Emergent Cosmos in Einstein-Cartan Theory

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Based on the Padmanabhan's proposal, the accelerated expansion of the universe can be driven by the difference between the surface and bulk degrees of freedom in a region of space, described by the relation $dV/dt = N_{sur} - N_{bulk}$ where N_{sur} and N_{bulk} are the degrees of freedom assigned to the surface area and the matter-energy content inside the bulk, respectively. In the present work, the dynamical effect of the Weyssenhoff perfect fluid with intrinsic spin and its corresponding spin degrees of freedom in the framework of Einstein-Cartan (EC) theory are investigated. Based on the modification of Friedmann equations due to the spin-spin interactions, a correction term for the Padmanabhan's relation including the number of degrees of freedom related to this spin interactions as $\Delta V/\Delta t = N_{sur} - N_{bulk} - N_{spin}$ is obtained where N_{spin} is the corresponding degree of freedom related to the intrinsics spin of the matter content of the universe. Moreover, the validity of the unified first law and the generalized second law of thermodynamics for the Einstein-Cartan cosmos are investigated.

Keywords: Padmanabhan's proposal, spin-spin interaction, Einstein-Cartan theory.

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I. INTRODUCTION

According to the current researches, one can obtain the gravitational field equations in the same way that the equations of an emergent phenomena like fluid mechanics or elasticity are derived [1–3]. In the framework of the emergent gravity model, the Einstein gravitational field equations can be derived from the thermodynamics principles with some extra assumptions [1, 4]. Therefore, Einstein field equations can be understood as spacetime equations of state [3]. By assuming the existence of a spacetime manifold, its metric and curvature, Padmanabhan has treated the Einstein field equations as an emergent phenomenon [5]. It has been proposed that in a cosmological context, the accelerated expansion of the universe [6] can be obtained from the difference between the surface and bulk degrees of freedom denoted by the relation $\Delta V/\Delta t = N_{sur} - N_{bulk}$ where N_{sur} and N_{bulk} are the corresponding degrees of freedom related to the surface area, matter-energy content (or dark matter (DM) and dark energy (DE)) inside the bulk space, respectively [7]. Different cosmological models have been proposed to explain the late time accelerated expansion of the universe [6]. One of these cosmological models is known as the dark energy model where the universe is supposed to be dominated by a dark fluid possessing a negative pressure [8–10] (for a review, see [11]). Violation of the strong energy condition is a feature of this dark fluid, i.e $\rho + 3p > 0$. On the other hand, the modified gravity theories, such as $f(R)$ gravity [12], $f(T)$ gravity [13], Weyl gravity [14], Gauss-Bonnet gravity [15], Lovelock gravity [16], Hořava-Lifshitz gravity [17], massive gravity [18], heterotic string theory [19] and braneworld scenarios [20], are another approaches for explaining the late time accelerated expansion of the universe. In these modified models, the additional terms in the gravitational Lagrangian play the role of an effective dark energy component with a geometric origin rather than an *ad hoc* introduction of the dark energy sector with unusual physical features. These cosmological models explaining the current accelerated expansion phase possess a series of conditions and constraints arising from various laws of physics such as thermodynamics laws [21] or astrophysical data. In this way, four laws of black hole mechanics driven from the classical Einstein field equations are implemented to explain the structure of spacetime and its relation with thermodynamical behaviour of the system [22, 23]. In the significant pioneering research, Jacobson proved that the classical general relativity (GR) behaves like thermodynamical system (for example, surface gravity could be understood similar to temperature in thermodynamical system) [24]. Then, the Einstein field equations were obtained from the relation of entropy and horizon area together with the Clausius relation $dQ = TdS$ where Q, S and T are the heat, the entropy and the temperature respectively. In this regard, where the connection between gravity and thermodynamics holds, the Friedmann equations are obtained by applying the first law of thermodynamics to the ap-

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parent horizon of the FLRW universe [25–35]. The second law of thermodynamics and its generalized version is also studied in different modified gravity models such as [36–45].

