

Precision Gravity Tests and the Einstein Equivalence Principle

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Abstract

General Relativity is today the best theory of gravity addressing a wide range of phenomena. Our understanding of physical laws, from cosmology to local scales, cannot be properly formulated without taking into account its concepts, procedures and formalism. It is based on one of the most fundamental principles of Nature, the Equivalence Principle, which represents the core of the Einstein theory of gravity describing, under the same standard, the metric and geodesic structure of the spacetime. The confirmation of its validity at different scales and in different contexts represents one of the main challenges of modern physics both from the theoretical and the experimental points of view.

A major issue related to this principle is the fact that we actually do not know if it is valid or not at quantum level. We are simply assuming its validity at fundamental scales. This question is crucial in any self-consistent theory of gravity.

Furthermore, recent progress on relativistic theories of gravity, including deviations from General Relativity at various scales, such as extensions and alternatives to the Einstein theory, have to take into account, besides the Equivalence Principle, new issues like Dark Matter and Dark Energy, as well as the validity of fundamental principles like local Lorentz and position invariance. The general trend is that high precision experiments are conceived and realized to test both Einstein's theory and its alternatives at fundamental level using established and novel methods. For example, experiments based on quantum sensors (atomic clocks, accelerometers, gyroscopes,

gravimeters, etc.) allow to set stringent constraints on well established symmetry laws (e.g. CPT and Lorentz invariance), on the physics beyond the Standard Model of particles and interactions, and on General Relativity and its possible extensions.

In this review, we discuss precision tests of gravity in General Relativity and alternative theories and their relation with the Equivalence Principle. In the first part, we discuss the Einstein Equivalence Principle according to its weak and strong formulation. We recall some basic topics of General Relativity and the necessity of its extension. Some models of modified gravity are presented in some details. This provides us the ground for discussing the Equivalence Principle also in the framework of extended and alternative theories of gravity. In particular, we focus on the possibility to violate the Equivalence Principle at finite temperature, both in the frameworks of General Relativity and of modified gravity. Equivalence Principle violations in the Standard Model Extension are also discussed. The second part of the paper is devoted to the experimental tests of the Equivalence Principle in its weak formulation. We present the results and methods used in high-precision experiments, and discuss the potential and prospects for future experimental tests.

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1 Introduction

General Relativity (GR) relies on the assumption that space and time are entangled into a unique structure, i.e. the spacetime. It is assigned on a pseudo-Riemannian manifold endowed with a Lorenzian signature. Dynamics has to reproduce, in the absence of a gravitational field, the Minkowski spacetime.

GR, as an extension of classical mechanics, has to match some minimal requirements to be considered a self-consistent theory of gravitation: it has to reproduce results of Newton's physics in the weak-energy limit, hence it must be able to explain dynamics related to the planetary motion and the gravitating structures such as stars, galaxies, clusters of galaxies. Moreover, it has to pass observational tests in the Solar System. These facts constitute the experimental foundation on which GR is based. They are usually called the "classical tests of GR" [1, 2].

Beside the above "mechanical issues", GR has to explain Galactic dynamics, considering baryonic constituents, like stars, planets, dust and gas, radiation. These components are tied together by the Newtonian potential, which is supposed to work at any Galactic scales. Also, GR has to address the large scale structure formation and dynamics. At cosmological scales, GR is required to address dynamics of the whole universe and correctly reproduce cosmological parameters like the Hubble expansion rate, the density parameters and the accelerated (decelerated) behavior of cosmic fluid. Cosmological and astrophysical observations actually probe only the standard baryonic matter, the radiation and the attractive overall interaction of gravity acting at all scales.

Furthermore, starting from Galileo, the free-fall of different bodies is assumed to be independent of the nature of massive bodies on the Earth. The free-fall acceleration is unique and implies that gravitational and inertial mass ratio is identical for different bodies. This experimental result is one of the foundations of Einstein's GR as well as of any metric theory of gravity. After Galileo's experiment from the leaning tower of Pisa, the free-fall acceleration uniqueness has been verified in many experiments, as widely discussed in the second part of this review. Summarizing, we can say that GR is based on four main assumptions:

The "*Relativity Principle*" - there is no preferred inertial frames, i.e. all frames (accelerated or not) are good frames for Physics.

The "*Equivalence Principle*" (EP) - inertial effects are locally indistinguishable from gravitational effects (which means the equivalence between the inertial and the gravitational masses). In other words, any gravitational field can be locally cancelled.

The "*General Covariance Principle*" - field equations must be "covariant" in form, i.e. they must be invariant under the action of any spacetime diffeomorphisms.

The "*Causality Principle*" - each point of space-time admits a universal notion of past, present and future.

On these bases, Einstein postulated that, in a four-dimensional spacetime manifold, the gravitational field is described by the metric tensor $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, with the same signature of Minkowski metric. The metric coefficients are the physical gravitational potentials. Moreover, spacetime is curved by the distribution of energy-matter sources, e.g., the distribution of celestial bodies.

An important historical remark is necessary at this point. E. Kretschmann, in 1917 [3], criticized the General Covariance Principle. In demanding General Covariance, he asserted that Einstein placed no constraint on the physical content of his theory. Kretschmann stressed that any spacetime theory could be formulated in a generally covariant way without any physical principle. In formulating GR, Einstein used tensor calculus. Kretschmann pointed out that this calculus allowed for general covariant formulations of theories while Einstein discussed general covariance as the form invariance of theory's

equations as soon as the spacetime coordinates are transformed. This can be considered as a "passive" point of view of General Covariance: if we have some system of fields, we can change our spacetime coordinate system as we like and the new descriptions of the fields in the new coordinate systems will still solve the theory's equations. The answer by Einstein was that the form invariance of the theory's equations also allows a second version, the so-called "active" General Covariance. It involves no transformation of the spacetime coordinate system. In fact, active General Covariance gives rise to the generation of new solutions of the equations of the theory in the same coordinate system once one solution is given. According to this approach, General Covariance Principle can be considered a physical principle.

The above principles require that the spacetime structure has to be determined by either one or both of the two following fields: a Lorentzian metric g and a linear connection Γ , assumed by Einstein to be torsionless because, at that time, the spin of particles was not considered a possible source for the gravitational field. The physical meaning of these two fields is the following: The metric g fixes the spacetime causal structure, that is the light cones. According to this statement, metric relations, i.e. clocks and rods, are possible. On the other hand, the connection Γ fixes the free-fall of objects, that is the local inertial observers according to the Equivalence Principle. Both fields, of course, have to satisfy some compatibility relations like the requirement that photons follow null geodesics of Γ . This means that Γ and g can be independent, *a priori*, but they are constrained, *a posteriori*, according to some physical restrictions which impose that Γ has to be the Levi-Civita connection of g . However, in more general approaches, Γ and g can be independent [4].

Despite the self-consistency and the solid foundation, there are several issues for GR, both from the theoretical and the experimental (observational) points of view. The latter clearly points out that GR is not capable of addressing Galactic, extra-galactic and cosmic dynamics unless a huge quantity of some exotic forms of matter-energy is considered. These ingredients are usually called *dark matter* and *dark energy* and constitute up to 95% of the total amount of cosmic matter-energy [5, 6].

On the other hand, instead of changing the source side of the Einstein field equations, a "geometrical view" can be taken into account to fit the missing matter-energy of the observed Universe. In such a case, the dark side could be addressed by extending GR including further geometric invariants into the Hilbert - Einstein Action besides the Ricci curvature scalar R . These effective Lagrangians can be justified at the fundamental level considering the quantization of fields on curved spacetimes [6]. However, at the present stage of the research, there is no final probe discriminating between dark matter-energy picture and extended (alternative) gravity¹. Furthermore, the bulk of observations to be considered is very large and then an effective Lagrangian or a single new particle, addressing the whole phenomenology at all astrophysical and cosmic scales, would be very difficult to find.

An important discussion is related to the choice of the dynamical variables. In formulating GR, Einstein assumed that the metric g is the fundamental object to describe gravity. The connection $\Gamma_{\mu\nu}^{\alpha} = \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\}_g$ is assumed, by construction, with no dynamics. Only g has dynamics. This means that the metric g determines, at the same time, the causal structure (light cones), the measurements (rods and clocks) and the free fall of test particles (geodesic structure). Spacetime is given by a couple of mathematical objects $\{\mathcal{M}, g\}$ constituted by a Riemann manifold and a metric. Einstein realized that gravity induces free fall and that the EP selects an object that cannot be a tensor because the connection Γ can be switched off and set to zero at least in a point. According to this consideration, Einstein was obliged to choose the Levi - Civita connection determined by the metric structure.

Alternatively, in the Palatini formalism a (symmetric) connection Γ and a metric g are assumed and

¹An important remark is useful at this point. With the term *Extended Gravity*, we mean any class of theories by which it is possible to recover Einstein GR as a particular case or in some post-Einsteinian limit as in the case of $f(R)$ gravity. With *Alternative Gravity*, we mean a class of theories which considers different approach with respect to GR, for example the Teleparallel Equivalent Gravity considering the torsion scalar instead of curvature scalar to describe dynamics.

these two fields can be varied independently. According to this picture, spacetime is a triple $\{\mathcal{M}, g, \Gamma\}$ where the metric determines causal structure while Γ determines the free fall. This means that, in the Palatini formalism, connections are differential equations determining dynamics. From this point of view, Γ is the Levi-Civita connection of g as an outcome of the field equations.

The connection is the fundamental field in the Lagrangian while the metric g enters the Lagrangian as the need to define lengths and distances to make experiments. It defines the causal structure but has no dynamical role. As a consequence, there is no reason to assume g as the potential of Γ .

With this consideration in mind, we discuss here the role of the EP in the debate of theories of gravity both from a theoretical and experimental point of view.

This review is organized as follows. In Section 2 we discuss the different formulations of the EP. After summarizing the main topics of GR and Quantum Field Theory (QFT) in curved spacetimes, we discuss metric theories of gravity considering possible extensions and modifications of GR. Motivations, both theoretical and experimental, suggesting generalizations of GR, are considered. Specifically, these theories have been introduced to account for shortcomings of GR, both at early and late phases of the Universe evolution: Cosmological Inflation, Dark Matter and Dark Energy represent the main issues of this debate. From the other side, GR is not a fundamental theory of physics because it should require the inclusion of quantum effects. It is then natural to ask whether the Equivalence Principle still holds in the framework of any modified gravity approach aimed to enclose quantum physics under the standard of gravitational interaction. According to this requirement, we discuss the possibility to violate the EP by considering QFT at finite temperature. Besides, violations of the EP also occur in the framework of the extensions of Standard Model of particles. Section 3 is essentially devoted to experimental tests. We present a wide class of experiments aiming to test the EP, in particular its weak formulation, with a high accuracy. These include free falling tests, measurements based on Earth-to-Moon and Earth-to-satellite distances, cold atoms and particles interferometry tests, spin-gravity coupling tests, matter-antimatter tests. Conclusions are drawn in Section 4.

2 The Foundation of the Equivalence Principle

The EP is related to the above considerations and plays a relevant role to discriminate among concurring theories of gravity. In particular, the role of g and Γ are related to the validity of EP. Specifically, precise measurements of EP could say if Γ is only Levi - Civita or if a more general connection, disentangled from g , is necessary to describe gravitational dynamics. Furthermore, possible violation of EP can put in evidence other dynamical fields like torsion discriminating among the fundamental structure of spacetime that can be Riemannian or not.

Summarizing, the relevance of EP comes from the following points:

- Competing theories of gravity can be discriminated according to the validity of EP;
- EP holds at classical level but it could be violated at quantum level;
- EP allows to investigate independently geodesic and causal structure of spacetime. If it is violated at fundamental level, such structures could be independent.

From a theoretical point of view, EP constitutes the foundation of metric theories. The first formulation of EP comes out from the formulation of gravity by Galileo and Newton, i.e. the Weak Equivalence Principle (WEP) which states that the inertial mass m_i and the gravitational mass m_g of physical objects are equivalent. The WEP implies that it is impossible to distinguish, locally, between the effects of a gravitational field from those experienced in uniformly accelerated frames using the straightforward observation of the free fall of physical objects.

The first generalization of WEP states that Special Relativity is locally valid. Einstein obtained, in the framework of Special Relativity, that mass can be reduced to a manifestation of energy and momentum. As a consequence, it is impossible to distinguish between an uniform acceleration and an external gravitational field, not only for free-falling objects, but whatever is the experiment. According to this observation, Einstein EP states:

- The WEP is valid.
- The outcome of any local non-gravitational test experiment is independent of the velocity of free-falling apparatus.
- The outcome of any local non-gravitational test experiment is independent of where and when it is performed in the universe.

One can define a "local non-gravitational experiment" as that performed in a small-size of a free-falling laboratory. Immediately, it is possible to realize that gravitational interaction depends on the curvature of spacetime. It means that the postulates of metric gravity theories have to be satisfied. Hence the following statements hold:

- Spacetime is endowed with a metric $g_{\mu\nu}$ that constitutes the dynamic variables.
- The world lines of test bodies are geodesics of the metric.
- In local freely falling frames, i.e. the local Lorentz frames, the non-gravitational laws of physics are those of Special Relativity.

One of the predictions of this principle is the gravitational red-shift, experimentally probed by Pound and Rebka [1]. It is worth noticing that gravitational interactions are excluded from WEP and Einstein EP.

To classify extended and alternative theories of gravity, the gravitational WEP and the Strong Equivalence Principle (SEP) is introduced. The SEP extends the Einstein EP by including all the laws of physics. It states:

- WEP is valid for self-gravitating bodies as well as for test bodies (gravitational WEP).
- The outcome of any local test is independent of the velocity of the free-falling apparatus.
- The outcome of any local test is independent of where and when it is performed in the universe.

The Einstein EP is recovered from SEP as soon as the gravitational forces are neglected. Several authors claim that the only theory coherent with SEP is GR and then WEP has to be deeply investigated.

A very important issue is the consistency of EP with respect to Quantum Mechanics. GR is not the only gravity theory and several alternatives have been investigated starting from the 60's [6]. Some of them are effective descriptions coming from quantum field theories on curved spacetime. Considering the spacetime as special relativistic at a background level, gravitation can be treated as a Lorentz-invariant perturbation field on the background. Assuming the possibility of GR extensions and alternatives, two different classes of experiments can be conceived:

- Tests for the foundations of gravity according to the various formulations of EP.
- Tests of metric theories where spacetime is endowed with a metric tensor and where the Einstein EP is assumed valid.

The difference between the two classes of experiments consists in the fact that EP can be postulated "a priori" or "recovered" from the self-consistency of the theory. What is clear is that, for several fundamental reasons, extra fields are necessary to describe gravity with respect to other interactions. Such fields can be scalar fields or higher-order corrections of curvature and torsion invariants [6]. According to these reasons, two sets of field equations can be considered: The first couples the gravitational field to non-gravitational fields, i.e. the matter distribution, the electromagnetic fields, etc. The second set of equations considers dynamics of non-gravitational fields. In the framework of metric theories, these laws depend only on the metric and this is a consequence of the Einstein EP. In the case where gravitational field equations are modified with respect to the Einstein ones, and matter field are minimally coupled with gravity, we are dealing with the *Jordan frame*. In the case where Einstein field equations are preserved and matter field are non-minimally coupled, we are dealing with the *Einstein frame*. Both frames are conformally related but the very final issue is to understand if passing from one frame to the other (and vice versa) is physically significant. Clearly, EP plays a fundamental role in this discussion. In particular, the main question is if EP is valid in any case or it is violated at quantum level.

2.1 The debate on gravitational theories

As discussed before, GR provides a comprehensive description of space, time, gravity, and matter under the same standard at macroscopic level. Einstein formulated it in such a way that space and time are dynamical and entangled quantities determined by the distribution and motion of matter and energy. As a consequence, GR is related to a new conception of the universe which can be considered as a dynamical system where precise physical measurements are possible.

In this perspective, cosmology is not only a philosophical branch of knowledge but can be legitimately incorporated into science. Investigating scientifically the universe has led, along the last century, to the formulation of the Standard Big Bang Model [7] which, in principle, matched most of the available cosmological observations until more or less twenty years ago.

Despite these successes, several shortcomings of Einstein's theory emerged recently at ultraviolet and infrared scales and scientists considered the hypothesis whether GR is the only fundamental theory of gravitational interaction. This new point of view comes from cosmology (infrared scales) and quantum field theory (ultraviolet scales). In the first case, the Big Bang singularity, the flatness, horizon, and monopole problems [8] led to the conclusion that a cosmological model based on GR and the Standard Model of particles is inadequate to describe the universe in extreme energy-curvature regimes. Furthermore, GR cannot work as a fundamental theory of gravity if a quantum description of spacetime is required. The Einstein theory is essentially a classical description. Due to these reasons, and, in particular due to the lack of a self-consistent quantum theory of gravity, various alternative and extensions of GR were proposed. The general approach is to formulate, at least, a semiclassical effective theory where GR and its positive results can be recovered in some limit (e.g. the weak field limit or the Solar System scales). A fruitful approach is the so-called *Extended Theories of Gravity* (ETGs) which have recently become a paradigm to study the gravitational interaction. Essentially they are based on corrections and extensions of Einstein's GR. The paradigm consists in adding higher order curvature invariants and minimally or non-minimally coupled scalar fields into the dynamics. In this sense, we can deal with effective gravity actions emerging from quantum field theory [6, 9].

Other reasons to modify GR are related to the issue of incorporating Mach's principle into the theory. GR is only partially Machian and allows solutions that are explicitly anti-Machian, e.g. the Gödel solution [10] or exact *pp*-waves [11].

Mach's principle states that local inertial frames are determined by the average motions of distant astronomical objects [12]. This implies that the gravitational coupling can be determined by the surrounding matter distribution and, therefore, becomes a spacetime function which can assume the form of a scalar field. As a consequence, inertia and Equivalence Principle are concepts that have to be

revised. Brans-Dicke theory [13] is the first alternative to GR and the first attempt to fully incorporate the Mach principle. It is considered the prototype of alternative theories of gravity and a straightforward GR extension. The gravitational “constant” is assumed “variable” and corresponds to a scalar field non-minimally coupled to geometry. This approach constitute a more satisfactory implementation of Mach’s principle than GR [13–15].

Furthermore, any scheme unifying fundamental interactions with gravity, such as superstrings, supergravity, or Grand-Unified Theories (GUTs) produces effective actions where non-minimal couplings to the geometry are present. Also higher order curvature invariants are present in general. They emerge as loop corrections in high-curvature regimes. This scheme has been adopted in quantizing matter fields on curved spacetimes and the result is that interactions between quantum fields and background geometry, or gravitational self-interactions give rise to corrections in the Hilbert-Einstein Lagrangian [16]. Furthermore, these corrections are unavoidable in the effective quantum gravity actions [17] and then GR extensions are necessary. All these models do not constitute a self-consistent quantum gravity theory, but are useful working schemes towards it.

To summarize, higher order curvature invariants like R^2 , $R^{\mu\nu}R_{\mu\nu}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$, $R\Box R$, $R\Box^k R$, or non-minimally couplings between matter fields and geometry such as $\phi^2 R$ have to be added to the gravitational Lagrangian as soon as quantum corrections are introduced. For example, these terms occur in the low-energy limit of string Lagrangian or in Kaluza-Klein theories where extra spatial dimensions are taken into account [18].

Moreover, from a conceptual viewpoint, there is no *a priori* reason to restrict the gravitational Lagrangian to a function, linear in the Ricci scalar R , minimally coupled to matter [19]. This concept is in agreement with the idea that there are no “exact” laws of physics. In this case, the effective Lagrangians of physical interactions would be given by the average quantities arising from the stochastic behaviour of fields at a microscopic level. This approach means that the local gauge invariances and the conservation laws are approximated and emerge only in the low-energy limit. In this perspective, fundamental constants of physics can be assumed variable.

Furthermore, besides fundamental physics motivations, ETGs are interesting in cosmology because they exhibit inflationary behaviours able to overcome shortcomings of Standard Big Bang model. The related inflationary scenarios are realistic and match current observations coming from the cosmic microwave background (CMB) [20, 21]. It can be shown that, by conformal transformations, the higher order and non-minimally coupled terms correspond to Einstein gravity plus one or more than one scalar field(s) minimally coupled to the curvature [22–24]. Specifically, after conformal transformations, higher order and non-minimally coupled terms appear as equivalent scalar fields in the Einstein field equations. For example, in the Jordan frame, a term like R^2 in the Lagrangian gives fourth order field equations, $R\Box R$ gives sixth order equations [25], $R\Box^2 R$ yields eighth order equations [26], and so on. After a conformal transformation, second order derivative terms corresponds to a scalar field:² specifically, fourth order gravity is conformally equivalent to Einstein gravity plus a scalar field; sixth order gravity is conformally equivalent to GR plus two scalar fields; and so on [27].

Furthermore, it is also possible to show that $f(R)$ gravity to the Einstein theory minimally coupled to an ideal fluid [28]. This feature is useful if multiple inflationary events are necessary for structure formation. In fact, an early stage could produce large-scale structure with very long wavelengths which after give rise to the observed clusters of galaxies. A later stage could select smaller scales observed as galaxies today [25]. The underlying philosophy is that any inflationary era is related to the dynamics of a related scalar field. Finally, these extended schemes could solve the graceful exit problem, avoiding the shortcomings of other inflationary models [29].

The revision of early cosmological scenarios, leading to inflation, can imply that a new approach is necessary also at late epochs: ETGs play a fundamental role also in today observed universe. In fact,

²Dynamics of any of these scalars fields is determined by a second order Klein-Gordon equation.

the increasing amount of observational data, accumulated over the past decades, has given rise to a new cosmological model, the so called *Concordance Model* or Λ -Cold Dark Matter (Λ CDM) model.

The Hubble diagram of type Ia Supernovae (hereafter SNeIa) was the first evidence that the universe is today undergoing an accelerated expansion phase. Furthermore, balloon-born experiments [30] determined the location of the first two Doppler peaks in the spectrum of CMB anisotropies. These features strongly suggest a spatially flat universe also if some recent data could question this result. If combined with constraints on matter density parameter Ω_M , these data point out that the universe is dominated by an un-clustered fluid, with negative pressure, usually referred to as *dark energy*. Such a fluid drives the accelerated expansion. This picture has been strengthened by other precise measurements on CMB spectrum and by the extension of the SNeIa Hubble diagram to redshifts higher than one.

A huge amount of papers appeared following these observational results. They present several models attempting to explain the cosmic acceleration. The simplest explanation is the well-known cosmological constant Λ . Although this ingredient provides the best-fit to most of the available astrophysical data [31], the Λ CDM model fails in explaining why the value of Λ is so tiny (120 orders of magnitude lower) if compared with the typical vacuum energy density predicted by particle physics, and why its present value is comparable to the matter density. This constitutes the so-called *coincidence problem*.

A possible solution is replacing the cosmological constant with a scalar field φ rolling slowly down a flat section of a potential $V(\varphi)$ and giving rise to the models known as *quintessence* [32, 33]. Also if it is successfully fitting data with many models, the quintessence approach to dark energy is still plagued by the coincidence problem since the dark energy and dark matter densities evolve in a different way and reach comparable values only during a very short time of the history of the universe. In particular, they coincide right at present era. In other words, the quintessence is tracking matter and evolves in the same way for a long time; at late times, it changes its behaviour and no longer tracks the dark matter but dominates dynamics as a cosmological constant. This is, specifically, the quintessence coincidence problem.

The origin of this quintessence scalar field is one of the big mystery of modern cosmology. Although several models have been proposed, a great deal of uncertainty is related to the choice of the scalar field potential $V(\varphi)$ necessary to achieve the late-time acceleration of the universe. The elusive nature of dark energy has led many authors to look for a different explanation of the cosmic acceleration without introducing exotic components. It is worth stressing that the present-day cosmic acceleration requires a negative pressure that has to dominate dynamics after the matter era. However, we do not anything about the nature and the number of the cosmic fluids filling the universe. This consideration suggests us that the accelerated expansion could be explained with a single cosmic fluid characterized by an equation of state acting like dark matter at high densities, and like dark energy at low densities. The relevant feature of these models, referred as *Unified Dark Energy* (UDE) or *Unified Dark Matter* (UDM) models, is that the coincidence problem is naturally solved. Examples are the Chaplyng gas [34], tachyon fields [35], and condensate cosmology [36]. These models are extremely interesting because they can be interpreted both in the framework of UDE models or as two-fluid models representing the dark matter and the dark energy regime. A main feature of this approach is that a generalized equation of state can be easily obtained and the fit of observational data can be achieved.

There is another approach to face the problem of the cosmic acceleration. As reported in [37], it is possible that the observed acceleration is not related to another cosmic ingredient, but rather the signal of a breakdown, at infra-red scales, of the law of gravitation as a scale invariant interaction. From this view point, modifying the Einstein-Friedmann equations, fitting the astrophysical data with models containing only standard matter and without exotic fluids is another approach. Examples in this direction are the Cardassian model [38] and Dvali-Gabadadze-Porrati (DGP) cosmology [39]. In the same conceptual framework, it is possible to find alternative approaches where a quintessential behavior is obtained by incorporating effective models coming from fundamental physics and giving rise to extended gravity actions. For example, a cosmological constant can be recovered as a consequence

of non-vanishing torsion fields. Also in this case, it is possible to build up models consistent with the SNeIa Hubble diagram and the Sunyaev - Zel'dovich effect in galaxy clusters [40]. SNeIa data can also be fitted by including higher-order curvature invariants. These models provide a cosmological component with negative pressure which is originated by the geometry of the universe. According to this picture, we do not need new particle counterparts to address the phenomenology.

