), (9) and (14) reduce to the Friedmann cosmology. (8) and (9) correspond to the Friedmann equation and the Raychaudhuri equation respectively, while (14) is the acceleration equation, which represent the Einstein frame of the theory.

Since the spin orientation of particles for ordinary matter is random, the macroscopic spacetime average of the spin vanishes, we suppose $s_{IJ}{}^{\lambda} = 0$, henceforth. Then, the equation (11) has the solutions

$$f = 0, \tag{15}$$

and

$$\begin{aligned} f^2 \;=\; & \frac{(\beta + 6\alpha)}{2\left(\beta \;+3\alpha\right)}\left(\dot{H} + \dot{h}\right) + \frac{3\left(\beta + 4\alpha\right)}{2\left(\beta \;+3\alpha\right)}H^2 + \frac{(5\beta + 18\alpha)}{2\left(\beta \;+3\alpha\right)}Hh + h^2 \\ & -\frac{\gamma}{\left(\beta \;+3\alpha\right)} + \frac{1}{8\left(\beta \;+3\alpha\right)}. \end{aligned} \tag{16}$$

The solution (15) has been investigated in [12]. We concentrate on the equation (16) now. Differentiating (16) gives

$$f\,\dot{f} = \frac{\beta + 6\alpha}{4\left(\beta + 3\alpha\right)}\left(\ddot{H} + \ddot{h}\right) + \frac{3\left(\beta + 4\alpha\right)}{2\left(\beta + 3\alpha\right)}H\,\dot{H} + \frac{5\beta + 18\alpha}{4\left(\beta + 3\alpha\right)}\,\dot{H}\,h + \frac{5\beta + 18\alpha}{4\left(\beta + 3\alpha\right)}H\,\dot{h} + h\,\dot{h}\,.$$

Substituting it and (16) into (10) gives (when $s_{01}{}^{1} = 0$)

$$2h\gamma = 0,$$

and then

$$h = 0. \tag{17}$$

In this case (8), (9), (12), (13) and (16) lead to

$$12\left(3\alpha + \beta\right)\left(\dot{H} + f^2\right)\left(\dot{H} + 2H^2 - f^2\right) - 3H^2 + 3f^2 - 24\gamma f^2 + \rho = 0,$$

$$4\left(3\alpha + \beta\right)\left(\dot{H} + f^2\right)\left(\dot{H} + 2H^2 - f^2\right) + 2\,\dot{H} + 3H^2 - f^2 - 8\gamma f^2 + p = 0,$$

$$\left(\beta + 6\alpha\right)\dot{H} + 3\left(\beta + 4\alpha\right)H^2 - 2\left(\beta \;+3\alpha\right)f^2 - 2\gamma + \frac{1}{4} = 0,$$

which further give

$$f^2 = \frac{\Lambda}{24\gamma} + \frac{\rho}{24\gamma} + \frac{3\left(4\alpha + \beta\right) - 16\gamma\left(3\alpha + \beta\right)}{48\beta\gamma}\left(\rho - 3p\right) + \frac{\left(3\alpha + \beta\right)\left(4\alpha + \beta\right)}{24\gamma\beta}\left(\rho - 3p\right)^2, \tag{18}$$

$$H^2 = \frac{(1-8\gamma)}{24\gamma}\Lambda + \frac{\rho}{24\gamma} - \frac{(8\gamma-1)(4\alpha+\beta)}{16\gamma\beta}(\rho-3p) + \frac{(3\alpha+\beta)(4\alpha+\beta)}{24\gamma\beta}(\rho-3p)^2, \tag{19}$$

$$\begin{aligned}\dot{H} &= \frac{16\gamma-1}{24\gamma}\Lambda - \frac{\rho}{24\gamma} \\ &\quad -\frac{3(4\alpha+\beta)-8\gamma(18\alpha+5\beta)}{48\beta\gamma}(\rho-3p) - \frac{(3\alpha+\beta)(4\alpha+\beta)}{24\gamma\beta}(\rho-3p)^2,\end{aligned} \tag{20}$$

where

$$\Lambda = \frac{3(1-8\gamma)}{4\beta}, \tag{21}$$

is the geometric cosmological constant coming from the terms $\beta R_{\mu\nu}R^{\mu\nu} + \gamma T^{\mu}{}_{\nu\rho}T_{\mu}{}^{\nu\rho}$ in (1). One finds that the higher order derivative $\ddot{H}$ and $\dot{f}$ in (10) disappear, whereas in [8] and [9] some restrictions on indefinite parameters have to be imposed in order to exclude higher order derivatives. We note that although we do not introduce a cosmological constant Λ in the action (1), it automatically emerges in these equations. In (18) the pseudoscalar torsion f is a function of ρ and p rather than a constant, in contrast to [5]. It should be noted that although (8) and (9) have the same form as the Friedmann equations, the solutions (19) and (20) are different. The reason is that in (8) and (9) ρ_g and p_g are functions of H, $\dot{H}$, h, $\dot{h}$, and f as indicated by (12) an (13). In other words, this is a different model from the ΛCDM model essentially.

(12) and (13) become now

$$\begin{aligned}\rho_g &= \frac{(1-8\gamma)}{8\gamma}\Lambda + \frac{(1-8\gamma)\rho}{8\gamma} + \frac{3(1-8\gamma)(4\alpha+\beta)}{16\beta\gamma}(\rho-3p) + \frac{(3\alpha+\beta)(4\alpha+\beta)}{8\beta\gamma}(\rho-3p)^2, &(22)\\ p_g &= -\frac{(1+8\gamma)}{24\gamma}\Lambda - \frac{(8\gamma+1)\rho}{24\gamma} - \frac{3(4\alpha+\beta)-8\beta\gamma}{48\beta\gamma}(\rho-3p) - \frac{(3\alpha+\beta)(4\alpha+\beta)}{24\beta\gamma}(\rho-3p)^2, &(23)\end{aligned}$$

which mean that *the geometrical dark energy includes the cosmological constant* Λ *but can not be identified with it.* The cosmological constant Λ is really a constant determined by β and γ as indicated by (20) while the geometrical dark energy ρ_g is a function of the density ρ and the pressure p of the matter. *The cosmological constant problem and the coincidence and fine tuning problem are relieved naturally, as shown in the next section.*

Substituting (22) and (23) into (14) yields

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} + \frac{3\alpha+\beta}{3\beta}(\rho-3p). \tag{24}$$

Furthermore, (18), (19), (20) and (24) mean that the vacuum universe has the torsion

$$f_{\text{vac}}^2 = \frac{\Lambda}{24\gamma}, \tag{25}$$

the curvature

$$R_{\text{vac}} = 6\dot{H} + 12H^2 - 3f^2 = \frac{\Lambda}{8\gamma}. \tag{26}$$

and the acceleration

$$\left(\frac{\ddot{a}}{a}\right)_{\text{vac}} = \frac{\Lambda}{3}. \tag{27}$$

This means that the cosmological constant is nothing but the intrinsic torsion or curvature of the vacuum universe.