On the other hand, a cosmological model is influenced by the choice of a matter field source which is coupled with the Einstein equations through its energy-momentum tensor. Usually, the matter source of the universe is considered as a perfect fluid or scalar fields [46]. Regarding the early stage of the Universe when its matter content possesses an additional intrinsic spin property, it is necessary to consider a classical spin fluid or even a massless or massive spinor fields as the matter source [47]. In 1923, Élie Cartan introduced a modification of the Einstein general theory of relativity (GR) which nowadays is known as Einstein-Cartan (EC) theory [48, 49]. In this framework, a relation between the intrinsic angular momentum of matter source and the spacetime torsion is introduced before introducing this intrinsic angular momentum as the spin into quantum theory by Goudsmit and Uhlenbeck in 1925. The classical spin can be introduced in general relativity in two distinct ways. As the first method, one can consider spin as a dynamical quantity without changing the Riemannian structure of the geometry of the background spacetime[50]. The spin introduced in this way is similar to the spin of quantum mechanics and the Dirac theory of the electron. In the second approach, as introduced by Cartan, the structure of spacetime is generalized to possess torsion as well as curvature by considering the metric and the non-symmetric affine connection as independent quantities [51]. This Riemann-Cartan geometry is usually denoted by U_4 in order to distinguish it from the Riemannian geometry. After Cartan's research, many other efforts have been made by Hehl [52], Trautman [53] and Kopczynski [54] to bring spin into the curved spacetime. This approach allowed one to define the torsion of spacetime and its connection with spin. In the context of EC theory, torsion does not appear as a dynamical quantity rather it can be represented in terms of the spin sources by matter fields with intrinsic angular momentum [52]. Most of the researches on the cosmological applications of the EC theory have been made with the semiclassical spin fluid possessing the energy density ρ_s, pressure p_s and spin density vector S^α which is orthogonal to the four velocity vector u^α of the spin fluid in the comoving reference frame of fluid. This generalization of the perfect fluid with spin is known as the Weyssenhoff fluid where its dynamics was comprehensively studied by Weyssenhoff and other researchers [55]. Similar to the other alternative theories of gravity, the cosmological solutions of the EC theory possessing the spin matter source and their influence on the structure and dynamics of the universe are extensively investigated. These studies include the effects of torsion and spinning matter in a cosmological setup and its possible role to solve the singularity problems, pre-Friedmann stages of evolution, inflationary expansion, the late time accelerated expansion of the Universe, rotation of the Universe and gravitational collapse and so on [56].

In this paper, we investigate the emergent universe scenario and its thermodynamical aspects in the framework of EC theory. By considering the modifications to Friedmann equations of the EC theory, we discuss on Padmanabhan's relation and thermodynamical features of the model. This paper is organized as follows. In section II, we review the EC theory. In section III, we study the issue of emergence of spacetime in the context of this model. In section IV, thermodynamics of the Einstein-Cartan universe is investigated. Finally, in the last section, our concluding remarks are represented. Also, we consider the units of $c = 1$ with metric signature $(+,-,-,-)$ of spacetime. Also, we use the signs [] and () for denoting antisymmetric and symmetric parts, respectively.

II. THE EINSTEIN-CARTAN MODEL

The Einstein-Cartan theory can be driven using the following action

$$\mathcal{S} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\tilde{R} + \int d^4x\sqrt{-g}\mathcal{L}_M, \quad (1)$$

where $\tilde{R}$ and $\mathcal{L}_m$ are the Ricci scalar associated to the asymmetric connection $\tilde{\Gamma}$ and the Lagrangian density of matter fields coupled to the gravity, respectively.

