

# Inflation by spin and torsion in Poincaré gauge theory of gravity

S. Akhshabi,\* E. Qorani,† and F. Khajenabi‡

*Department of Physics, Golestan University, P. O. Box 49138-15759, Gorgan, IRAN*

In Poincaré gauge theory of gravity, in addition to mass-energy content, spin is also a source for gravitational interactions. Although the effects of spin are negligible at low energies, they can play a crucial role at very early universe when the spin density was very high. In this paper by choosing a suitable Lagrangian for Poincaré gauge theory of gravity, and suitable energy-momentum and spin density tensors, we show that the effects of spin and torsion can lead to an inflationary phase without the need for any additional fields. No fine tuning of parameters is required in this setup. We also calculate the scalar spectral index at the end of inflation and show that it agrees with the most recent observational data.

PACS numbers: 98.80.-k, 98.80.Jk, 04.50.+h, 98.80.Es

## I. INTRODUCTION

Based on the pioneering works of Hermann Weyl, today, the standard model of particle physics is described by a gauge theory of the non-Abelian  $SU(3) \times SU(2) \times U(1)$  group. The main idea of gauge theories is that global symmetries are not compatible with field theories and must be replaced with local ones. For keeping physics invariant under local symmetries, we have to introduce some compensating fields which then will describe the fundamental interactions. This scheme applied to the symmetry group of standard model, accurately describe three of the four fundamental interactions, namely electromagnetic, weak and strong nuclear force. The same framework can also be applied to gravitational interaction. In the absence of gravity, the physical world is characterized by the theory of special relativity which has the global Poincaré transformations as its symmetry group. Localizing the Poincaré transformations and demanding that the Lagrangian remains invariant under the new local transformations, introduces two new fields which turn out to be tetrad and spin connection fields. These new fields (or similarly their field strengths, curvature and torsion tensors) contain all the information about the gravitational interaction. The resulting theory is called Poincaré gauge theory of gravity (PGT), which contains general relativity as a special case. The geometry of this theory is described by Riemann-Cartan space-time which has both curvature and torsion [1].

Throughout the paper, the Greek indices  $\mu, \nu, \dots$  run over 0, 1, 2, 3 and refer to the spacetime coordinates and Latin letters  $i, j, \dots$  run over 0, 1, 2, 3 and refer to the local Lorentz (or tangent space) coordinates. In PGT tetrad and spin connection are dynamical variables and fundamental geometric structures of the theory. The tetrad field is given as the components of a set of 4 linearly in-

dependent vectors  $e_i = e_i^\mu \partial_\mu$  which form a basis in the tangent space on every point of the manifold. The dual of this basis  $\vartheta^i = e^i_\mu dx^\mu$  are coframes. Spin connection  $\Gamma_{\mu i}^j$  which is assumed to exist as an independent field variable is related to the usual holonomic linear connection  $\Gamma_{\mu\nu}^\rho$  by the relation

$$\Gamma_{\mu i}^j = e^\nu_i e_\rho^j \Gamma_{\mu\nu}^\rho + e^\nu_i \partial_\mu e_\nu^j \quad (1)$$

The spacetime metric is not an independent dynamical variable here and is related to the tetrad through the relations

$$g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu, \quad (2)$$

$$e_i \cdot e_j = \eta_{ij} \quad (3)$$

The inverse of the tetrad is defined by  $e_i^\mu e_\mu^j = \delta_i^j$ .

Torsion and curvature are given in terms of tetrad and spin connection as

$$T_{\mu\nu}^i = 2(\partial_{[\mu} e_{\nu]}^i) + \Gamma_{[\mu|j}^i e_{|\nu]}^j, \quad (4)$$

$$R_{\mu\nu i}^j = 2(\partial_{[\mu} \Gamma_{\nu]}^j + \Gamma_{[\mu|k}^j \Gamma_{|\nu]}^k), \quad (5)$$

