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Non-linear diffusion of cosmic rays

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Abstract

The propagation of cosmic rays in the interstellar medium after their release from the sources – supernova remnants – can be attended by the development of streaming instability. The instability creates MHD turbulence that changes the conditions of particle transport and leads to a non-linear diffusion of cosmic rays. We present a self-similar solution of the equation of non-linear diffusion for particles ejected from a SNR and discuss how obtained results may change the physical picture of cosmic ray propagation in the Galaxy.

Keywords: Cosmic rays; Supernova remnants; Interstellar turbulence; Streaming instability

1. Introduction

The cosmic rays are relativistic charged particles and they not always can be considered as test particles moving in given regular and random interstellar magnetic fields (Ginzburg, 1966). In particular, the cosmic ray streaming instability may amplify the MHD waves in background plasma and affect the level of interstellar turbulence that controls particle scattering on pitch-angle and determines the cosmic ray transport. The self-consistent approach is needed when this collective effect is essential (Skilling, 1971). This is the situation in the studies of diffusive shock acceleration in SNRs. The acceleration is accompanied by strong streaming instability of cosmic rays at the shock precursor region (Bell, 1978, 2005; McKenzie and Völk, 1982; Lucek and Bell, 2000). The general leakage of cosmic rays from the Galaxy can also occur with a development of streaming instability (Wentzel, 1969; Kulsrud and Pearce, 1969; Kulsrud and Cesarsky, 1971; Holmes, 1975; Ptuskin et al., 1997; Farmer and Goldreich, 2004). In the present work, we consider the intermediate stage of cosmic ray propagation when the cloud of energetic particles accelerated after a SN burst has left the source but not yet completely mixed with the background cosmic rays produced by other SNR. The point that is essential for our consideration is a relatively large gradient of cosmic ray density within a few hundred parsecs of the source that leads to the development of streaming instability.

2. Non-linear diffusion

The propagation of cosmic rays with energies 10^9–10^{15} eV is usually described in the diffusion approximation; see Berezinskii et al. (1990). The simplified equation for cosmic ray diffusion coefficient along the average magnetic field at the resonant scattering of particles by random magnetic field is $D = \frac{4vr_g}{3\pi U(k_r)}$, where $v \approx c$, r_g, and $k_r = 1/r_g$ are the particle velocity, the particle Larmor radius, and the resonant wave number respectively, see Bell (1978). The function $U(k)$ characterizes the spectral distribution of random magnetic field: $(\delta B)^2 = B^2 \int \frac{dk}{k} U(k)$, where the average magnetic field $B = 5 \times 10^{-6}$ G and $B \gg \delta B$.

In the problem considered in the present work, the diffusion flux of cosmic rays amplifies the amplitude of random resonant MHD waves with the growth rate $\Gamma_{cr}(k_r) = \frac{16\pi^2 V_a v p^4}{3B^2 U(k_r)} \nabla f$ (e.g. Lagage and Cesarsky, 1983), where it is assumed that the instability threshold is significantly exceeded, V_a is the Alfven velocity in the interstellar medium, and $f(t, \mathbf{r}, p)$ is the particle distribution function on

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momentum p (the total density of cosmic ray particles is $4\pi \int dpp^2 f$).

The wave amplification is balanced by the wave dissipation with some decrement Γ_{dis}. The selection of the appropriate mechanism of dissipation that determines Γ_{dis} is ambiguous because of the diversity and complexity of non-linear processes of wave interactions in the magnetized space plasma. To be specific, we assume the Kolmogorov type of non-linear dissipation for the MHD turbulence and use a simple equation for the efficiency of non-linear dissipation $\Gamma_{\text{K}}(k) = (2C_{\text{K}})^{-3/2} kV_{\text{a}}\sqrt{U(k)}$, where $C_{\text{K}} \approx 3.6$, see Ptuskin and Zirakashvili (2003).