The amount of cosmological models which are viable candidates to explain the observed accelerated expansion is too wide to be reported here. This overabundance points out that only a few number of cosmological tests is available to discriminate between competing approaches, so it is clear that there is a high degeneracy of models. It is important to stress that both SNeIa Hubble diagram and angular size-redshift relation of compact radio sources are distance-based probes of the cosmological model and, therefore, systematic errors and biases could be iterated. According to this consideration, it is interesting to search for tests based on time-dependent observables. For example, we can take into account the *lookback time* to distant objects. This quantity discriminates among different cosmological models. The lookback time is estimated as the difference between the age of the universe and the age of a given object at redshift z . This estimate becomes realistic when the object is a galaxy observed in more than one photometric band because its color is determined by the age as a consequence of stellar evolution. Hence, it is possible to obtain the galaxy age by measuring its magnitude in different bands and then using stellar evolutionary codes to best reproduce the observed colors.

In general, in the case of weak-field limit, which essentially coincides with Solar System scales, ETGs are expected to reproduce GR which is precisely tested at these scales [1]. Even this limit is a matter of debate because several theories do not reproduce exactly the Einstein theory in its Newtonian limit but, in some sense, generalize it giving rise to Yukawa-like corrections to the Newtonian potential which could be physically relevant already at Galactic scales [41–46].

As a general remark, relativistic gravity theories give rise to corrections to the weak-field gravitational potentials which, at the post-Newtonian level and in the Parametrized Post-Newtonian (PPN) formalism, constitute a test bed for these theories [1]. Furthermore, the *gravitational lensing astronomy* [47] provide additional tests over small, large, and very large scales which can provide measurements on the variation of the Newton constant [48], the potential of galaxies, clusters of galaxies, and other features of gravitating systems. In principle, such data can be capable of confirming or ruling out any alternative to GR.

In ETGs, the Einstein field equations can be modified in two ways: *i*) the geometry part can be non-minimally coupled to some scalar field, and/or *ii*) higher than second order derivatives of the metric can appear. In the former case, we deal with scalar-tensor theories of gravity; in the latter, with higher order theories of gravity. Combinations of non-minimally coupled and higher order components can also emerge.

From the mathematical viewpoint, the problem of reducing more general theories to the Einstein theory has been widely discussed. Through a Legendre transformation on the metric, higher order theories with Lagrangians satisfying some regularity conditions assume the form of GR with (possibly multiple) scalar field(s) as sources the gravitational field (*e.g.*, [19, 49, 50]). The formal equivalence between models with variable gravitational coupling and Einstein gravity via conformal transformations is also well known [51]. This gave rise to the debate on whether the mathematical equivalence between different conformal representations is also a physical equivalence [52, 53].

Another important issue is the Palatini approach: this problem was first proposed by Einstein himself, but it was called the Palatini approach because the Italian mathematician Attilio Palatini formalized it [4]. The main idea of this formalism is considering the connection $\Gamma_{\alpha\beta}^{\mu}$ as independent of the metric $g_{\mu\nu}$. It is well known that the Palatini formulation of GR is equivalent to the metric formulation [7]. This result follows from the fact that the field equations for connection $\Gamma_{\alpha\beta}^{\mu}$, also if assumed independent of the metric, yield the Levi-Civita connection of $g_{\mu\nu}$ in GR. Therefore, the Palatini variational principle in the Einstein theory gives exactly the same field equations of the metric variational

principle. However, the situation changes if we consider ETGs formulated as functions of curvature invariants, such as $f(R)$, or as scalar-tensor theories. There, the Palatini and the metric variational principles give rise to different field equations that could describe different physics. The relevance of the Palatini formulation has been recently highlighted according to cosmological applications [54].

Another crucial problem is related to the Newtonian potential in alternative gravity and its relations with the conformal factor [55]. From a physical point of view, considering the metric and the connection as independent fields corresponds to decoupling the metric structure of spacetime from the geodesic structure (with the connection being, in general, different from the Levi-Civita connection of the metric). The causal structure of spacetime is governed by $g_{\mu\nu}$, while the spacetime trajectories of particles are governed by $\Gamma_{\alpha\beta}^{\mu}$.

The decoupling of causal and geodesic structures enlarges the spacetime geometry and generalizes the metric formalism. This metric-affine structure can be immediately translated, by means of the Palatini field equations, into a bi-metric structure. In addition to the physical metric $g_{\mu\nu}$, a second metric $h_{\mu\nu}$ is present which is related, in the case of $f(R)$ gravity, to the connection. As a matter of fact, the connection $\Gamma_{\alpha\beta}^{\mu}$ turns out to be the Levi-Civita connection of this second metric $h_{\mu\nu}$ and provides the geodesic structure of spacetime.

If we consider non-minimal couplings in gravitational Lagrangian, the further metric $h_{\mu\nu}$ is related to the coupling. According to the Palatini formalism, non-minimal couplings and scalar fields entering the evolution of the gravitational field are related by the metric structure of spacetime³

2.2 Einstein's General Relativity

The Newton theory of gravity was the issue that Einstein needed to recover in the weak field limit and slow motion. In Newton formulation, space and time are absolute entities and require particles to move, in a preferred inertial frame, along curved trajectories, the curvature of which (*i.e.*, the acceleration) is a function of the intensity of the sources through the “forces”. According to this requirements, Einstein postulated that the gravitational forces have to be described by the curvature of the metric tensor $g_{\mu\nu}$ related to the line element $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$ of a four-dimensional spacetime manifold. This metric has the same signature of the Minkowski metric (the *Lorentzian signature* here assumed to be $(-, +, +, +)$). Einstein postulated also that spacetime curves onto itself and that curvature is locally determined by the distribution of the sources, that is by the four-dimensional generalization of the matter stress-energy tensor (another rank-two symmetric tensor) $T_{\mu\nu}^{(m)}$ of continuum mechanics.

Once a metric $g_{\mu\nu}$ is assigned, curvature is given by the Riemann (or curvature) tensor

$$R_{\alpha\beta\mu}{}^{\nu} = \Gamma_{\alpha\beta,\mu}^{\nu} - \Gamma_{\beta\mu,\alpha}^{\nu} + \Gamma_{\alpha\mu}^{\sigma}\Gamma_{\sigma\beta}^{\nu} - \Gamma_{\beta\mu}^{\sigma}\Gamma_{\sigma\alpha}^{\nu} \quad (1)$$

where the commas denote partial derivatives. Its contraction

$$R_{\alpha\mu} \equiv R_{\alpha\beta\mu}{}^{\beta}, \quad (2)$$

is the *Ricci tensor*, while the contraction

$$R \equiv R^{\mu}{}_{\mu} = g^{\mu\nu}R_{\mu\nu} \quad (3)$$

is the *Ricci curvature scalar* of $g_{\mu\nu}$. Einstein initially derived the field equations $R_{\mu\nu} = \frac{\kappa}{2}T_{\mu\nu}^{(m)}$, where $\kappa = 8\pi G$ (in units in which $c = 1$) is the gravitational coupling constant. These equations turned out to be inconsistent as pointed out by Levi-Civita. Furthermore Hilbert stressed that they do not derive from an action principle [57]. In fact, there is no action reproducing them exactly through a variation.

³Due to these features, the Palatini approach could play a main role in clarifying the physical aspects of conformal transformations [56].

Einstein's answer was that he realized that the equations were physically inconsistent, since they were incompatible with the continuity equation deemed to be satisfied by reasonable forms of matter.

Assuming matter consisting of perfect fluids with stress-energy tensor

$$T_{\mu\nu}^{(m)} = (P + \rho) u_\mu u_\nu + P g_{\mu\nu}, \quad (4)$$

where u^μ is the four-velocity of the particles, P and ρ the pressure and energy density of the fluid, respectively, the continuity equation requires $T_{\mu\nu}^{(m)}$ to be covariantly constant, *i.e.*, to satisfy the conservation law

$$\nabla^\mu T_{\mu\nu}^{(m)} = 0, \quad (5)$$

where ∇_α denotes the covariant derivative operator of the metric $g_{\mu\nu}$. In fact, $\nabla^\mu R_{\mu\nu}$ does not vanish, except in the trivial case $R \equiv 0$. Einstein concluded that the field equations are

$$G_{\mu\nu} = \kappa T_{\mu\nu}^{(m)} \quad (6)$$

where

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad (7)$$

is the *Einstein tensor* of $g_{\mu\nu}$. These equations can be derived also by minimizing an action containing R and satisfy the conservation law (5) since

$$\nabla^\mu G_{\mu\nu} = 0, \quad (8)$$

holds as a contraction of the Bianchi identities that the curvature tensor of $g_{\mu\nu}$ satisfies [7].

Specifically, the Lagrangian that, if varied, produces the field equations (6) is the sum of a ‘‘matter’’ Lagrangian density $\mathcal{L}^{(m)}$, whose variational derivative is

$$T_{\mu\nu}^{(m)} = \frac{\delta \mathcal{L}^{(m)}}{\delta g^{\mu\nu}}, \quad (9)$$

and of the gravitational (*Hilbert-Einstein*) Lagrangian density

$$\mathcal{L}_{HE} = \sqrt{-g} g^{\mu\nu} R_{\mu\nu} = \sqrt{-g} R, \quad (10)$$

where g is the determinant of the metric $g_{\mu\nu}$.

Einstein's choice was arbitrary but it was certainly the simplest. As clarified by Levi-Civita in 1919, curvature is not a purely metric notion but it is also related to the linear connection of parallel transport and covariant derivative. In some sense, this idea is the precursor of ‘‘gauge-theoretical framework’’ [58], following the pioneering work by Cartan of 1925.

After, it was clarified that the principles of relativity, equivalence, and covariance, together with causality, require only that the spacetime structure can be determined by a Lorentzian metric $g_{\mu\nu}$ and a linear connection $\Gamma_{\mu\nu}^\alpha$, assumed to be torsionless for the sake of simplicity. The metric fixes the causal structure of spacetime (the light cones) as well as its metric relations measured by clocks and rods and the lengths of four-vectors. The connection determines the laws of free fall, that is the four-dimensional spacetime trajectories followed by locally inertial observers. These observers must satisfy some compatibility relations including the requirement that photons follow null geodesics, so that $\Gamma_{\mu\nu}^\alpha$ and $g_{\mu\nu}$ can *a priori* be independent, but constrained *a posteriori* by the physics. These physical constraints, however, do not necessarily impose that $\Gamma_{\mu\nu}^\alpha$ is the Levi-Civita connection of $g_{\mu\nu}$ [6].

2.3 Quantum Gravity motivations

A challenge of modern physics is constructing a theory capable of describing the fundamental interactions of nature under the same standard. This goal has led to formulate several unification schemes which attempt to describe gravity together with the other interactions. All these schemes describe the fields under the conceptual apparatus of Quantum Mechanics. This is based on the assumption that the states of physical systems are described by vectors in a Hilbert space \mathcal{H} and the physical fields are linear operators defined on domains of \mathcal{H} . Till now, any attempt to incorporate gravity into this scheme is failed or revealed unsatisfactory. The main problem is that gravitational field describes, at the same time, the gravitational degrees of freedom and the spacetime background where these degrees of freedom are defined.

Owing to the difficulties of building up a self-consistent theory unifying interactions and particles, the two fundamental theories of modern physics, GR and Quantum Mechanics, have been critically re-analyzed. On the one hand, we assume that matter fields (bosons and fermions) come out from superstructures (*e.g.*, Higgs bosons or superstrings) that, undergoing certain phase transitions, generate the known particles. On the other hand, it is assumed that the geometry interacts directly with quantum matter fields which back-react on it. This interaction necessarily modifies the standard gravitational theory, that is the Hilbert-Einstein Lagrangian. This fact leads to the ETGs.

From the point of view of cosmology, the modifications of GR provide inflationary scenarios of remarkable interest. In any case, a condition that such theories have to respect in order to be physically acceptable is that GR is recovered in the low-energy limit.

Although conceptual progresses have been made assuming generalized gravitational theories, at the same time mathematical difficulties have increased. The corrections into the Lagrangian enlarge the (intrinsic) non-linearity of the Einstein equations, making them more difficult to study because differential equations of higher order than second are often obtained and because it is extremely difficult to separate geometry from matter degrees of freedom. To overcome these difficulties and try to simplify the field equations, one often looks for symmetries of dynamics and identifies conserved quantities which, often, allow to find out exact solutions.

The necessity of quantum gravity was recognized at the end of 1950s, when physicists tried to deal with all interactions at a fundamental level and describe them under the standard of quantum field theory. The first attempts to quantize gravity adopted the canonical approach and the covariant approach, which had been already applied with success to electromagnetism. In the first approach applied to electromagnetism, one takes into account electric and magnetic fields satisfying the Heisenberg uncertainty principle and the quantum states are gauge-invariant functionals, generated by the vector potential, defined on 3-surfaces labeled with constant time. In the second approach, one quantizes the two degrees of freedom of the Maxwell field without (3+1) decomposition of the metric, while the quantum states are elements of a Fock space [59]. These procedures are fully equivalent. The former allows a well-defined measure, whereas the latter is more convenient for perturbative calculations such as the computation of the S -matrix in Quantum Electrodynamics (QED).

These methods have been adopted also in GR, but several difficulties arise in this case due to the fact that GR cannot be formulated as a quantum field theory on a fixed Minkowski background. To be specific, in GR the geometry of background spacetime cannot be given a priori: spacetime is itself the dynamical variable. To introduce the notions of causality, time, and evolution, one has to solve equations of motion and build up the related spacetime. For example, to know if a particular initial condition will give rise to a black hole, it is necessary to evolve it by solving the Einstein equations. Then, taking into account the causal structure of the solution, one has to study the asymptotic metric at future null infinity in order to assess whether it is related, in the causal past, with that initial condition. This problem become intractable at quantum level. Due to the uncertainty principle, in non-relativistic quantum mechanics particles do not move along well-defined trajectories and one can only calculate

the probability amplitude $\psi(t, x)$ that a measurement at a given time t detects a particle at the spatial point x . In the same way, in quantum gravity, the evolution of an initial state does not provide a given spacetime (that is a metric). In absence of a spacetime, how is it possible to introduce basic concepts as causality, time, scattering matrix, or black holes?

Canonical and covariant approaches provide different answers to these issues. The first is based on the Hamiltonian formulation of GR and is adopting a canonical quantization procedure. The canonical commutation relations are those that lead to the Heisenberg uncertainty principle; in fact, the commutation of operators on a spatial 3-manifold at constant time is assumed, and this 3-manifold is fixed in order to preserve the notion of causality. In the limit of asymptotically flat spacetime, motions related to the Hamiltonian have to be interpreted as time evolution (in other words, as soon as the background becomes the Minkowski spacetime, the Hamiltonian operator assumes again its role as the generator of time translations). The canonical approach preserves the geometric structure of GR without introducing perturbative methods.

On the other hand, the covariant approach adopts quantum field theory concepts. The basic idea is that the shortcomings mentioned above can be circumvented by splitting the metric $g_{\mu\nu}$ into a kinematic part $\eta_{\mu\nu}$ (usually flat) and a dynamical part $h_{\mu\nu}$. That is

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} . \quad (11)$$

The background geometry is given by the flat metric tensor and it is the same of Special Relativity and standard quantum field theory. It allows to define concepts of causality, time, and scattering. The procedure of quantization is applied to the dynamical field, considered as a little deviation of the metric from the flat background. Quanta are particles with spin two, i.e. *gravitons*, which propagate in Minkowski spacetime and are defined by $h_{\mu\nu}$. Substituting $g_{\mu\nu}$ into the GR action, it follows that the gravitational Lagrangian contains a sum whose terms contains a different orders of approximation, the interaction of gravitons and, eventually, terms describing matter-graviton interaction (if matter is present). These terms are analyzed by the standard perturbation approach of quantum field theory.

These quantization approaches were both developed during the 1960s and 1970s. In the canonical approach, Arnowitt, Deser, and Misner developed the Hamiltonian formulation of GR using methods proposed by Dirac and Bergmann. In this scheme, the canonical variables are the 3-metric on the spatial 3-manifolds obtained by foliating the 4-dimensional manifold. It is worth noticing that this foliation is arbitrary. Einstein's field equations give constraints between the 3-metrics and their conjugate momenta and the evolution equation for these fields is the so-called *Wheeler-DeWitt (WDW) equation*. In this picture, GR is the dynamical theory of the 3-geometries, that is the *geometrodynamics*. The main problems arising from this formulation are that the quantum equations involve products of operators defined at the same spacetime point and, furthermore, they give rise to distributions whose physical meaning is unclear. In any case, the main question is the absence of the Hilbert space of states and, as consequence, the probabilistic interpretation of the quantities is not exactly defined.

The covariant quantization is much similar to the physics of particles and fields, because, in some sense, it has been possible to extend QED perturbation methods to gravitation. This allowed the analysis of mutual interaction between gravitons and of the matter-graviton interactions. The Feynman rules for gravitons and the demonstration that the theory might be, in principle, unitary at any order of expansion was achieved by DeWitt.

Further progress was reached by the Yang-Mills theories, describing the strong, weak, and electromagnetic interactions of particles by means of symmetries. These theories are renormalizable because it is possible to give the particle masses through the mechanism of spontaneous symmetry breaking. According to this principle, it is natural to try to consider gravitation as a Yang-Mills theory in the covariant perturbation approach and search for its renormalization. However, gravity does not fit into this scheme; it turns out to be non-renormalizable if we consider the graviton-graviton interactions

(two-loops diagrams) and graviton-matter interactions (one-loop diagrams).⁴ In any case, the covariant method allows to construct a gravity theory which is renormalizable at one-loop level in the perturbation series [16].

Due to the non-renormalizability of gravity at higher orders, the validity of the approach is restricted to the low-energy domain, that is, to infrared scales, while it fails at ultraviolet scales. This implies that the theory of gravity is unknown near or at Planck scales. On the other hand, sufficiently far from the Planck epoch, GR and its first loop approximation describe quite well gravitation. In this context, it makes sense to add higher order and non-minimally coupled terms to the Hilbert-Einstein action. Furthermore, if the free parameters are chosen appropriately, the theory has a better ultraviolet behaviour and it is asymptotically free. Nevertheless, the Hamiltonian of these theories is not bounded from below and they are unstable. Specifically, unitarity is violated and probability is not conserved.

Another approach to the search for quantum gravity comes from the study of the electroweak interaction. Here, gravity is treated neglecting the other fundamental interactions. The unification of electromagnetism and weak interaction suggests that it could be possible to obtain a consistent theory if gravity is coupled to some kind of matter. This is the basic idea of Supergravity. In this kind of theories, divergences due to the bosons (in this case the 2-spin gravitons) are cancelled exactly by those due to the fermions. In this picture, it is possible to achieve a renormalized theory of gravity. Unfortunately, the approach works only up to two-loop level and for matter-gravity couplings. The corresponding Hamiltonian is positive-definite and the theory is unitary. However, including higher order loops, the infinities appear and renormalizability is lost.

Perturbation methods are also adopted in string theories. In this case, the approach is different from the previous one since particles are replaced by extended objects which are the fundamental strings. The physical particles, including the spin two gravitons, correspond to excitations of the strings. The only free parameter of the theory is the string tension and the interaction couplings are uniquely determined. As a consequence, string theory contains all fundamental physics and it is considered a possible *Theory of Everything*. String theory is unitary and the perturbation series converges implying finite terms. This feature follows from the characteristic that strings are intrinsically extended objects, so that ultraviolet divergencies, appearing at small scales or at large transferred impulses, are naturally cured. This means that the natural cutoff is given by the string length, which is of Planck size l_P . At larger scales than l_P , the effective string action can be written as non-massive vibrational modes, that is, in terms of scalar and tensor fields. This constitutes the *tree-level effective action*. This approach leads to an effective theory of gravity non-minimally coupled with scalar fields, which are the so-called *dilaton fields*.

In conclusion, we can summarize the previous considerations: 1) a unitary and renormalizable theory of gravity does not yet exist⁵. 2) In the quantization program of gravity, two approaches are used: the *covariant approach* and the *perturbation approach*. They do not lead to a self-consistent quantum gravity. 3) In the low-energy regime, with respect to the Planck energy, GR can be improved by introducing, into the Hilbert-Einstein action, higher order terms of curvature invariants and non-minimal couplings between matter and gravity. The approach leads, at least at one-loop level, to a consistent and renormalizable theory.

⁴Higher order terms in the perturbative series imply an infinite number of free parameters. At the one-loop level, it is sufficient to renormalize only the effective constants G_{eff} and Λ_{eff} which, at low energy, reduce to the Newton constant G_N and the cosmological constant Λ .

⁵It is worth to mention that recently it has been shown that an infinite derivative theory of covariant gravity, which is motivated from string theory, see [60,61], can be made ghost free and also singularity free [62,63] (see Refs. [64–67] for some applications).

2.4 Emergent gravity and thermodynamics of spacetime

Recently, several theoretical approaches towards the so-called *emergent gravity* theories have been proposed. The main idea is that, given the lack of experimental data for quantum gravity at high-energies, it is worth approaching gravity from low-energies considering some effective theories. Gravity emerges from fundamental constituents, as a sort of “atoms of spacetime”, with metric and affine-connection being collective variables similar to hydrodynamics, where a fluid description emerges from an aggregate of microscopic particles. Emergent gravity attempts the reconstruction of microscopic system underlying classical gravity. It is possible to constrain the microscopic features of the fundamental constituents by requiring that the emergent gravity is similar to GR in weak field limit. This picture, however, questions the principles constituting the foundations of gravitational theories.

A related research line is that of *analogue models*: if gravity emerges as a collective system made of microscopic quantum constituents, it could be possible to model it with the help of physical systems where an effective metric and a connection govern the dynamics. For example one can study the Hawking radiation coming from black holes adopting acoustic analogues (the so called “dumb holes”) [68–70], or Bose-Einstein condensates (see [71] for a review of analogue models). If an effective metric is generated, it is a kinematic, in the sense that field equations are not generated by it. However, some results are able to generate a theory of scalar gravity [72] and progresses are possible in this direction. A standard feature for emergent spacetimes is that they exhibit Lorentz invariance at low-energies. The Lorentz symmetry is broken in the ultraviolet limit where the fundamental quantum constituents of gravity cannot be avoided.

We mention these approaches here because they question the foundations of gravitational theory and do not state that GR is the only theory to be reproduced at large scale in coarse-graining: the message is that different theories with similar features are possible as well.

Another approach is based on the idea that gravity could be reproduced through a sort of *spacetime thermodynamics*. This means that the Einstein field equations should be derived through local thermodynamics at equilibrium. Using thermodynamics on the Rindler horizons associated to the worldlines of physical observers and assuming the relation $S = A/4$ between entropy and horizon area (which should be more fundamental than the Einstein field equations) Jacobson was able to derive the Einstein equations as an equation of state derived for an ideal gas. The implication of this result is that it does not make sense to quantize the field equations to learn about quantum gravity. The philosophy is that by quantizing the equation of state of an atomic hydrogen gas, we do not learn anything about the hydrogen atom and its energy levels. From this perspective, if a similar thermodynamics of spacetime approach is applied to $f(R)$ gravity, it is then necessary to consider near-equilibrium thermodynamics in order to derive the field equations. This demonstrates that GR is just a state of gravity corresponding to a given thermodynamic equilibrium and, when this equilibrium is perturbed, near-equilibrium configurations correspond to alternative theories of gravity. According to this approach, this justify the study of ETGs.

A result with a conceptual similar meaning is found in scalar-tensor cosmologies: they should relax to GR during the evolution of the universe at recent epochs. This is another hint that GR could be only a particular state of equilibrium, while an entire spectrum of theories should be considered at higher energy excitations.

These results are very speculative and require further studies; however, they stress the necessity to think about gravity outside of the strict GR scheme and hint to the fact that much more work needs to be done before claiming for a self-consistent theory of gravity also at lower energies.

2.5 Kaluza-Klein theories

The attempt to construct a unified theory of GR and electromagnetism was first proposed by Kaluza [73] (for a review, see [74–77]). He showed that the electromagnetism and the gravitation interactions can be described by making use of a single *metric* tensor if an additional spatial dimension is introduced. In a Universe with 5-dimensions, the element line reads $ds^2 = G_{AB}(x, y)dx^A dx^B$, where $x^A = (x^\mu, y)$, being y the additional dimension (here $A, B = 0, 1, 2, 3, 4$, and x^μ , with $\mu = 0, 1, 2, 3$, the usual four dimensional coordinates). In matrix form, the 5-dimensional metric tensor assumes the form $G_{AB} = \begin{pmatrix} g_{\mu\nu} & g_{\mu 4} \\ g_{4\nu} & g_{44} \end{pmatrix}$.