They also satisfy the Bianchi identities

$$\nabla_{[\mu} T_{\nu\rho]}^i \equiv R_{[\mu\nu\rho]}^i, \quad (6)$$

$$\nabla_{[\mu} R_{\nu\rho]}^{ij} \equiv 0. \quad (7)$$

General structure and physically acceptable Lagrangians of PGT have been extensively studied in the literature, see for example [2–4]. In [4] it has been argued that in the class of theories described by  $R+R^2$  Lagrangian, one can evaluate the torsion in terms of the spin tensor. The general Lagrangian of Poincaré gauge theory of gravity is a quadratic function of curvature and torsion [1]

$$L_G = c_4 R + L_T + L_R + c_0 \quad (8)$$

\*Electronic address: s.akhshabi@gu.ac.ir

†Electronic address: e.qorani@stu.gu.ac.ir

‡Electronic address: f.khajenabi@gu.ac.ir

Here in this paper, we will only include even parity terms in the Lagrangian, in this case we have

$$L_T = c_1 T_{ijk} T^{ijk} + c_2 T_{ijk} T^{jik} + c_3 T_i T^i \quad (9)$$

$$L_R = c_5 R^2 + c_6 R_{ij} R^{ij} + c_7 R_{ij} R^{ji} + c_8 R_{ijkl} R^{ijkl} \\ + c_9 R_{ijkl} R^{klij} + c_{10} (\varepsilon_{ijkl} R^{ijkl})^2 \quad (10)$$

However cosmological implications of also including the parity violating terms have been studied in [5, 6]. Field equations are obtained by variation of Lagrangian with respect to dynamical variables, tetrad field and spin connection (or equivalently curvature and torsion). They take the form [2, 7, 8]

$$\nabla_\nu H_i^{\mu\nu} - E_i^\mu = \mathcal{T}_i^\mu, \quad (11)$$

$$\nabla_\nu H_{ij}^{\mu\nu} - E_{ij}^\mu = S_{ij}^\mu, \quad (12)$$

where we have defined

$$H_i^{\mu\nu} := \frac{\partial e L_G}{\partial \partial_\nu e_\mu^i} = 2 \frac{\partial e L_G}{\partial T_{\nu\mu}^i}, \quad (13)$$

$$H_{ij}^{\mu\nu} := \frac{\partial e L_G}{\partial \partial_\nu \Gamma_\mu^{ij}} = 2 \frac{\partial e L_G}{\partial R_{\nu\mu}^{ij}}, \quad (14)$$

and

$$E_i^\mu := e^\mu_i e L_G - T_{iv}^j H_j^{\nu\mu} - R_{iv}^{jk} H_{jk}^{\nu\mu}, \quad (15)$$

$$E_{ij}^\mu := H_{[ij]}^\mu. \quad (16)$$

The source terms here are energy-momentum and spin density tensors respectively and are defined by

$$\mathcal{T}_i^\mu := \frac{\partial e L_M}{\partial e_\mu^i}, \quad (17)$$

$$S_{ij}^\mu := \frac{\partial e L_M}{\partial \Gamma_\mu^{ij}}, \quad (18)$$

where  $L_M$  is the matter Lagrangian and  $e$  is the determinant of the tetrad.

So there are two field equations in Poincaré gauge theory of gravity and their sources are energy-momentum and spin tensors. For a thorough review of gauge theories of gravity including PGT and its cosmological solutions see [7].

## II. COSMOLOGY OF POINCARÉ GAUGE THEORY OF GRAVITY

In this paper we consider a Lagrangian with the form

$$L_g = c_1 T_{ijk} T^{ijk} + c_2 T_{ijk} T^{jik} + c_3 T_i T^i + c_4 R + c_5 R^2 \quad (19)$$

To find the field equations, first we should determine the suitable form of energy-momentum, spin and torsion tensors by using large-scale homogeneity and isotropy of the