The steady state condition $\Gamma_{\text{cr}} = \Gamma_{\text{dis}}$ allows to obtain the self-consistent expression for diffusion coefficient and to write down the corresponding non-linear equation of diffusion in the following form:

$$\frac{\partial f}{\partial t} - \nabla D \nabla f = 0, \quad D = \frac{\kappa}{|\nabla f|^{2/3}}, \tag{1}$$

where $\kappa = \frac{(vr_g)^{1/3} B^{4/3}}{2^{5/3} 3^{1/3} \pi^{7/3} C_{\text{K}} p^{8/3}}$. Here we ignore the contribution of background interstellar turbulence produced by external sources and neglect the possible convective transport of cosmic rays.

Let us analyze a one-dimensional propagation of energetic particles ejected in a tube of magnetic field lines of cross-section S from a SNR at the moment $t = 0$. The magnetized particles move along magnetic field lines symmetrically in both directions from the source located at $x = 0$ (the coordinate x is directed along the regular magnetic field). We consider the static magnetic field which is constant in absolute magnitude and may have a considerable random component. In this case the total cross-section S of a magnetic flux tube remains constant ($S = \text{const}$) along its length x as a consequence of the equation $\nabla \mathbf{B} = 0$ but the tube geometry evolves. The tube experiences continuous shape evolution stretching in one direction and contracting in other, see Rechester and Rosenbluth (1978). The wandering of initially close magnetic field lines leads to their divergence and eventually destroys the tube of correlated field lines over distance of the order of $X_{\text{c}} \sim L/A^2 \sim 200$ pc, where $L \sim 100$ pc is the main scale of the interstellar random field and $A^2 \sim 0.5$ is the squared ratio of the random to the total magnetic field strength (here the extended spectrum of random field at scales smaller then L was assumed; the value of X_{c} increases if the random field is concentrated solely at the main scale L). Thus the concept of magnetic flux tube can be used up to scales of a few hundred parsecs.

The conserved number of particles in a flux tube at a given particle momentum is equal to ηS, where $\eta = 2\int_0^\infty dxf$. We search for a self-similar solution of non-linear Eq. (1) in the form $f = \frac{\eta}{\kappa^{3/2} t^{3/2}} \Phi(z)$, where the variable $z = \frac{\eta x}{\kappa^{3/2} t^{3/2}}$ is introduced, and the function $\Phi(z)$ can be determined by solving Eq. (1). As a result, we obtain the following solution for the cosmic ray distribution function in the magnetic flux tube:

$$f(p,x,t) = \frac{1}{\sqrt{\frac{\Gamma^8(1/4)}{3^3 \Gamma^4(1/2)} \cdot \frac{\kappa^3 t^3}{\eta^4} + \frac{3^3}{2^4} \cdot \frac{x^4}{\kappa^3 t^3}}}, \tag{2}$$

where $\Gamma(s) = \int_0^\infty dt e^{-t} t^{s-1}$ is the Euler's gamma function.

The solution (2) shows that an observer at the position $x = x_*$ registers the maximum of cosmic ray intensity at the moment $t = t_*$ determined by the equation $x_* = \frac{2\Gamma^2(1/4)}{3^{3/2}\Gamma(1/2)} \cdot \frac{\kappa^{3/2}}{\eta} t_*^{3/2}$. This law of particle propagation differs from the case of an ordinary diffusion with constant diffusion coefficient when $x_* \sim t_*^2$.

Using Eq. (2), one can find the diffusion coefficient:

$$D = \kappa \left| \frac{\partial f}{\partial x} \right|^{-2/3} = \frac{2^2 \Gamma^8(1/4) \cdot \kappa^6 t^5}{3^5 \Gamma^4(1/2) \cdot \eta^4 x^2} + \frac{3x^2}{4t}. \tag{3}$$

The expression on the right-hand side of Eq. (3) reaches its' minimum value $D_{**} = \frac{3\Gamma^{4/3}(1/4) \cdot \kappa x_{**}^{4/3}}{2^{1/3} 5^{5/6} \Gamma^{2/3}(1/2) \cdot \eta^{2/3}}$ at $x_{**} = \frac{2 \cdot 5^{1/4} \Gamma^2(1/4) \cdot \kappa^{3/2}}{3^{3/2} \Gamma(1/2) \cdot \eta} t_{**}^{3/2}$; the last relation between x_{**} and t_{**} differs little from the relation between x_* and t_*.