The asymmetric connection $\tilde{\Gamma}^\mu{}_{\alpha\beta}$ can be written in terms of the Levi-Civita connection $\Gamma^\mu_{\alpha\beta}$ as

$$\tilde{\Gamma}^\mu{}_{\alpha\beta} = \Gamma^\mu{}_{\alpha\beta} + K^\mu{}_{\alpha\beta}, \quad (2)$$

where $K^\mu{}_{\alpha\beta}$, known as the "contorsion tensor", which is related to the torsion ($Q_{\alpha\beta}{}^{\mu} := \tilde{\Gamma}_{[\alpha\beta]}{}^{\mu}$) as [52]

$$K^\mu{}_{\alpha\beta} := \frac{1}{2}\left(Q^\mu{}_{\alpha\beta} - Q_\alpha{}^\mu{}_\beta - Q_\beta{}^\mu{}_\alpha\right). \quad (3)$$

Using the variation of the action with respect to the metric $g_{\mu\nu}$ and contorsion tensor $K^\mu_{\alpha\beta}$, one can find the dynamical equations of motion of the theory as follows[52]

$$\begin{aligned} G^{\mu\nu} &- \left(\tilde{\nabla}_\alpha + 2Q_{\alpha\beta}{}^{\beta}\right)\left(T^{\mu\nu\alpha} - T^{\nu\alpha\mu} + T^{\alpha\mu\nu}\right) \\ &= 8\pi G T^{\mu\nu}, \\ T^{\mu\nu\alpha} &= 8\pi G \tau^{\mu\nu\alpha}, \end{aligned} \quad (4)$$

where $G^{\mu\nu}$ and $\tilde{\nabla}_\alpha$ are the Einstein tensor and covariant derivative based on the asymmetric connection $\tilde{\Gamma}^{\mu}_{\alpha\beta}$, respectively, and $T_{\mu\nu}{}^{\alpha}$ is defined in terms of the torsion tensor $Q_{\mu\nu}{}^{\alpha}$ as

$$T_{\mu\nu}{}^{\alpha} = Q_{\mu\nu}{}^{\alpha} + \delta^{\alpha}_{\mu} Q_{\nu\beta}{}^{\beta} - \delta^{\alpha}_{\nu} Q_{\mu\beta}{}^{\beta}. \quad (5)$$

We also define

$$\begin{aligned} T^{\mu\nu} &:= \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_M}{\delta g_{\mu\nu}}, \\ \tau^{\mu\nu\alpha} &:= \frac{1}{\sqrt{-g}} \frac{\delta \mathcal{L}_M}{\delta K_{\alpha\nu\mu}}, \end{aligned} \quad (6)$$

as the energy-momentum tensor and the spin-density tensor, respectively. Combining equations (4) and (5), one can obtain the Einstein field equations with a modification in the energy-momentum as

$$G^{\mu\nu} = 8\pi G (T^{\mu\nu} + \tau^{\mu\nu}), \quad (7)$$

where

$$\begin{aligned} \tau^{\mu\nu} &= -4\tau^{\mu\alpha}{}_{[\beta}\tau^{\nu\beta}{}_{\alpha]} - 2\tau^{\mu\alpha\beta}\tau^{\nu}{}_{\alpha\beta} + \tau^{\alpha\beta\mu}\tau_{\alpha\beta}{}^{\nu} \\ &\quad + \frac{1}{2} g^{\mu\nu} \left(4\tau_{\lambda}{}^{\alpha}{}_{[\beta}\tau^{\lambda\beta}{}_{\alpha]} + \tau^{\alpha\beta\lambda}\tau_{\alpha\beta\lambda} \right), \end{aligned} \quad (8)$$

is the correction term for the energy-momentum tensor generated by the spacetime torsion [55]. If the torsion, or spin, vanishes then $\tau^{\alpha\beta}$ vanishes and the standard Einstein field equations ($G^{\mu\nu} = 8\pi G T^{\mu\nu}$) is recovered. Suppose that the Lagrangian $\mathcal{L}_M$ represents a fluid of spinning particles in the early Universe minimally coupled to the metric and the torsion of the U_4 theory. Then, for this spin fluid, the canonical spin tensor $\tau^{\mu\nu\alpha}$ can be written as follows [55]

$$\tau^{\mu\nu\alpha} = \frac{1}{2} S^{\mu\nu} u^{\alpha}, \quad (9)$$

where $S^{\mu\nu}$ is the antisymmetric spin density tensor and u^{α} is the four velocity vector of the spin fluid [55]. The Frenkel condition which arises by varying the Lagrangian of the sources [55] requires $S^{\mu\nu} u_{\nu} = 0$. This condition further restricts the torsion tensor to be traceless.