Universe (cosmological principle). In Einstein-Cartan theory, the special case of Poincaré gauge theory of gravity, Weyssenhoff fluid is usually used to describe spin fluid [9, 10]. However, it can be shown that the Weyssenhoff fluid description of the spin tensor is not compatible with cosmological principle [11–13]. In cosmological solutions in Riemann-Cartan spacetime, where connection is assumed to be independent of the metric, both the metric and connection should satisfy the killing equation [11]

$$L_\xi g_{\mu\nu} = 0 \quad ; \quad L_\xi \Gamma^\mu_{\nu\rho} = 0 \quad (20)$$

where  $L$  is the Lie derivative in the direction of  $\xi$ . By the above argument the only non-zero components of spin tensor are [13]

$$q(t) = S_{011} = S_{022} = S_{033} = -S_{i0i} \quad (21)$$

$$s(t) = S_{123} = S_{312} = S_{231} = S_{[123]} \quad (22)$$

Likewise, for torsion tensor

$$h(t) = T_{110} = T_{220} = T_{330} = -T_{i0i} \quad (23)$$

$$f(t) = T_{123} = T_{312} = T_{231} = T_{[123]} \quad (24)$$

Where due to cosmological principle,  $q$ ,  $s$ ,  $h$  and  $f$  can only depend on time. Moreover, similar to standard cosmology, we assume that the energy-momentum tensor has the form of a perfect fluid.

The dual basis (or tetrad) is assumed to be homogeneous and isotropic FRW type in the form of

$$\vartheta^0 = dt \quad , \quad \vartheta^A = a(t) \left(1 + \frac{1}{4} k r^2\right)^{-1} dx^A \quad (25)$$

$$(k = 0, \pm 1) \quad , \quad (A = 1, 2, 3)$$

Using equation (2), this gives the usual FRW metric

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (26)$$

By using the above definitions and equations (13-16) the field equations are obtained by varying the Lagrangian with respect to tetrad and spin connection

$$12c_5(M^2 - N^2) + \sigma(2hH - h^2) - (c_1 + 3c_2)f^2 - 2c_4N \\ = -\frac{1}{3}\rho \quad (27)$$

$$12c_5(M^2 - N^2) + \sigma(3h^2 - 4hH - 2\dot{h}) + (c_1 + 3c_2)f^2 \\ + 2c_4(2M + N) = -p \quad (28)$$

$$2(2c_4 - \sigma)h - 24c_5(\dot{\varphi} - 2h\varphi) = -q \quad (29)$$

$$2(c_4 - 2c_1 - 6c_2)f + 24c_5f\varphi = -s \quad (30)$$

Where we have defined

$$H = h + \frac{\dot{a}}{a} \quad ; \quad M = \dot{H} + \frac{\dot{a}}{a}H$$

$$N = H^2 + \frac{k}{a^2} - \frac{1}{4}f^2 \quad ; \quad F = \frac{1}{2}(f + \frac{\dot{a}}{a}f)$$

$$\varphi = M + N = \frac{1}{6}R \quad ; \quad \psi = fH + F = \frac{1}{12}R_{ijkl}\varepsilon^{ijkl}$$

$$\sigma = c_1 + 3c_3$$

Here a dot denotes differentiation with respect to time and  $a(t)$  is the scale factor in FRW line element. The Bianchi identities (6) and (7) have been used to simplify the above equations. The trace of tetrad field equation is

$$(c_1 + 3c_2 - \frac{1}{2}c_4)f^2 = \frac{1}{6}(\rho - 3p) + (\sigma - 2c_4)(\dot{h} + h^2 + 3\frac{\dot{a}}{a}h) - 2c_4(\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 + \frac{k}{a^2}) \quad (31)$$

Furthermore, fluid equation describing the conservation of energy-momentum and spin tensors is

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 3(sF - qM) \quad (32)$$

Here we have used the definitions and notations of reference [13] (see also [14]). In addition to the above equations, one should also specify a suitable form of the equation of state  $p = \omega\rho$ . The first two of the field equations, e.g equations (27) and (28) can be regarded as the PGT analogues to Einstein field equations of general relativity and can be obtained by variation of gravitational and matter Lagrangians with respect to the tetrad field. They have components of the energy-momentum tensor as their source. The next two field equations (29) and (30) however, are unique to PGT and are obtained by variation with respect to the spin connection field. The spin tensor components which are the source of these two equations, can be regarded as negligible in late-time cosmological dynamics due to very low density of spinning matter. However, in the early universe, when the densities were extremely high, they can play a crucial role in the dynamics. It should be noted that the spin of