III. The state equation of the geometrical dark energy

(22) and (23) gives the state equation of the dark energy:

$$w_g = \frac{p_g}{\rho_g} = \frac{-2\left(1+8\gamma\right)\beta\Lambda - 2\left(8\gamma+1\right)\beta\rho - \left(3\left(4\alpha+\beta\right)-8\beta\gamma\right)\left(\rho-3p\right) - 2\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)\left(\rho-3p\right)^2}{6\left(1-8\gamma\right)\beta\Lambda + 6\left(1-8\gamma\right)\beta\rho + 9\left(1-8\gamma\right)\left(4\alpha+\beta\right)\left(\rho-3p\right) + 6\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)\left(\rho-3p\right)^2}. \tag{28}$$

The source matter includes ordinary baryon matter, dark matter and radiation:

$$\rho = \rho_{\mathrm{m}} + \rho_{\mathrm{r}}, p_{\mathrm{m}} = 0, p = \frac{1}{3}\rho_{\mathrm{r}}. \tag{29}$$

The equation (8) can be written as

$$\Omega = \Omega_{\mathrm{r}} + \Omega_{\mathrm{m}} + \Omega_g = 1,$$

where

$$\Omega_{\mathrm{m}} := \frac{\rho_{\mathrm{m}}}{3H^2}, \Omega_{\mathrm{r}} := \frac{\rho_{\mathrm{r}}}{3H^2}, \Omega_g := \frac{\rho_g}{3H^2}, \tag{30}$$

are the dimensionless density parameters of the matter, the radiation and the geometrical dark energy, respectively.

Suppose

$$\alpha = -\frac{\beta}{2}. \tag{31}$$

(22), (23) and (24) become

$$\rho_g = \frac{1-8\gamma}{8\gamma}\Lambda + \frac{\left(1-8\gamma\right)\rho_{\mathrm{r}}}{8\gamma} - \frac{1-8\gamma}{16\gamma}\rho_{\mathrm{m}} + \frac{\beta}{16\gamma}\rho_{\mathrm{m}}^2, \tag{32}$$

$$p_g = -\frac{1+8\gamma}{24\gamma}\Lambda - \frac{\left(1+8\gamma\right)\rho_{\mathrm{r}}}{24\gamma} + \frac{1-8\gamma}{48\gamma}\rho_{\mathrm{m}} - \frac{\beta}{48\gamma}\rho_{\mathrm{m}}^2, \tag{33}$$

$$w_g = \frac{p_g}{\rho_g} = \frac{-2\left(1+8\gamma\right)\Lambda - 2\left(1+8\gamma\right)\rho_{\mathrm{r}} + \left(1-8\gamma\right)\rho_{\mathrm{m}} - \beta\rho_{\mathrm{m}}^2}{6\left(1-8\gamma\right)\Lambda + 6\left(1-8\gamma\right)\rho_{\mathrm{r}} - 3\left(1-8\gamma\right)\rho_{\mathrm{m}} + 3\beta\rho_{\mathrm{m}}^2}. \tag{34}$$

and

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - \frac{1}{6}\left(\rho - 3p\right) = \frac{\Lambda}{3} - \frac{1}{6}\rho_{\mathrm{m}}. \tag{35}$$

(30) and (19) give

$$\Omega_{\mathrm{m}} = \frac{16\gamma\rho_{\mathrm{m}}}{2\left(1-8\gamma\right)\Lambda + 2\rho_{\mathrm{r}} + \left(24\gamma-1\right)\rho_{\mathrm{m}} + \beta\rho_{\mathrm{m}}^2}, \tag{36}$$

$$\Omega_g = \frac{2\left(1-8\gamma\right)\Lambda + 2\left(1-8\gamma\right)\rho_{\mathrm{r}} - \left(1-8\gamma\right)\rho_{\mathrm{m}} + \beta\rho_{\mathrm{m}}^2}{2\left(1-8\gamma\right)\Lambda + 2\rho_{\mathrm{r}} + \left(24\gamma-1\right)\rho_{\mathrm{m}} + \beta\rho_{\mathrm{m}}^2}. \tag{37}$$

From the observed data

$$\rho_{\mathrm{crit}} = 1.88h^2 \times 10^{-29} gcm^{-3} = 7.2402 \times 10^{-58} cm^{-2},$$

$$\Omega_{\mathrm{m}} = 0.3, \Omega_{\mathrm{r}} = 1.8035 \times 10^{-4}\Omega_{\mathrm{m}} \tag{38}$$

$$w_g = -1, \tag{39}$$

using (31), (34) and (36) we can determine the parameters

$$\begin{aligned} \alpha &= -4.1969 \times 10^{56} cm^2, \\ \beta &= 8.3937 \times 10^{56} cm^2, \\ \gamma &= 0.0576. \end{aligned} \tag{40}$$

Then (21), (25), (26) and (27) give

$$\Lambda = 4.8179 \times 10^{-58} cm^{-2}, \tag{41}$$

$$f_{\mathrm{vac}}^2 = 3.4852 \times 10^{-58} cm^{-2}, \tag{42}$$

$$R_{\mathrm{vac}} = 1.0456 \times 10^{-57} cm^{-2}, \tag{43}$$

and

$$\left(\frac{\ddot{a}}{a}\right)_{\mathrm{vac}} = 1.606 \times 10^{-58} cm^{-2} = 1.4425 \times 10^{-37} s^{-2}. \tag{44}$$

The value given by (41) can be compared with the observed datum

$$\Lambda^{(\mathrm{obs})} = 8\pi G \rho_\Lambda^{(\mathrm{obs})} = 8\pi G \left(10^{-12} GeV\right)^4 \sim 8\pi G \times 2 \times 10^{-10} erg/cm^3 = 4.1574 \times 10^{-58} cm^{-2}.$$

Since

$$\rho_{\mathrm{r}} = \frac{\overline{\rho}_{\mathrm{r}}}{a^4}, \overline{\rho}_{\mathrm{r}} = \rho_{\mathrm{r},a=1}, \rho_{\mathrm{m}} = \frac{\overline{\rho}_{\mathrm{m}}}{a^3}, \overline{\rho}_{\mathrm{m}} = \rho_{\mathrm{m},a=1},$$

using (40) and

$$\overline{\rho}_{\mathrm{m}} = 0.3\rho_{\mathrm{crit}}, \overline{\rho}_{\mathrm{r}} = 1.8035 \times 10^{-4} \overline{\rho}_{\mathrm{m}}$$

the state equation of the dark energy (34) can be written as

$$w_g(a) = \frac{p_g}{\rho_g} = \frac{-0.78766 - 6.4044 \times 10^{-5} a^{-4} + 6.5538 \times 10^{-2} a^{-3} - 2.216 \times 10^{-2} a^{-6}}{0.87221 + 7.0919 \times 10^{-5} a^{-4} - 0.19661 a^{-3} + 6.6481 \times 10^{-2} a^{-6}}, \tag{45}$$

or

$$w_g(z) = \frac{-0.78766 - 6.4044 \times 10^{-5} (1+z)^4 + 6.5538 \times 10^{-2} (1+z)^3 - 2.216 \times 10^{-2} (1+z)^6}{0.87221 + 7.0919 \times 10^{-5} (1+z)^4 - 0.19661 (1+z)^3 + 6.6481 \times 10^{-2} (1+z)^6}. \tag{46}$$

Figure 1 plots the evolution history of $w_g(a)$ given by (45).