The solutions (2) and (3) obtained above for f and D were derived for the burst of a single source (SNR) in an infinite magnetic flux tube. With the multiple random SNR bursts, there exists some region limited by the characteristic size Δx and the characteristic time Δt around every source where this source dominates i.e. where it determines the value of cosmic ray gradient $\frac{\partial f}{\partial x}$. Let us assume that the SN bursts occurs in the galactic disc of the total thickness $2H (H = 150\text{pc}, H \gg \sqrt{S})$ with the frequency ν_{sn} per unit disk area (we suppose that $\nu_{\text{sn}} = 50$ (kpc^2 Myr)$^{-1}$ in the Solar System vicinity). The rough estimate gives the relation $\nu_{\text{sn}} H^{-1} S \Delta x \Delta t = 1$ (see Lagutin and Nikulin, 1995 for mathematical analysis of the problem of determination of Δx and Δt in the ensemble of random SN bursts).

The effective diffusion coefficient D_{ef} created in the galactic disk by the streaming instability of cosmic rays released from many randomly distributed SNRs can be estimated as $D_{\text{ef}} \approx D_{**}(x_{**} = \Delta x, t_{**} = \Delta t)$ that results in the evaluation

$$D_{\text{ef}} \approx \left(\frac{6\Gamma^{12}(1/4)}{5^{7/2} \Gamma^6(1/2)} \frac{H^4 \kappa^9}{\nu_{\text{sn}}^4 S^4 \eta^6} \right)^{1/5}. \tag{4}$$

The one-dimensional diffusion at the scales $\leqslant \Delta x$ was assumed in the derivation of Eq. (4) that requires the condition $\Delta x \leqslant X_{\text{c}}$ to be fulfilled. If the opposite condition $\Delta x > X_{\text{c}}$ is valid, the one-dimensional diffusion switches to the three-dimensional one at distance X_{c} that leads to the rapid dilution of cosmic rays expelled from a given SNR in the background cosmic rays produced by other numerous SNR. The generated effective diffusion coefficient in the galactic disk is estimated in this case as $D_{\text{ef,c}} \approx D(x = X_{\text{c}}, t = \Delta t_{\text{c}})$ where Δt_{c} is determined by the equation $\nu_{\text{sn}} H^{-1} S X_{\text{c}} \Delta t_{\text{c}} = 1$. It gives then

$$D_{\text{ef,c}} \approx \frac{4\Gamma^8(1/4)}{3^5 \Gamma^4(1/2)} \frac{H^5 \kappa^6}{\nu_{\text{sn}}^5 S^5 X_{\text{c}}^7 \eta^4}. \tag{5}$$

We present now the corresponding numerical estimates. The value of $\eta(p)$ is related to the total energy $W = 10^{50} W_{50}$ erg of accelerated relativistic particles at the source (mainly protons with energies from 10^9 to $\sim 10^{15}$ eV) as $\eta(p) = \frac{(\gamma_s - 2) \cdot W}{4\pi p^2 S c p_0^2} \left(\frac{p}{p_0}\right)^{-\gamma_s}$, where, $p_0 = mc$, m is the proton mass, $\gamma_s > 2$ is the exponent of the power-law source spectrum we assume that $\gamma_s = 2.1$, the particle velocity $v = c$, and the typical value $W_{50} \sim 1$–3.

According to the theory of diffusive shock acceleration, the cosmic rays are accelerated up to some maximum momentum $p_{\max}(t)$ at a given SNR age t (and the corresponding shock radius $R(t)$) and the particles with $p > p_{\max}(t)$ cannot be confined in the source region even if they were accelerated earlier. The particles with $p \sim p_{\max}(t)$ run-away from a decelerating SNR envelope. The efficient acceleration ceases at $t = 10^4 \ldots 10^5$ yr and the particles, which are confined inside the SNR by this time, came out into the interstellar medium, see Berezhko and Krymsky (1988), Ptuskin and Zirakashvili (2005). Thus in principle the cross-section area $S = \pi R^2$ in our equations is a function of momentum p. We assume that the momentum of runaway particles is decreasing with SNR radius as $R = 30(p/p_0)^{-\mu}$ pc. The exponent $\mu > 0$ is not well determined both observationally and theoretically. The theory of strong MHD turbulence in the shock precursor based on the Bell instability (Bell, 2004) leads to $\mu \sim 0.5$ at energies $> 10^{13}$ eV and this regime is probably changing to $\mu \sim 0.1$ at smaller energies (Ptuskin and Zirakashvili, 2005). Below we consider cosmic rays with energies less than 10^{12} eV and take the value $\mu = 0.1$.