From the metric tensor, one construct all geometric quantities such as the Riemann tensor, the Ricci tensor and scalar curvature and then the field equations. The components of the metric tensor are typically written in the following form: $G_{44} = \phi$, $G_{4\mu} = \kappa\phi A_\mu$, $G_{\mu\nu} = g_{\mu\nu} + \kappa^2\phi A_\mu A_\nu$. The fields $g_{\mu\nu}(x, y)$, $A_\mu(x, y)$, and $\phi(x, y)$ transform as a tensor, a vector, and a scalar under diffeomorphisms (four-dimensional general coordinate transformations), respectively. The field ϕ is the dilaton field. Is it then natural to write down the The Einstein-Hilbert action in Kaluza-Klein five-dimensional gravity $S_{HE} = \frac{1}{2\kappa_5^2} \int d^5X R_5$, where κ_5 represents the five-dimensional coupling constant while R_5 the five-dimensional scalar curvature. The field equations of gravity and electromagnetism can be derived from the usual variational principles.

The extra dimension y is imposed to be become compact [78]. Hence y must satisfy the boundary condition $y = y + 2\pi R$. This implies that the fields $\mathcal{F}_A(x, y) = \{g_{\mu\nu}(x, y), A_\mu(x, y), \phi(x, y)\}$ are periodic in y and may be expanded in a Fourier series as follows

$$\mathcal{F}_A = \sum_{n=-\infty}^{+\infty} \mathcal{F}_{An} e^{iny/R},$$

where R is the radius of the compactified dimension. The equations of motion are

$$\square_5 \mathcal{F}_A = \left(\square_4 + \frac{n^2}{R^2} \right) \mathcal{F}_A = 0,$$

where $\square_5 = \square_4 - \partial^y \partial_y$ and $\square_4 = \partial^\mu \partial_\mu$ is the usual 4-dimensional D'Alembert operator. Comparing with the Klein-Gordon equation, one infers that only the massless zero modes $n = 0$ is observable at our present energy, while all the excited states (Kaluza-Klein states) have a mass and charge given by $m \sim |n|/R$ and $q \sim \kappa n/R$ [17], with n the mode of excitation. In 4-dimensions, all these excited states would appear with mass or momentum $\sim \mathcal{O}(n/R)$. The natural radius of compactification is the Planck length $R = l_{Pl} = 1/M_{Pl}$.

Concerning the number of degree of freedom present in the Kaluza-Klein theory, owing to the fact that the metric is a 5×5 symmetric tensor, there are 15 independent components [77]. The gauge fixings reduce the number of independent degrees of freedom to 5 (in 4-dimensions there are only 2 degrees of freedom for a massless graviton). Therefore the theory does contain particles other than just ordinary four dimensional graviton. The zero-mode of five-dimensional graviton contains a four-dimensional massless graviton with 2 physical degrees of freedom, a four-dimensional massless gauge boson with 2 physical degrees of freedom, and a real scalar with 1 physical degree of freedom. The non-zero mode of five-dimensional graviton is massive and has 5 physical degrees of freedom.

Kaluza-Klein theory, although flawed and is in contradiction with experimental data, has represented, nonetheless, an important model for building up the unification the forces of nature. Many modified version of the Kaluxa-Klein theory, in fact, have been proposed in which higher and extremely small extra dimensions have been taken into account. The higher dimensional unification approaches mainly studied in literature are [77] : 1) The Compactified Approach; 2) The Projective Approach. 3) The Noncompactified Approach.

The violation of the equivalence principle in Kaluza-Klein theories has been discussed in [79].

2.6 Quantum field theory in curved spacetime

In this Section we point out that any attempt to formulate quantum field theory on curved spacetime necessarily leads to modifying the Hilbert-Einstein action. This means adding terms containing non-linear invariants of the curvature tensor or non-minimal couplings between matter and the curvature originating in the perturbative expansion.

At high energies, a description of matter as a hydrodynamical perfect fluid is inadequate: an accurate description asks for quantum field theory formulated on a curved spacetime. Since, at scales comparable to the Compton wavelength of particles, matter has to be quantized, one can adopt a semiclassical description of gravitatu where the Einstein field equations assume the form

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \langle T_{\mu\nu} \rangle, \quad (12)$$

where the Einstein tensor $G_{\mu\nu}$ is on the left hand side while the right hand contains the expectation value of quantum stress-energy tensor which is the source of the gravitational field. Specifically, if $|\psi\rangle$ is a quantum state, then $\langle T_{\mu\nu} \rangle \equiv \langle \psi | \hat{T}_{\mu\nu} | \psi \rangle$, where $\hat{T}_{\mu\nu}$ is the quantum operator associated with the classical energy-momentum tensor of the matter field with a regularized expectation value.

If the background is curved, quantum fluctuations of matter fields give, even in absence of classical matter and radiation, non-vanishing contributions to $\langle T_{\mu\nu} \rangle$ like it happens in QED [16]. If matter fields are free, massless and conformally invariant, these corrections are

$$\langle T_{\mu\nu} \rangle = k_1 {}^{(1)}H_{\mu\nu} + k_3 {}^{(3)}H_{\mu\nu}. \quad (13)$$

Here $k_{1,3}$ are numerical coefficients and

$${}^{(1)}H_{\mu\nu} = 2R_{;\mu\nu} - 2g_{\mu\nu}\square R + 2R^{\sigma\tau}R_{\sigma\tau}g_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2, \quad (14)$$

$${}^{(3)}H_{\mu\nu} = R^\sigma{}_\mu R_{\nu\sigma} - \frac{2}{3}RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^{\sigma\tau}R_{\sigma\tau} + \frac{1}{4}g_{\mu\nu}R^2. \quad (15)$$

${}^{(1)}H_{\mu\nu}$ is a tensor derived by varying the local action,

$${}^{(1)}H_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} (\sqrt{-g} R^2). \quad (16)$$

It is divergence free, that is ${}^{(1)}H_{\mu;\nu}^\nu = 0$.

Infinites coming from $\langle T_{\mu\nu} \rangle$ are removed by introducing an infinite number of counterterms in the Lagrangian density of gravitation. The procedure yields a renormalizable theory. For example, one of these terms is $CR^2\sqrt{-g}$, where with C indicates a parameter that diverges logarithmically. Eq. (12) cannot be generated by a finite action because the gravitational field would be completely renormalizable, that is, it would eliminate a finite number of divergences to make gravitation similar to QED. On the contrary, one can only construct a truncated quantum theory of gravity. The parameter used for the expansion in loop is the Planck constant \hbar . It follows that the truncated theory at the one-loop level contains all terms of order \hbar , that is the first quantum correction. Some points have to be stressed now: 1) Matter fields are *free* and, if the Equivalence Principle is valid at quantum level, all forms of matter couple in the same way to gravity. 2) The *intrinsic* non-linearity of gravity naturally arises, and then a number of loops are necessary to take into account self-interactions interactions between matter and gravitation. In view of removing divergences at one-loop order, one has to renormalize the gravitational coupling G_{eff} and the cosmological constant Λ_{eff} . One-loop contributions of $\langle T_{\mu\nu} \rangle$ are the quantities introduced above, that is ${}^{(1)}H_{\mu\nu}$ and ${}^{(3)}H_{\mu\nu}$. Furthermore, one has to consider

$${}^{(2)}H_{\mu\nu} = 2R^\sigma{}_{\mu;\nu\sigma} - \square R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\square R + R^\sigma{}_\mu R_{\sigma\nu} - \frac{1}{2}R^{\sigma\tau}R_{\sigma\tau}g_{\mu\nu}. \quad (17)$$

In a conformally flat spacetime, one has ${}^{(2)}H_{\mu\nu} = \frac{1}{3}{}^{(1)}H_{\mu\nu}$ [16], so that only the first and third terms of $H_{\mu\nu}$ are present in (13). The tensor ${}^{(3)}H_{\mu\nu}$ is conserved only in conformally flat spacetimes and it cannot be obtained by varying a local action. The trace of the energy-momentum tensor is null for conformally invariant classical fields while, one finds that the expectation value of the tensor (13) has non-vanishing trace. This result gives rise to the so-called *trace anomaly* [16].

By summing all the geometric terms in $\langle T_{\rho}^{\rho} \rangle_{ren}$, deduced by the Riemann tensor and of the same order, one derives the right hand side of (13). In the case in which the background metric is conformally flat, it can be expressed in terms of Eqs. (14) and (15). We conclude that the trace anomaly, related to the geometric terms emerges because the one-loop approach formulates quantum field theories on curved spacetime.⁶

Masses of the matter fields and their mutual interactions can be neglected in the high curvature limit because $R \gg m^2$. On the other hand, matter-graviton interactions generate non-minimal couplings in the effective Lagrangian. The one-loop contributions of such terms are comparable to those given by the trace anomaly and generate, by conformal transformations, the same effects on gravity.

The simplest effective Lagrangian taking into account these corrections is

$$\mathcal{L}_{NMC} = -\frac{1}{2}\nabla^{\alpha}\varphi\nabla_{\alpha}\varphi - V(\varphi) - \frac{\xi}{2}R\phi^2, \quad (18)$$

where ξ is a dimensionless constant. The stress-energy tensor of the scalar field results modified accordingly but a conformal transformation can be found such that the modifications related to curvature terms can be cast in the form of a matter-curvature interaction. The same argument holds for the trace anomaly. Some Grand Unification Theories lead to polynomial couplings of the form $1 + \xi\phi^2 + \zeta\phi^4$ that generalize the one in (18). An exponential coupling $e^{-\alpha\varphi}R$ between a scalar field (dilaton) φ and the Ricci scalar appears in the effective Lagrangian of strings.

Field equations derived by varying the action \mathcal{L}_{NMC} are

$$\begin{aligned} (1 - \kappa\xi\phi^2) G_{\mu\nu} &= \kappa \left\{ \nabla_{\mu}\phi\nabla_{\nu}\phi - \frac{1}{2}g_{\mu\nu}\nabla^{\alpha}\phi\nabla_{\alpha}\phi - V g_{\mu\nu} \right. \\ &\quad \left. + \xi [g_{\mu\nu}\square(\phi^2) - \nabla_{\mu}\nabla_{\nu}(\phi^2)] \right\}, \end{aligned} \quad (19)$$

$$\square\phi - \frac{dV}{d\phi} - \xi R\phi = 0. \quad (20)$$

The non-minimal coupling of the scalar field is similar to that derived for the 4-vector potential of curved space in Maxwell theory. See below Eq. (36).

Several motivations can be provided for the non-minimal coupling in the Lagrangian \mathcal{L}_{NMC} . A nonzero ξ is generated by first loop corrections even if it does not appear in the classical action [16,80–83]. Renormalization of a classical theory with $\xi = 0$ shifts this coupling constant to a value which is small [84,85]. It can, however, affect drastically an inflationary cosmological scenario and determine its success or failure [86–89]. A non-minimal coupling is expected at high curvatures [82,83]. Furthermore, non-minimal coupling solves potential problems of primordial nucleosynthesis [90] and, besides, the absence of pathologies in the propagation of φ -waves requires conformal coupling for all non-gravitational fields [91–95]⁷.

⁶Eqs. (14) and (15) can include terms containing derivatives of the metric of order higher than fourth (fourth order being the R^2 term) if all possible Feynman diagrams are included. For example, corrections such as $R\square R$ or $R^2\square R$ can be present in ${}^{(3)}H_{\mu\nu}$ implying equations of motion that contain sixth order derivatives of the metric. Also these terms can be treated by making use of conformal transformations [25].

⁷Note, however, that the distinction between gravitational and non-gravitational fields becomes representation-dependent in ETGs, together with the various formulations of the EP [96].

The conformal value $\xi = 1/6$ is fixed at infrared scales of renormalization group [97–102]. Non-minimally coupled scalar fields have been used in inflationary scenarios [103–112]. The approach adopted was considering ξ as a free parameter to fix problems of specific inflationary scenarios [89, 113]. Cosmological reheating with strong coupling $\xi \gg 1$ has also been studied [107, 114, 115] and considered in relation with wormholes [116–118], black holes [119, 120], and boson stars [121–123]. The coupling ξ is not, in general, a free parameter but depends on the particular scalar field φ considered [82, 83, 88, 89, 113, 124–126].

2.7 Higher-order gravitational theories

Let us take into account higher order theories and their relations to scalar-tensor gravity [127]. The first straightforward generalizaion of GR is

$$\mathcal{L} = \sqrt{-g} f(R), \quad (21)$$

The variation with respect to $g^{\mu\nu}$ yields the field equations

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu f'(R) + g_{\mu\nu} \square f'(R) = 0, \quad (22)$$

with $f' \equiv df(R)/dR$. Equation (22) is a fourth-order field equations (in metric formalism). It is convenient to introduce the new set of variables

$$p = f'(R) = f'(g_{\mu\nu}, \partial_\sigma g_{\mu\nu}, \partial_\sigma \partial_\rho g_{\mu\nu}), \quad (23)$$

$$\tilde{g}_{\mu\nu} = p g_{\mu\nu}. \quad (24)$$

This choice links the *Jordan frame* variable $g_{\mu\nu}$ to the *Einstein frame* variables $(p, \tilde{g}_{\mu\nu})$, where p is some auxiliary scalar field. The term ‘‘Einstein frame’’ comes from the fact that the transformation $g \rightarrow (p, \tilde{g})$ allows to recast Eqs. (22) in a form similar to the Einstein field equations of GR. In absence of matter, hence $T_{\mu\nu}^{(m)} = 0$, the Einstein equations in are

$$\tilde{G}_{\mu\nu} = \frac{1}{p^2} \left[\frac{3}{2} p_{,\mu} p_{,\nu} - \frac{3}{4} \tilde{g}_{\mu\nu} \tilde{g}^{\alpha\beta} p_{,\alpha} p_{,\beta} + \frac{1}{2} \tilde{g}_{\mu\nu} (f(R) - Rp) \right]. \quad (25)$$

These equation can be rewritten in a more attractive way by defining $\varphi = \sqrt{\frac{3}{2}} \ln p$, which implies

$$\tilde{G}_{\mu\nu} = \left[\varphi_{,\mu} \varphi_{,\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \varphi_{,\sigma} \varphi^{,\sigma} - \tilde{g}_{\mu\nu} V(\varphi) \right], \quad (26)$$

where

$$V(\varphi) = \frac{Rf'(R) - f(R)}{2f'^2(R)} \Big|_{R=R(p(\varphi))}. \quad (27)$$

The curvature $R = R(p(\varphi))$ is inferred by inverting the relation $p = f'(R)$ (provided $f''(R) \neq 0$). The field equation (26) can be obtained from the Lagrangian (21) rewritten in terms of φ and the tilded quantities

$$\mathcal{L} = \sqrt{-\tilde{g}} \left(\frac{1}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \varphi_\mu \varphi_\nu - V(\varphi) \right). \quad (28)$$

which has the same form of Einstein gravity minimally coupled to a scalar field in presence of a self-interaction potential. Equation Eq. (28) clearly suggests that why the set of variables $(\tilde{g}_{\mu\nu}, p)$ is called Einstein frame [52, 53, 128].

A comment is in order. As we have seen, in the vacuum, one can pass from the Einstein frame to the Jordan frame. However, in the presence of matter fields, a caution is required since particles and photons have to be dealt in different ways. In the case of photons, their worldlines are geodesics both in the Jordan frame and in the Einstein frame. This is not the case for massive particles since their geodesic in the Jordan frame are no longer transformed into geodesic in the Einstein frame, and vice-versa, and therefore, the two frames are not equivalent. The consequence is that the physical meaning of conformal transformations is not straightforward, although the mathematical transformations are, in principle, always possible. These considerations extend to any higher-order or non-minimally coupled theory.

2.8 Some aspects of the Equivalence Principle

As we have mentioned, in the previous Sections, the formulation of the EP is the equivalence between inertial and gravitational mass $m_I = m_G$ (Galileo’s experiment), which implies that all bodies fall with the same acceleration, independently of their mass and internal structure, in a given gravitational field (universality of free fall or WEP). A more precise statement of WEP is [1]

“If an uncharged body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition”

This formulation of WEP was enlarged by Einstein adding a new fundamental part: according to which in a local inertial frame (the freely-falling elevator) not only the laws of mechanics behave in it as if gravity were absent, but *all* physical laws (except those of gravitational physics) have the same behaviour. The current terminology refers to this principle as the *Einstein Equivalence Principle* (EEP). A more precise statement is [1]

“The outcome of any local non-gravitational test experiment is independent of the velocity of the (free falling) apparatus and the outcome of any local non-gravitational test experiment⁸ is independent of where and when in the universe it is performed”.

From the EEP it follows that the gravitational interaction must be described in terms of a curved spacetime, that is the postulates of the so-called metric theories of gravity have to be satisfied [1]:

1. spacetime is endowed with a metric $g_{\mu\nu}$;
2. the world lines of test bodies are geodesics of that metric;
3. in local freely falling frames (called *local Lorentz frames*), the non-gravitational laws of physics are those of Special Relativity.

These definitions characterize the most fundamental feature of GR, hence the Equivalence Principle, as well as the physical properties that allow to discriminate between GR and other metric theories of gravity. In the ETGs some additional features arise because these definitions depend on the conformal representation of the theory adopted. More precisely, in scalar-tensor gravity, massive test particles in the Jordan frame follow geodesics, satisfying the WEP, but the same particles deviate from geodesic motion in the Einstein frame (a property referred to as non-metricity of the theory). This difference shows that the EP is formulated in a representation-dependent way [96]. This serious shortcoming has not yet been addressed properly; for the moment we proceed ignoring this problem.

In what follows we shall discuss some specific features related to the Equivalence Principle:

⁸A “local non-gravitational experiment” is defined as an experiment performed in a small size freely falling laboratory, in order to avoid the inhomogeneities of the external gravitational field, and in which any gravitational self-interaction can be ignored. For example, the measurement of the fine structure constant is a local non-gravitational experiment, while the Cavendish experiment is not.

- Let us assume that WEP is violated. Let us assume, for example, that the inertial masses (m_{Ii}) in a system differ from the passive ones,

$$m_{Pi} = m_{Ii} \left(1 + \Sigma_A \eta^A \frac{E^A}{m_{Ii} c^2} \right), \quad (29)$$

where E^A is the internal energy of the body connected to the A-interaction and η^A is a dimensionless parameter quantifying the violation of the WEP. It is then convenient to introduce a new dimensionless parameter (the *Eötvös ratio*) considering, for example, two bodies moving with accelerations

$$a_i = \left(1 + \Sigma_A \eta^A \frac{E^A}{m_{Ii} c^2} \right) g \quad (i = 1, 2); \quad (30)$$

where g is now the acceleration of gravity. Then we define the Eötvös ratio as

$$\eta = 2 \frac{|a_1 - a_2|}{|a_1 + a_2|} = \Sigma_A \eta^A \left(\frac{E_1^A}{m_{I1} c^2} - \frac{E_2^A}{m_{I2} c^2} \right). \quad (31)$$

The measured value of η provides information on the WEP-violation parameters η^A . Experimentally, the equivalence between inertial and gravitational masses is strongly confirmed [1].

- The minimal coupling prescriptions. In electrodynamics the interaction is introduced replacing the partial derivative with the covariant derivative $\partial_\mu \rightarrow \mathcal{D}_\mu \equiv \partial_\mu + ieA_\mu$ [129] (see also [59]). A similar scheme is used to introduce the gravitational interaction

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu}, \quad \partial_\mu \rightarrow \nabla_\mu, \quad \sqrt{-\eta} d^4x \rightarrow \sqrt{-g} d^4x, \quad (32)$$

Here $\eta_{\mu\nu}$ is the flat Minkowski metric and $g_{\mu\nu}$ is the Riemannian one, while η and g are their determinants [130–132].

Consider the Maxwell equations in a curved spacetime

$$F^{\alpha\beta}{}_{;\beta} = 4\pi J^\alpha, \quad F_{\alpha\beta;\gamma} + F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} = 0, \quad (33)$$

and the four-vector potential A^μ related to the Maxwell field by $F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha$. In this framework, however, a problem arises. Using the above-mentioned rule one obtains two possible equations from the first of eqs. (33):

$$A^{\beta;\alpha}{}_{;\beta} - A^{\alpha;\beta}{}_{;\beta} = 4\pi J^\alpha, \quad (34)$$

or

$$A^{\beta;\alpha}{}_{;\beta} - A^{\alpha;\beta}{}_{;\beta} + R^\alpha{}_\beta A^\beta = 4\pi J^\alpha; \quad (35)$$

while the second of eqs. (33) yields, using the Lorentz gauge $\nabla_\mu A^\mu = 0$,

$$(\Delta_{dR} A)^\alpha = 4\pi J^\alpha, \quad (36)$$

where

$$(\Delta_{dR} A)^\alpha = -\square A^\alpha + R^\alpha{}_\beta A^\beta \quad (37)$$

and Δ_{dR} is the de Rham vector wave operator. Now the question is: both Maxwell equations for the four-potential A^μ are obtained using the “comma goes to semicolon” rule, but which is the correct one? The answer is: the one obtained using the de Rham operator. As consequence, we see that “correspondence rules” are not sufficient to write down equations in curved space from

known physics in flat space when second derivatives are involved (that is, in most situations of physical interest). In such cases, extra caution is needed⁹

The minimal coupling prescription here discussed is connected with the mathematical formulation of the EEP (actually, to implement the EEP one needs to put in special-relativistic form the laws under consideration and then proceed to find the general-relativistic formulation, switching on gravity. In other words, we have to apply minimal coupling prescriptions with the caveat already discussed).

- The last point is strictly related with the scalar-tensor theories of gravity, do these theories satisfy the EEP?

To address this question one has to generalize the above two principles and introduce new concepts. Following Will [1], one introduces the notion of “purely dynamical metric theory”, i.e. a theory in which *the behaviour of each field is influenced to some extent by a coupling to at least one of the other fields in the theory* [1]. In this respect, GR is a purely dynamical theory, as well as the Brans-Dicke theory since the equations for the metric involve the scalar field, and vice-versa.

In these theories, the calculations of the metric is done in two stages: 1) the assignment of boundary conditions “far” from the local system; 2) infer the solutions of equations for the fields generated by the local system. Owing to the coupling of the metric with fields (for given boundary conditions), the latter will influence the metric. This implies that local gravitational experiments can depend on where the lab is located in the universe, as well as on its velocity relative to the external world. One of the consequence of such a new physical scenario is that in a Brans-Dicke theory, and more generally in Scalar Tensor Theories, the gravitational coupling “constant” turns out to depend on the asymptotic value of the scalar field.

All these considerations are strictly related to the *Strong Equivalence Principle* (SEP) [1]:

- (i) “WEP is valid for self-gravitating bodies as well for test bodies;
- (ii) the outcome of any local test experiment is independent of the velocity of the (freely falling) apparatus;
- (iii) the outcome of any local test experiment is independent on where and when in the universe it is performed” [1].

The SEP differs from the EEP because it includes the self-gravitating interactions of bodies (such as planets or stars), and because of experiments involving gravitational forces (*e.g.*, the Cavendish experiment). SEP reduces to the EEP when gravitational forces are ignored. In connection with the SEP, many authors have conjectured that the only theory compatible with the Strong Equivalence Principle is GR (that is $SEP \rightarrow GR$ – only).

2.9 The Schiff conjecture

The Schiff conjecture represents one of the most important topic related to the foundations of the gravitational physics. Its original formulation asserts that *every theory of gravity that satisfies the WEP and is relativistic necessarily satisfies the EEP, and is consequently a metric theory of gravity*. Hence $WEP \Rightarrow EEP$. Later, Will proposed a slight modification of Schiff conjecture: *every theory of*

⁹As stressed, for example, in [15], such a prescription does not work for interactions which do not have a “Minkowskian” counterpart. These interactions are expressed in terms of the Riemann tensor or some function of it and occur, for example, in the study of the free fall of a particle with spin: the corresponding equations of motion (Papapetrou equations) involve a contribution in which the spin tensor couples to the Riemann tensor [15]. Such a contribution can not be obtained from the prescriptions given above. This motion is described by the corrected geodesic equation [7].

gravity that satisfies WEP and the principle of universality of gravitational red shift (UGH) necessarily satisfies EEP. Hence in such a case $WEP + UGR \rightarrow EEP$.

Let us discuss in some details these topics. Notice that the correctness Schiff's conjecture implies that the Eötvös and the gravitational red-shift experiments would provide a direct empirical confirmation of the EEP, with the consequence that gravity can be interpreted as a geometrical (curved space-time) phenomenon. The relevance of such a fundamental aspect of the gravitational physics led to different mathematical approaches to prove the Schiff conjecture. These frameworks encompass all metric theories, as well as non metric theories of gravity. Lightman and Lee [133, 134] proved Schiff's conjecture in the framework of the so called $TH\epsilon\mu$ formalism. They consider the motion of a charged particles (electromagnetic coupling) in a static spherically symmetric gravitational field $U = GM/r$

$$S_{TH\epsilon\mu} = - \sum_a m_a \int dt \sqrt{T - H v_a^2} + \sum_a e_a \int dt v_a^\mu A_\mu(x_a^\mu) + \frac{1}{2} \int d^4x \left(\epsilon \mathbf{E}^2 + \frac{\mathbf{B}^2}{\mu} \right),$$

where $m_a, e_a, v_a^\mu \equiv \frac{dx_a^\mu}{dt}$ represent the mass, the charge and the velocity of the particle a . The parameters $TH\epsilon\mu$ do depend on the gravitational field U , that is they essentially account for the response of the electromagnetic fields to the external potential, and may vary from theory to theory. A metric theory must satisfy the relation $\epsilon = \mu = \sqrt{\frac{H}{T}}$ for all U . In the case of non-metric theories, the parameters $TH\epsilon\mu$ may depend on the species of particles or on the field coupling to gravity. The metric is given by $ds^2 = T(r)dt^2 - H(r)(dr^2 + r^2d\Omega)$. Lightmann and Lee showed in [133] that the rate of fall of a test body made up of interacting charged particles does not depend on the structure of the body (WEP) *if and only if* $\epsilon = \mu = \sqrt{\frac{H}{T}}$. This implies $WEP \Rightarrow EEP$, satisfying hence the Schiff conjecture. Will generalized the Dirac equation in $TH\epsilon\mu$ formalism, and computed the gravitational red-shift experienced by different atomic clocks showing that the red-shift is independent on the nature of clacks (Universality of Gravitational Red-shift (UGR)) *if and only if* $\epsilon = \mu = \sqrt{\frac{H}{T}}$ [135]. Therefore $UGR \Rightarrow EEP$, verifying in such a way another aspect of the Schiff conjecture (see also [136]).