Thus, one can separate the energy-momentum tensor as

$$T^{\mu\nu} = T_P{}^{\mu\nu} + T_S{}^{\mu\nu}, \quad (10)$$

where $T_P{}^{\mu\nu}$ and $T_S{}^{\mu\nu}$are the usual perfect fluid and the intrinsic-spin fluid part as

$$\begin{aligned} T_P{}^{\mu\nu} &= (\rho + p) u^{\mu} u^{\nu} - p g^{\mu\nu}, \\ T_S{}^{\mu\nu} &= u^{(\mu} S^{\nu)\alpha} u^{\beta} u_{\alpha;\beta} + \nabla_{\alpha} (u^{(\mu} S^{\nu)\alpha}) \\ &\quad + Q_{\alpha\beta}{}^{(\mu} u^{\nu)} S^{\beta\alpha} - u^{\beta} S^{\alpha(\nu} Q^{\mu)}_{\alpha\beta} \\ &\quad - \omega^{\alpha(\mu} S^{\nu)}{}_{\alpha} + u^{(\mu} S^{\nu)\alpha} \omega_{\alpha\beta} u^{\beta}, \end{aligned} \quad (11)$$

ω being the angular velocity corresponding to the intrinsic spin, and ∇_{μ} represents the covariant derivative associated to the symmetric Levi-Civita connection $\Gamma^{\mu}_{\alpha\beta}$. Usually, it is supposed that spin density tensor $S_{\mu\nu}$ is associated with the quantum mechanical spin of microscopic particles in the early universe [56]. Thus, for such an unpolarized spinning fluid, we have $< S_{\mu\nu} >= 0$. We also define

$$< S_{\mu\nu} S^{\mu\nu} >:= 2\sigma^2, \quad (12)$$

where σ is known as spin density scalar. From the microscopical viewpoint a randomly oriented gas of fermions is the source for the spacetime torsion. However, we have to treat this issue from a macroscopic viewpoint, that means we need to perform suitable spacetime averaging. In this respect, the average of the spin density tensor vanishes, $\langle S_{\mu\nu} \rangle = 0$. But, even with vanishing this term at macroscopic level, the square of spin density tensor $S_{\mu\nu} S^{\mu\nu}$ which represents the spin-spin interaction of particles contributes to the total energy-momentum tensor. Then, we obtain

$$\begin{aligned} &< \tau^{\mu\nu} >= 4\pi G \sigma^2 u^{\mu} u^{\nu} + 2\pi G \sigma^2 g^{\mu\nu}, \\ &< T_S{}^{\mu\nu} >= -8\pi G \sigma^2 u^{\mu} u^{\nu}. \end{aligned} \quad (13)$$

Therefore, the Einstein field equations (7) read as

$$G^{\mu\nu} = 8\pi G \Theta^{\mu\nu}, \quad (14)$$

where $\Theta^{\mu\nu}$ represents the effective macroscopic limit of matter fields defined as

$$\begin{aligned} \Theta^{\mu\nu} &:= < T^{\mu\nu} > + < \tau^{\mu\nu} > \\ &= (\rho + p + \rho_s + p_s) u^{\mu} u^{\nu} - (p + p_s) g^{\mu\nu} \\ &= \left(\rho + p - 4\pi G \sigma^2 \right) u^{\mu} u^{\nu} \\ &\quad - \left(p - 2\pi G \sigma^2 \right) g^{\mu\nu}. \end{aligned} \quad (15)$$

Then, one may consider the following forms for the total energy density and pressure which support the field equations

$$\rho_{tot} = \rho + \rho_s, \qquad p_{tot} = p + p_s, \quad (16)$$

where $\rho_s = p_s = -2\pi G \sigma^2$. From this equation, it is seen that $p_s/\rho_s = 1$ and consequently the spin matter behaves as a fictitious fluid with an equation like that of the Zeldovich stiff matter with negative pressure and energy density leading to the gravitational repulsion effect. Also, it is seen that the spin-spin interaction contribution to the energy-momentum tensor is of the second order in the gravitational coupling constant $8\pi G$.