In observation and experiments it is conventional to phrase constraints or projected constraints on $w(z)$ in terms of a linear evolutional model [15]:

$$w(a) = w_0 + w_a(1-a) = w_{\mathrm{p}} + w_a(a_{\mathrm{p}} - a),$$

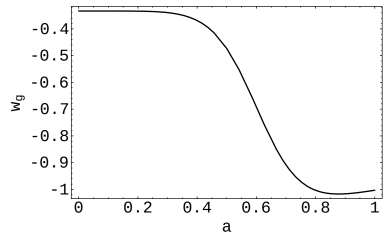

FIG. 1: The evolution of $w_g(a)$.

where w_0 is the value of w at $z = 0$ $(a = 1)$, and $w_{\rm p}$ is the value of w at a "pivot" redshift $z_{\rm p}$. For typical data combinations, $z_{\rm p} \approx 0.5$. To this end we give the linear approximation of (45). When $a = 1$,

$$w_{g0} = -1,$$

and

$$\frac{dw_g}{da}|_{a=1} = 0.19936, \tag{47}$$

so we have

$$w_g(a) = w_{g0} + \frac{dw_g}{da}|_{a=1}(a-1) = -1 - 0.19936(1-a). \tag{48}$$

When $z_{\rm p} = 0.5$, $a = \frac{2}{3}$,

$$w_{g{\rm p}} = -0.84781, \tag{49}$$

and

$$\frac{dw_g}{da}|_{a=\frac{2}{3}} = -0.7653, \tag{50}$$

then we have

$$w_{g{\rm p}}(a) = w_{g{\rm p}} + \frac{dw_g}{da}|_{\rm p}\left(a - \frac{2}{3}\right) = -0.8478 + 0.7653\left(\frac{2}{3} - a\right). \tag{51}$$

Using (35) and $\rho_{\rm m} = \overline{\rho}_{\rm m}/a^3$, one finds that when

$$a = a_{\rm trans} = \left(\frac{2\beta\overline{\rho}_{\rm m}}{3(1-8\gamma)}\right)^{\frac{1}{3}} = 0.60859, \tag{52}$$

$$z_{\rm trans} = 0.64314, \tag{53}$$

the expansion of the universe transforms from deceleration to acceleration. Using (37) one can compute that when

$$a = 0.75817, z = 0.31897, \Omega_g = 0.5, \tag{54}$$

the universe transforms from the matter dominating phase into the dark energy dominating phase. In a flat ΛCDM universe with (Ω_m, Ω_Λ) = (0.3, 0.7) acceleration begins at $z = 0.67$, while dark energy doesn't dominate the energy density of the universe until $z = 0.33$ [16].

IV. An exact analytic solution of cosmological equation

The equation (19) can be solved in two cases as follows. In the radiation dominated era

$$\rho_{\rm r} = \frac{\overline{\rho}_{\rm r}}{a^4}, \overline{\rho}_{\rm r} = \rho_{{\rm r},a=1} = {\rm const},\ p_{\rm r} = \frac{1}{3}\rho_{\rm r},$$

(19) reads

$$H^2 = \frac{(1-8\gamma)^2}{32\gamma\beta} + \frac{\overline{\rho}_{\rm r}}{24\gamma a^4}, \tag{55}$$

and can be rewritten

$$\frac{da}{dt} = a\sqrt{\frac{(1-8\gamma)^2}{32\gamma\beta} + \frac{\overline{\rho}_{\rm r}}{24\gamma a^4}}. \tag{56}$$

Its integration gives

$$\ln\left(a^2 + \sqrt{a^4 + \frac{4\beta\overline{\rho}_{\rm r}}{3(1-8\gamma)^2}}\right) - \ln\sqrt{\frac{4\beta\overline{\rho}_{\rm r}}{3(1-8\gamma)^2}} = \frac{(1-8\gamma)}{2\sqrt{2\gamma\beta}}t, \tag{57}$$

or

$$\begin{aligned} a &= \left(\frac{\beta\overline{\rho}_{\rm r}}{3(1-8\gamma)^2}\right)^{\frac{1}{4}} \sqrt{\left(e^{\frac{(1-8\gamma)}{2\sqrt{2\gamma\beta}}t} - e^{-\frac{(1-8\gamma)}{2\sqrt{2\gamma\beta}}t}\right)} \\ &= \left(\frac{\beta\overline{\rho}_{\rm r}}{3(1-8\gamma)^2}\right)^{\frac{1}{4}} \sqrt{2\sinh\frac{(1-8\gamma)}{2\sqrt{2\gamma\beta}}t}. \end{aligned} \tag{58}$$

In the matter dominated era

$$\rho_{\rm m} = \frac{\overline{\rho}_{\rm m}}{a^3}, \overline{\rho}_{\rm m} = \rho_{{\rm m},a=1} = {\rm const},\ p = 0,$$

(19) reads

$$H^2 = \frac{(1-8\gamma)^2}{32\gamma\beta} + \frac{12\alpha+5\beta-24\gamma(4\alpha+\beta)}{48\gamma\beta}\frac{\overline{\rho}_{\rm m}}{a^3} + \frac{(3\alpha+\beta)(4\alpha+\beta)}{24\gamma\beta}\frac{\overline{\rho}_{\rm m}^2}{a^6}, \tag{59}$$

and then

$$\frac{da}{dt} = a\sqrt{\frac{(1-8\gamma)^2}{32\gamma\beta} + \frac{12\alpha+5\beta-24\gamma(4\alpha+\beta)}{48\gamma\beta}\frac{\overline{\rho}_{\rm m}}{a^3} + \frac{(3\alpha+\beta)(4\alpha+\beta)}{24\gamma\beta}\frac{\overline{\rho}_{\rm m}^2}{a^6}}. \tag{60}$$

Its integration gives

$$\begin{aligned} &\ln\left(\sqrt{a^6 + 2\frac{12\alpha+5\beta-24\gamma(4\alpha+\beta)}{3(1-8\gamma)^2}\overline{\rho}_{\rm m}a^3 + \frac{4(3\alpha+\beta)(4\alpha+\beta)}{3(1-8\gamma)^2}\overline{\rho}_{\rm m}^2} + a^3 + \frac{12\alpha+5\beta-24\gamma(4\alpha+\beta)}{3(1-8\gamma)^2}\overline{\rho}_{\rm m}\right) \\ &- \ln\left(\sqrt{\frac{4(3\alpha+\beta)(4\alpha+\beta)}{3(1-8\gamma)^2}\overline{\rho}_{\rm m}^2} + \frac{12\alpha+5\beta-24\gamma(4\alpha+\beta)}{3(1-8\gamma)^2}\overline{\rho}_{\rm m}\right) \\ = &\ 3\frac{(1-8\gamma)}{\sqrt{32\gamma\beta}}t, \end{aligned} \tag{61}$$