Using Eq. (2), the distribution of cosmic ray density defined as $n_{cr} = 4\pi p^3 f$ can be presented as

$$n_{cr} = \frac{10^{-10}\ \mathrm{cm}^{-3}}{\sqrt{0.5 \cdot W_{50}^{-4} (p/p_0)^{2.6} (t/1\ \mathrm{Myr})^3 + 10^3 (p/p_0)(x/1\ \mathrm{kpc})^4 (t/1\ \mathrm{Myr})^{-3}}}. \tag{6}$$

The self-consistently calculated diffusion coefficient (3) is equal to

$$D = (1 \cdot 10^{26} \cdot W_{50}^{-4} (p/p_0)^{1.6} (x/1\ \mathrm{kpc})^{-2} (t/1\ \mathrm{Myr})^5 + 2 \cdot 10^{29} \cdot (x/1\ \mathrm{kpc})^2 (t/1\ \mathrm{Myr})^{-1})\mathrm{cm}^2/\mathrm{s}. \tag{7}$$

The assumption of weak turbulence $B > \delta B$ used in the equation for diffusion coefficient is fulfilled at $x > 0.1$ pc.

The time dependence of cosmic ray distribution in the magnetic flux tube is characterized by the relation

$$x_* \approx 150 \cdot W_{50}^{-1} (p/p_0)^{0.4} (t_*/1\ \mathrm{Myr})^{3/2} \mathrm{pc}. \tag{8}$$

Some applications of obtained results are discussed below.

3. Discussion

In the approximation of one-dimensional diffusion an individual SNR has pronounced effect on the cosmic ray density in the region of the size $\Delta x \approx 570 \cdot W_{50}^{-2/5} (p/p_0)^{0.28}$ pc during the time $\Delta t \approx 1.9 \cdot W_{50}^{2/5} (p/p_0)^{-0.08}$ Myr after the injection of accelerated cosmic rays into the interstellar medium. These values are substituted by $X_c \sim L/A^2 \sim 200$ pc and $\Delta t_c \approx 5.4 \cdot (p/p_0)^{0.2}$ Myr if Δx exceeds X_c.

The corresponding effective diffusion coefficient for protons and ions of charge Z is estimated in these two cases with the use of Eqs. (4) and (5) correspondently:

$$D_{ef} \approx 5 \cdot 10^{28} \cdot W_{50}^{-6/5} (p/Zp_0)^{0.64}\ \mathrm{cm}^2/\mathrm{s}, \tag{9}$$

and

$$D_{ef,c} \approx 1 \cdot 10^{31} \cdot W_{50}^{-4} (p/Zp_0)^{2.6}\ \mathrm{cm}^2/\mathrm{s}. \tag{10}$$

Before comparing the calculated values (9) and (10) with the average empirical diffusion coefficient of cosmic rays in the Galaxy one should take into account the isotropisation of local diffusion under the effect of large-scale random magnetic field (Jokipii and Parker, 1969). The strongly magnetized cosmic ray particles move along the magnetic field lines and experience a one-dimensional diffusion with some diffusion coefficient D inside the domains of the size X_c where the correlated tube of magnetic field lines exists. The locally one-dimensional diffusion becomes the three-dimensional one at the scales larger than X_c, i.e. at a few hundred parsecs. The resulting large scale diffusion coefficient perpendicular to the average magnetic field can be estimated as $D_\perp \sim A^4 D \sim 0.3D$, see Berezinskii et al. (1990) and Casse et al. (2001) for more extended discussion and references. This perpendicular diffusion coefficient mainly determines the leakage of cosmic rays from the galactic disk across the regular spiral magnetic field. The found in this way from the empirical plain diffusion model (Ptuskin et al., 2006) diffusion coefficient along the magnetic field lines is

$$D_{emp} \approx 4 \cdot 10^{28} (p/Zp_0)^{0.6}\ \mathrm{cm}^2/\mathrm{s} \tag{11}$$

at magnetic rigidities $pc/Z = 3 \ldots 100$ GV.