particles in the early universe were most probably randomly oriented and as a result will probably cancel out when averaged over a macroscopic region. However, even if this was the case, many of the corrections in the field equations are quadratic in spin and will not vanish by averaging procedure. As a result even in macroscopic limit, one should expect some new dynamics due to effects of spin in the early universe [15, 16]. The field equations (27-30) together with the equation of state give us five equations for seven unknown functions  $q, s, h, f, \rho, p$  and the scale factor  $a(t)$ . To proceed further in solving the equations, one need to specify an exact model for describing the spin density tensor. Here one could proceed in several ways. One way is to specify exact equation of state parameter for the spin functions  $q$  and  $s$  in the form of  $q = \omega_q\rho$  and  $s = \omega_s\rho$ . This will bring down the number of independent parameters to five and allow us to solve the system of equations, however the choice of equation of state parameters in this method is quite arbitrary and we will not employ it here. Instead one could treat spin functions  $q$  and  $s$  as macroscopically averaged quantities and perform the averaging procedure to get a specific relation between  $q, s$  and  $\rho$ . One assumes a universe filled with unpolarized particles of spin  $\frac{1}{2}$ , then using the averaging procedure given in references [16, 17] we have

$$\sigma^2 = \frac{1}{2}\langle S_{ijk}S^{ijk} \rangle = \frac{1}{8}\hbar^2 A_\omega^{-2/(1+\omega)}\rho^{2/(1+\omega)} \quad (33)$$

where  $\omega$  is the equation of state of the perfect fluid and  $A_\omega$  is a dimensional constant depending on  $\omega$ . Applying this relation to the spin tensor given by (21) and (22), gives us the relation between  $q, s$  and  $\rho$ . Assuming that  $q$  and  $s$  are macroscopically averaged quantities, equation (33) gives

$$q^2 + s^2 = \frac{1}{48}\hbar^2 A_\omega^{-2/(1+\omega)}\rho^{2/(1+\omega)} \quad (34)$$

One can also follow the method described in reference [12] and describe the spinning particles quantum mechanically, i.e. with a Dirac field. In this method the macroscopic average of the spin density is obtained by the relativistic Wigner function formalism. The macroscopic spin density tensor is given in this model by

$$S^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\tau} S_\tau \quad (35)$$

where  $\epsilon^{\mu\nu\rho\tau}$  is four-dimensional Levi-Civita tensor and  $S_\tau$  is macroscopic spin vector field defined in equation (3.25) in reference [12]. Applying equation (35) to (21) and (22) and noting that the spin tensor defined in (35) is a completely antisymmetric tensor, we conclude that the spin function  $q(t)$  should be zero in this model. In the next section we solve the set of equations (27-30) together with the equation of state and relation (34) to determine seven unknown functions. To bring the number of equations to seven, we impose the condition  $M = N$ , which ensures that the tetrad field equation leads to the usual

Einstein's equation of GR in the appropriate limit [13]. Equations (31) and (32) can be used to simplify the field equations.

We are interested in the dynamics of the early universe, specially the inflation era. The analysis is done using the most general assumptions.

### III. DYNAMICS OF EARLY UNIVERSE AND INFLATIONARY EPOCH

Usually studying the role of spin in the dynamics of early universe has been done in the framework of Einstein-Cartan theory using a Weyssenhoff fluid to describe the spin density tensor [16, 18–21]. It has been shown that by using a suitable spin fluid in Einestein-Cartan theory, we get to an inflationary epoch, however in this framework the fine tuning of parameters is required [16]. Also inflationary solutions in PGT has been studied in [22] but with the assumption of vanishing spin tensor. Here we show that in PGT using the Lagrangian in the form of (11), the effects of spin and torsion cause the Universe to enter an inflationary epoch with a wide range of parameters. Also, using the assumption of vanishing spin tensor at late times, it has been shown that Poincaré gauge theory of gravity is also able to explain present time accelerated expansion of the universe [5, 8, 23–25].