On the other hand, the combination of equations (24) and (25) gives the following generalization of the covariant energy conservation law including the spin term

$$\frac{d}{dt} (\rho - 2\pi G \sigma^2) = -3H (\rho + p - 4\pi G \sigma^2), \quad (17)$$

where we can consider the filling matter field as a unpolarized fermionic perfect fluid with the barotropic equation of state $p = \omega\rho$. By decomposition of the matter source in equation (10), we can treat the above conservation law for two non-interacting fluids. Therefore, it gives

$$\rho = \rho_0 a^{-3(1+\omega)}, \tag{18}$$

where ρ_0 is energy density at present time. On the other hand, we obtain

$$\sigma^2 = \frac{1}{2} < S_{\mu\nu}S^{\mu\nu} >, \tag{19}$$

where using the averaging procedure [57] and the equations (17) and (18) lead to

$$\sigma^2 = \frac{\hbar}{8} A_\omega^{\frac{-2}{1+\omega}} \rho^{\frac{2}{1+\omega}}, \tag{20}$$

and

$$\rho_s = 2\pi\sigma^2 = \rho_{0s} a^{-6}, \tag{21}$$

in which A_ω is a dimensional constant depending on ω and $\rho_{0s} = \frac{\hbar}{8} A_\omega^{\frac{-2}{1+\omega}} \rho_0^{\frac{2}{1+\omega}}$. Also, we note that if $< \sigma >= 0$ then $< \sigma^2 >$ is just the square of the dispersion of the spin density distribution around its average value as

$$\begin{aligned} < (\Delta\sigma)^2 > &= < (< \sigma > - \sigma)^2 > \\ &= << \sigma >< \sigma > - 2\sigma < \sigma > + \sigma^2 > \\ &= < \sigma^2 > . \end{aligned} \tag{22}$$

III. EMERGENCE OF SPACETIME IN EINSTEIN-CARTAN THEORY

We consider a homogeneous and isotropic universe described by the FLRW metric

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right). \tag{23}$$

where $k = 0, \pm 1$ represents spatial curvature of the universe. Then, using equations (14) and (15), Friedmann equations will be

$$H^2 = \frac{8\pi}{3}(\rho - 2\pi\sigma^2), \tag{24}$$

$$\dot{H} + H^2 = \frac{-4\pi}{3}(\rho + 3p - 8\pi\sigma^2). \tag{25}$$

Multiplying equation (25) by $-4\pi H^{-4}$, we get

$$-4\pi\frac{\dot{H}}{H^4} = \frac{4\pi}{H^2} + \frac{16\pi^2(\rho + 3p)}{3H^4} - \frac{128\pi^3\sigma^2}{3H^4}. \tag{26}$$

Assuming $V = 4\pi H^{-3}/3$ as the spherical volume with Hubble radius H^{-1}, namely the Hubble volume, we have

$$\frac{dV}{dt} = -4\pi\frac{\dot{H}}{H^4} = \frac{4\pi}{H^2} + \frac{16\pi^2(\rho + 3p)}{3H^4} - \frac{128\pi^3\sigma^2}{3H^4}. \tag{27}$$

On the other hand, according to Padmanabhan's idea, the number of degrees of freedom on the spherical surface of Hubble radius H^{-1} is given by [7]

$$N_{\rm sur} = \frac{A}{L_P^2} = \frac{4\pi}{L_P^2 H^2}, \tag{28}$$

where L_p is the Planck length and $A = 4\pi H^{-2}$ represents the area of the Hubble horizon. In this regard, the area law $S = A/4L_p^2$ leads to[1]