and then

$$
\begin{aligned}
a = \{ & \frac{1}{2}\left(\sqrt{\frac{4\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{3\left(1-8\gamma\right)^{2}}\overline{\rho}_{\mathrm{m}}^{2}}+\frac{12\alpha+5\beta-24\gamma\left(4\alpha+\beta\right)}{3\left(1-8\gamma\right)^{2}}\overline{\rho}_{\mathrm{m}}\right)e^{\frac{3(1-8\gamma)}{4\sqrt{2\gamma\beta}}t} \\
& -\frac{12\alpha+5\beta-24\gamma\left(4\alpha+\beta\right)}{3\left(1-8\gamma\right)^{2}}\overline{\rho}_{\mathrm{m}} \\
& +\frac{36\alpha+13\beta-48\gamma\left(1+4\gamma\right)\left(4\alpha+\beta\right)}{18\left(1-8\gamma\right)^{4}}\beta\overline{\rho}_{\mathrm{m}}\left(\sqrt{\frac{4\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{3\left(1-8\gamma\right)^{2}}}+\frac{12\alpha+5\beta-24\gamma\left(4\alpha+\beta\right)}{3\left(1-8\gamma\right)^{2}}\right)^{-1}e^{-\frac{3(1-8\gamma)}{4\sqrt{2\gamma\beta}}t}\}^{\frac{1}{3}}
\end{aligned}
\tag{62}
$$

In the case (40), equations (58) and (62) become

$$
a = 0.67617\sqrt{2\sinh 2.7417\times 10^{-29}t}, \tag{63}
$$

and

$$
a = \left(0.17801e^{4.1125\times 10^{-29}t}-9.8074\times 10^{-2}e^{-4.1125\times 10^{-29}t}-7.9934\times 10^{-2}\right)^{\frac{1}{3}}, \tag{64}
$$

where the time t is in cm. This is a new exact analytic cosmological solution which resembles but differs from the ΛCDM solution [17].

Now we can evaluate the age of the universe using (40), (57) and (61). In the radiation dominating era $z \gtrsim 3000$ [17], we have the equation

$$
\ln\left(a^{2}+\sqrt{a^{4}+\frac{4\beta\overline{\rho}}{3\left(1-8\gamma\right)^{2}}}\right)-\ln\sqrt{\frac{4\beta\overline{\rho}}{3\left(1-8\gamma\right)^{2}}}=\frac{\left(1-8\gamma\right)}{2\sqrt{2\gamma\beta}}t.
$$

Choosing $z = 3000$, then $a = 1/3001$, this equation gives

$$
t = 3.298\times 10^{23}cm = 3.4895\times 10^{5}Years. \tag{65}
$$

In the matter dominating era $z \lesssim 3000$, we have the equation

$$
\begin{aligned}
& \ln\left(\sqrt{1+2\frac{\left(24\gamma-1\right)}{3\left(1-8\gamma\right)^{2}}\beta\overline{\rho}a_{2}^{3}+\frac{2}{3\left(1-8\gamma\right)^{2}}\left(\beta\overline{\rho}\right)^{2}}+a_{2}^{3}+\frac{\left(24\gamma-1\right)}{3\left(1-8\gamma\right)^{2}}\beta\overline{\rho}\right) \\
& -\ln\left(\sqrt{1+2\frac{\left(24\gamma-1\right)}{3\left(1-8\gamma\right)^{2}}\beta\overline{\rho}a_{1}^{3}+\frac{2}{3\left(1-8\gamma\right)^{2}}\left(\beta\overline{\rho}\right)^{2}}+a_{1}^{3}+\frac{\left(24\gamma-1\right)}{3\left(1-8\gamma\right)^{2}}\beta\overline{\rho}\right) \\
= & = 3\frac{\left(1-8\gamma\right)}{\sqrt{32\gamma\beta}}\left(t_{2}-t_{1}\right),
\end{aligned}
$$

Choosing

$$
\begin{aligned}
z_{1} &= 3000, a_{1}=\frac{1}{3001}, \\
z_{2} &= 0, a_{2}=1,
\end{aligned}
$$

we have

$$
t_{2}-t_{1} = 1.6383\times 10^{28}cm = 1.7334\times 10^{10}Years = 17.334Gy. \tag{66}
$$

V. Perturbation theory

In order to discriminate lots of dark energy models, it is interested to seek the additional information other than the background expansion history of the universe [18]. Now we discuss the dynamics of linear perturbations and the structure growth of universe.

A. Cosmological perturbations of gravitational potentials and the torsion

The perturbed equations can be derived by straightforward and tedious calculations, following the approach of [19]. The computer software Maple has been applied to work out the lengthy calculations. We focus on the scalar perturbations, since they are sufficient to reveal the basic features of the theory, allowing for a discussion of the growth of matter overdensities. The perturbed vierbein reads

$$\begin{aligned} e^0_\mu &= \delta^0_\mu\,(1+\phi)\,, e^a_\mu = a\delta^a_\mu\,(1-\psi)\,, \\ e^\mu_0 &= \delta^\mu_0\,(1-\phi)\,, e^\mu_a = \frac{1}{a}\delta^\mu_a\,(1+\psi)\,. \end{aligned} \tag{67}$$

in which we have introduced the scalar modes ϕ and ψ as functions of t. This induces a metric perturbation of the known form, namely

$$ds^2 = a^2(\eta)[-(1+2\phi)d\eta^2 + (1-2\psi)\gamma_{ij}dx^i dx^j], \tag{68}$$

in the longitudinal gauge and the conformal time η.

In order to preserve the global homogeneity and isotropy of the spacetime the perturbations are assumed to be small. It has been proved [6] that only two scalar torsion modes h and f are physically acceptable and no-ghosts. On the basis of the above theoretical tests (e.g., "no-ghosts" or "no-tachyons"), we use (7) to give the linear perturbation of the nonvanishing torsion components

$$\delta T_{ij0} = \delta_{ij}a^2\delta h, \delta T_{ijk} = 2\epsilon_{ijk}a^3\delta f, \quad i,j,k,... = 1,2,3.$$

In the case (16), $h=0$, we have

$$\delta T_{ij0} = 0, \delta T_{ijk} = 2\epsilon_{ijk}a^3\xi, \quad i,j,k,... = 1,2,3. \tag{69}$$

where $\xi = \delta f$.

The unperturbed field equation (2) can be written as

$$G^\mu{}_\nu = T^\mu{}_\nu + T_{(g)}{}^\mu{}_\nu,$$

where $G^\mu{}_\nu$ is the Einstein tensor, $T^\mu{}_\nu$ is the energy-momentum of the ordinary matter and the radiation, $T_{(g)}{}^\mu{}_\nu$ is the energy-momentum of the "geometric dark energy" given by (4). The equations of motion for small perturbations linearized on the background metric are

$$\delta G^\mu{}_\nu = \delta T^\mu{}_\nu + \delta T_{(g)}{}^\mu{}_\nu. \tag{70}$$

For scalar type metric perturbations with a line element given in (68) (in conformal time), the perturbed field equations can be obtained following the approach of [19].