#### A. Numerical results

To find scale factor and other parameters we set

$$k = 0, \quad c_1 = 0.025, \quad c_2 = 0.05,$$

$$c_3 = -0.1 \quad c_4 = 0.5, \quad c_5 = 10^{-4}$$

This choice corresponds to the so-called 'scalar torsion mode' which has spin and parity  $0^+$  [26]. For describing density and pressure of perfect fluid we use the equation of state of radiation-dominated Universe  $p = \frac{1}{3}\rho$  in the following numerical demonstrations, however any physically acceptable equation of state parameter leads to the same cosmological dynamics at the very early universe. We also use equation (33-34) with  $A_\omega = 1$  to describe macroscopically averaged spin tensor parameters. The condition  $M = N$  leads to the following equation

$$5.25 \dot{h} + 4.33 \frac{\ddot{a}}{a} + 2.33 \frac{\dot{a}^2}{a^2} + 11.75 h \frac{\dot{a}}{a} + 3.25 h^2 = 0 \quad (36)$$

Then, the system of equations (27-30) can be manipulated to give the following relation between the scale factor  $a(t)$  and torsion function  $h(t)$

$$h = \frac{1}{2} \frac{\ddot{a}a + \dot{a}^2}{a\dot{a}} \quad (37)$$

For numerical analysis of equations, we have used two sets of initial values

$$a(0) = 0.001, \quad \dot{a}(0) = 1, \quad h(0) = 1$$

$$a(0) = 0.001, \quad \dot{a}(0) = 1, \quad h(0) = -1$$

However as can be seen from the figures, both sets of initial values will yield almost similar results. Figures (1) shows the qualitative evolution of the scale factor  $a(t)$  and density function  $\rho(t)$  versus the cosmic time. As we see in figure 1, the effects of spin and torsion causes the universe to enter an inflationary epoch where the scale factor grows quasi-exponentially with a positive acceleration. There is no need for any additional scalar (or any other) field in this setup. The question of graceful exit from inflation should be considered in subsequent studies, however figures (2) show that two torsion functions  $h(t)$  and  $f(t)$  and also two spin tensor functions  $q(t)$  and  $s(t)$  all approach zero at late times. Consequently, the effects of spin tensor in the large-scale dynamics of the universe will only be relevant at the very early times. However, one should note that due to the fundamental difference between the field equations of PGT and their general relativistic counterparts, torsion can also effects the late-time dynamics of the universe, as has been shown in [5, 8, 23, 24]

One of the most important tests for any inflationary model is its ability to generate nearly scale-invariant spectrum of scalar perturbations at the end of inflation. The scalar spectral index is given by [27]

$$n_s(k) - 1 \equiv \frac{d \ln A_s^2}{d \ln k} \quad (38)$$

where  $A_s$  is the amplitude of scalar perturbations at the time of horizon crossing

$$A_s(k) \equiv \frac{4}{5} \frac{H^2}{m_{pl}^2 |H'|} \Big|_{k=aH} \quad (39)$$

and

$$k = aH \exp[-N] \quad (40)$$

$N$  is the number of e-folding which can be calculated by

$$N \equiv \int H(t) dt \quad (41)$$

Which we have obtained  $N = 35.11$  at the end of the inflation.  $A_s(k)$  and  $n(k)$  are shown in figure (7) and (8). As can be seen in figure (3) the scalar spectral index approaches unity at the end of inflation which is compatible with observation that predicts nearly scale-invariant but slightly red-tilted spectrum [28].