The value of effective diffusion coefficient (9) is close to the empirical value (11) at $W_{50} = 1$. The effective diffusion coefficient (10) has much stronger momentum dependence than (11) but the value of $D_{ef,c}$ at GeV energies is close to the empirical value (11) at $W_{50} = 3$. It is worth to emphasize that Eq. (9) most probably underestimates the effective diffusion coefficient created by the streaming instability of cosmic rays since it assumes that the individual magnetic flux tube is not destroyed up to the distance of a few hundred parsecs from the source. On the other hand, Eq. (10) strongly overestimates the value of the effective diffusion coefficient since the generation of turbulence by escaping particles beyond two hundred parsecs from the source is disregarded here. We plan more comprehensive consideration of the problem in a future work.

Even if the approximation used to derive Eqs. (5) and (10) is appropriate and the level of self-consistently generated MHD turbulence is not essential for the global diffusion of cosmic rays in the Galaxy, it is essential for the cosmic ray propagation in the immediate vicinity of the source. Eq. (8) allows estimating the matter thickness X_m (in units g/cm^2) traversed while particles leave the source region in this case as $X_m \approx 2.6 \cdot W_{50}^{2/3} n_\mathrm{ism} (p/Zp_0)^{-0.27}$ g/cm^2, where n_ism is the density of interstellar hydrogen. It may change the standard interpretation of the data on secondary nuclei in cosmic rays where only the matter thickness gained by cosmic rays after their exit from the source regions is taken into account (see also discussion by Ptuskin and Soutoul (1990) and Biermann et al. (2001); recall that the total matter thickness traversed by galactic cosmic rays is $\sim 25(p/Zp_0)^{-0.6}$ g/cm^2 at $pc/Z > 4$ GV).

4. Conclusion

The generation of MHD turbulence by relativistic particles injected in the interstellar medium from a SNR leads to the non-linear diffusion with the diffusion coefficient which depends on the gradient of particle density. The self-similar solution which describes the non-stationary evolution of the cloud of relativistic particles confined in the magnetic field flux tube after a SN burst was obtained in the present work. Compared to the ordinary diffusion with constant preset diffusion coefficient the considered non-linear transport is characterized by a relatively slow expansion of the particle distribution around the source.

This mechanism of turbulence production by multiple supernovae can be important for cosmic ray transport in the Galaxy. The question is of particular interest in view of the known problem with maintenance of needed spectrum of interstellar turbulence by external sources, e.g. Yan and Lazarian (2002).

It must be emphasized that the form of non-linear diffusion equation essentially depends on the dissipation mechanism that limits the amplitude of exciting waves. The results presented in the present paper refer to the case of non-linear wave dissipation of Kolmogorov type when $\Gamma_\mathrm{dis} = \Gamma_\mathrm{K}(k) \sim kV_\mathrm{a}\sqrt{U(k)}$. Another possible dissipation mechanism typical of the theory of weak MHD turbulence is characterized by the dissipation rate $\Gamma_\mathrm{dis} \sim kV_\mathrm{a}U(k)$, e.g. Zirakashvili (2000). The use of the last equation leads to the relation $D \sim |\nabla f|^{-1/2}$ which is different from (1). The character of non-linear propagation of cosmic rays is also different in this case and it is characterized by the relation $x_* \sim t_*$.

It is essential for the application of obtained results that the cosmic-ray streaming instability is heavily suppressed in the regions of neutral interstellar gas where the wave dissipation is due to the ion-neutral collisions in thermal background plasma (Kulsrud and Cesarsky, 1971).

The allied mechanism of generation of interstellar turbulence (Fedorenko and Ostryakov, 1987) is worth mentioning in addition to our present consideration. The accelerating cosmic rays produce turbulence inside an expanding SNR envelope. This turbulence evolves till the arriving of a "fresh" shock wave from a next nearby SN burst that supplies a new portion of energy and thus maintains some level of turbulence in the galactic disk.

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