W.-T. Ni was able to provide a counterexample to Schiff's conjecture by considering the coupling between a pseudoscalar field ϕ with the electromagnetism field $\mathcal{L}_{\phi F} \sim \phi \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}$, where $\epsilon^{\alpha\beta\gamma\delta}$ is the completely anti-symmetric Levi-Civita symbol [137]. In [138–140] the Schiff conjecture is analyzed in the framework of gravitational non-minimally coupled theories. More specifically, the total Lagrangian density considered is given by $\mathcal{L}_{NMC} = \frac{R}{16\pi G} + \mathcal{L}_M + \mathcal{L}_I(\psi^A, g_{\mu\nu})$, where $\mathcal{L}_I(\psi^A, g_{\mu\nu})$ is the Lagrangian density of some field ψ^A non-minimally coupled to gravity [139, 140], while $\mathcal{L}_I = \chi^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}$ in [138], where $\chi^{\alpha\beta\gamma\delta}$ depends on matter, for example $\chi^{\alpha\beta\gamma\delta} = \bar{\psi} \sigma^{\alpha\beta} \psi \bar{\psi} \sigma^{\gamma\delta} \psi, \psi^{\alpha\mu} \psi^{\beta\nu} - \psi^{\beta\mu} \psi^{\alpha\nu}$, where ψ is a spin-half field and $\psi^{\alpha\beta}$ is a (nongravitational) spin-2 field. Both results show that these gravitational theories are in general, incompatible with Schiff's conjecture.

These counterexample indicate that a rigorous proof of such a conjecture is impossible. However, some powerful arguments of plausibility can be formulated. One of them is based upon the assumption of energy conservation [141]. Following [142], consider a system in a quantum state $|A\rangle$ that decays in a state $|B\rangle$, with the emission of a photon with frequency ν . The quantum system falls a height H in an external gravitational field $gH = \Delta U$, so that the system in state B falls with acceleration g_B and the photon frequency is shifted to ν' . Assuming a violation of the WEP, the acceleration g_A and g_B of the system A and B are

$$g_A = g \left(1 + \frac{\alpha E_A}{m_A} \right), \quad g_B = g \left(1 + \frac{\alpha E_B}{m_B} \right), \quad E_B - E_A = h\nu$$

that is they depend on that portion of the internal energy of the states. Here ν is frequency of the quantum emitted by the system $|A\rangle$. The conservation of energy implies that there must be a corresponding violation of local position invariance in the frequency shift given by $\frac{\nu' - \nu}{\nu} = (1 + \alpha)\Delta U$, where ν' is the frequency of the quantum at the bottom of the trajectory. The Eötvös parameter is (for $m_A \sim m_B \sim m$)

$$\eta = \frac{|g_B - g_A|}{|g_B + g_A|} \simeq \frac{\alpha(E_A - E_B)}{m}.$$

The Schiff conjecture is still nowadays an argument of a strong scientific debate and deep scrutiny.

2.10 Mach's principle and the variation of G

Following Bondi [12] there are, at least in principle, two entirely different ways of measuring the rotational velocity of Earth. The first is a purely terrestrial experiment (*e.g.*, a Foucault pendulum), while the second is an astronomical observation consisting of measuring the terrestrial rotation with respect to the fixed stars. In the first type of experiment the motion of the Earth is referred to an idealized inertial frame in which Newton's laws are verified. However, a unique general relativistic approach to define rotations has been introduced by Pirani considering the bouncing photons [143, 144] (see also [145]). In the second kind of experiment the frame of reference is connected to a matter distribution surrounding the Earth and the motion of the latter is referred to this matter distribution. In this way we face the problem of Mach's principle, which essentially states that the local inertial frame is determined by some average motion of distant astronomical objects [12, 15].¹⁰ Trying to incorporate Mach's principle into metric gravity, Brans and Dicke constructed a theory alternative to GR [13]. Taking into account the influence that the total matter has at each point (constructing the "inertia"), these two authors introduced, together with the standard metric tensor, a new scalar field of gravitational origin as the effective gravitational coupling. This is why the theory is referred to as a "scalar-tensor" theory; actually, theories in this spirit had already been proposed years earlier by Jordan, Fierz, and Thiery (see the book [147]). An important ingredient of this approach is that the gravitational "constant" is actually a function of the total mass distribution, that is of the scalar field, and is actually variable. In this picture, gravity is described by the Lagrangian density

$$\mathcal{L}_{BD} = \sqrt{-g} \left[\varphi R - \frac{\omega}{\varphi} \nabla^\mu \varphi \nabla_\mu \varphi + \mathcal{L}^{(m)} \right], \quad (38)$$

where ω is the dimensionless Brans-Dicke parameter and $\mathcal{L}^{(m)}$ is the matter Lagrangian including all the non-gravitational fields. As stressed by Dicke [51], the Lagrangian (38) has a property similar to one already discussed in the context of higher order gravity. Under the conformal transformation $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega = \sqrt{G_0 \varphi}$, the Lagrangian (38) is mapped into

$$\mathcal{L} = \sqrt{-\tilde{g}} \left(\tilde{R} + G_0 \tilde{\mathcal{L}}^{(m)} + G_0 \tilde{\mathcal{L}}^{(\Omega)} \right), \quad (39)$$

where

$$\tilde{\mathcal{L}}^{(\Omega)} = -\frac{(2\omega + 3)}{4\pi G_0 \Omega} (\nabla^\alpha \sqrt{\Omega})(\nabla_\alpha \sqrt{\Omega}), \quad (40)$$

and $\tilde{\mathcal{L}}^{(m)}$ is the conformally transformed Lagrangian density of matter. In this way the total matter Lagrangian $\tilde{\mathcal{L}}_{tot} = \tilde{\mathcal{L}}^{(m)} + \tilde{\mathcal{L}}^{(\Omega)}$ has been introduced. The field equations are now written in the form of

¹⁰An interesting discussion on this topic, also connected with different theories of space, both in philosophy and in physics, is found in Dicke's contribution "The Many Faces of Mach" in *Gravitation and Relativity* [146]. This discussion presents also the problematic position that Einstein had on Mach's principle.

Einstein-like equations as

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = G_0\tilde{\tau}_{\mu\nu}, \quad (41)$$

where the stress-energy tensor is now the sum of two contributions,

$$\tilde{\tau}_{\mu\nu} = T_{\mu\nu}^{(m)} + \Lambda_{\mu\nu}(\Omega). \quad (42)$$

Dicke noted that this new (tilded, or Einstein frame) form of the scalar-tensor theory has certain advantages over the theory expressed in the previous (non-tilded, or Jordan frame) form; the Einstein frame representation, being similar to the Einstein standard description is familiar and easier to handle in some respects. But, in this new form, Brans-Dicke theory also exhibits unpleasant features. If we consider the motion of a spinless, electrically neutral, massive particle, we find that in the conformally rescaled world its trajectory is no longer a geodesic. Only null rays are left unchanged by the conformal rescaling. This is a manifestation of the fact that the rest mass is not constant in the conformally transformed world and the equation of motion of massive particles is modified by the addition of an extra force proportional to $\nabla^\mu\Omega$ [51]. Photon trajectories, on the other hand, are not modified because the vanishing of the photon mass implies the absence of a preferred physical scale and photons stay massless under the conformal rescaling, therefore their trajectories are unaffected.

This new approach to gravitation has increased the relevance of theories with varying gravitational coupling. They are of particular interest in cosmology since, as we discuss in detail in the following chapters, they have the potential to circumvent many shortcomings of the standard cosmological model. We list here the Lagrangians of this type which are most relevant for this review.

- The low-energy limit of the bosonic string theory [148–150] produces the Lagrangian

$$\mathcal{L} = \sqrt{-g} e^{-2\phi} (R + 4g^{\mu\nu}\phi_\mu\phi_\nu - \Lambda). \quad (43)$$

- The general scalar-tensor Lagrangian is

$$\mathcal{L}_{ST} = \sqrt{-g} \left[f(\varphi)R - \frac{\omega(\varphi)}{2} g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi - V(\varphi) \right], \quad (44)$$

where $f(\varphi)$ and $\omega(\varphi)$ are arbitrary coupling functions and $V(\varphi)$ is a scalar field potential. The original Brans-Dicke Lagrangian is contained as the special case $f(\varphi) = \varphi$, $\omega(\varphi) = \omega_0/\varphi$ (with ω_0 a constant), and $V(\varphi) \equiv 0$.

- A special case of the previous general theory is that of a scalar field non-minimally coupled to the Ricci curvature, which has received so much attention in the literature to deserve a separate mention,

$$\mathcal{L}_{NMC} = \sqrt{-g} \left[\left(\frac{1}{16\pi G} - \frac{\xi}{2} \right) R - \frac{1}{2} g_{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\varphi) \right], \quad (45)$$

where ξ is a dimensionless non-minimal coupling constant. This explicit non-minimal coupling was originally introduced in the context of classical radiation problems [151] and, later, conformal coupling with $\xi = 1/6$ was discovered to be necessary for the renormalizability of the $\lambda\varphi^4$ theory on a curved spacetime [16, 152]. The corresponding stress-energy tensor (“improved energy-momentum tensor”) and the relevant equations will be discussed later. In particular, the theory is conformally invariant when $\xi = 1/6$ and either $V \equiv 0$ or $V = \lambda\varphi^4$ [16, 131, 152, 153].

All these theories exhibit a non-constant gravitational coupling. The Newton constant G_N is replaced by the effective gravitational coupling

$$G_{eff} = \frac{1}{f(\varphi)}, \quad (46)$$

in eq. (44) which, in general, is different from G_N (we use ϕ as the generic function describing the effective gravitational coupling). In string theory or with non-minimally coupled scalars, such functions are specified in (43) and (45). In particular, in spatially homogeneous and isotropic cosmology, the coupling G_{eff} can only be a function of the epoch, *i.e.*, of the cosmological time.

We stress that all these scalar-tensor theories of gravity do not satisfy the SEP because of the above mentioned feature: the variation of G_{eff} implies that local gravitational physics depends on the scalar field via ϕ . We have then motivated the introduction of a stronger version of the Equivalence Principle, the SEP. General theories with such a peculiar aspect are called *non-minimally coupled theories*. This generalizes older terminology in which the expression “non-minimally coupled scalar” referred specifically to the field described by the Lagrangian \mathcal{L}_{NMC} of (45), which is a special case of (44).

Let us consider, as in (44), a general scalar-tensor theory in presence of “standard” matter with total Lagrangian density $\phi R + \mathcal{L}^{(\phi)} + \mathcal{L}^{(m)}$, where $\mathcal{L}^{(m)}$ describes ordinary matter. The dynamical equations for this matter are contained in the covariant conservation equation $\nabla^\nu T_{\mu\nu}^{(m)} = 0$ for the matter stress-energy tensor $T_{\mu\nu}^{(m)}$, which is derived from the variation of the total Lagrangian with respect to $g^{\mu\nu}$. In other words: concerning standard matter, everything goes as in GR (*i.e.*, $\eta_{\mu\nu} \rightarrow g_{\mu\nu}$, $\partial_\mu \rightarrow \nabla_\mu$) following the minimal coupling prescription. What is new in these theories is the way in which the scalar and the metric degrees of freedom appear: now there is a direct coupling between the scalar degree of freedom and a function of the tensor degree of freedom (the metric) and its derivatives (specifically, with the Ricci scalar of the metric $R(g, \partial g, \partial^2 g)$). Then, confining our analysis to the cosmological arena, we face two alternatives. The first is

$$\lim_{t \rightarrow \infty} G_{eff}(\phi(t)) = G_N; \quad (47)$$

this is the case in which standard GR cosmology is recovered at the present time in the history of the universe. The second possibility occurs if the gravitational coupling is not constant today, *i.e.*, G_{eff} is still varying with the epoch and $\dot{G}_{eff}/G_{eff}|_{\text{now}}$ (in brief \dot{G}/G) is non-vanishing.

In many theories of gravity, then, it is perfectly conceivable that G_{eff} varies with time: in some solutions G_{eff} does not even converge to the value observed today. What do we know, from the observational point of view, about this variability? There are three main avenues to analyze the variability of G_{eff} : the first is *lunar laser ranging* (LLR) monitoring the Earth-Moon distance; the second is information from solar astronomy; the third consists of data from binary pulsars. The LLR consists of measuring the round trip travel time and thus the distances between transmitter and reflector, and monitoring them over an extended period of time. The change of round trip time contains information about the Earth-Moon system. This round trip travel time has been measured for more than twenty-five years in connection with the Apollo 11, 14, 15, and the Lunakhod 2 lunar missions. Combining these data with those coming from the evolution of the Sun (the luminosity of main sequence stars is quite sensitive to the value of G) and the Earth-Mars radar ranging, the current bounds on \dot{G}/G allow at most 0.4×10^{-11} to 1.0×10^{-11} per year [154]. The third source of information on G -variability is given by binary pulsars systems. In order to extract data from this type of system (the prototype is the famous binary pulsar PSR 1913+16 of Hulse and Taylor [155]), it has been necessary to extend the post-Newtonian approximation, which can be applied only to a weakly (gravitationally) interacting n -body system, to strongly (gravitationally) interacting systems. The order of magnitude of \dot{G}/G allowed by these strongly interacting systems is $2 \times 10^{-11} \text{ yr}^{-1}$ [154].

A general remark is necessary at this point. According to the Mach Principle, gravity can be considered as an average interaction given by the distribution of celestial bodies. This means that the same gravitational coupling can be related to the spacetime scale, then supposing a variation of G_N is an issue to make more Machian the theory. From an experimental point of view, this fact reflects on the uncertainties of the measurements of G_N and it could constitute a test for any alternative theory of gravity with respect to GR.

Finally it is worth noticing that there exist also Higgs-scalar-tensor theories (see for example, [156–158]) where inertia and gravity are strongly related. Such theories have been introduced to solve the issues raised in the Brans-Dicke theory where the observational results, coming from the Mercury perihelion shift, are not matched. In view of this shortcoming, Dicke postulated the existence of a mass-quadrupole momentum giving rise to an oblateness correction of the Sun shape. Since this feature was not detected, Higgs-scalar-tensor theories were deemed necessary.

2.11 Violation of the weak equivalence principle and quintessence

In the previous Sections, we have pointed out that over the last years several, observations led to the conclusions that the observed Universe is dominated by some form of (homogeneously distributed) DE. In modified gravity the DE can be described by introducing one or more than one scalar fields coupled (minimally or non-minimally) to gravity. A candidate for DE is quintessence (the energy density associated a scalar field that evolves slowly in time) [159–163]. In this scenario, fundamental coupling constants do depend on time even in late cosmology [159, 164–167]. This because, as we have seen, it is usual that in modified models of gravity the fundamental coupling constants may depend on the scalar field, that vary during the Universe evolution. Clearly, an observation of a possible time-variation of fundamental constants could be a signal in favour of quintessence, and more generally, of modified theories of gravity, since no such time dependence would be connected to DE in the case in which the latter is described by a cosmological constant.¹¹

It is expected that, in a quintessence scenario, the gauge couplings may vary owing to the coupling between the field $\phi(x)$ and the kinetic term for the gauge fields in a GUT [20]. For example, for the electromagnetic field one has $L_F = \frac{1}{4}Z_F(\phi(x))F^2$, where $F^2 = F_{\mu\nu}F^{\mu\nu}$. Such a coupling preserves all symmetries and makes the renormalized gauge coupling $g \sim Z_F^{-1/2}$ dependent on time through the evolution of the field $\phi(x)$ [174]. As argued in [174], the coupling of the field $\phi(x)$ with matter induces a new gravity-like force that does depend on the composition of the test bodies. In this respect a violation of the equivalence principle arises [175].

Along these lines, very recently it have been proposed new and general models in which a light scalar field (playing the role of scalar Dark Matter) is introduced in the gravity action (similar to Eq. (72)). In the most and simplest general case, in fact, the light scalar field couples non-universally to the standard matter fields, leading as a consequence to a violation of the Einstein equivalence principle (EEP). As discussed in the previous Sections, the scalar fields are predicted in high dimensional theories, in particular in string theory with the dilaton and the moduli fields [148, 176, 177]. It is worth to mention that these models based on light scalar field provide galactic and cosmological predictions for low masses, ranging from 10^{-24} eV to 10^{-22} eV (see for example, Refs. [178–181]. Here we recall the total action in which a microscopic modeling for the coupling between the scalar field and standard matter has been conveniently introduced [182, 183]

$$S = \int d^4x [\mathcal{L}_{NMC} + \mathcal{L}_{SM} + \mathcal{L}_{int}], \quad (48)$$

where \mathcal{L}_{NMC} is the Lagrangian density (72), \mathcal{L}_{SM} the Lagrangian density of the Standard Model, and finally \mathcal{L}_{int} is the Lagrangian density of the interaction, which can be of two form [182–184]

$$\mathcal{L}_{int} = \phi^a \left[\frac{d_e^{(a)}}{4\mu_0} F^2 - \frac{d_g^{(a)}\beta_3}{2g_3} (F^A)^2 - \sum_{i=e,u,d} (d_{m_i}^{(a)} + \gamma_{m_i} d_g^{(a)}) m_i \bar{\psi}_i \psi_i \right] \quad (49)$$

¹¹A low value of the electromagnetic fine structure constant α_{em} was reported [168] for absorption lines in the light from distant quasars. The data are consistent with a variation $\Delta\alpha_{em}/\alpha_{em} \simeq -0.7 \times 10^{-5}$ for a cosmological red-shift $z \approx 2$. Such a result has renewed the interest on the variation of fundamental couplings (see for example [166, 167, 169–173]).

Here $a = 1$ and $a = 2$ correspond to the linear and quadratic [184–186] coupling between scalar and matter field, respectively, while $F_{\mu\nu}$ and $F_{\mu\nu}^A$ are the electromagnetic and the gluon strength tensors, μ_0 the magnetic permeability, g_3 the QCD gauge coupling, β_3 the β function for the running of g_3 , m_i the mass of the fermions (electron and light quarks u, d), γ_{m_i} the anomalous dimension giving the energy running of the masses of the QCD coupled fermions, and finally d^a are the constants characterizing the interaction between the light scalar field and the different matter sectors. The main consequence of the model based on (48) is that the constants of nature turn out to be linearly or quadratically depending on the scalar field [182, 183]. For the electromagnetic fine structure constant α_{EM} , the masses m_i of the fermions, and the QCD energy scale Λ_3 , one obtains

$$\begin{aligned}\alpha_{EM}(\phi) &= \alpha_{EM} \left[1 + \frac{d_e^{(a)} \phi^a}{a} \right] \\ m_i(\phi) &= m_i \left[1 + \frac{d_{m_i}^{(a)} \phi^a}{a} \right] \quad i = e, u, d \\ \Lambda_3(\phi) &= \Lambda_3 \left[1 + \frac{d_g^{(a)} \phi^a}{a} \right]\end{aligned}$$

with $a = 1, 2$ for the linear and the quadratic coupling. The dependence of the particle masses on the scalar field suggests to study tests of the universality of free fall. Following [187], one gets that the differential acceleration between two bodies A and B located at the same position in a gravitational field generated by a body C , is

$$\Delta \mathbf{a} \equiv \mathbf{a}_A - \mathbf{a}_B = -[\alpha_A(\phi) - \alpha_B(\phi)][\nabla\phi + \mathbf{v}\dot{\phi}], \quad (50)$$

where $\alpha_{A,B} = \frac{\partial \ln m_{A,B}(\phi)}{\partial \phi}$ and \mathbf{v} the particle velocity. Using the expressions for the scalar field derived in the case of a spherically symmetric extended body with radius R and constant matter density with mass M , one infers the explicit expression for the Eötvös parameter η (Eq. (64)) [187]

$$\eta = 2 \frac{|\mathbf{a}_A - \mathbf{a}_B|}{|\mathbf{a}_A + \mathbf{a}_B|} = \begin{cases} \Delta \tilde{\alpha}^{(1)} s_C^{(1)} e^{-r/\lambda_\phi} \left(1 + \frac{r}{\lambda_\phi} \right) & \text{(linear coupling)} \\ \Delta \tilde{\alpha}^{(2)} s_C^{(2)} \frac{\phi_0}{2} \left(1 - s_C^{(2)} \frac{GM_C}{r} \right) & \text{(quadratic coupling)} \end{cases} \quad (51)$$

Here $\Delta \tilde{\alpha}^{(a)} = \tilde{\alpha}_A^{(a)} - \tilde{\alpha}_B^{(a)}$, with $a = 1, 2$ and $\tilde{\alpha}^{(a)}$ is a combination of the coefficients $d_{e,m,g}^{(a)}$ and the dilatonic charges associated to the bodies A and B , $s_C^{(1)} = 3\tilde{\alpha}_C^{(1)} \frac{x \cosh x - \sinh x}{x^3}$, with $x = \frac{R}{\lambda_\phi}$ ($\lambda_\phi = m_\phi^{-1}$ is the Compton wavelength of the scalar field), and $s_C^{(2)} = \tilde{\alpha}_C^{(2)} J_\pm(y)$, with $y = \sqrt{3|\tilde{\alpha}_C^{(2)}|GM_A/R_A}$ and $J_\pm = \pm 3 \frac{y - \tanh y}{y^3}$, and finally ϕ_0 is the amplitude of the scalar field.

An interesting aspect of these results is that in the neighborhood region of a central body and in the limit of strong coupling, for the quadratic coupling Eq. (51) assumes the form $\eta \simeq \Delta \tilde{\alpha}^{(2)} s_C^{(2)} \phi_0^2 \frac{h}{R_C + h}$, where h is the altitude with respect to the radius R_C . On the other hand, for small coupling and far from the gravitational source, one gets $\eta \simeq s_C^{(2)} \Delta \tilde{\alpha}^{(2)} \phi_0^2$, that is the Eötvös parameter is independent on the location of the two masses. As argued in [187], this particular forms of the Eötvös parameter could be potentially tested in dedicated experiments.

Finally we comment the possibility to violate the Einstein equivalence principle by measuring the frequency ratio between two clocks located at the same position and working on different atomic transition. Defining $Y = X_A/X_B$, where $X_{A,B}$ are the specific transitions for each clocks, one finds [187]

$$\frac{Y(t, \mathbf{x})}{Y_0} = \begin{cases} K + \Delta\kappa^{(1)} \left[\phi_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{x} + \delta) - s_A^{(1)} \frac{GM_A}{r} e^{-r/\lambda_\phi} \right] & \text{(linear coupling)} \\ K + \Delta\kappa^{(2)} \frac{\phi_0^2}{2} \left[\left(1 - s_A^{(2)} \frac{GM_A}{r} \right)^2 + \cos(2\omega t + 2\delta) \left(1 - s_A^{(2)} \frac{GM_C}{r} \right)^2 \right] & \text{(quadratic coupling)} \end{cases} \quad (52)$$

where K is an unobservable constant and $k^{(a)}$, $a = 1, 2$ depend on the constants $d_{e,m,g}^{(a)}$.

2.12 Equivalence principle in screening mechanisms

As extensively discussed in the previous Sections, the introduction of the extended theories of gravity have been motivated by the necessity to explain the observed cosmic acceleration, hence to provide a "geometric" interpretation of the DE. In these models, gravity is modified on large distances. However, although modifications to GR must be relevant on large scales, they are strongly constrained in Solar System (in what follows we shall refer to [188]). In fact, any deviation is subdominant in Solar System tests by a factor $\lesssim 10^{-5}$, and the latter is further reduced in some specific theories (in [189, 190] is discussed the case of theory that predicts strong violations of the weak equivalence principle for which deviations are constrained by a factor $\lesssim 10^{-15}$). As an example of extended theories of gravity, consider once again the Brans-Dicke gravity (the scalar field ϕ couples to gravity and is parameterized by the parameter ω_{BD}). In the non-relativistic limit, one finds the equation of motion for ϕ

$$\nabla^2 \phi = -\frac{8\pi G\rho}{2 + 3\omega_{BD}} \quad (53)$$

from which one derives the PPN parameter $|\gamma - 1| = (2 + \omega_{BD})^{-1}$. The Cassini bound $|\gamma - 1| < 2.11 \times 10^{-5}$ [191] implies $\omega_{BD} > 4 \times 10^4$. From (53) it follows that the effective coupling to matter is $\alpha_{eff} \sim 1/\omega_{BD} \lesssim 10^{-4}$. As a consequence, any Brans-Dicke like modifications of GR must be subdominant on all scales by a factor $\sim 10^4$, hence such theories are cosmologically irrelevant. A similar conclusions follows if one assumes that the scalar field is massive, so that the field equation (53) gets modified a $(\nabla^2 + m^2)\phi = -8\pi G\alpha\rho$, yielding, for a a static, spherically symmetric body, a Yukawa-like potential $V(r) = \frac{GM}{r} (1 + 2\alpha^2 e^{-mr})$ (experiments constrained Yukawa-like potentials on distances ranging from the Earth-Moon scale [191, 192] to micron scales [193, 194], so that $m > (\mu\text{m})^{-1}$ is required to evade Solar system tests).