The cosmic fluid includes radiation, baryonic matter and dark matter, $\rho_{\rm m} = \rho_b + \rho_d$, we have

$$\rho = \rho_{\rm m} + \rho_{\rm r} = \rho_b + \rho_d + \rho_r, p = p_r = \frac{1}{3}\rho_r. \tag{71}$$

Since

$$\rho_b \propto \frac{1}{a^3}, \rho_r \propto \frac{1}{a^4}, \tag{72}$$

we suppose

$$\rho_d \propto \frac{1}{a^n}. \tag{73}$$

Then we have

$$\rho_r = r\rho_b, \rho_d = v\rho_b, \tag{74}$$

where

$$r \propto a^{-1}, v \propto a^{3-n}. \tag{75}$$

The equation

$$\delta G^0{}_0 = -\delta\rho - \delta\rho_g \tag{76}$$

takes the form

$$\begin{aligned}
& 2a^{-2}\left(3\mathcal{H}\left(\mathcal{H}\phi + \psi'\right) - \nabla^2\psi\right) \\
& + \left(\frac{3\left(1-8\gamma\right)}{16\beta\gamma} + \frac{12\alpha + 5\beta - 16\gamma\left(3\alpha+\beta\right)}{8\beta\gamma}\left(1+v\right)\rho_b + \frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{4\gamma\beta}\left(1+v\right)^2\rho_b^2\right)\psi \\
= & -\frac{1+r+v}{3\gamma}\rho_b\delta - \frac{12\alpha+5\beta-72\alpha\gamma-20\beta\gamma}{8\beta\gamma}\left(1+v\right)\rho_b\delta - \frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{2\beta\gamma}\left(1+v\right)^2\rho_b^2\delta,
\end{aligned} \tag{77}$$

where the growth of the baryonic matter density perturbation $\delta := \delta\rho_b/\rho_b$, $\mathcal{H} := a'/a = aH$, prime denotes derivative with respect to the conformal time η.

The equation

$$\delta G^i{}_j = \left(\delta p_r + \delta p_g\right)\delta^i{}_j, \tag{78}$$

reads

$$\begin{aligned}
& -2a^{-2}\{\left[\left(2\mathcal{H}' + \mathcal{H}^2\right)\phi + \mathcal{H}\phi' + \psi'' + 2\mathcal{H}\psi' + \frac{1}{2}\nabla^2\left(\phi-\psi\right)\right]\delta^i{}_j - \frac{1}{2}\partial_i\partial_j\left(\phi-\psi\right)\} + 2\left(f^2\psi + f\xi\right)\delta^i{}_j \\
= & \{\frac{1}{3}r\rho_b\delta - \frac{\left(8\gamma+1\right)\left(1+r+v\right)}{24\gamma}\rho_b\delta + \frac{8\beta\gamma - 3\left(4\alpha+\beta\right)}{48\beta\gamma}\left(1+v\right)\rho_b\delta \\
& -\frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{12\beta\gamma}\left(1+v\right)^2\rho_b^2\delta\}\delta^i{}_j,
\end{aligned} \tag{79}$$

where $i=1,2,3$.

The equation

$$\begin{aligned} G^0{}_i &= R^0{}_i \\ &= T^0{}_i - 4\alpha R^0{}_i R - \beta\left(2R^{\rho 0}R_{\rho i} + 2R^{\rho\sigma}R^0{}_{\rho i\sigma}\right) \\ &\quad +\gamma\left(4e_I{}^0\partial_\nu\left(e^{I\lambda}T_{i\lambda}{}^\nu\right) - 4e^K{}_\tau T_i{}^{0\nu}\partial_\nu e_K{}^\tau - 4T^{\lambda 0}{}_\tau T_{\lambda i}{}^\tau\right), \end{aligned} \tag{80}$$

yields

$$\begin{aligned} & 2a^{-2}\left[\mathcal{H}\phi+\psi'\right]_{,i} - a^{-1}\left(8\gamma-1-4\alpha\left(1+r+v\right)\rho_b\right)\psi'_{,i} - 2a^{-1}\left(1+\frac{4}{3}r+v\right)\rho_b V_{,i} \\ & -4\beta a^{-1}\{\frac{(8\gamma-1)(4\gamma+1)}{16\beta\gamma} - \frac{12\alpha+5\beta-8\gamma(3\alpha+2\beta)}{24\beta\gamma}(1+v)\rho_b \\ & -\frac{(3\alpha+\beta)(4\alpha+\beta)}{12\gamma\beta}(1+v)^2\rho_b^2\}\mathcal{H}\phi_{,i} + 8\beta a^{-1}\mathcal{H}f^2\psi_{,i} + 8a^{-1}\mathcal{H}f\xi_{,i} \\ = {} & 0. \end{aligned} \tag{81}$$

In commoving orthogonal coordinates, the three-velocity of baryonic matter vanishes, $V_b^i=0$ [20]. For the potential V of the three-velocity field of the dark matter, the perturbed conservation law

$$\delta\left(\nabla_\mu T^\mu{}_\nu + \nabla_\mu T_g{}^\mu{}_\nu\right) = 0, \tag{82}$$

leads to the equation

$$\dot{V}_{,i} + \left(\frac{\frac{4}{3}\dot{r}+\dot{v}}{1+\frac{4}{3}r+v} + H\right)V_{,i} = 0, \tag{83}$$

when $\nu=i$.

In the case (15) and (16), using (67) and (69) we obtain the perturbation of the equation (3)

$$2f\xi = \left(\frac{1+r+v}{24\gamma} + \frac{3(4\alpha+\beta)-16\gamma(3\alpha+\beta)}{48\beta\gamma}(1+v) + \frac{(3\alpha+\beta)(4\alpha+\beta)}{12\gamma\beta}(1+v)^2\rho_b\right)\rho_b\delta. \tag{84}$$

In the Fourier space k, from (77) and (79) we obtain the equations of ϕ and ψ,

$$\begin{aligned} & 2a^{-2}\left(3\mathcal{H}\left(\mathcal{H}\phi+\psi'\right)+k^2\psi\right) \\ & +\left(\frac{3(1-8\gamma)}{16\beta\gamma} + \frac{12\alpha+5\beta-16\gamma(3\alpha+\beta)}{8\beta\gamma}(1+v)\rho_b + (3\alpha+\beta)\frac{4\alpha+\beta}{4\gamma\beta}(1+v)^2\rho_b^2\right)\psi \\ = {} & -\frac{1+r+v}{3\gamma}\rho_b\delta - \frac{12\alpha+5\beta-72\alpha\gamma-20\beta\gamma}{8\beta\gamma}(1+v)\rho_b\delta - \frac{(3\alpha+\beta)(4\alpha+\beta)}{2\beta\gamma}(1+v)^2\rho_b^2\delta, \end{aligned} \tag{85}$$

and

$$\begin{aligned} & -2a^{-2}\{\left[\left(2\mathcal{H}'+\mathcal{H}^2\right)\phi+\mathcal{H}\phi'+\psi''+2\mathcal{H}\psi'-\frac{1}{2}k^2(\phi-\psi)\right]\delta^i{}_j - \frac{1}{2}\partial_i\partial_j(\phi-\psi)\} + 2\left(f^2\psi+f\xi\right)\delta^i{}_j \\ = {} & \{\frac{1}{3}r\rho_b\delta - \frac{(8\gamma+1)(1+r+v)}{24\gamma}\rho_b\delta + \frac{8\beta\gamma-3(4\alpha+\beta)}{48\beta\gamma}(1+v)\rho_b\delta - \frac{(3\alpha+\beta)(4\alpha+\beta)}{12\beta\gamma}(1+v)^2\rho_b^2\delta\}\delta^i{}_j. \end{aligned} \tag{86}$$