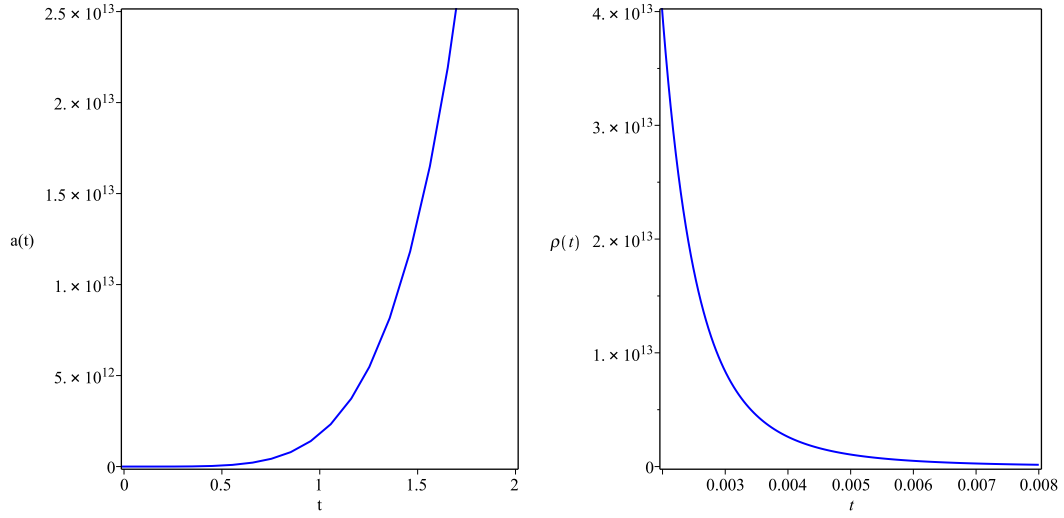


FIG. 1: Evolution of scale factor  $a(t)$  and the matter density  $\rho(t)$  as function of time.