These two examples show that solar system tests constraint these models with the consequence that they do not have any cosmological relevance because the force must either be too weak, or too short ranged. Such difficulties are avoided by screening mechanisms by nonlinear modifications of the Poisson equation. The modifications are such that deviations from GR in the Solar system are dynamically suppressed, without requiring a fine-tuning of the mass or the coupling to matter. Screening mechanisms studied in literature are:

- Chameleon screening [195, 196] (the mass of the field changes dynamically mediating short ranged forces in the Solar System but may have effects on cosmological scales).
- Symmetron screening [195, 196] (the coupling to matter varies dynamically so that it is uncoupled in the Solar System and may induces deviations from GR on cosmological scales).

- Vainshtein’s mechanism [197] (nonlinear kinetic terms alter the field profile sourced by massive bodies. In such a case fifth forces are highly suppressed in the Solar System, while on cosmological scales, theories that exhibit this mechanism can self-accelerate without a cosmological constant, which makes them interesting alternatives to Λ CDM cosmologies).

An interesting aspect of screening mechanisms, is that they may violate the equivalence principle [198] (see also [199]). For example, in chameleon theories one can define a scalar charge for an object [198]

$$Q_i = M_i \left(1 - \frac{M_i(r_s)}{M_i} \right)$$

so that the force on an object due to an externally applied chameleon field is $F_{Ch} = \alpha Q_i \nabla \phi_{ext}$ (this is analogous to the gravitational charge M so that $F_{grav} = M \nabla \phi_N^{ext}$ where ϕ_N^{ext} is an external Newtonian potential). Two objects of different masses and internal compositions will have different scalar charges and will therefore fall at different rates in an externally applied chameleon field, signifying a breakdown of the weak equivalence principle (WEP). The chameleon force between two bodies, A and B, is [200]

$$F_{AB} = \frac{GM_A M_B}{r^2} (1 + 2\alpha Q_A Q_B e^{-m_{eff} r})$$

and as a result of this the PPN parameter γ is $\gamma = \frac{2}{1 + 2\alpha Q_A Q_B e^{-m_{eff} r}} - 1$ (see also [201–203]). Here A refers to the body responsible for the deflection/time delay of light while the body B is a separate body used to measure the mass of the body A (for example, for light bending by the Sun one would take A as the Sun and B as the Earth).

2.13 Long-range forces and spin-gravity coupling terms

In this Section we discuss the possibility that the spin of particles can be present in gravitational potentials. There are essentially some reasons for searching long-range forces that are depending on spin of particles: 1) The role of spin in gravitation (see for example [204–206]). 2) The interaction associated with the exchange of a light or massless pseudoscalar boson or similar interactions [207–212].

In fact, new particles predicted in theories that extend the standard model may induce modifications to spin-spin interaction between fermions [213]. As an example, we recall the pseudoscalar fields, such as the axion [212], and the axial-vector fields, such as paraxions [214] and extra Z bosons [213, 215], the first associated with theories with spontaneously broken symmetries [207–209], the latter in new gauge theories (these new particles, predicted also in string theories [216], are typically introduced to explain the DE [217, 218] and the DM [219]). 3) A number of Kaluza-Klein theories [220, 221] and supersymmetric theories [222], in the low-energy limit, predict couplings in which the spins of particles are involved.

As an example we report the Yukawa-like potential between fermions in the case in which they exchange a (new) vector or axial vector¹² A [212, 213]

$$V_A(r) = \xi_A \mathbf{s}_1 \cdot \mathbf{s}_2 \frac{e^{-r/\lambda}}{r}, \quad (54)$$

¹²It is worth to recall that gravitational interactions between two objects that do not conserve the discrete symmetries were proposed in [205]

$$U(r) = \frac{GM}{r} \left[\alpha_1 \frac{\mathbf{s}^{(1)} \cdot \hat{\mathbf{r}}}{r} + \alpha_2 \frac{\mathbf{s}^{(1)} \cdot \mathbf{v}}{r} + \alpha_3 \mu \hat{\mathbf{r}} \cdot \mathbf{v} \right],$$

where $\alpha_{1,2,3}$ are generic coefficients, M is the total mass, μ the reduced mass, \mathbf{r} the relative displacement, \mathbf{v} the relative velocity, and $\mathbf{s}^{(1)}$ is the intrinsic spin of one of the objects (see also [223, 224]).

where $\xi_A = \frac{g_A^{(e)} g_A^{(e)}}{4\pi}$ is the dimensionless axial-vector coupling constant between the electrons, \mathbf{s}_1 and \mathbf{s}_2 represent the spins of the electrons, and r is the inter-particle separation, while the dipole-dipole potential between electrons corresponding to the exchange of a new axion-like pseudoscalar particle P is [212, 213]

$$V_P(r) = \xi_P \frac{e^{-r/\lambda}}{4m_e^2} \left[\mathbf{s}_1 \cdot \mathbf{s}_2 \left(\frac{4\pi}{3} \delta^3(r) + \frac{1}{\lambda r^2} + \frac{1}{r^3} \right) - (\mathbf{s}_1 \cdot \hat{\mathbf{r}})(\mathbf{s}_2 \cdot \hat{\mathbf{r}}) \left(\frac{1}{\lambda^2 r} + \frac{3}{\lambda r^2} \frac{3}{r^3} \right) \right], \quad (55)$$

where $\xi_P = \frac{g_P^{(e)} g_P^{(e)}}{4\pi}$ is the dimensionless pseudoscalar coupling constant between the electrons, m_e the mass of the electron mass, and $\hat{\mathbf{r}} = \mathbf{r}/r$.

The general analysis of long-range forces between macroscopic objects (polarized spin medium) mediated by light particles that include spin and velocity terms have been performed in [212, 213]. Experiments aimed to tests such new terms will be discussed in next Sections.

2.14 The equivalence principle in Poincare Gauge Theory and Torsion

As we have seen the equivalence principle states that the effect of gravity on matter is locally equivalent to the effect of a non-inertial reference frame in special relativity. The dynamical content of the equivalence principle can be understood by considering an inertial frame in¹³ M_4 , in which matter field ϕ is described by the Lagrangian $L_M(\phi; \partial_i \phi)$. Passing to a noninertial frame, L_M transforms into $\sqrt{-g} L_M(\phi; \nabla_i \phi)$, with $\nabla_i = e_i^\mu (\partial_\mu + \omega_\mu)$ the covariant derivative (this is the minimal substitution discussed in the previous Section). The gravitational field (equivalent to the non-inertial reference frame) appears in the quantities $\sqrt{-g}$ and ∇_i , and can be eliminated on the whole spacetime by reducing to the global inertial frame, while for real gravitational fields one has that they can be eliminated only locally. For introducing a real gravitational field, hence, Einstein replaced M_4 with a Riemann space V_4 . However, also a Riemann-Cartan space U_4 could have been chosen [225].

Another formulation of the Equivalence Principle asserts that the effect of gravity on matter can be locally eliminated by a suitable choice of reference frame, and matter behaves following the laws of Special Relativity [225], i.e. *at any point P in spacetime an orthonormal reference frame \mathbf{e}_i can be chosen such that $\omega^{ij}{}_\mu = 0$ and $e_i^\mu = \delta_i^\mu$ at P* . The important consequence of this statement is that it holds not only in GR (i.e. V_4), but also in Poincare Gauge Theory (i.e. U_4) [226, 227]. The Equivalence Principle is not violated in manifolds with torsion, fitting in natural way into a U_4 geometry of spacetime. It holds in V_4 , as well as in T_4 . Notes however that in more general geometries, characterized by a symmetry of the tangent space higher than the Poincare group, the usual form of the Equivalence Principle can be violated, and local physics differs from Special Relativity [225, 228].

2.15 The violation of the equivalence principle for charged particles

Let us discuss now the tests of Universality of Free Fall (UFF) for charged particles. The interest for these studies follows from the fact that, in some frameworks, a violation of the UFF is related with charge non-conservation [229]. Considering a connection of UFF and Universality of the Gravitational Red-shift (UGR) [166], the most favourable model for a violation of the UGR is a time dependent fine-structure constant caused by a time-varying electron charge. Therefore, tests of the UFF for charged matter can be interpreted as UGR tests, too.

¹³In this Section, we follow the notation in [225]. A space $(L_4; g)$ with the most general metric compatible linear connection Γ is called Riemann-Cartan space U_4 . If the torsion vanishes, a U_4 becomes a Riemannian space V_4 of GR; if, alternatively, the curvature vanishes, a U_4 becomes Weitzenbock's teleparallel space T_4 . The condition $R^\alpha{}_{\beta\gamma\chi} = 0$ transforms a V_4 into a Minkowski space M_4 , and $T^\alpha{}_{\beta\gamma} = 0$ transforms a T_4 into an M_4 .

To test the validity of the EP is analogue to test the minimal coupling procedure, hence to search for an anomalous coupling of the gravitational field (as an extension of the standard minimal coupling procedure, previously discussed). In the non-relativistic regime, the Hamiltonian of a charged particle in a gravitational field is given by

$$H = -\frac{\hbar^2}{2m} \left(\nabla + \frac{iq}{\hbar c} \mathbf{A} \right)^2 + mU + \kappa qU + \lambda q^2U, \quad (56)$$

where κ and λ are free parameters with dimensions $[\kappa] = [mass/charge]$ and $[\lambda] = [mass/charge^2]$ respectively. In the Hamiltonian (56), one can define an effective mass $m_{eff} = (m + \kappa q)U$ that can be interpreted as the charge dependence of the gravitational mass. Since the charge of a particle is related to spacetime symmetries through the CPT theorem, the problem of violation of EP for charged particles assumes a particular interest for anomalous charge couplings.

The stability of a non-pointlike electron requires an effective dependence on the square of the electron charge [230]. Furthermore, the generalized Maxwell equations, in general, violate the UFF in a way in which appears once more the square of the charge [137]. According to these considerations, it makes sense to take into account a general model having chargedependent inertial and gravitational masses [231]. One can choose the parameters in such a way that neutral systems, made up of bound charged particles, exactly fulfill the UFF while isolated charged particles may violate it. Thus, one can introduce an Eötvös coefficient that depends on the charge of particles, i.e. $\eta = \eta_0 + \kappa_1 \frac{q_1}{m_1} - \kappa_2 \frac{q_2}{m_2}$. Here only the linear charge dependence is considered and η_0 indicates the ordinary Eötvös parameter for the masses. These considerations then suggest a comparison between the free fall of a charged and a neutral particle described by $\eta = \eta_0 + \kappa \frac{q}{m}$. By shielding all electromagnetic fields, neutral and charged particles, without internal structure, must fall following the same path. Let us note that, in this framework, experiments in space seem to be favoured in order to reduce the disturbances induced by the stray fields [231].

2.16 Equivalence principle violation via quantum field theory

In this Section we discuss the EP violation in a QFT and GR framework [232, 233] (for modified gravity, see for example Refs. [234, 235] and Ref. [236] for the generalized uncertainty principle). The system consists of an electron with mass m_0 (the renormalized mass of the particle when the temperature is zero) in thermal equilibrium with a photon heat bath. The aim of the analysis is the evaluation of electron's gravitational and inertial mass in the low-temperature limit (namely, $T \ll m_0$). The presence of a non-zero temperature is crucial since $m_g = m_i$ for $T = 0$.

The gravitational and inertial masses are derived by adopting a Foldy–Wouthuysen transformation [237] on the Dirac equation which allows to derive a Schrödinger equation (non-relativistic limit of particles with spin half) in which the expression for the mass is easily recognizable.

In order to operationally define the inertial mass, one applies an electric field to charged particle and study the consequent acceleration [232, 233]. One has therefore to evaluate the finite temperature (radiative) corrections to the electromagnetic vertex. After the renormalization procedure and taking into account the finite temperature contributions, one obtains [232, 233]

$$\left(\not{p} - m_0 - \frac{\alpha}{4\pi^2} \not{I} \right) \psi = e \Gamma_\mu A^\mu \psi. \quad (57)$$

Here we have used the notation $\not{p} \equiv \gamma^\mu a_\mu$, α is the fine-structure constant, γ^μ are the Dirac matrices, A^μ is the electromagnetic four-potential, and the quantity I_μ is defined as

$$I_\mu = 2 \int d^3k \frac{n_B(k)}{k_0} \frac{k_\mu}{\omega_p k_0 - \mathbf{p} \cdot \mathbf{k}}, \quad (58)$$

with $k_\mu = (k_0, \mathbf{k})$ and where ω_p and \mathbf{p} are connected by $\omega_p = \sqrt{m_0^2 + |\mathbf{p}|^2}$. In Eq. (58) $n_B(k)$ represents the Bose-Einstein distribution:

$$n_B(k) = \frac{1}{e^{\beta k} - 1}, \quad (59)$$

where $\beta = 1/k_B T$, with k_B being the Boltzmann constant. Finally, Γ_μ accounts for the finite temperature corrections to the electromagnetic vertex

$$\Gamma_\mu = \gamma_\mu \left(1 - \frac{\alpha}{4\pi^2} \frac{I_0}{E} \right) + \frac{\alpha}{4\pi^2} I_\mu. \quad (60)$$

Applying the Foldy–Wouthuysen transformation, Eq. (57) reduces to a Schrödinger-like equation

$$\begin{aligned} i \frac{\partial \psi_s}{\partial t} &= \left[m_0 + \frac{\alpha \pi T^2}{3m_0} + \frac{|\mathbf{p}|^2}{2 \left(m_0 + \frac{\alpha \pi T^2}{3m_0} \right)} + e\phi + \frac{\mathbf{p} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{p}}{2 \left(m_0 + \frac{\alpha \pi T^2}{3m_0} \right)} + \dots \right] \psi_s \\ &= H \psi_s \end{aligned} \quad (61)$$

To identify the inertial mass one calculates the acceleration

$$\mathbf{a} = -[H, [H, \mathbf{r}]] = \frac{e\mathbf{E}}{m_0 + \frac{\alpha \pi T^2}{3m}}$$

from which one identifies the inertial mass

$$m_i = m_0 + \frac{\alpha \pi T^2}{3m_0}. \quad (62)$$

This relation shows that the difference between the inertial mass of an electron at finite temperature and m_0 is due exclusively to the thermal radiative correction of Eq. (62). The fact that the inertial mass m_i increases with T is expected since it represents the increased inertia needed to travel through the background heat bath.

An analogous procedure can be also performed for the gravitational mass m_g . Calculations of Refs. [232, 233] rely on the weak field approximation, i.e., to first order in the gravitational field (see Eq. (11)), and consider the radiative corrections calculated in flat space. The Dirac equation that takes into account the gravitational interaction reads [232, 233]

$$\left(\not{p} - m_0 - \frac{\alpha}{4\pi^2} \not{I} \right) \psi = \frac{1}{2} h_{\mu\nu} \tau^{\mu\nu} \psi, \quad (63)$$

where $\tau^{\mu\nu}$ is the renormalized stress-energy tensor while $h_{\mu\nu} = 2\phi_g \text{diag}(1, 1, 1, 1)$, with ϕ_g the gravitational potential. Once again, a Foldy–Wouthuysen transformation yields the Schrödinger-like equation

$$\begin{aligned} i \frac{\partial \psi_s}{\partial t} &= \left[m_0 + \frac{\alpha \pi T^2}{3m_0} + \frac{|\mathbf{p}|^2}{2 \left(m_0 + \frac{\alpha \pi T^2}{3m_0} \right)} + \left(m_0 - \frac{\alpha \pi T^2}{3m_0} \right) \phi_g \right] \psi_s, \\ &= H_g \psi_s \end{aligned} \quad (64)$$

The calculation of the acceleration gives

$$\mathbf{a} = -[H_g, [H_g, \mathbf{r}]] = \frac{m_0 - \alpha \pi T^2 / 3m_0}{m_0 + \alpha \pi T^2 / 3m_0}$$

from which the identification of the gravitational mass gives

$$m_g = \left(m_0 - \frac{\alpha\pi T^2}{3m_0} \right). \quad (65)$$

Clearly, there is no difference between m_g and m_i at zero temperature, so that only radiative corrections render the violation of the equivalence principle feasible. In principle, this result would yield a violation of the equivalence principle in an Eötvös-type experiment, although at accessible temperatures the effect is small. In fact, from Eqs. (62) and (65) one gets

$$\frac{m_g}{m_i} = 1 - \frac{2\alpha\pi T^2}{3m_0^2}, \quad (66)$$

in the first-order approximation in T^2 . At temperature of the order $T \sim 300\text{K}$ the corrections is $\sim 10^{-17}$. We point once more that these results hold in the approximation $T \ll m_e$. Equation (66) is a direct consequence of the fact that Lorentz invariance of the finite temperature vacuum is broken, which means that it is possible to define an absolute motion through the vacuum (i.e. the one at rest with the heat bath). The case of gravitational coupling of leptons in a medium has been studies in [238, 239].

2.17 Equivalence principle violation via modified geodesic equation

Let us now discuss a different method, proposed in [240], that reproduces the previous results, in particular Eq. (66).

The starting point is the analysis of a charged test particle of renormalized mass at zero temperature m_0 in thermal equilibrium with a photon heat bath in the low-temperature limit $T \ll m_0$. The dispersion relation reads [232]

$$E = \sqrt{m_0^2 + |\mathbf{p}|^2 + \frac{2}{3}\alpha\pi T^2}, \quad (67)$$

which can be easily identified with the first-order correction in T^2 that descends from the finite temperature analysis. The stress-energy tensor $T^{\mu\nu}$ related to the test particle, whose world line can be contained in a narrow “world tube” in which $T^{\mu\nu}$ is non-vanishing. The conservation equation for the stress-energy tensor can be integrated over a three-dimensional hyper-surface Σ and defined as:

$$\int_{\Sigma} d^3x' \sqrt{-g} T^{\mu\nu}(x') = \frac{p^\mu p^\nu}{E}, \quad (68)$$

where p^μ is the four-momentum and $E = p^0$ the energy, given by $E = \int_{\Sigma} d^3x' \sqrt{-g} T^{00}(x')$. These equations hold in the limit where the world tube radius goes to zero [241].

As shown in [232], the source of gravity, at finite temperature and in weak-field approximation, turns out to be (in the rest frame of the heat bath)

$$\Xi^{\mu\nu} = T^{\mu\nu} - \frac{2}{3}\alpha\pi \frac{T^2}{E^2} \delta^\mu_0 \delta^\nu_0 T^{00}, \quad (69)$$

where $\Xi^{\mu\nu}$ contains not only the information on the Einstein tensor $G^{\mu\nu}$, but also thermal corrections to it¹⁴

¹⁴Eq. (69) is explicitly derived after the choice of the privileged reference frame at rest with the heat bath. The latter give rises to a Lorentz invariance violation of the finite temperature vacuum. In fact, in the tangent space (flat space), one cannot consider a Minkowski vacuum anymore owing to the fact that it is replaced by a thermal bath. As a consequence, Lorentz group is no longer the symmetry group of the local tangent space to the Riemannian manifold, even though general covariance still holds there. According to this, one can proceed keeping in mind that the situation under investigation is slightly different from the usual GR scheme [240].

The generalization of (69) to a curved space-time is [240]

$$\Xi^{\mu\nu} = T^{\mu\nu} - \frac{2}{3}\alpha\pi\frac{T^2}{E^2}e^{\mu}_{\hat{0}}e^{\nu}_{\hat{0}}T^{\hat{0}\hat{0}}, \quad (70)$$

where $e^{\mu}_{\hat{0}}$ denotes the vierbein field and the hatted indexes are the ones related to the flat tangent space. The Einstein field equations are hence given by $G^{\mu\nu} = \Xi^{\mu\nu}$. The Bianchi identity $\nabla_{\nu}G^{\mu\nu} = 0$ implies

$$\nabla_{\nu}T^{\mu\nu} = \nabla_{\nu}\left(\frac{2}{3}\alpha\pi\frac{T^2}{E^2}e^{\mu}_{\hat{0}}e^{\nu}_{\hat{0}}T^{\hat{0}\hat{0}}\right), \quad (71)$$

so that, using $\dot{x}^{\mu} \equiv dx^{\mu}/ds$ and $E = m\dot{x}^{\hat{0}} = m\dot{x}^{\rho}e_{\rho}^{\hat{0}}$ one gets [240]

$$\ddot{x}^{\mu} + \Gamma_{\alpha\nu}^{\mu}\dot{x}^{\alpha}\dot{x}^{\nu} = \frac{2}{3}\alpha\pi T^2 \left[\frac{\dot{x}^{\nu}\partial_{\nu}e^{\mu}_{\hat{0}}}{mE} - \frac{e^{\mu}_{\hat{0}}\left(\ddot{x}^{\nu}e_{\nu}^{\hat{0}} + \dot{x}^{\nu}\dot{x}^{\beta}\partial_{\beta}e_{\nu}^{\hat{0}}\right)}{E^2} + \frac{\Gamma_{\alpha\nu}^{\mu}e^{\alpha}_{\hat{0}}e^{\nu}_{\hat{0}}}{m^2} \right]. \quad (72)$$

Eq. (72) represents a generalization of the geodesic equation to the case in which the temperature is non-vanishing.

2.18 Application to the Schwarzschild metric

We now analyze Eq. (72) for the Schwarzschild geometry. The metric tensor is given by

$$g_{\mu\nu} = \text{diag}\left(e^{\nu}, -e^{\lambda}, -r^2, -r^2\sin^2\theta\right), \quad (73)$$

where

$$e^{\nu} = e^{-\lambda} = 1 - 2\phi = 1 - \frac{2M}{r}.$$

For our purpose, we shall consider only radial motion ($\dot{\vartheta} = \dot{\varphi} = 0$). The non-vanishing vierbeins for the Schwarzschild metric are $e^0_{\hat{0}} = e^{-\frac{\nu}{2}}$, $e^1_{\hat{1}} = e^{-\frac{\lambda}{2}}$.

The geodesic equation for $\mu = 0$ is (here $' \equiv \partial/\partial r$)

$$\ddot{t} + \nu'\dot{t} = -\frac{2}{3}\alpha\pi T^2 \left[\frac{\dot{r}\nu'}{2mE} + \frac{\ddot{t} + \frac{\dot{r}\nu'}{2}e^{\frac{\nu}{2}}}{E^2} \right] e^{-\frac{\nu}{2}}, \quad (74)$$

and since $\frac{E}{m} = \dot{x}^{\hat{0}} = \dot{x}^{\alpha}e_{\alpha}^{\hat{0}} = \dot{t}e^{\nu/2}$, Eq. (74) can be cast in the form

$$\left(1 + \frac{2\alpha\pi T^2}{3E^2}\right) (\ddot{t} + \nu\dot{t}) = 0. \quad (75)$$

The radial contribution can be computed involving Eq. (72) for $\mu = 1$

$$\ddot{r} + \frac{\nu'}{2} \left(\dot{t}^2 e^{\nu-\lambda} - \dot{r}^2 - \frac{2\alpha\pi T^2}{3m^2} e^{-\lambda} \right) = 0. \quad (76)$$

An integration of Eq. (76) gives [240]

$$e^{\lambda}\dot{r}^2 - e^{\nu}\dot{t}^2 - \frac{2\alpha\pi T^2}{3m^2}\nu = \text{const.} \quad (77)$$

The constant is determined from the condition of normalization of the 4-velocity $\dot{x}^\mu \dot{x}_\mu = -1$, i.e.

$$e^\lambda \dot{r}^2 - e^\nu \dot{t}^2 = -1. \quad (78)$$

In the limit of vanishing gravitational field (namely, $\nu, \lambda \rightarrow 0$ as $r \rightarrow \infty$), Eq. (78) reduces to $\dot{r}_\infty^2 - \dot{t}_\infty^2 = -1$, which compared with Eq. (77), implies

$$e^\lambda \dot{r}^2 - e^\nu \dot{t}^2 - \frac{2\alpha\pi T^2}{3m^2} \nu = -1. \quad (79)$$

In the weak-field approximation and owing to Eq. (79), it is immediate to find that Eq. (76) results modified as:

$$\ddot{r} = -\frac{M}{r^2} \left(1 - \frac{2\alpha\pi T^2}{3m^2} \right). \quad (80)$$

To first-order approximation in T^2 , as in the previous QFT treatment, one obtains

$$\frac{m_g}{m_i} = 1 - \frac{2\alpha\pi T^2}{3m_0^2},$$

which is exactly Eq. (66).