$$N_{sur} = 4S. \tag{29}$$

Also, the bulk degrees of freedom obey the equipartition law of energy

$$N_{\rm bulk} = \frac{2|E|}{k_B T}, \tag{30}$$

where E, k_B and T are the energy inside of the bulk, the Boltzmann constant and the temperature of the bulk space, respectively. In the following of the paper, we use natural unit ($k_B = c = G = L_p = 1$) for the sake of simplicity. We also consider the temperature associated with the Hubble horizon as the Hawking temperature $T = H/2\pi$, and the energy contained inside the Hubble volume in Planck units $V = 4\pi/3H^3$ as the Komar energy

$$E_{\rm Komar} = |(\rho + 3p)|V. \tag{31}$$

Based on the novel idea of Padmanabhan, the cosmic expansion which is conceptually equivalent to the emergence of space is related to the difference between the number of degrees of freedom in the holographic surface and the ones in the corresponding emerged bulk [7]. Equations (30) and (31) with Hawking temperature will give the bulk degrees of freedom as

$$N_{bulk} = -\epsilon\frac{2(\rho + 3p)V}{k_B T} \tag{32}$$

where $\epsilon = +1$ denotes $(\rho + 3p) < 0$ and $\epsilon = -1$ if $(\rho + 3p) > 0$. Based on Padmanabhan's assumption the universe can be divided as matter, respecting the strong energy condition $\rho + 3p > 0$, and dark energy,

[1] The area law for Hubble horizon $S = A/4L_p^2$ is used as the saturation of the Bekenstein limit [58].

violating the strong energy condition $\rho + 3p < 0$. Hence, the bulk degrees of freedom reads as

$$N_{bulk} = N_m - N_{de} \tag{33}$$

where the indexes "m" and "de" represent matter and dark energy, respectively. Then, we have

$$N_m - N_{de} = -\frac{16\pi^2}{3}\frac{(\rho+3p)_{tot}}{H^4}, \tag{34}$$

Then, using the equation (27), the Padmanabhan's equation can be written as follows

$$\begin{aligned}\frac{dV}{dt} &= N_{sur} - N_{bulk} - N_{spin} \\ &= N_{sur} + N_m - N_{de} - N_{spin}, \end{aligned}\tag{35}$$

where N_{sur} and N_m and N_{de} are given by equations (28) and (30) while the degrees of freedom related to the spin of matter content is given by

$$N_{spin} = \frac{16\pi V\sigma^2}{T}. \tag{36}$$

Equation (35) indicates that there are four modes of degrees of freedom for a cosmos filled by the dark energy fluid and the matter content possessing spin-spin interactions. For such a universe, other than the surface degrees of freedom, the bulk degrees of freedom and the ones related to the dark energy, there are additional degrees of freedom which lie in its spin sector. Thus, using equations (36) and (21), the spin degrees of freedom will be

$$N_{spin} = \frac{8\rho_{0s}V}{T}a^{-6}. \tag{37}$$

This relation shows that the spin degrees of freedom is vanishing at late time. This is because, the spin density and consequently its contribution to Eq.(35) is very weak at low energy limits, i.e at the late times of the Universe, in contrast to the high energy limits in the very early universe where the evolution of universe can be considerably affected by it.