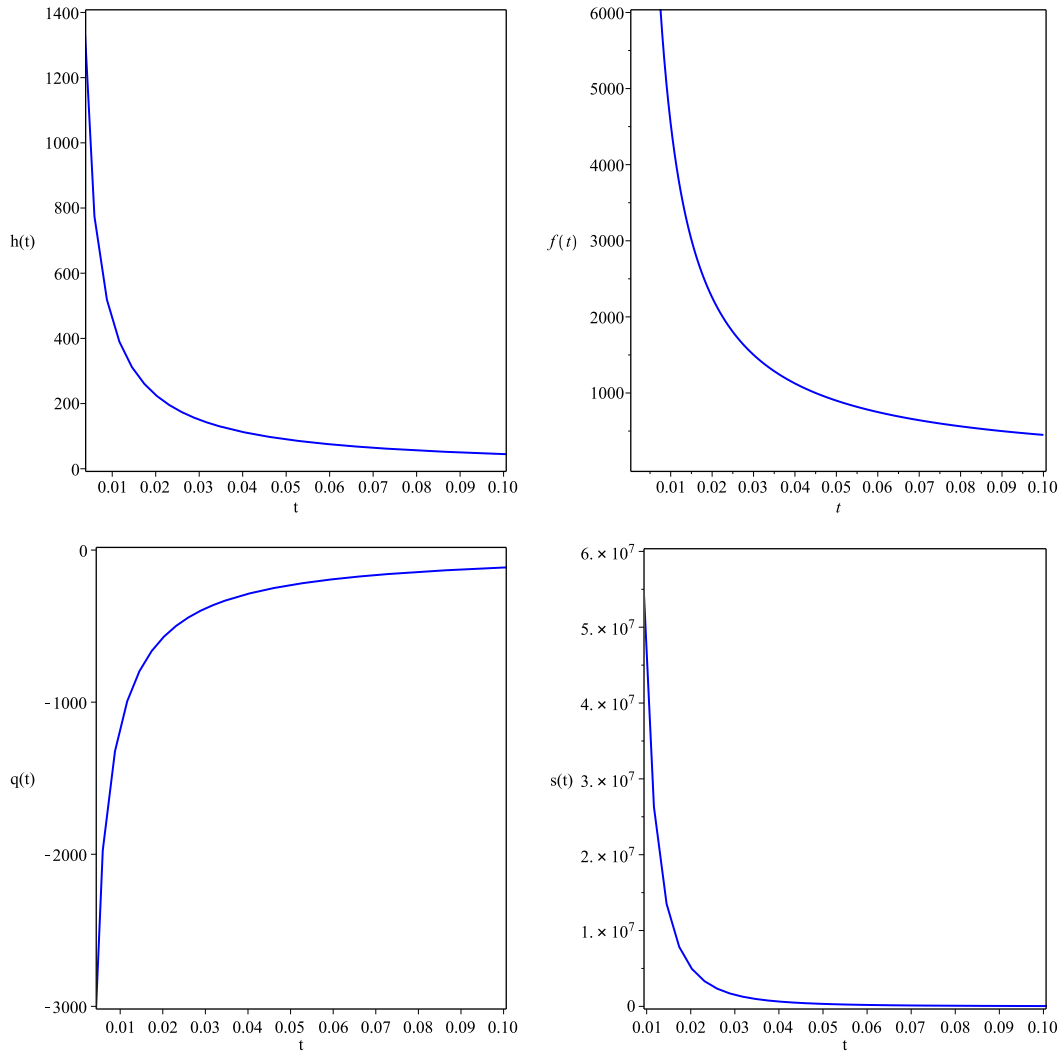


FIG. 2: Evolution of torsion functions  $h(t)$  and  $f(t)$  and spin density functions  $q(t)$  and  $s(t)$  as functions of time. They all approach zero at late times as expected.

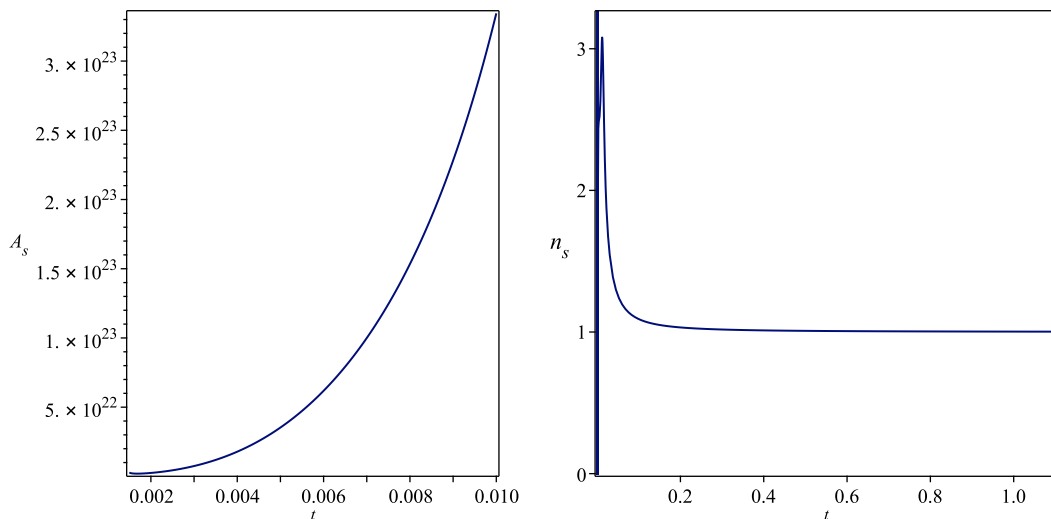


FIG. 3: The amplitude of scalar perturbations  $A_s$  and The scalar spectral index  $n_s$  as function of time.  $n_s$  is almost 1 at the time  $t=1.0001$  which is the end of inflation as calculated by equation (25).

#### IV. CONCLUSION

Recently a cosmological model based on the Poincaré gauge theory with quadratic Lagrangian has been constructed [8]. This model with suitable Lagrangian parameters, can explain the accelerated expansion of the universe without the need for 'dark energy'. By extending the work to the early universe, with a non-zero spin density tensor, we numerically solved the set of field equations of PGT, and found the evolution of geometric and matter parameters at the time of inflation. High density of particles with spin can be considered as a source of torsion in early universe. Although, because of random orientation of spin of particles, the expectation value of spin tensor may be zero, nevertheless the expectation value of the square of the spin tensor may not be so. As a result, the effects of torsion play a crucial role in high densities. In this paper by choosing a suitable Lagrangian for Poincaré gauge theory of gravity and suitable spin and torsion tensors, we have shown that the effects of spin

and torsion can lead to an inflationary epoch with exponential expansion in early universe without the need for any additional fields. One of the most fundamental test of any cosmological model is the issue of cosmological perturbations. The scalar spectral index in this framework also approaches the Harrison-Zel'dovich spectrum which is perfectly consistent with recent observations. The number e-foldings at the end of the inflation is also of the order of magnitude required to solve the problems with standard big bang cosmology. Here in PGT, in addition to perturbation modes originating from a perturbed coframe, there are also independent connection modes. By studying the dynamics and spectrum of extra modes, we can test the validity of the model against current observational data and compare the results to the standard models. For example, the extra modes generating from a propagating spin connection, may result in some identifiable signature in the anisotropy and non-Gaussianity spectrum of the cosmic microwave background radiation.