2.19 Application to the Brans-Dicke metric

In the case of the Brans-Dicke action, the action reads

$$S_{BD} = \int d^4x \sqrt{-g} \left(\varphi R - \omega \frac{1}{\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \mathfrak{L}_{\text{matter}}(\psi) \right). \quad (81)$$

where

$$\varphi = \frac{1}{16\pi G_{\text{eff}}}, \quad (82)$$

and such a result is traduced in the introduction of a new “effective” gravitational constant that has to be identified with the scalar field. Here one assumes that φ is spatially uniform, and it must vary slowly with cosmic time (this is consistent with experimental data)

Field equations derived from Eq. (81) are

$$2\varphi G_{\mu\nu} = T_{\mu\nu} + T_{\mu\nu}^\varphi - 2(g_{\mu\nu} \nabla_\mu \nabla_\nu) \varphi, \quad (83)$$

and

$$\square \varphi = \zeta^2 T, \quad (84)$$

where $\zeta^{-2} = 6 + 4\omega$ and $T = g^{\mu\nu} T_{\mu\nu}$. The symbol \square denotes the usual D’Alembert operator. In Eq. (83), $T_{\mu\nu}$ and $T_{\mu\nu}^\varphi$ are extracted by varying $\mathfrak{L}_{\text{matter}}$ and the kinetic term of S_{BD} , respectively. As expected, field equations for the metric tensor becomes the ones derived by GR in the limit $\varphi = \text{const} = 1/16\pi G$.

The field equations admit a static and isotropic solution so that the line element is:

$$ds^2 = e^v dt^2 - e^u [dr^2 + r^2 (d\vartheta^2 + \sin^2\vartheta d\Phi^2)], \quad (85)$$

with

$$e^v = e^{2\alpha_0} \left(\frac{1 - \frac{B}{r}}{1 + \frac{B}{r}} \right)^{\frac{2}{\lambda}}, \quad e^u = e^{2\beta_0} \left(1 + \frac{B}{r} \right)^4 \left(\frac{1 - \frac{B}{r}}{1 + \frac{B}{r}} \right)^{\frac{2(\lambda-C-1)}{\lambda}}, \quad (86)$$

with α_0 , β_0 , B , C and λ being constants that can be connected to the free parameter of the theory ω . Since it is a scalar-tensor theory, a solution for φ must also be found; in the considered case, the outcome turns out to be

$$\varphi = \varphi_0 \left(\frac{1 - \frac{B}{r}}{1 + \frac{B}{r}} \right)^{-\frac{C}{\lambda}}, \quad (87)$$

where φ_0 is another constant.

Repeating the previous analysis leading to (80), for BD theory one gets

$$\ddot{r} = -\frac{v'}{2} \left\{ 1 + (e^{-v} - 1) \left(\frac{\lambda B}{r} - C \right) - \frac{2\alpha\pi T^2}{3m^2} \left[1 + v - \left(\frac{\lambda B}{r} - C \right) v \right] \right\} e^{-u}. \quad (88)$$

From Eq. (88), one observes that there is not only the radiative correction to the ratio m_g/m_i , but also another contribution which exclusively depends on ω and that correctly vanishes in the limit $\omega \rightarrow \infty$, that is when GR is recovered. The evaluation of the second quantity of Eq. (88) allows to put a lower bound to the parameter of the Brans-Dicke theory. In fact, in the weak field regime, imposing $|(m_g - m_i)/m_i| < 10^{-14}$ [242] and using [243]

$$\alpha_0 = \beta_0 = 0; \quad C = -\frac{1}{2 + \omega}; \quad B = \frac{GM\lambda}{2}; \quad \lambda = \sqrt{\frac{2\omega + 3}{2\omega + 4}}. \quad (89)$$

one infers

$$\omega > \frac{2GM}{r} \cdot 10^{14}, \quad (90)$$

which is the final expression for the lower bound of the Brans-Dicke parameter in the weak-field approximation. For the Earth $M_{\oplus} = 5.97 \cdot 10^{24} \text{ Kg}$; $R_{\oplus} = 6.37 \cdot 10^6 \text{ m}$. so that [244]

$$\omega > 1.40 \cdot 10^5, \quad (91)$$

that is similar to a bound recently obtained [142], which gives $\omega > 3 \cdot 10^5$. For the sake of completeness, it is useful to look at a table that contains a prediction of the most reliable bounds for ω [245].

Table 1: This table includes expected bounds on the parameter ω from different experiments (see [245] and references therein).

Detector	System	Expected bound on ω
aLIGO	$(1.4 + 5) M_{\odot}$	~ 100
Einstein Telescope	$(1.4 + 5) M_{\odot}$	$\sim 10^5$
Einstein Telescope	$(1.4 + 2) M_{\odot}$	$\sim 10^4$
eLISA	$(1.4 + 400) M_{\odot}$	$\sim 10^4$
LISA	$(1.4 + 400) M_{\odot}$	$\sim 10^5$
DECIGO	$(1.4 + 10) M_{\odot}$	$\sim 10^6$
Cassini	Solar System	$\sim 10^4$

2.20 Standard Model Extensions and the Weak Equivalence Principle

As stated before, the EEP asserts that in any local Lorentz frame about any point in spacetime, the laws of physics are described by the special relativity (including the standard model of particle physics) [130].

As widely believed, general relativity and the standard model can be considered as the low energy limit of some fundamental theory of physics at high energy scales, that, in turn, might give rise to violations of EEP at some scale [246–248], although its exact form is not well defined. In this framework, the standard model extension (SME) [248] represent a flexible and widely applied [249] context for describing violations of EEP. The SME is an effective field theory that extend the standard model action by adding new terms that break local Lorentz invariance and other tenets of EEP [250]. In this model, the energy conservation, gauge invariance, and general covariance are preserved. As in other models [246], EEP violation in the SME can manifest in different ways (for example, it may be strongly suppressed in normal matter relative to antimatter [250]).

In the framework of SME, in Ref. [251] the authors show that EEP violation in antimatter can be constrained by means of tests in which bound systems of normal matter are used. More specifically, an anomaly that violates the WEP for free particles generates anomalous gravitational redshifts in the energy of systems in which they are bound. For a nuclear shell model one can estimate the sensitivity of a variety of atomic nuclei to EEP violation for matter and antimatter.

Focusing on conventional matter (made up of protons, neutrons, and electrons), the spin-independent violations of EEP in the SME acting on a test particle of mass m^w are described by the action [250] (see also [249, 252])

$$S = - \int \left[\frac{m^w \sqrt{[g_{\mu\nu} - 2(\tilde{c}^w)_{\mu\nu}] dx^\mu dx^\nu}}{1 + \frac{5}{3}(\tilde{c}^w)_{00}} + (\tilde{a}^w)_\mu dx^\mu \right], \quad (92)$$

where the superscript $w = p, n, e$ (for proton, neutron, or electron) indicates the type of particle in question, $g_{\mu\nu}$ is the metric tensor, dx^μ is the interval between two points in spacetime. The $(\tilde{c}^w)_{\mu\nu}$ tensor describes a fixed background field that modifies the effective metric that the particle experiences, and thus, its inertial mass relative to its gravitational mass. The four vector $(\tilde{a}^w_{eff})_\mu = ((1 - \alpha U)(\tilde{a}^w_{eff})_0, (\tilde{a}^w_{eff})_j)$, where U is the Newtonian potential, represents the particles coupling to a field with a nonmetric interaction α with gravity. As $(\tilde{a}^w_{eff})_\mu$ is CPT odd [248], this term enters with opposite sign in the action for an antiparticle \tilde{w} . Both $(\tilde{c}^w)_{\mu\nu}$ and $(\tilde{a}^w)_\mu$ vanish if general relativity is valid. For convenience, Eq. (92) includes an unobservable scaling of the particle mass by $1 + \frac{5}{3}(\tilde{c}^w)_{00}$. Consider the isotropic subset of the model [250], i.e. $(\tilde{c}^w)_{\mu\nu}$ is diagonal and traceless, and the spatial terms in the vector $(\tilde{a}^w_{eff})_\mu$ vanish. In the nonrelativistic, Newtonian limit, the single particle Hamiltonian produced by the action (92) is given by

$$H = \frac{1}{2} m^w v^2 - m_g^w U \quad (93)$$

where the effective gravitational mass m_g^w is given by

$$m_g^w = m^w \left[1 - \frac{2}{3}(\tilde{c}^w)_{00} + \frac{2\alpha}{m^w}(\tilde{a}^w_{eff})_0 \right]. \quad (94)$$

Experimentally observable EEP violations are proportional to the particles gravitational to inertial mass ratio

$$\frac{m_g^w}{m^w} = 1 - \frac{2}{3}(\tilde{c}^w)_{00} + \frac{2\alpha}{m^w}(\tilde{a}^w_{eff})_0 \equiv 1 + \beta^w \quad (95)$$

and are described by the parameter β^w [252]. From Eq. (95), it follows that $(\tilde{c}^w)_{00}$ and $(\tilde{a}^w_{eff})_0$ are responsible for violations of the WEP, an aspect of EEP [142], since they produce particle-dependent rescalings of the effective gravitational potential. In addition, EEP violation is not apparent in the nonrelativistic motion of a free particle if $\alpha(\tilde{a}^w_{eff})_0 = \frac{m^w}{3}(\tilde{c}^w)_{00}$, although it remains manifest in the motion of the antiparticle \tilde{w} , for which $\beta^{\tilde{w}} = -\frac{2\alpha}{m^w}(\tilde{a}^w_{eff})_0 - \frac{2}{3}(\tilde{c}^w)_{00}$, a limit discussed in [250]. The antimatter anomaly $\beta^{\tilde{w}}$ does contribute to tests involving nongravitationally bound systems of matter, owing to the anomalous gravitational red-shift produced by $(\tilde{c}^w)_{00}$ in the energies of bound systems (for details, see [252]).

2.21 Strong Equivalence principle in modified theories of gravity

As we have discussed in the previous Sections, in modified or alternative theories of gravity, General Relativity is generalized including extra degrees of freedoms, such as scalar, vector or tensor fields, higher orders terms in the scalar invariants, and so on [1]. Typically, in these models the new degree of freedoms couple non-minimally with, referring to Section 2, scalar curvature. More explicitly, this is the case of the Brans-Dicke theory, the prototype of scalar tensor-theories, in which the scalar field ϕ couples minimally to scalar curvature R , so that the action reads (81). The effects of the non-minimal coupling is, in some regime, to generate new (gravitational) interactions among masses, modifying in a different way the values of the perturbations of the metric (weak-field approximation) $h_{00} = -2GM_G/r$, related to the gravitational mass M_G , and h_{ij} , related to the inertial mass M_I [253, 254], i.e. $M_I = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (h_{i,j}^i - h_{j,i}^i) dS^j$. In General Relativity, since $h_{ij} = -2GM_G/r$ ($h_{00} = h_{ij}$, one gets $M_I = M_G$, while in Brans-Dicke theory (and hence in more general theories of gravity), since $h_{00} \neq h_{ij}$ (weak-field limit of (86)), one gets $M_G = M_I + f(\omega, E_\phi)$, where $f(\omega, E_\phi)$ depends on the parameter ω and the self-energy of the scalar field E_ϕ [253, 254].

3 Experimental tests of the weak equivalence principle

The WEP has been experimentally verified to remarkable accuracy. This is made possible by the fact that the universality of free fall (UFF) can be tested in null experiments, as the physical quantity of interest is the relative acceleration between two free falling proof masses. If the gravitational mass m_g of a body differs from its inertial mass m_I , the acceleration a of the body in a gravitational field g is given by $\mathbf{a} = (m_g/m_I)\mathbf{g}$. Experiments determine upper limits to the differential acceleration $|a_1 - a_2|$ between two free falling test masses of different composition. Possible violations of WEP are then quantified by the Eötvös parameter defined in Eq. (31)

$$\eta = 2 \left| \frac{a_1 - a_2}{a_1 + a_2} \right|. \quad (96)$$

Tests with increasing accuracy correspond to decreasing upper limits on η . As long as UFF is valid, the differential acceleration and thus η must be null within experimental uncertainties. As for any null experiment, no specific model is required to obtain the physical quantity of interest by comparing with the measured signal.

Various kinds of null experiments are possible to test WEP, differing in the magnitude of the potential signal and in the impact of noise sources and systematic effects. In the following of this paper we describe past, ongoing and future WEP test experiments by grouping them into three main classes. Section 3.1 describes experiments in which the test masses are macroscopic bodies. In section 3.2 we present UFF tests by the observation of celestial bodies and their movement with respect to each other. In section 3.3 we discuss experiments with microscopic test masses, i.e. atoms, molecules, and elementary particles. WEP tests can be also classified according to different criteria. Sections 3.1 and 3.3 include ground laboratory tests as well as experiments in space. Macroscopic proof masses in ground experiments can be either suspended or left in free fall. The differences between experimental classes are discussed in the following sections.

It is worth mentioning that other experiments, that strictly speaking cannot be considered as tests of WEP, deeply rely on it for their validity. Relevant examples are the measurement of the Newtonian gravitational constant G performed with freely falling samples [255, 256] or the comparison of different gravimeters for metrological purposes [257].

3.1 Lab experiments with macroscopic masses

Laboratory WEP tests based on macroscopic masses are either performed with freely falling masses or with suspended masses. The latter class of experiments compares the acceleration experienced by two masses of different composition as they fall in the gravity field of the Earth. In this case, the signal to be detected, namely a non-zero differential acceleration resulting from a WEP violation, is maximum as it is proportional, via the Eötvös parameter, to the full gravitational acceleration of the Earth. Unfortunately, the typical free fall time on Earth cannot be longer than a few seconds to keep the height of the instrument within a reasonable size. This imposes a major limitation to the measurement sensitivity. In addition, free-fall experiments are very much dependent from the initial conditions (position and velocity) of the test masses as they are released and therefore to external perturbations acting on the instrument.

Experiments with suspended masses are done, with a few exceptions, using a torsion balance, with test masses of different composition suspended at the opposite ends of the beam. When the beam is oriented along the East-West direction, the differential acceleration responsible for a WEP violation is proportional to the centrifugal acceleration, which provides a driving signal for a WEP violation about three orders of magnitude smaller than in a free-fall experiments. Despite the lower signal, torsion balances are today providing the best laboratory tests of the Weak Equivalence Principle due to the long measurement time at equilibrium and to the excellent control of systematic effects that they can offer by spinning the instrument around its axis.

Experiments with suspended and freely falling macroscopic masses are described in the next sections.

3.1.1 Tests with suspended masses

As a cornerstone of mechanical theories, WEP has been experimentally investigated since the dawn of modern age. First experimental tests of the UFF date back to the early 1600's, when Galileo Galilei compared the oscillation periods of two simple pendulums with different composition [258]. Considering that the two masses are in free fall along the tangent to the trajectory of their respective oscillation, Galilei managed to test the UFF with an accuracy at the 10^{-3} level [259]. Newton repeated the experiment to test the equivalence of inertial and gravitational mass with similar precision [260], and two centuries later Bessel improved it to an accuracy of 2×10^{-5} [261] with a more precise determination of the pendulum length, and by comparing many different materials including gold, silver, lead, quartz, marble, clay, loadstone, water. Pendulum WEP tests were also performed on radioactive materials in the early XX century: Thomson [262] reached a 5×10^{-4} precision for radon, and Southern [263], achieved a 5×10^{-6} precision for uranium oxide. Further evolution of this method led to the remarkable precision of 3×10^{-6} in the experiment of H. H. Potter in 1923 [264]. Simple pendulum experiments are intrinsically limited by the large impact of dissipative damping forces from suspension and from air, as well as by geometrical asymmetries between the two pendulums to be compared. Moreover, with increasing precision the anharmonic terms of the pendulum dynamics become relevant, and the period depends on the amplitude which then has to be controlled with high precision.

A breakthrough occurred in the late 19th century, due to the intuition of Eötvös to employ a Cavendish torsion balance (more precisely, Boy's modification) to compare inertial and gravitational mass in a null experiment. Eötvös' first series [265], published in 1890, reached a precision of 5×10^{-8} . Two decades later, with D. Pekar and E. Fekete, Eötvös improved it to 3×10^{-9} [266]. In Eötvös' experiments, the inertial acceleration is given by the centrifugal force due to the Earth's rotation, while the gravitational acceleration is the component of g necessary to compensate it. Two masses of different composition are suspended at opposite ends of the torsion balance beam; the centrifugal forces on the two weights due to the Earth's rotation are balanced against a component of the Earth's gravitational field. A WEP violation would produce a rotation of the torsion balance: if the ratio of passive gravitational mass to inertial mass should differ from one test mass to the other, there would

be a torque tending to twist the torsion balance. When the beam is oriented along the East-West direction, the differential acceleration responsible for a WEP violation is proportional to the centrifugal acceleration $a_c = \Omega_{\oplus}^2 R_{\oplus} \cos \theta \sin \theta$, where R_{\oplus} and Ω_{\oplus} are the radius of our planet and the angular velocity of its rotation motion, and θ is the latitude at the instrument location. For $\theta = \pi/4$, the centrifugal acceleration amounts to 16.8 mm/s^2 providing a driving signal for a WEP violation about 600 times smaller than in a free-fall experiments. One major limitation is given by the fact that a potential violation would produce a static (DC) signal. The effect of a non-zero signal can be detected by exchanging the position of the two masses, so that the sense of the twist from WEP violation would be reversed.

Improved versions of Eötvös' experiment were designed to produce an AC signal from potential WEP violations, by spinning the instrument around its axis. The spin motion introduces a modulation of the WEP violating signal without intervening on the balance configuration [267,268]. This is achieved mainly in two different ways: by locking the torsion balance on the gravitational field of Sun at equilibrium with the inertia of the Earth that rotates around it, so that the violation signal is modulated by the Earth's spin with a 24 h period; or by actively rotating the torsion balance around the suspension wire to up-convert possible violation signals from DC to the rotation frequency. Main advantage of the former method is the natural modulation of the potential signal without potential systematics and technical noises from active mechanical rotation of the apparatus. The main advantage of the latter method is the higher modulation frequency of the signal, allowing to remove many low-frequency noise sources and systematics. In particular, mechanical losses due to internal damping are lower at higher frequencies, and up-conversion brings the signal to a region of reduced thermal noise. Combinations of different up-conversions have also been designed, e.g. with rotating torsion balances in the field of the Sun.

Earth rotation offers a natural platform for spinning a torsion balance with daily period against the Sun. If the beam is aligned with the north-south direction, a WEP-violating differential acceleration would produce a maximum torque when the Sun is at the astronomical horizon. The horizontal component of the gravitational acceleration toward the Sun is at most 6 mm/s^2 . Thus the signal for UFF tests in the gravitational field of the Sun is smaller than for tests in the Earth's field by about a factor $3/8$. Dicke's torsion balance experiment provided the first UFF test in the field of the Sun [267], reaching 10^{-11} , followed by Braginsky and Panov down to 10^{-12} . The latter experiment provided the best estimate of the Eötvös parameter for nearly 30 years. A variant of the torsion balance is obtained by replacing the suspension wire with a so-called fluid fiber, introduced by Keiser and Fallor at the end of the 1970s [269]. In this kind of setup, test masses made of hollow metal bodies float on fluids, and their position is controlled by an electrostatic system. A potential WEP violation, inducing a differential acceleration between the solid and the liquid, is measured on the control signal needed to keep the test masses in constant position. WEP tests with fluid fibers were performed in 1979 with an accuracy of 10^{-10} [269] and in 1982 with an accuracy of 4×10^{-11} [270]. The potential accuracy of such method was estimated at levels between 10^{-13} and 10^{-14} [271]. A similar experiment was performed by Thieberger in 1986 by observing the horizontal drift of a hollow copper sphere floating freely in water [272]. The driving horizontal gravitational force was generated by placing the setup near a steep cliff; a differential acceleration between copper and water would result in a drift velocity of the sphere. Indeed Thieberger measured a net differential acceleration, indicating a potential WEP violation with $\eta < 1.3 \times 10^{-11}$ arising from a fifth force. A comparative experiment of the same kind was performed with a more symmetric setup by Bizzeti et al. in 1989 [273]; no WEP violation was found, up to an accuracy of 2.4×10^{-12} .

A disadvantage with the Sun as source is a weaker driving signal as compared to that in the field of the Earth. Spinning a torsion balance by means of a uniformly rotating turntable allows the Earth to be used as the attractor [274]. Moreover, driving force modulation can be kept at higher frequencies, reducing the thermal noise [275] and disentangling the WEP violating signal from other effects, e.g. temperature

variations, that naturally occur at the diurnal frequency, including thermal effects, microseismicity, local mass motions. Such effects originate from the Sun through radiative heating of the Earth’s surface and atmosphere, with a typical thermal time delay, rather than by gravitational interaction. Though Earth tidal forces have no daily periodicity and can be neglected, gravity gradients originating from solar tides occur mostly at twice the diurnal frequency, resulting in a spurious WEP violation of the order 10^{-12} for a balance arm of 15 cm.

Torsion balance experiments provided the best limits on potential WEP violations for ground tests so far (see [274], Table 3). Such experiments have confirmed UFF both in the field of the Sun, up to about 10^{-12} , and in the field of the Earth, up to about 10^{-13} , as well as in the field of local source masses, up to about 5×10^{-12} .

The most precise torsion balance to date was realised by the so called Eöt-Wash research group. A first experiment in 1989 provided a WEP test in the gravitational field of the Earth with 1×10^{-11} accuracy [276]. In the same year, a test with 5.3×10^{-12} accuracy was done in the field of a local mass distribution by placing the torsion balance near a river lock, resulting in a 12 min periodic modulation of $\sim 2 \times 10^8$ kg of water with known distribution as an attractor for copper and lead test masses [277]. The precision was improved to 1.9×10^{-12} in 1994 with test masses made of Beryllium and of an Aluminum/Copper alloy in the field of the Earth, and later to 1.2×10^{-12} with Si and Al+Cu test masses in the field of the Sun. In 1999 the Eöt-Wash group measured the differential acceleration of a Cu test mass toward a Pb attractor to be $a_{\text{Cu}} - a_{\text{Pb}} < (1.0 \pm 2.8) \times 10^{-15} \text{ m/s}^2$. Comparing to the corresponding gravitational acceleration of $9.2 \times 10^{-7} \text{ m/s}^2$ this leads to $\eta < 10^{-8}$ [278]. An experiment performed in 2001 in the field of the Sun [279] improved the result of Braginsky and Panov to $\eta < 10^{-13}$. In 2008 the same group obtained $\eta < (0.3 \pm 1.8) \times 10^{-13}$ with Be and Ti test masses in the field of the Earth [280]. The latter result represents the most accurate WEP test on ground.

Another recent torsion balance experiment provided a WEP test at the 10^{-13} level on chiral masses [281], using a pair of left-handed and right-handed quartz crystals.

Current experiments with torsion balances are mainly limited by systematic effects arising from gravity gradients coupling to geometrical asymmetries in the the torsion pendulum, and thus producing differential directions for the forces on the test bodies. Several environmental parameters can produce effects that mimic a WEP-violating signal. Tilts of the rotation axis with respect to local vertical, couple the pendulum to gravity gradients; the same applies to temperature fluctuations, thermal gradients, and magnetic fields. The main bias terms can be subtracted to some extent using the method described in [282, 283]. For each driving term, the corresponding parameter is modulated with large amplitude to calibrate its effect on the WEP-violating signal; calibration factors and measured parameters are combined in post-processing of the actual WEP data to correct for the contribution of bias driving terms. Gravity gradients can be measured with a gradiometer and compensated with a suitable configuration of local source masses. Rotating the compensation system by 180° about its vertical principal axis doubles the effect of ambient gradient. This allows to determine the corresponding systematic error on the torsion balance WEP test, which is measured from the ratio of the torsion balance and gradiometer signals in the two compensator positions. Additional sources of systematic errors originate from fibre twisting due to residual tilts of the setup or wobbles of the rotary axis in combination with asymmetries in the upper suspension point. Such effects can be corrected by carefully measuring the residual tilt, e.g with a dual-axis tilt sensor placed above the upper attachment of the fibre and beneath the pendulum, and controlling the rotation axis to be along the vertical direction. Temperature gradients and magnetic effects are usually mitigated by multi-stage passive shielding. Changes in the balance spinning frequency ω_s are another source of systematic errors. A spurious signal would be proportional to the component of the rate variation at the Fourier frequency ω_s . As the corresponding torque scales as ω_s^2 , the effect can be measured by operating the torsion pendulum at different spinning frequencies, and can be mitigated by choosing the lowest spinning rate compatible with the technical noise floor.

A modern variant of Galileo’s simple pendulum WEP test in the field of the Sun has been proposed

recently, [284]. The experiment is based on a differential accelerometer with zero baseline, measuring the relative acceleration of two test masses of different materials suspended on a pendulum. Ensuring a precise centering of the test masses the system should provide a high degree of attenuation of the local seismic noise. With a cryogenic differential accelerometer under vacuum, the experiment should provide a WEP test with 10^{-14} precision.