When $i\neq j$, (86) leads to

$$\phi=\psi, \tag{87}$$

agreeing with GR but in contrast to $f(R)$ theory [21]. Then we have the equations of ψ:

$$\begin{aligned}
&2a^{-2}\left(3\mathcal{H}\left(\mathcal{H}\psi+\psi'\right)+k^2\psi\right)\\
&+\left(\frac{3\left(1-8\gamma\right)}{16\beta\gamma}+\frac{12\alpha+5\beta-16\gamma\left(3\alpha+\beta\right)}{8\beta\gamma}\left(1+v\right)\rho_b+\left(3\alpha+\beta\right)\frac{4\alpha+\beta}{4\gamma\beta}\left(1+v\right)^2\rho_b^2\right)\psi\\
=&-\frac{1+r+v}{3\gamma}\rho_b\delta-\frac{12\alpha+5\beta-72\alpha\gamma-20\beta\gamma}{8\beta\gamma}\left(1+v\right)\rho_b\delta-\frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{2\beta\gamma}\left(1+v\right)^2\rho_b^2\delta,
\end{aligned}\tag{88}$$

$$\begin{aligned}
&-2a^{-2}\left[\left(2\mathcal{H}'+\mathcal{H}^2\right)\psi+\psi''+3\mathcal{H}\psi'\right]+2\left(f^2\psi+f\xi\right)\\
=&\ \frac{1}{3}r\rho_b\delta-\frac{\left(8\gamma+1\right)\left(1+r+v\right)}{24\gamma}\rho_b\delta+\frac{8\beta\gamma-3\left(4\alpha+\beta\right)}{48\beta\gamma}\left(1+v\right)\rho_b\delta\\
&-\frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{12\beta\gamma}\left(1+v\right)^2\rho_b^2\delta.
\end{aligned}\tag{89}$$

One of the methods to measure the cosmic growth rate is redshift-space distortion that appears in clustering pattern of galaxies in galaxy redshift surveys. In order to confront the models with galaxy clustering surveys, we are interested in the modes deep inside the Hubble radius. In this case we can employ the quasistatic approximation on sub-horizon scales, under which, $\partial/\partial\eta\sim\mathcal{H}\ll k$. Then the perturbation equations (88), (89) give

$$\begin{aligned}
&\left(2a^{-2}k^2-\frac{3\left(1-8\gamma\right)}{16\beta\gamma}-\frac{12\alpha+5\beta-16\gamma\left(3\alpha+\beta\right)}{8\beta\gamma}\left(1+v\right)\rho_b-\left(3\alpha+\beta\right)\frac{4\alpha+\beta}{4\gamma\beta}\left(1+v\right)^2\rho_b^2\right)\psi\\
=&\left(\frac{1+r+v}{3\gamma}\rho_b+\frac{12\alpha+5\beta-72\alpha\gamma-20\beta\gamma}{8\beta\gamma}\left(1+v\right)\rho_b+\frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{2\beta\gamma}\left(1+v\right)^2\rho_b^2\right)\delta,
\end{aligned}\tag{90}$$

and

$$\begin{aligned}
&-2a^{-2}\left[\left(2\mathcal{H}'+\mathcal{H}^2\right)\psi+\psi''+3\mathcal{H}\psi'\right]+2\left(f^2\psi+f\xi\right)\\
=&\ \frac{1}{3}r\rho_b\delta-\frac{\left(8\gamma+1\right)\left(1+r+v\right)}{24\gamma}\rho_b\delta+\frac{8\beta\gamma-3\left(4\alpha+\beta\right)}{48\beta\gamma}\left(1+v\right)\rho_b\delta\\
&-\frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{12\beta\gamma}\left(1+v\right)^2\rho_b^2\delta.
\end{aligned}\tag{91}$$

The equation (90) gives the expression of gravitational potential ψ. In the case $\alpha=-\frac{\beta}{2}$, if

$$a^{-2}k^2\gg\rho_b,\left|\alpha\rho_b\right|\gg 1,\tag{92}$$

it reduces to the Poisson equation

$$\frac{k^2}{a^2}\psi=-4\pi G_{eff}\rho_b\delta,\tag{93}$$

where

$$G_{eff}=\frac{1}{16\pi\gamma}\left(1+v\right)^2\alpha\rho_b\tag{94}$$

is the effective gravitational coupling constant. In the framework of GR, G_{eff} is equivalent to the gravitational constant $G=1$.

B. Equation of the structure growth and its solution

Since different theoretical models can achieve the same expansion history of universe, several methods should be used to discriminate the different models. The study on the growth of matter density perturbations may become the useful tool due to that theories with the same expansion history can have a different cosmic growth history. The perturbation quantities can be easily related to the cosmic observations [22].

Using (59), the equations (8) and (9) can be rewritten as

$$\begin{aligned} H^2 &= \frac{1}{3}\left(\rho_b + \rho_{other}\right), & (95)\\ \dot{H} &= -\frac{1}{2}\left(\rho_b + \rho_{other} + p_{other}\right), & (96) \end{aligned}$$

where

$$\begin{aligned} \rho_{other} &= \rho_r + \rho_d + \rho_g, \\ p_{other} &= p_r + p_g. \end{aligned} \tag{97}$$

We introduce the perturbations of ρ_b, ρ_{other}, p_{other} and H [23]:

$$\begin{aligned} \rho_b &\longrightarrow (1+\delta)\,\rho_b, \rho_{other} \longrightarrow \rho_{other} + \delta\rho_{other}, \\ p_{other} &\longrightarrow p_{other} + \delta p_{other}, H \longrightarrow H + \delta H, \end{aligned}$$

with

$$\delta H \equiv \frac{1}{3a}\nabla \cdot \mathbf{u}, \quad \mathbf{u} = \nabla V. \tag{98}$$

Following the approach of [23] and [19], using (95), (96) and the perturbed conservation law

$$\delta\left(\nabla_\mu T^\mu{}_\nu + \nabla_\mu T_g{}^\mu{}_\nu\right) = 0, \tag{99}$$

we obtain the equation for the growth of the baryonic matter density perturbation δ:

$$\begin{aligned} &\rho_b\,\ddot{\delta} + \frac{\dot{\rho}_b\,\rho_{other} - \rho_b\,\dot{\rho}_{other} + \dot{\rho}_b\,p_{other} - \rho_b\,\dot{p}_{other}}{\rho_b + \rho_{other} + p_{other}}\,\dot{\delta} \\ &- \left(\ddot{\rho}_{other} - \frac{3}{2}\rho_b\rho_b - 3\left(\rho_{other} + p_{other}\right)\rho_b - \frac{3}{2}\left(\rho_{other} + p_{other}\right)^2\right)\delta \\ &+ \frac{2\,\dot{\rho}_b + 2\,\dot{\rho}_{other}}{\rho_b + \rho_{other} + p_{other}}\,\dot{p}_{other}\,\delta + \frac{2\,\dot{\rho}_b + 2\,\dot{\rho}_{other}}{\rho_b + \rho_{other} + p_{other}}\,\dot{\rho}_{other}\,\delta \\ &- \frac{\left(\dot{\rho}_b + \dot{\rho}_{other}\right)^2 + \left(\dot{\rho}_b + \dot{\rho}_{other}\right)\dot{p}_{other}}{\left(\rho_b + \rho_{other} + p_{other}\right)^2}\left(\rho_{other} + p_{other}\right)\delta \\ &+ \delta\ddot{\rho}_{other} - \frac{2\,\dot{\rho}_b + 2\,\dot{\rho}_{other} + \dot{p}_{other}}{\rho_b + \rho_{other} + p_{other}}\,\delta\dot{\rho}_{other} - \frac{\dot{\rho}_b + \dot{\rho}_{other}}{\rho_b + \rho_{other} + p_{other}}\,\delta\dot{p}_{other} \\ &- \left(3\left(\rho_b + \rho_{other} + p_{other}\right) - \frac{\left(\dot{\rho}_b + \dot{\rho}_{other}\right)^2}{\left(\rho_b + \rho_{other} + p_{other}\right)^2} - \frac{\left(\dot{\rho}_b + \dot{\rho}_{other}\right)\dot{p}_{other}}{\left(\rho_b + \rho_{other} + p_{other}\right)^2}\right)\left(\delta\rho_{other} + \delta p_{other}\right) \\ = {}& 0. \end{aligned} \tag{100}$$

Up to now, the complete set of equations that describes the general linear perturbations has been obtained. It provides enough information about the behaviors of the perturbation and can be compared with the results of the ΛCDM model.

(71), (74) and (75) yield

$$\begin{aligned}\dot{\rho}_b &= -3H\rho_b, \dot{\rho}_r = -4H\rho_r, \dot{\rho}_d = -nH\rho_d,\\ \dot{p}_r &= -\frac{4}{3}H\rho_r,\\ \dot{r} &= -rH, \dot{v} = (3-n)\,vH, \end{aligned} \tag{101}$$

and then (97), (22) and (23) give

$$\begin{aligned}\rho_{other} &= \frac{3\left(8\gamma-1\right)^2}{32\beta\gamma} + \left(r + v - \frac{\left(8\gamma-1\right)\left(1+r+v\right)}{8\gamma} - \frac{3\left(8\gamma-1\right)\left(4\alpha+\beta\right)}{16\beta\gamma}\left(1+v\right)\right)\rho_b\\ &\quad + \frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{8\beta\gamma}\left(1+v\right)^2\rho_b^2, \end{aligned} \tag{102}$$

$$\begin{aligned}p_{other} &= \frac{64\gamma^2-1}{32\beta\gamma} + \left(\frac{1}{3}r - \frac{\left(8\gamma+1\right)\left(1+r+v\right)}{24\gamma} + \frac{8\beta\gamma - 3\left(4\alpha+\beta\right)}{48\beta\gamma}\left(1+v\right)\right)\rho_b\\ &\quad - \frac{\left(3\alpha+\beta\right)\left(4\alpha+\beta\right)}{24\beta\gamma}\left(1+v\right)^2\rho_b^2, \end{aligned} \tag{103}$$

Using (102) and (103), by straightforward and tedious calculations, the equation (100) can be written as

$$\left(1+r+v+A+D\rho_b\right)\rho_b\,\ddot{\delta} + M\left(r,v,\rho_b\right)H\,\dot{\delta} + N\left(r,v,\rho_b\right)H^2\delta + Q\left(r,v,\rho_b\right)\delta = 0, \tag{104}$$

where A, D, $M\left(r,v,\rho_b\right)$, $N\left(r,v,\rho_b\right)$, and $Q\left(r,v,\rho_b\right)$ are given in Appendix.

Supposing

$$n = 3, \tag{105}$$

in the case (31) and (92), i.e. when $\rho_d \propto a^{-3}$, $\beta = -2\alpha$, and $|\alpha\rho_b| \gg 1$, the equation (104) becomes

$$\ddot{\delta} - 22H\,\dot{\delta} + 3H^2\delta = 0. \tag{106}$$

Introduce the logarithmic time variable

$$N = \ln a. \tag{107}$$

(106) takes the form

$$\frac{d^2\delta}{dN^2} - 23\frac{d\delta}{dN} + 3\delta = 0,$$

and gives the solution

$$\begin{aligned}\delta &= \delta_{0+}a^{\frac{1}{2}\left(23+\sqrt{517}\right)} + \delta_{0-}a^{\frac{1}{2}\left(23-\sqrt{517}\right)}\\ &\approx \delta_{0+}a^{22.869} + \delta_{0-}a^{0.13118}, \end{aligned} \tag{108}$$

which can be compared with the result in GR[24].

VI. CONCLUSIONS

A cosmology of Poincare gauge theory has been developed. We focus on the case including a pseudoscalar scalar torsion function f as suggested by Baekler, Hehl and Nester [25]. The cosmic equations has been derived. The analytic solutions has been obtained. Although we do not introduce a cosmological constant in the action it automatically emerges in the derivation of the cosmological equations and then is endowed with intrinsic character. It is nothing but the intrinsic torsion and curvature of the vacuum universe. The dark energy is identified with the geometry of the spacetime. Now we are returning to the original idea of Einstein and Wheeler: gravity is a geometry [26]. The cosmological constant puzzle and the coincidence and fine tuning problem are solved naturally. The point is that the dark energy is the functions of the density and pressure of the cosmic fluid and includes the cosmological constant but can not be identified with it. The analytic expressions of the state equation and the density parameters of the matter and the geometric dark energy are derived and used to determine the values of α , β and γ. Then a theoretical value of the cosmological constant is computed and compared with the observed datum. An analytic integral of the cosmological equation is obtained and used to evaluate the age of the universe which can be compared with observed data. The full equations of linear cosmological perturbations and the solutions are obtained. In addition, the behavior of perturbations for the sub-horizon modes relevant to large-scale structures is discussed. This model can be distinguished from others by considering the evolution of matter perturbations and gravitational potentials.

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Appendix A: The growth of structures in linear perturbation theory

In the following, we give the derivation of the equation (104):

Letting

$$\begin{aligned} A &= -\frac{(8\gamma-1)(1+r+v)}{8\gamma} - \frac{3(8\gamma-1)(4\alpha+\beta)}{16\beta\gamma}(1+v)\,, \\ B &= -\frac{(8\gamma+1)(1+r+v)}{24\gamma} + \frac{8\beta\gamma-3(4\alpha+\beta)}{48\beta\gamma}(1+v)\,, \\ D &= \frac{(3\alpha+\beta)(4\alpha+\beta)}{4\beta\gamma}(1+v)^2\,. \end{aligned} \tag{A1}$$

$$\begin{aligned} E &= -\frac{(8\gamma-1)((3-n)v-r)}{8\gamma} - \frac{3(8\gamma-1)(4\alpha+\beta)}{16\beta\gamma}(3-n)v, \\ F &= -\frac{(8\gamma+1)((3-n)v-r)}{24\gamma} + \frac{8\beta\gamma-3(4\alpha+\beta)}{48\beta\gamma}(3-n)v, \\ K &= \frac{(3\alpha+\beta)(4\alpha+\beta)}{2\beta\gamma}(1+v)(3-n)v, \\ L &= \frac{(8\gamma-1)\left((3-n)^2 v+r\right)}{8\gamma} + \frac{3(8\gamma-1)(4\alpha+\beta)}{16\beta\gamma}(3-n)^2 v, \end{aligned} \tag{A2}$$