- 
- [1] M. Blagojevic, *Gravitation and Gauge Symmetries* (IoP Publishing, Bristol, 2002)
- [2] K. Hayashi and T. Shirafuji, *Prog. Theor. Phys.* 64, 866 (1980); 64, 883 (1980); 64, 1435 (1980); 64, 2222 (1980); 65, 525 (1981); 66, 318 (1981); 66, 2258 (1981).
- [3] E. Sezgin and P. van Nieuwenhuizen, *Phys. Rev. D* 21, 3269 (1980)
- [4] R. Rauch, *Phys. Rev. D* 26, 931 (1982)
- [5] P. Baekler, F. W. Hehl, and J. M. Nester, *Phys. Rev. D* 83, 024001 (2011)
- [6] F. -H. Ho and J. M. Nester, *J. Phys. Conf. Ser.* 330, 012005 (2011)
- [7] M. Blagojevic and F. W. Hehl (eds.), *Gauge Theories of Gravitation. A Reader with Commentaries* (Imperial College Press, London, 2013)
- [8] K.-F. Shie, J. M. Nester, and H.-J. Yo, *Phys. Rev. D* 78, 023522 (2008)
- [9] J. Weyssenhoff, and A. Raabe, *Acta Phys. Pol.* 9,7 (1947)
- [10] Y N Obukhov and V A Korotky, *ClassQuantum Grav.* 4, 16331657 (1987)
- [11] M. Tsamparlis, *Phys. Lett. A* 75, 272 (1979)
- [12] G.G.A. Bauerle and CHR.J. Hanveld, *Physica* 121A (1983) 541-551
- [13] H. Goenner and F. Muller-Hoissen, *Class. Quantum Grav.* 1, 651672 (1984)
- [14] V. I Kudin, A. V Minkevich and F. I Fedorov, *Vesci Akad.*

- Nauk BSSR. Ser. Fiz.-Mat. Nauk 4 59 ( 1981)
- [15] F. W. Hehl, P. von der Heyde, and G. D. Kerlick Phys. Rev. D 10, 1066 (1974)
- [16] M. Gasperini, Phys. Rev. Lett. 56, 2873-2876 (1986)
- [17] I. S. Nurgaliev and V. N. Ponomarev, Phys. Lett. 1308, 378 (1983)
- [18] N. J. Poplawski, Phys. Lett. B 694:181-185 (2010)
- [19] S. Capozziello, S. Carloni, and A. Troisi, Recent Res. Dev. Astron. Astrophys. 1, 625 (2003)
- [20] M. Demianski, R. de Ritis, G. Platania, P. Scudellaro, and C. Stornaiolo, Phys. Rev. D 35, 11811184 (1987)
- [21] C. G. Boehmer, Acta Phys. Polon. B36 2841-2845, (2005)
- [22] A.V Minkevich, Class. Quantum Grav. 23, 4237 (2006) ; Mod. Phys. Lett. A26, 259(2011); Int. J. Mod. Phys. A31 1641011, (2016)
- [23] H. Chen, F.-H. Ho, J. M. Nester, C.-H. Wang, and H.-J. Yo, J. Cosmology Astropart. Phys. 10, 027 (2009)
- [24] X.-c. Ao, X.-z. Li and P. Xi, Phys. Lett. B 694, 186-190 (2010)
- [25] A.V Minkevich, Class. Quantum Grav. 24, 5835 (2007); Phys. Lett. B678, 423 (2009)
- [26] H.J. Yo and J.M. Nester, Int. J. Mod. Phys. D8, 459 (1999)
- [27] James E. Lidsey, Andrew R. Liddle, Edward W. Kolb, Edmund J. Copeland, Tiago Barreiro, Mark Abney, Rev.Mod.Phys.69:373-410 (1997)
- [28] P. Ade et al and Planck Collaboration, Astronomy and Astrophysics 594, A13 (2016)