3.1.2 Freely falling masses

Unlike in torsion balances, mass drop WEP tests are done by leaving two test masses in free fall at the same time, and measuring their relative displacement as a signature of differential acceleration. This method was never used in high-precision experiments until the late 1980s. The legendary UFF test by Galileo was indeed never done by dropping masses from the Pisa leaning tower, but rather using pendulums

Free fall experiments in evacuated tubes, which are nowadays popular in science teaching and outreach, date back to the 17th century, when Boyle performed free fall tests with feathers or pieces of paper [285]. Similar experiments with coins and feathers are reported in 1717, when Desaguliers demonstrated the UFF to King George I and to the Royal Society led by Newton [286]. In 1971 Astronaut Scott performed a UFF test on the Moon during the Apollo 15 mission, by dropping an Aluminum hammer and a falcon feather from a height of about 1.6 m and observing them hit the ground simultaneously [287].

A revival of UFF tests with freely falling test masses occurred during the 1980's, in the attempt to improve Eötvös' WEP tests [288]. After Dicke's and Braginsky's experiments it was clear that substantial progress required a rotation of the torsion balance; however the control of systematic effects in rotating the setup in the laboratory, that was necessary for tests in the field of the Earth, was considered extremely challenging. On the contrary, the recent progress in laser interferometry ranging made mass drop tests more attractive. Moreover, free fall tests have the advantage of a much higher driving acceleration ($g \simeq 9.8 \text{ m/s}^2$ in mass drop tests versus $g < 1.69 \cdot 10^{-2} \text{ m/s}^2$ on the torsion balance). A first precision mass drop WEP test was performed by Worden in 1982 at the 10^{-4} uncertainty level, with a test mass constrained to 1D motion by means of a magnetic bearing [289]. This result was improved in 1984 by Sakuma with an accuracy of 1×10^{-8} [290]. In 1986, by observing the rotation of a freely falling disc made of two halves of different materials, Cavasinni et al. confirmed the validity of WEP with 1×10^{-10} accuracy [291]. A high-precision mass drop experiment with two separate test masses was performed for the first time in 1987 by Niebauer et al. [292]. Measuring the position of freely falling Uranium and Copper test masses with an interferometer, they proved the WEP up to 5×10^{-10} . The same experiment with different materials was repeated by Kuroda and Mio in 1989, reaching an accuracy of 1×10^{-10} [293, 294]. The disc experiment of [291] was repeated in 1992 by Carusotto et al. with Aluminum and Copper, reaching an accuracy of 7.2×10^{-10} [295, 296].

So far, the precision of drop tests was limited to a few parts in 10^{10} , in spite of the 600-fold larger driving signal strength compared to torsion balances. The improvements over Eötvös' result was limited to one order of magnitude, in contrast to the much more sophisticated technologies employed. More recently this has been overwhelmed by the Eöt-Wash rotating balance. As discussed in [292, 295, 296] the main limitations of mass drop experiments are from errors in initial conditions at release coupling to the gravity gradient of the Earth to provide a differential acceleration error that mimics a violation signal. In experiments with separate test masses, a laser interferometer tracks the differential trajectory

$$\delta x(t) = \delta x_0 + \delta v_0 t(1 + \gamma_v t^2/6) + \gamma_h t^2/2 \quad (97)$$

where δX_0 , δv_0 , γ_v , γ_h represent the initial differential displacement, initial differential velocity, vertical gravity gradient, and horizontal gravity gradient. The effect of vertical gradient is partly removed by fitting the measured trajectory with Eq. 97. Further mitigation of the systematic error from vertical

velocity differences is obtained by alternating the order in which the objects are dropped. Errors arising from the horizontal gravity gradient are mitigated by alternating the position of the two masses. For experiments with disk test masses, a major systematic error arises from the disk precession around its angular momentum. The effect can be partly corrected by measuring the two components of the angular momentum in the disk plane just after the release.

A way to improve drop tests is by increasing the free fall time, since the effect of a violation increases quadratically with time. Experiments on balloons [297] and sounding rockets [298] have been proposed, which would allow a free fall time of several tens of seconds. The effect of gravity gradients, which would be a major source of systematic errors, can be separated from potential WEP violation signals by spinning the system around a horizontal axis: in such way the signal from gravity gradient appears at twice the rotation frequency while a violation signal would be at the rotation frequency.

In principle there is still more potential in mass drop experiments, especially with a longer free fall time and sensors with a higher resolution. Similar tests are possible on ground in facilities such as the drop tower of the ZARM center at the University of Bremen, where a free fall time of 4.74 s is achievable (9.3 s using the catapult). A mass drop WEP test with 10^{-7} accuracy was performed in 2001 with highly sensitive SQUID sensors [299]. In free fall experiments the control of starting conditions for positions and velocities of the test masses is crucial. An Electrostatic Positioning System (EPS) was developed to this purpose [300]. For optimal conditions an accuracy of 10^{13} is expected with this setup.

Another free fall experiment is the project Principle Of the Equivalence Measurement (POEM) at the Harvard-Smithsonian Center for Astrophysics [301, 302]. Two test masses in 0.5 m distance in a co-moving vacuum chamber were bounced on a kind of trampoline 0.9 m up and down several times. To test the WEP, the shifting between the test masses is measured. In principle, with a time average over several bounces a sensitivity of 5×10^{-13} can be reached and an improvement on ground up to 1×10^{-14} is possible.

Free fall experiments with microscopic test masses enable in principle a much better control over systematic effects. They are discussed in section 3.3.

3.1.3 Experiments with macroscopic masses in space

Testing the Weak Equivalence Principle in a ground based laboratory has some obvious limitations that can be overcome by going to space.

As discussed in the previous sections, experiments with freely falling masses exploit the full gravitational acceleration of the Earth to maximize the strength of a WEP violation; unfortunately, their sensitivity is hampered by the short measurement time achievable in a ground-based laboratory and their accuracy is limited by the poor control of systematic effects depending on the initial conditions (position and velocity) of the test masses. On the contrary, experiments based on masses suspended on a torsion balance allow for a long integration time and provide a much better control of systematic effects, but their sensitivity is limited by the driving gravitational signal being three orders of magnitude lower than the Earth gravity acceleration.

The next evolutionary step of these instruments is clearly space. The laboratory inside a freely falling spacecraft is indeed the ideal environment to push WEP tests to their ultimate limits.

Under weightlessness conditions, the classical free fall experiment can still benefit from a WEP violating signal proportional to the local acceleration of gravity (at the spacecraft height). More importantly, the experiment can now be executed in a compact apparatus, with the test masses still in the spacecraft reference frame, where their relative motion can be observed over a long and unperturbed measurement time. The reduced volume of the instrument, compared to the much larger drop towers or free-fall capsules used on Earth, allows to better control the experiment against external perturbations.

A torsion balance in space would as well benefit from the full gravitational acceleration as a driving signal, gaining almost three orders of magnitude on the effect to be measured with respect to an Earth-

based experiment having the same sensitivity to differential accelerations.

Concentric test masses are the common denominator of all the instruments proposed for a space test of the Weak Equivalence Principle. This is the case for the space missions that will be described in this section, MICROSCOPE, STEP and GG, which is the natural evolution of a torsion balance for space. Such a design is possible in space because of the extremely small coupling forces needed to control the masses position under weightlessness conditions. Small coupling constants directly translate into higher instrument sensitivity to differential accelerations and therefore to WEP violations.

More importantly, a spinning spacecraft can be used to introduce a modulation of the differential acceleration resulting from a WEP violation, both for a free-fall and a torsion balance experiment, to distinguish the WEP violating signal from other effects appearing at different frequencies. In this case, as the platform is rotating with the instrument itself, the mass distribution in the immediate vicinity of the test bodies does not introduce any signal modulation as soon as there are no moving parts or changes in the mass distribution of the spacecraft.

Still, gravity gradients remain one of the predominant sources of systematic error imposing an ad-hoc design of the experimental setup and the test masses. Test masses of different shape couple differently to gravity gradients due their different multipole moments. This effect produces a differential acceleration competing with a violation of the Weak Equivalence Principle. Test masses shall therefore be designed to approach the shape of a gravitational monopole or to have matching gravitational multipole moments [303]. As a consequence, manufacturing processes shall ensure precise control on the shape of the masses and the material itself shall be selected to be highly homogeneous and easily machinable [304]. On the other hand, the gravity environment generated by the spacecraft surrounding the test masses, and thus primarily interacting with them, can also be controlled. As already demonstrated by the LISA Pathfinder mission, a protocol-based measurement of the mass and the distance of all satellite parts can ensure a balance of the gravitational accelerations at the sub nms^{-2} level [305]. Such techniques have proven to be very effective in reducing the systematic errors introduced by gravity gradients.

Differential accelerometers based on a nested test mass design are also affected by the radiometer effect. The infrared radiation of the Earth is absorbed by the satellite and consequently by the instrument housing, thus producing a temperature gradient that depends on the satellite's orientation with respect to the Earth-to-satellite direction. Due to the residual gas around the test masses, this temperature gradient is responsible for a differential acceleration that is directly proportional to the pressure of the residual gas and to the temperature gradient and that cannot be distinguished from a WEP violating signal. The thermal design of the spacecraft and the instrument head as well as the design of the vacuum system enclosing the test masses is therefore important to minimize this effect. In STEP, where cryogenic temperatures are reached in a He dewar, the residual gas pressure and the temperature gradients can be better controlled. In [306] the radiometer effect is calculated for MICROSCOPE, STEP and GG and discussed with respect to the specific design of the three instruments.

In space, the test masses of the differential acceleration sensor can accumulate charges due to the interaction with high energy charged particles travelling through the solar system. In the presence of stray charges, the source mass interacts with the caging mechanism and readout system via Coulomb forces introducing noise and bias on its position and on the measurement signal readout. Different methods can be used to discharge the masses and counteract this effect. MICROSCOPE stray charges are managed via a thin ($0.7 \mu\text{m}$) gold wire connected to a sole plate and driven by a control voltage [304]. A different discharging system has been demonstrated in space by the LISA Pathfinder mission. In this case, an ultra-violet lamp illuminating both the test masses and the surrounding environment is used to generate a current of photoelectrons [307] that can be tuned to null the charge of the test mass itself. In this way, the corresponding noise and bias can be reduced to negligible levels.

Finally, external perturbations can be accurately controlled in space. Stabilization loops can be implemented to reduce temperature fluctuation at the instrument head below $100 \mu\text{K}$ [308]. The Newtonian noise, generated by fluctuations of terrestrial gravity and representing one of the most

important limitations of ground-based tests of the Weak Equivalence Principle, is totally absent on a spacecraft. In space, other perturbations, such as air drag or solar radiation pressure, can introduce noise and bias affecting the WEP test. However, several drag free systems have already demonstrated their ability to reduce residual accelerations below $3 \times 10^{-11} \text{ms}^{-2} \text{Hz}^{-1/2}$, as in the MICROSCOPE mission [308], or even better, down in the $10^{-15} \text{ms}^{-2} \text{Hz}^{-1/2}$ regime, as in LISA Pathfinder [309, 310].

The MICROSCOPE (MICROSatellite à trainée Compensée pour l’Observation du Principe d’Equivalence) mission provides the most accurate test of the Weak Equivalence Principle [304, 308]. The satellite was launched from Kourou on 25 April 2016 on a Soyuz rocket and injected into a dawn-dusk Sun-synchronous orbit with an altitude of 710 km. The spacecraft embarks two differential accelerometers, each of them based on two hollow cylinders. They are aligned along the symmetry axis, precisely centered, and kept in their equilibrium position by capacitive electrodes. The two differential accelerometers only vary for the test masses composition: Pt:Rh alloy for both cylinders of the reference sensor unit (SUREF); Pt:Rd and Ti:Al:V for the inner and outer test mass of the unit sensitive to WEP violations (SUEP). The test masses have been precisely machined to a relative difference between the momenta of inertia smaller than 10^{-3} and a density homogeneity better than 0.1%, thus reducing differential accelerations due to gravity gradients to negligible levels [304]. The same set of electrodes provides measurement and control of both the position and the attitude of the test masses. They are machined on a silica substrate to ensure high position stability. The voltage applied at the electrodes, which is proportional to the force exerted on the test masses to keep them centered, represents the main data output of the instrument from which the differential acceleration between the test masses is extracted. Once the test masses are correctly aligned, if General Relativity holds, zero differential acceleration at both the SUREF and SUEP sensor heads shall be read. The magnetic environment is controlled by a magnetic shield surrounding the complete payload and modelled by a finite element calculation. The tight housing allows the sensors to operate in the 10^5 Pa regime, where the radiometer effect is strongly reduced. Radiometer effect and radiation pressure disturbances are kept below the damping introduced by the $7 \mu\text{m}$ wire connecting the test masses and the cage to control electrical charging effects [311]. Cold gas thrusters actuated by the accelerometers’ measurements reduce the effect of air drag and, more generally, of non-gravitational forces acting on the spacecraft. The drag-free control system relies on the linear and angular accelerations measured at one of the test masses. Residual accelerations below $3 \times 10^{-11} \text{ms}^{-2} \text{Hz}^{-1/2}$ could be measured in closed-loop configuration. This result is about a factor 10 better than originally specified. Star tracker measurements are also used to determine the spacecraft attitude. To increase the modulation frequency of the gravitational signal provided by the Earth, the satellite is rotated (~ 1 mHz) around the orthogonal direction to the sensitive axis of the differential accelerometers. The SUREF and SUEP power spectral density of differential acceleration measurements along the axial direction (sensitive axis of the instrument) are $5.6 \times 10^{-11} \text{ms}^{-2} \text{Hz}^{-1/2}$ and $1.8 \times 10^{-11} \text{ms}^{-2} \text{Hz}^{-1/2}$, respectively, at the modulation frequency of a few mHz expected for the WEP violation. This floor level is limited by the damping noise of the thin gold wire connected to the test masses to control charging effects. Systematic errors are currently dominated by thermal effects, which could be evaluated to $< 67 \times 10^{-11} \text{ms}^{-2}$. The instrument sensitivity was determined by applying temperature variations both at the electronics and at its baseplate. Measurements revealed that the sensor unit temperature coefficient is 2 orders of magnitude higher than expected. This issue is still under investigation. On the positive side, the temperature stability of the instrument baseplate and the electronics was measured to be better than $20 \mu\text{K}$ over 120 orbits, about 2 orders of magnitude smaller than initial estimates, thus mitigating temperature-related effects. The contribution of self-gravity and stray magnetic fields to the measurement error was estimated from finite element models and found negligible. After analyzing the data corresponding to 120 satellite orbits, an Eötvös parameter of $[-1 \pm 9 \text{ (stat)} \pm 9 \text{ (syst)}] \times 10^{-15}$ could be estimated for titanium and platinum [308], improving by one order of magnitude previous results obtained from torsion balance [274] and lunar laser ranging [312] experiments. This measurement also establishes new constraints to modifications of the Newton’s law

of gravity by a Yukawa-like coupling and improves existing constraints on WEP violations by a light scalar field [313]. The MICROSCOPE mission has been decommissioned on 18 October 2018, after accumulating about 1900 orbits of science data on the SUEP sensor, 900 on the SUREF sensor, and 300 orbits for calibration. This also includes 750 orbits of measurements for characterizing the on-board temperature sensors and further reduce the systematic effects due to temperature variations. After the first results reported in 2017, the complete data set delivered by the MICROSCOPE mission is still under scrutiny to improve both the statistical and systematic error on the WEP test, hopefully going below the 1×10^{-15} accuracy level.

The STEP (Satellite Test of the Equivalence Principle) mission concept is similar to MICROSCOPE, but it relies on a completely different technology, which is expected to push the accuracy of WEP tests down to 1 part in 10^{18} [314, 315]. The STEP mission was selected for a phase A study in 1990. An engineering model of the accelerometer to test the technology was built in 2004. The payload is composed of 4 differential accelerometers (DAs) operating simultaneously with the following combination of test masses: Be and Pt:Ir for DA1; Be and Nb for DA2; Nb and Pt:Ir for DA3; Be and Pt:Ir for DA4. DA1 to DA3 measurements allow to check that the sum of the differential acceleration measured at the 3 sensor heads between the three materials - Be, Pt:Ir, and Nb - is zero thus providing control on measurements systematics. DA1 and DA4 differ for the shape of the test masses and for their coupling to Earth and spacecraft gravity. As for MICROSCOPE, the differential accelerometers are composed of two hollow masses with cylindrical symmetry, precisely centred and aligned along their axis. STEP DAs are arranged in a helium dewar and operated at 2 K. The cryogenic environment is providing very good thermal and mechanical stability for the DAs operation, ultra-high vacuum and reduced thermal noise from gas damping, excellent shielding from external magnetic fields, reduced radiation pressure effects due to temperature gradients. More importantly, it allows to use SQUID technology for high-sensitivity position readout and for generating the weak reaction force that centers the test masses along the axial direction. The gas generated by boiling helium is used by thrusters to stabilize the spacecraft against non gravitational accelerations. In addition, when in drag-free, common mode accelerations can be measured with respect to the spacecraft reference frame to $10^{-15} - 10^{-12} \text{ ms}^{-2}$, well below the MOND (MOdified Newtonian Dynamics) acceleration scale a_0 . Based on this performance, STEP has recently been proposed for a test of MOND theories and of the Strong Equivalence Principle [316]. As discussed in section 3.1.2, alternative WEP tests in microgravity are also possible on ground laboratories such as the Bremen drop tower [299].

Galileo Galilei (GG) is an alternative proposal designed to test the Einstein WEP to better than 1 part in 10^{17} [317, 318]. Differently from MICROSCOPE and STEP, GG can be considered as the space version of a beam balance. The test masses are two hollow cylinders of different composition that are weakly coupled by means of mechanical suspensions. Once properly set into equilibrium by piezo actuators, the beam of the balance is aligned along the symmetry axis of the cylinders, thus defining the plane orthogonal to this direction as the sensitive plane of the instrument. This configuration provides a rejection of common mode acceleration noise as high as 10^5 , thus drastically relaxing the level of drag control required at the spacecraft. Rapid rotation of the instrument is important to reduce the thermal noise in the detection of WEP violations and to efficiently decouple it from systematic effects appearing at different frequencies. In GG, the spin axis of the spacecraft coincides with the symmetry axis of the instrument. Therefore, after initial spin up, the spacecraft co-rotates with the cylindrical test masses around the principal axis of the system and it is passively stabilized to very fast rotation rates (~ 1 Hz) by angular momentum conservation. Due to the high mass of the GG test cylinders and the large gap between them, thermal noise due to gas damping and to the radiometer effect become two orders of magnitude smaller than in MICROSCOPE. Finally, the displacement of the GG test masses is read by a laser interferometer gauge, which provides very low noise and fast integration times. An accuracy budget of the GG instrument and an evaluation of the systematic effects is provided in [317]. A laboratory demonstrator of the space instrument has been built and it is presently under test. On the ground the

instrument has reached a sensitivity to differential acceleration measurements of $\sim 7 \times 10^{-11} \text{ ms}^{-2}$ (at $1.7 \times 10^{-4} \text{ Hz}$ upconverted by rotation to 0.2 Hz), currently limited by Newtonian noise, mainly tilt, acting on the ball bearings [319]. An optimized design based on low noise air bearings, low coupling joints, and a laser interferometer readout system is under study to push the instrument performance down to the $10^{-16} - 10^{-15}$ regime.

3.2 Tests based on the measurement of the Earth-to-Moon and Earth-to-satellite distance

Lunar Laser Ranging (LLR) experiments are performed since 1969, when the first array of corner cube reflectors was positioned on the Moon by Apollo 11. A review of LLR tests of gravity can be found in [320]. To date, 5 arrays of retro-reflectors are operational on the Moon surface and routinely used for ranging experiments: Apollo 11, 14 and 15, Lunokhod 1 and 2. Among them, Apollo 15 is the one with the largest lidar cross section and therefore the most widely used for LLR (about 75% of normal point data). In a LLR measurement, a short laser pulse is fired by a ground-based Satellite Laser Ranging (SLR) station towards one of the Moon corner cube reflector arrays. The back reflection is collected by the SLR station and the interval between the fire time and the reception time is recorded. Round-trip travel time measurements are then fitted to a model of the solar system ephemeris including tidal effects, relativistic effects, propagation in the atmosphere, plate motion, etc. The SLR stations mostly contributing to LLR data are the Observatoire de la Côte d'Azur (OCA) in France, the McDonald Laser Ranging System (MLRS) and the Apache Point Observatory Lunar Laser-ranging Operation (APOLLO), both in the US. Thanks to the 3.5 m diameter telescope and to the array of high-efficiency avalanche detectors, the APOLLO station has today reached millimeter ranging precision and accuracy to the Moon [321,322]. To this level, effects like regolith motion, thermal expansion of the retro-reflectors array, oceans and atmosphere loading effects start to become relevant.

Earth and Moon are two celestial bodies freely falling in the gravitational field of the Sun (primary body). If the Universality of Free Fall principle is violated, their accelerations towards the Sun are different, thus introducing a polarization of the lunar orbit [323]. This effect manifests itself with the appearance of a modulation of the Earth-Moon distance (LLR measurements) along the Earth-Sun direction at the synodic period (29.53 day). For a relative differential acceleration between the Earth and the Moon of $\Delta a/a$, the perturbation δr to the Earth-Moon distance expressed in meters is given by [324]

$$\delta r = -2.9427 \times 10^{10} \frac{\Delta a}{a} \cos D \text{ [m]}, \quad (98)$$

where D is the synodic angle. After the first LLR test of the Equivalence Principle in 1976 [325,326], the accuracy of relative differential acceleration measurements $\Delta a/a$ between Earth and Moon has progressively improved to 1.4×10^{-13} [312,327,328], to recently reach 5×10^{-14} [329]. This result could be obtained after modelling the effects of the gravitational interaction of the Sun and the planets on the Moon, now treated as an extended body. High order terms of the Earth-Moon gravitational interaction and the effect of solid Earth tides on the Moon orbit were also improved.

Celestial bodies have non negligible gravitational self-energy. This was already clear in 1968 [323], when Nordtvedt proposed to use the Earth-Moon system to test the Strong Equivalence Principle. In this case, the relative differential acceleration responsible for a violation of the Universality of Free Fall principle can be expressed as

$$\frac{\Delta a}{a} = \eta_{CD} + \eta_{SEP} \left(\frac{U_E}{M_E c^2} - \frac{U_M}{M_M c^2} \right), \quad (99)$$

where η_{CD} is the composition-dependent violation parameter, η_{SEP} is the Nordtvedt parameter measuring SEP violations, U and M represent the gravitational self-energy of the test body and its mass.

Therefore, to exclude any cancellation effect between a composition-dependent WEP violation and an equal and opposite SEP violation, an independent test of the Weak Equivalence Principle based on test masses having similar composition to the Earth and Moon interior, but with negligible gravitational self-energy is required. The experiment, performed in 1999 with the torsion balance apparatus of the Washington group, confirmed the validity of WEP for two test bodies reproducing the Earth and the Moon composition to 1.4×10^{-13} [242, 279]. Combined with with LLR measurements, this test can be used to constrain SEP violations. The best estimate of the Nordtvedt parameter based on laser ranging measurements of the Earth-Moon system has reached an uncertainty of 1.1×10^{-4} [329].

The Universality of Free Fall can also be tested by tracking satellites orbiting around the Earth, e.g. LAGEOS, LAGEOS II and LARES [330]. As also discussed in [331], the sensitivity of a test based on the Earth-LAGEOS system in the gravitational field of the Sun is a factor 300 worse than for the Earth-Moon system. Indeed, LAGEOS and LARES satellites are much closer to the Earth compared to the Moon. As a consequence, the effects of the Sun gravitational potential on the Moon orbit are significantly stronger than for a satellite orbiting the Earth at low altitude. Even if not competitive with LLR experiments, SLR tests still remain of interest to evaluate the impact of different systematic errors in the final result. As an example, non-gravitational perturbations, which play a major role in the determination of the LAGEOS and LARES orbits, are completely negligible for the Moon. On the contrary, some gravitational perturbations (tidal effects) are important for the Moon and negligible for an Earth-orbiting satellite. Finally, SLR measurements could also be combined with LLR measurements in a grand-fit procedure to better estimate common parameters thus improving LLR and interplanetary ranging [332].

Similar tests can be performed by ranging other gravitating bodies in the solar system. The MESSENGER mission with its Doppler tracking measurements collected over 7 years allows the precise determination of Mercury's ephemeris. This wealth of data has recently been used to test the Strong Equivalence Principle with reduced uncertainty [333]. Spacecraft and planet orbits are numerically integrated to provide a global solution from which parameters relevant for General Relativity, planetary physics, and heliophysics can be extracted. This analysis is today constraining the Nordtvedt parameter to 7×10^{-5} .

3.3 Tests with microscopic particles: atoms, molecules, neutrons, anti-matter

This section is devoted to a review of the tests of the WEP with microscopic particles, mainly atoms.

Based on recent advances in cold atom optics, atomic sensors, namely atom interferometers [334, 335] and atomic clocks [336, 337], established themselves as new powerful tools for precision measurements and fundamental tests in physics [338].

Atom interferometry enabled the realization of precision tests of the WEP that were previously performed only with macroscopic classical masses. As will be clear from the data reported in this review, the sensitivity of atomic experiments did not reach yet the one of the classical experiments but predictions are that similar or even higher levels of sensitivity will be obtained both in Earth laboratories and in experiments in space.