(89), (90) and (91) give

$$\begin{aligned} \rho_{other} &= \frac{3\left(1-8\gamma\right)^2}{32\beta\gamma} + \left(r+v+A\right)\rho_b + \frac{D}{2}\rho_b^2, \\ p_{other} &= \frac{64\gamma^2-1}{32\beta\gamma} + \left(\frac{1}{3}r+B\right)\rho_b - \frac{D}{6}\rho_b^2. \end{aligned} \tag{A3}$$

Then we compute

$$\begin{aligned} \dot{\rho}_{other} &= -\left(vn+4r\right)H\rho_b + EH\rho_b + \frac{K}{2}H\rho_b^2, \\ \dot{p}_{other} &= -\frac{4}{3}rH\rho_b + FH\rho_b - \frac{K}{6}H\rho_b^2, \end{aligned} \tag{A4}$$

$$\begin{aligned} \ddot{\rho}_{other} &= \left(n^2v+16r-L-3E\right)H^2\rho_b \\ &\quad -\frac{1}{2}\left(n+3\right)KH^2\rho_b^2 + \frac{K}{2}\frac{\left(3-n\right)v}{\left(1+v\right)}H^2\rho_b^2 \\ &\quad +\left(E-\left(vn+4r\right)\right)\dot{H}\rho_b + \frac{K}{2}\dot{H}\rho_b^2, \end{aligned} \tag{A5}$$

$$\begin{aligned} \delta\rho_{other} &= \left(r+v+A\right)\rho_b\delta + D\rho_b^2\delta, \\ \delta p_{other} &= \left(\frac{1}{3}r+B\right)\rho_b\delta - \frac{D}{3}\rho_b^2\delta, \end{aligned} \tag{A6}$$

$$\begin{aligned} \dot{\delta\rho}_{other} &= -\left(vn+4r\right)H\rho_b\delta + \left(r+v\right)\rho_b\dot{\delta} + A\rho_b\dot{\delta} + D\rho_b^2\dot{\delta} - \left(3A-E\right)H\rho_b\delta - \left(2D-K\right)H\rho_b^2\delta, \\ \dot{\delta p}_{other} &= -\frac{4}{3}rH\rho_b\delta + \frac{1}{3}r\rho_b\dot{\delta} + B\rho_b\dot{\delta} - \frac{D}{3}\rho_b^2\dot{\delta} - \left(3B-F\right)H\rho_b\delta + \left(2D-\frac{K}{3}\right)H\rho_b^2\delta, \end{aligned} \tag{A7}$$

and

$$\begin{aligned} \ddot{\delta\rho}_{other} &= \left(r+v+\left(A+D\rho_b\right)\right)\rho_b\ddot{\delta} \\ &\quad -2\left[\left(nv+4r\right)-2\left(E-3A+\left(K-6D\right)\rho_b\right)\right]H\rho_b\dot{\delta} \\ &\quad +\left[n^2v+16r+9A-6E-L+\left(36D-12K+\frac{1+2v}{1+v}K\left(3-n\right)\right)\rho_b\right]H^2\rho_b\delta \\ &\quad +\left(E-3A-\left(vn+4r\right)+\left(K-6D\right)\rho_b\right)\dot{H}\rho_b\delta. \end{aligned} \tag{A8}$$

Substituting these into (100) yields (104):

$$\left(1+r+v+A+D\rho_b\right)\rho_b\ddot{\delta} + M\left(r,v,\rho_b\right)H\dot{\delta} + N\left(r,v,\rho_b\right)H^2\delta + Q\left(r,v,\rho_b\right)\delta = 0, \tag{A9}$$

where

$$\begin{aligned} M\left(r,v,\rho_b\right) &= -\left(3+4r+2vn-3v+9A-3B-4E\right)\rho_b + \left(4K-22D\right)\rho_b^2 \\ &\quad + \frac{\left(-\left(1+r+v+A\right)\rho_b + D\rho_b^2\right)\left(-vn-\frac{16}{3}r+E+F-3\right)}{\frac{(8\gamma-1)(16\gamma-1)}{16\beta\gamma} + \left(1+\frac{4}{3}r+v+A+B\right)\rho_b + \frac{1}{3}D\rho_b^2}\rho_b \\ &\quad + \frac{-\frac{1}{3}\left(1+r+v+A\right)\rho_b + \frac{1}{3}D\rho_b^2}{\frac{(8\gamma-1)(16\gamma-1)}{16\beta\gamma} + \left(1+\frac{4}{3}r+v+A+B\right)\rho_b + \frac{1}{3}D\rho_b^2}K\rho_b^2, \end{aligned} \tag{A10}$$

$$N\left(r,v,\rho_b\right) \;=\; -\frac{\frac{3(8\gamma-1)(16\gamma-1)}{16\beta\gamma}+3A\rho_b+\left(D-\frac{K}{2}\right)\rho_b^2}{\frac{(8\gamma-1)(16\gamma-1)}{16\beta\gamma}+\left(1+\frac{4}{3}r+v+A+B\right)\rho_b+\frac{1}{3}D\rho_b^2}\left(3+\frac{16}{3}r+vn-E-F\right)\rho_b$$
$$+\left(-9B+36D\rho_b-3E+\frac{1}{2}\left(n-19\right)K\right)\rho_b+\frac{2+3v}{2\left(1+v\right)}\left(3-n\right)K\rho_b^2$$
$$+\frac{\frac{(8\gamma-1)(16\gamma-1)}{16\beta\gamma}+A\rho_b+\frac{1}{3}\left(D-\frac{K}{2}\right)\rho_b^2}{\frac{(8\gamma-1)(16\gamma-1)}{16\beta\gamma}+\left(1+\frac{4}{3}r+v+A+B\right)\rho_b+\frac{1}{3}D\rho_b^2}K\rho_b^2, \tag{A11}$$

and

$$Q\left(r,v,\rho_b\right) \;=\; +\frac{3}{2}\left(\frac{\left(8\gamma-1\right)\left(16\gamma-1\right)}{16\beta\gamma}\right)^2-\frac{3}{2}\frac{\left(8\gamma-1\right)\left(16\gamma-1\right)}{16\beta\gamma}\rho_b$$
$$-\left(\frac{\left(8\gamma-1\right)\left(16\gamma-1\right)}{16\beta\gamma}D+\frac{3}{2}\left(1+\frac{4}{3}r+v+A+B\right)^2+\frac{3}{2}\left(1+\frac{4}{3}r+v+A+B\right)\right)\rho_b^2$$
$$-\left(2\left(1+\frac{4}{3}r+v+A+B\right)+\frac{1}{2}\right)D\rho_b^3-\frac{1}{2}D^2\rho_b^4$$
$$-3A\,\dot{H}\,\rho_b\delta+\left(\frac{K}{2}-6D\right)\dot{H}\,\rho_b^2. \tag{A12}$$

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