On the other hand, although the Universe is not pure de Sitter, however it evolves toward an asymptotically de Sitter phase. Then, in order to reach the holographic equipartition, we demand $dV/dt \rightarrow 0$ in the equation (35) which requires $N_{sur} = N_{bulk} + N_{spin}$. To understand the feature of N_{spin}, it is better to look at equation (35) without this term. Following the discussion of Padmanabhan, one can consider that N_{bulk} includes two parts. The first one is related to the normal matter sector respecting the strong energy condition and the second one related to the dark energy sector violating the strong energy condition[7]. This provides the possibility of dividing the degrees of freedom of the bulk into two parts, one arising from the degrees of freedom of dark energy leading to acceleration and the other one arising from the degrees of freedom of normal matter leading to deceleration. Then, equation (35), without N_{spin} term, takes the form of $\frac{dV}{dt} = N_{sur} + N_m - N_{de}$. Therefore, there is no hope for reaching the holographic equipartition for a universe without a dark energy sector [4]. In reference [59], the Padmanabhan's emergent scenario is investigated in a general braneworld setup. It is found that the Padmanabhan's relation takes the form $\frac{dV}{dt} = N_{sur} - N_{bulk} - N_{extr}$ where N_{extr} is referred to the degree of freedom related to the extrinsic geometry of a four dimensional brane embedded in a higher dimensional ambient space, while N_{sur} and N_{bulk} are exactly the same as before. Moreover, it is shown that one can avoid of the term N_{de} denoting dark energy which has been previously proposed by Padmanabhan. This is because, the geometrical component N_{extr} arising from the brane extrinsic geometry, representing a geometrical dark energy [60], can play the role of N_{de}. However, in the framework of EC theory, the spin term cannot completely play the role of dark energy or cosmological constant leading to the satisfaction of holographic equipartition law, because the corresponding degrees of freedom in equation (35) are vanishing at late time, see equation (37), leading to $\frac{dV}{dt} > 0$ in the absence of dark energy. Then, unlike in [59], although the spin sector in EC framework plays an important role in the early stages of universe with a repulsive gravitational effect, at late times the cosmological constant or dark energy term proposed by Padmanabhan is required to achieve the holographic equipartition in this model. This fact is in agreement with the result obtained in [61] where the luminosity distance is implemented to test the models using the supernovae type Ia observations. There, the authors showed that although a cosmological model with a spin fluid is admissible but the cosmological constant is still required to explain the accelerating expansion rate of the universe. Consequently, the spin fluid can not be considered as an alternative to the cosmological constant description of dark energy.

IV. THERMODYNAMICS OF AN EINSTEIN-CARTAN COSMOLOGY

In recent years the connection between gravitation and thermodynamics have received much attention, see for example [1] and [3], where the first and second laws of thermodynamics are vastly investigated. Through this section, first we obtain the unified first law of thermodynamics based on the (0,0) component of Einstein field equations introduced by Hayward [62] and [64]. Then, we investigate the generalized second law of thermodynamics for the Einstein-Cartan cosmos.

A. Unified First Law of Thermodynamics

The Hubble horizon H^{-1} can be understood as an apparent horizon of the flat FLRW universe [2][65]. By calculating the derivative of $\frac{1}{\tilde{r}_A} = H$ respects to cosmic time, we easily have $-d\tilde{r}_A/\tilde{r}_A^3 = \dot{H}H dt$. Also, by implementing the modified Friedmann equation (24), we obtain

$$\frac{-1}{\tilde{r}_A^3}\frac{d\tilde{r}_A}{dt} = \dot{H}H = \frac{4\pi}{3}\frac{d}{dt}(\rho - 2\pi\sigma^2) = \frac{4\pi}{3}(\dot{\rho} + \dot{\rho}_{spin}). \tag{38}$$

One can simplify this equation using the generalized conservation law in Eq.(17) as follows

$$d\tilde{r}_A = 4\pi\tilde{r}_A^3 H\left(\rho_t + p_t\right)dt, \tag{39}$$

where ρ_t and p_t are total energy density and pressure including both of the normal and spin sectors. The Hawking-Bekenstein entropy is $S = A/4 = \pi\tilde{r}_A^2$. Therefore, using this entropy expression and Eq. (39), we get

$$\frac{1}{2\pi\tilde{r}_A}dS = d\tilde{r}_A = 4\pi\tilde{r}_A^3 H\left(\rho_t + p_t\right)dt. \tag{40}$$

On the other hand, the temperature can be obtained as [3]

$$T_H = \frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{41}$$

Then, combining the equations (40) and (41) leads to

$$T_H dS = 4\pi\tilde{r}_A^3 H\left(\rho_t + p_t\right)dt - 2\pi\tilde{r}_A^2\left(\rho_t + p_t\right)d\tilde{r}_A\ . \tag{42}$$

We also have the total intrinsic energy as

$$dE = -4\pi\tilde{r}_A^3 H\left(\rho_t + p_t\right)dt + 4\pi\tilde{r}_A^2\rho_t d\tilde{r}_A, \tag{43}$$

as well as the work density [62–64] as follows

$$W \equiv -\frac{1}{2}T^{\alpha\beta}h_{\alpha\beta} = \frac{1}{2}(\rho_t - p_t), \tag{44}$$

where $T^{\alpha\beta}$ is the effective energy-momentum tensor of the EC cosmos. Therefore, the unified first law of thermodynamics can be obtained in a straightforward manner by combining the equations (42), (43) and (44) as

$$dE = -T_H dS + WdV. \tag{45}$$

In addition, from the equation (40), we have

$$\dot{S} = -2\pi\left[\dot{H}/H^3\right] \tag{46}$$

for the surface entropy. Therefore, from equations (40) and (46) it is seen that if the null energy condition holds, i.e. $\rho_t + p_t \geq 0$, the surface entropy always increases in the expanding universe and we have $\dot{H} \leq 0$.

B. Generalized Second Law of Thermodynamics (GSL)

In order to studying the generalized second law of thermodynamics, we consider the Gibbs equation

$$T_H dS_b = d\left(\rho_t V\right) + p_t dV = Vd\rho_t + \left(\rho_t + p_t\right)dV \tag{47}$$

for total matter content inside the bulk where we used the subscript "b" to denote the entropy of inside the bulk [66–68]). By combining the definition of Hubble volume and equations (24) and (25), we obtain

$$T_H dS_b = \frac{\dot{H}}{H^4}(\dot{H} + H^2). \tag{48}$$

Then, the total entropy can be divided into two parts, the total entropy inside the bulk S_b and the part related to the surface S as $S_t \equiv S + S_b$. By combining the modified Friedmann equations (24) and (25) and (46), we have

$$T_H\frac{dS_t}{dt} = \frac{\dot{H}^2}{2H^4}. \tag{49}$$

Consequently, for an accelerating expanding universe with $H > 0$, the generalized second law of thermodynamics always holds in the framework of the Einstein-Cartan cosmology.

V. CONCLUSION

According to the Padmanabhan's emergent proposal, the accelerated expansion of the universe can be driven by the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space. The dynamical emergent equation is represented by the relation $dV/dt = N_{sur} - N_{bulk}$ where N_{sur} and N_{bulk} are the degrees of freedom assigned to the surface area and the matter-energy

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[2] The dynamical apparent horizon, i.e. $\tilde{r}_A = a(t)\,r$, can be obtained from the equation $h^{\alpha\beta}\partial_\alpha\tilde{r}\partial_\beta\tilde{r} = 0$ where $h^{\alpha\beta}$ is the non-spherical part of the FLRW metric.

[3] The Apparent horizon temperature can be calculated by $T_{\rm H} = \frac{|\kappa|}{2\pi}$ where $\kappa = \frac{1}{2\sqrt{-h}}\partial_\alpha\left(\sqrt{-h}h^{\alpha\beta}\partial_\beta\tilde{r}\right) = -\frac{1}{\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right) = -\frac{\tilde{r}_A}{2}\left(2H^2 + \dot{H}\right)$ and h is the determinant of the non-spherical part of FLRW metric.

content inside the bulk, respectively. In the present work, spin degrees of freedom in the framework of Einstein-Cartan (EC) theory are investigated. In this regard, based on the modification of Friedmann equations due to the spin-spin interactions, a correction term for the Padmanbhan's relation including the number of degrees of freedom related to this spin interactions is obtained as $\Delta V/\Delta t = N_{sur} - N_{bulk} - N_{spin}$, where N_{spin} is the corresponding degree of freedom related to the intrinsics spin of the matter content of the universe. It is shown that although the spin degrees of freedom can play an important role in the early stages of universe, but for the late times the cosmological constant or dark energy term proposed by Padmanabhan is required to achieve the holographic equipartition in this model. Also, the unified first law of thermodynamics for the Einstein-Cartan cosmos is obtained. Finally, it is shown that for an accelerating expanding universe, the generalized second law of thermodynamics always holds in the framework of this cosmological model.

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