An important advantage of using atoms is, in a properly designed apparatus, the control of possible systematics thanks to the well known and reproducible properties of the atoms, the possibility of realizing an atomic probe of extremely small size and precisely controlling its position, the potential immunity from stray field effects, and the availability of different states and different isotopes that in some cases allows the rejection of common-mode spurious effects and/or a cross-check of the results. Perhaps still more important is that new kinds of tests are possible that exploit the specific quantum features of atomic probes: qualitatively new experiments can be performed with test masses having well-defined properties in terms of, e.g., proton and neutron number, spin, internal quantum state,

bosonic or fermionic nature.

In the final part of this section, tests with neutrons, with charged particles, and with anti-matter are also described because of their fundamental interest but the precision achieved so far in these cases is still much lower compared to the other tests.

It can be expected that in the future the development of matter-wave interferometry with molecules will enable also the comparison of the free fall for such systems with different conformations, different internal states, different chiralities; this will not be discussed in the present review because sensitivities are still too low to be significant in this context but preliminary results and a discussion of future prospects can be found in [339].

It should be noticed that all experiments so far were performed with systems consisting of particles of the first elementary particle family and that direct tests for particles of the second and the third families are missing until now (see, e.g., [340] and references therein).

3.3.1 Precision measurements of gravity with atom interferometry

The idea of an atom interferometer can be easily understood from the analogy with an optical interferometer: using suitable atom optics made of material structures or, more often, laser light, an atomic wave packet is split, reflected and recombined: at the output, interference can be observed. In a more general view, it can be considered as a quantum interference effect arising from the different paths connecting the initial and final states of a system. Any effect affecting in a different way the different paths will produce a change in the interference pattern at the output; by detecting this change, the effect can be measured.

In most experiments, the best performances have been achieved using atom interferometry schemes in which the wavepackets of freely falling samples of cold atoms are split and recombined using laser pulses in a Raman [341, 342] or Bragg [343, 344] configuration. The effect of gravity leads to a phase change $\Delta\phi = kgT^2$ where k is the effective wave-vector of the light used to split and recombine the wave packet, g is gravity acceleration, and T is the time of free-fall of the atom between the laser pulses. This corresponds indeed to the free-fall distance measured in terms of the laser wavelength.

Other schemes were developed to measure g based on Bloch oscillations ([345] and references therein). In this case, the atoms are not falling freely under the effect of gravity but the combined effect of gravity and the periodical potential produced by the laser standing wave leads to oscillations in momentum space with a frequency $\nu_{BO} = mg\lambda/2h$, where m is the atomic mass, λ is the wavelength of the laser producing the lattice and h is Planck's constant. By measuring the frequency of the Bloch oscillation ν_{BO} , g is determined. This method can also be interpreted as the measurement of the gravitational potential difference between adjacent lattice wells which are separated by $\lambda/2$. A few wells are filled with ultracold atoms so that the gravimeter has a sub-millimeter size down to a few micrometers. For this reason, it was also proposed as a method to test the $1/r^2$ Newtonian law for gravity at micrometric distances [345, 346].

Atom interferometers enabled precise measurements of several physical effects; in particular, in gravitational physics, the measurement of gravity acceleration [342, 347–354], gravity gradient [351, 355–360] and curvature [361, 362], determination of the gravitational constant G [256, 356, 363–368], investigation of gravity at microscopic distances [345, 346, 369], search for dark energy and exotic forces [370, 371], applications to geodesy, geophysics, engineering prospecting, inertial navigation [372–374].

In [375], experiments measuring g with different atom interferometry methods [345, 347, 376] were reinterpreted as measurements of the Einstein's gravitational redshift, thus claiming an improvement in precision by 4 orders of magnitude with respect to the Gravity Probe A test reported in [377]. This paper started a controversy on such an interpretation and on the nature of the phase shift measurement in an atom interferometer [252, 375, 378–385].

Atom interferometers and optical atomic clocks were proposed for the detection of gravitational

waves on ground and in space [386–395] and the first prototypes are presently under construction [396–398].

Experiments in space based on cold atom sensors were proposed since long [399–402], the required technology development is in progress [403], and proof-of-principle experiments were recently performed [404].

After the early observation of free fall of atoms using long beams of potassium and cesium atoms [405], several experiments have compared the free fall of different atoms to test the WEP, as described in detail in the following: ^{85}Rb vs ^{87}Rb [406–408], ^{39}K vs ^{87}Rb [409], the bosonic ^{88}Sr vs the fermionic ^{87}Sr [410], atoms in different spin orientations [410, 411]. The relative accuracy of these measurements, reaching so far 10^{-8} - 10^{-9} , is expected to improve by several orders of magnitude in the near future thanks to the rapid progress of atom-optical elements based on multi-photon momentum transfer [412, 413] and of large-scale facilities providing a few seconds of free fall during the interferometer sequence [414–416]. Experiments testing the free fall of anti-hydrogen are in progress [417–419]. WEP tests with a precision $\sim 10^{-15}$ using atom interferometers in space were proposed [399–401].

Concerns have been raised on the potential of atom interferometry for high precision tests of the WEP [420]. A scheme to overcome these possible limitations was proposed in [421] and experimentally demonstrated in [422, 423].

3.3.2 Atoms vs macroscopic objects

In different experiments, gravity acceleration for atoms has been compared with the one for macroscopic masses.

Already in the first demonstration of a high-precision atom interferometry gravimeter with Cs atoms, a classical gravimeter based on a Michelson optical interferometer with a vertical arm containing a freely falling corner-cube was used for comparison. The atom gravimeter was realized with a Raman interferometry scheme. An uncertainty of $\Delta g/g = 3 \times 10^{-9}$ was achieved with a free-fall time $2T = 320$ ms. The comparison between the two gravimeters was interpreted as the demonstration that the macroscopic glass mirror falls with the same acceleration, to within 7 parts in 10^9 , as the quantum-mechanical Cs atom [347].

In [257, 424], comparisons between a mobile Raman atom gravimeter with ^{87}Rb and classical absolute gravimeters were performed with comparable uncertainties.

A conceptually different scheme was used in [425]. The experiment was based on Bloch oscillations of Sr atoms in a vertical optical lattice. In order to increase the sensitivity, in this work a method to measure the frequency of higher harmonics of the Bloch frequency was adopted. The value of the acceleration measured with this atomic sensor was compared with the one obtained in the same lab with a classical FG5 gravimeter. The two values agreed within 140 ppb.

3.3.3 Different isotopes

Experiments were performed testing WEP for different isotopes of an atomic species. Compared to the experiments discussed in the following in which different atomic species are compared, these are somehow simpler; the similar masses and nearby transition frequencies make the apparatus and the control of systematics less complex.

In [406], an atom interferometer based on the diffraction of atoms from standing optical waves acting as effective absorption gratings was used to compare the two stable isotopes of rubidium, ^{85}Rb and ^{87}Rb , with a relative accuracy of 1.7×10^{-7} . In this work, a test for a possible difference of the free fall acceleration as a function of relative orientation of nuclear and electron spin was also performed with a differential accuracy of 1.2×10^{-7} by comparing interference patterns for ^{85}Rb atoms in two

different hyperfine ground states (see sect. 3.3.5). A comparable precision for the differential free fall measurement of ^{85}Rb and ^{87}Rb was later obtained in [407] using Raman atom interferometry.

About one order of magnitude improvement in precision was obtained in [408]. A four-wave double-diffraction Raman transition scheme was used for the simultaneous dual-species atom interferometer to compare ^{85}Rb and ^{87}Rb . The value obtained for the Eötvös parameter is $\eta = (2.8 \pm 3.0 \times 10^{-8})$.

Ongoing experiments in large baseline interferometers aim to a final precision in the 10^{-15} range and beyond [414–416]. Limiting factors due to the gravity gradients were discussed [420] and possible solutions were proposed in [421] and demonstrated in [422, 423]. In [423] the gravity gradient compensation in a long duration and large momentum transfer dual-species interferometer with ^{85}Rb and ^{87}Rb allowed to reach a relative precision of $\Delta g/g \approx 6 \times 10^{-11}/\text{shot}$ or $3 \times 10^{-10}/\sqrt{Hz}$ that makes such a WEP test realistically feasible at the 10^{-14} level.

In [410], the WEP was tested for the bosonic and fermionic isotopes of strontium atoms, namely, ^{88}Sr and ^{87}Sr . As in [425], gravity acceleration for the two isotopes was determined by measuring the frequency of the Bloch oscillations for the atoms in a vertical optical lattice. By detecting the coherent delocalization of matter waves induced by an amplitude modulation of the lattice potential at a frequency corresponding to a multiple of the Bloch frequency, the limit obtained in this work for the Eötvös parameter is $\eta = (0.2 \pm 1.6 \times 10^{-7})$. As discussed in the following, the results of this experiment are also relevant as a WEP test for bosons vs fermions and for the search of spin-gravity coupling.

3.3.4 Different atomic species

Recently, experiments were performed to test WEP with different atomic species. This requires the development of more complex experimental setups and a more difficult control of systematics. The theoretical framework to interpret the experimental results can be found in Refs. [426–428].

The possibility of a test using rubidium and potassium atoms was discussed in [429]. The first results were reported in [409]; in this work, ^{87}Rb and ^{39}K were compared using two Raman interferometers. The result was an Eötvös ratio $\eta = (0.3 \pm 5.4 \times 10^{-7})$ mainly limited by the quadratic Zeeman effect and the wave front curvature of the Raman beams. The choice of atomic species in this paper was compared with others in terms of sensitivity to possible violations of the EP predicted by a dilaton model [426] and by standard-model extensions [427].

The ongoing activity for a test with rubidium and ytterbium atoms in a 10-m baseline atom interferometer was discussed in [428] with the goal to reach an Eötvös ratio in the $10^{-12} - 10^{-13}$ range.

3.3.5 Atoms in different quantum states

While it can be argued that some of the experiments with atoms described above are not qualitatively different from the ones performed with macroscopic classical systems as far as the physics which is tested is concerned, the experiments described in this section take full advantage of the quantum nature of the atoms as probes of the gravitational interaction.

- Atoms in different energy eigenstates and in superposition states

The mass-energy relation $E = mc^2$ in special relativity implies that the internal energy of a system affects its mass. It is then of interest to verify the validity of the equivalence of the inertial and gravitational mass for systems in different internal quantum states. This was theoretically discussed in [430, 431] and possible experimental tests with atoms in different internal states were proposed. In particular, the importance of tests involving atoms in superpositions of the internal energy eigenstates was highlighted because this corresponds to a genuine quantum test. Another possible experimental test of the quantum formulation of the equivalence principle was proposed in [432].

A first experimental test of the equivalence principle in this quantum formulation was reported in [433]. A Bragg atom interferometer was used to compare the free fall of ^{87}Rb atoms prepared in two hyperfine states $|1\rangle = |F = 1, m_F = 0\rangle$ and $|2\rangle = |F = 2, m_F = 0\rangle$, and in their coherent superposition $|s\rangle = (|1\rangle + e^{i\gamma}|2\rangle)/\sqrt{2}$. In order to increase the measurement sensitivity, the atom interferometer was operated at the 3rd Bragg diffraction order, corresponding to $6\hbar k$ total momentum transfer between the atoms and the radiation field.

The comparison of the free-fall acceleration for atoms in the $|1\rangle$ and $|s\rangle$ states led to the first experimental upper bound of 5×10^{-8} for the parameter corresponding to a violation of the WEP for a quantum superposition state.

Based on models in which WEP violations increase with the energy difference between the internal levels [426], in this paper the prospect to use states with an energy separation larger than the hyperfine splitting was also proposed considering optically separated levels in strontium for which the relevant atom interferometry schemes were already demonstrated [434–436].

The comparison of gravity acceleration for atoms in the $|1\rangle$ and $|2\rangle$ hyperfine states led to an Eötvös ratio $\eta_{1-2} = (1.0 \pm 1.4 \times 10^{-9})$ that corresponds to an improvement by about two orders of magnitude with respect to the previous limit set in [406]. A further improvement by a factor of 5 in the precision of this test was recently reported in [437], approaching the 10^{-10} level.

- Atoms in entangled states

In [438] a quantum test of the WEP with entangled atoms was proposed. In the proposed experiment, a measurement of the differential gravity acceleration between the two atomic species would be performed by entangling two atom interferometers operating on the two species. The example of ^{85}Rb and ^{87}Rb was analyzed in detail showing that an accuracy better than 10^{-7} on the Eötvös parameter can be achieved.

Although no theoretical model is available predicting a WEP violation in the presence of entanglement, this is clearly a case of a purely quantum system to be further investigated.

The free fall of particles in quantum states without a classical analogue and in particular for Schrödinger cat states in the configuration space was studied theoretically in [439].

- Atoms in different spin states

As discussed above (see in particular Sect. 2.13), spin-gravity coupling and torsion of space-time were extensively investigated theoretically [6, 440, 441].

Different experiments were performed using macroscopic test masses [281, 283, 441], atomic magnetometers [442, 443], and by measuring hyperfine resonances in trapped ions [444]. The differential free-fall experiments with atoms in different hyperfine states are also relevant in this frame [406, 433, 437].

Recently, experiments were performed using atom interferometry to search for the coupling of the atomic spin with gravity.

In [410], the experimental comparison of the gravitational interaction for the bosonic isotope of strontium ^{88}Sr , which has zero total spin in its ground state, with that of the fermionic isotope ^{87}Sr , which has a half-integer nuclear spin $I = 9/2$, was performed based on the measurement of the frequency of Bloch oscillations for the atoms in a vertical optical lattice under the effect of Earth's gravity.

A modified gravitational potential including a possible violation of WEP and the presence of a spin-dependent gravitational mass was considered in the form $V_{g,A}(z) = (1 + \beta_A + kS_z)m_Agz$, where m_A is the rest mass of the atom, β_A is the anomalous acceleration generated by a nonzero difference between gravitational and inertial mass due to a coupling with a field with nonmetric interaction with gravity, k is a model-dependent spin-gravity coupling strength, and S_z is the projection of the atomic spin along gravity direction.

As already described above, the Bloch frequency corresponds to the site-to-site energy difference induced by the gravitational interaction; by measuring the frequency of Bloch oscillations for ^{88}Sr and ^{87}Sr an Eötvös parameter $(0.2 \pm 1.6) \times 10^{-7}$ was obtained. Since the frequency of Bloch oscillations depends on the mass of the particle, in the analysis of the data the m^{88}/m^{87} mass ratio was taken into account which is known with a relative precision $\sim 10^{-10}$. The analysis of the Bloch resonance spectrum for ^{87}Sr provided an upper limit for the spin-gravity coupling strength $k = (0.5 \pm 1.1) \times 10^{-7}$. This result also sets a bound for an anomalous acceleration and a spin-gravity coupling for the neutron either as a difference in the gravitational mass depending on the spin direction or as a coupling to a finite-range interaction [441, 442].

In [411], a Mach-Zehnder-type Raman atom interferometer was used to compare the gravity acceleration of freely-falling ^{87}Rb atoms in different Zeeman sublevels $m_F = +1$ and $m_F = -1$, corresponding to opposite spin orientations. The experiment required a special care to control the high sensitivity of these states to magnetic field inhomogeneity. The Eötvös parameter obtained in this experiment was $(0.2 \pm 1.6) \times 10^{-7}$. The data were also interpreted as providing an upper limit of $5.4 \times 10^{-6} \text{ m}^{-2}$ for a possible gradient field of the spacetime torsion.

In [339], based on recent advances of matter-wave interferometry with large molecules, the prospect of a test of WEP for molecules with opposite chiralities was proposed.

It should be noted that a complete analysis connecting theoretically the models tested in the different experiments performed so far in this frame is still missing.

- Atoms in a Bose-Einstein condensate

Possible differences in the gravitational interaction for bosons and fermions were investigated theoretically [445] and tested experimentally [410].

Violations of the WEP for atoms in a quantum state such as a Bose-Einstein condensate were discussed in ([446, 447] and references therein). Since in quantum physics particles are described by an extended wave packet, the validity of the WEP which refers to point-like particles can be questioned. A model based on spacetime fluctuations allows to predict a possible difference in the observed free fall for different particles because the different spatial extensions of the wavefunction of particles of different masses would lead to an averaging of the metric fluctuations over different spatial volumes. Also, the metric fluctuations would produce decoherence.

Such elusive effects, if ever observable, would require atom interferometers with extremely high sensitivity, that is, a very long evolution time. For this and other scientific goals, the technology needed to perform experiments in microgravity is being developed [300, 401, 404, 448, 449] as described in detail in the following.

3.3.6 Experiments with atoms in microgravity

The ultimate performance of atomic sensors for WEP tests can be reached in a space-based laboratory. In space atoms can rely on a very quiet environment where Newtonian noise is absent and microvibrations and non-gravitational accelerations can be reduced to very low levels. Very long and unperturbed free fall conditions can be obtained, allowing atomic wavepackets to evolve, sense the space-time metric, and record its signature in their phase. At the same time, very long and unperturbed interaction times between the atomic ensemble and the interrogation fields can be achieved. This is translating into a significant increase of the instrument sensitivity and a better control of the systematic errors.

As an example, the phase accumulated in a Mach-Zehnder interferometer, $\Delta\phi = kgT^2$, is directly proportional to the square of the free evolution time T between the three laser pulses of the interferometry sequence. The typical duration of an atom interferometry sequence on the ground is $2T \approx 1 \text{ s}$, which corresponds to a free fall distance of about 10 m in the gravity field of the Earth. In space, both

the atoms and the instrument platform are in free fall and interrogation times of $2T = 10$ s or longer can be achieved, improving the instrument sensitivity by a factor 100 or more with respect to a similar instrument operated on the ground.

Achieving a free evolution time of $2T \approx 10$ s on the ground would require an atomic fountain apparatus with several hundred meters of free-fall length, showing another important aspect of atom-based sensors designed for space compared to their laboratory counterpart, i.e. the compactness. In space, atoms interrogation can take place in a small vacuum chamber with a typical size of a few liters. This volume can be better controlled against external perturbations, such as temperature, magnetic fields, etc. As an example, the development of large size mu-metal shields to accurately control the external magnetic field along the free evolution trajectories of a long atomic fountain (10 m or longer) remains a non negligible technology challenge.

Finally, a technique to counteract the effect of gravity gradients has recently been developed [421] and experimentally demonstrated [422], reducing to a negligible level one important source of instability and systematic error in precision measurements by atom interferometry.

STE-QUEST (Space-Time Explorer and QUantum Equivalence Space Test) is a mission designed to test different aspects of the Einstein Equivalence principle in space [401]. The STE-QUEST scientific objectives include an atom interferometry test of the Weak Equivalence Principle, an absolute measurement of the Einstein's gravitational redshift, and tests of Standard Model Extension (SME). Here, we will only focus on the WEP test. The on-board instrument dedicated to this measurement is a differential atom interferometer. Originally designed to compare the free fall of the 85 and 87 rubidium isotopes, the instrument has recently been re-adapted to operate on potassium and rubidium that, due to the larger difference in neutrons and protons, are expected to provide higher sensitivity in the detection of a WEP violation [450]. The two atomic ensembles would be cooled down to very cold temperatures (100 pK regime) and simultaneously interrogated in the atom interferometry sequence by using the double-diffraction technique [451]. The simultaneous interrogation provides rejection ratio of common mode acceleration noise (e.g. air drag and mechanical vibrations), which can vary from 10^{-9} for ^{85}Rb - ^{87}Rb simultaneous interferometers [403] to $< 10^{-3}$ for the ^{87}Rb - ^{39}K couple. The requirements on the control of non-gravitational acceleration acting on the spacecraft are therefore very modest for a ^{85}Rb - ^{87}Rb differential interferometer and significantly more stringent for the ^{87}Rb - ^{39}K one, but still well within the available technology as demonstrated in the MICROSCOPE [308] and LISA Pathfinder missions [310]. A design description of the STE-QUEST differential atom interferometer can be found in [452]. The expected error budget is presented in [403]. The instrument will be able to measure differential accelerations down to $8 \times 10^{-15} \text{ms}^{-2}$ corresponding to a WEP test at the 1×10^{-15} level.

A similar instrument has also been proposed for a WEP test on the International Space Station (ISS) [453]. The ISS is a harsh environment for what concerns non gravitational accelerations, rotations, tilt noise, and mechanical vibrations. The instrument is therefore designed to ensure optimal control on systematic errors and high rejection of common mode effects. The differential accelerometer compares the free fall acceleration of ^{85}Rb and ^{87}Rb atomic samples in a symmetric configuration with two separate source regions. Bragg lasers tuned to the wavelength for which the two rubidium isotopes have the same polarizability are used to simultaneously interrogate the atomic samples in the interferometric sequence. This approach ensures a very high suppression of laser noise and common mode acceleration noise. The instrument will be accommodated on a rotating platform to control gravity gradient effects.

The SAGE (Space Atomic Gravity Explorer) mission proposal [402] has the scientific objective to investigate gravitational waves, dark matter, and other fundamental aspects of gravity such as the WEP as well as the connection between gravitational physics and quantum physics using optical atomic clocks and atom interferometers based on ultracold strontium atoms.

Several experiments and test activities are currently in progress to demonstrate the maturity of atom-based sensors for space operation and to evaluate the ultimate stability and accuracy that can be reached in differential acceleration measurements for WEP tests.

In 23 January 2017, the MAIUS-1 experiment was launched in a sounding rocket to a height of 243 km. During the lift-off phase and the 360 min of free-fall conditions, 110 experiments involving atoms cooling and manipulation were performed. They include laser cooling and trapping of atoms, observation of the BEC phase transition, BEC transport on the atom chip, and study of BEC collective oscillations under weightlessness conditions [404]. This experiment demonstrates the building blocks of future atom interferometry experiments in space.

The Cold Atom Lab (CAL) is a multiuser facility launched to the ISS in 21 May 2018. The CAL instrument is designed to produce ultracold atomic samples of ^{39}Rb , and ^{41}K [454] down to quantum degeneracy. In the microgravity environment of the ISS, it is possible to decompress the atomic traps to very low levels thus achieving ultra-low densities and picokelvin temperatures. The experiment will test different atomic sources for atom interferometry in weightlessness conditions.

Significant progress has also been achieved by making use of microgravity facilities available on the ground, in particular the Bremen drop tower and the zero-gravity parabolic airplane.

Mach-Zehnder interferometry experiments on a Bose-Einstein condensate have been performed in the Bremen drop tower [455]. The drop tower capsule was operated both in drop and catapult mode, providing a free fall duration of 4.7 s and 9.4 s, respectively. The atom interferometer could then demonstrate a shot-noise limited resolution of $6.2 \times 10^{-11} \text{ ms}^{-2}$ in the drop mode and $5.5 \times 10^{-12} \text{ ms}^{-2}$ in the catapult mode. With this performance, a sensitivity of a few parts in 10^{13} for a WEP test should be possible in less than 100 drops (also see [447]). Unfortunately, the study of systematic effects of a WEP test would result very unpractical in the drop tower facility.

A WEP test on ^{87}Rb and ^{39}K has been performed in the microgravity conditions of an airplane in parabolic flight. The Eötvös ratio was measured to 3.0×10^{-4} , limited by the noisy acceleration environment ($10^{-2}g \text{ Hz}^{-1/2}$). This result, certainly not competitive with respect to other WEP tests, remains important as it demonstrates the possibility of using correlated interference fringes to perform a WEP test with an accuracy two orders of magnitude below the level of ambient vibration noise. The experiment could therefore confirm the expected rejection to common mode vibration for a ^{87}Rb - ^{39}K differential interferometer [429, 456].

3.3.7 Tests with neutrons

As for the atoms, the first low-precision measurements of gravity acceleration for neutrons were performed by measuring the drop of collimated beams of thermal neutrons [457, 458]. They were also interpreted as tests of the universality of free fall. In [458], a test of a possible dependence of neutron acceleration on the two vertical neutron-spin projections $\pm\frac{1}{2}$ was performed finding no difference within the experimental sensitivity.

After the first observation of gravitationally induced interference in a neutron interferometer [459], tests of WEP were performed using neutron interferometers reaching a precision of 10^{-3} [460, 461], an accelerated interferometer [462], and by a slow neutron gravity refractometer with a quoted relative uncertainty of 3×10^{-4} [463, 464].

More recently, gravity acceleration for neutrons was measured with a cold neutron interferometer [465] and with a spin-echo spectrometer [466] with a relative precision of 10^{-3} .

In [467] prospects to achieve a relative precision $\Delta g/g \sim 10^{-5}$ using a three phase-grating moiré large area neutron interferometer were discussed proposing also a measurement of the value of the gravitational constant G with neutrons.

3.3.8 Tests with antimatter and with charged particles

In principle, gravitation for antimatter may obey different laws than for ordinary matter. Measuring and comparing the gravitational properties of matter and antimatter may probe different aspects of SME [250] and quantum vacuum [468]. Theoretical considerations based on energy conservation in the gravitational field and on arguments from QFT constrain the validity of the WEP for antimatter up to an accuracy of 10^{-14} [469, 470]. Nevertheless, these arguments are indirect and need some experimental validation by comparing the effect of gravity on antiparticles on and their corresponding ordinary matter particles.

Experiments to test gravity on elementary particles and antiparticles were proposed and developed since the 1960's. Electrically charged antiparticles (e.g. positrons and antiprotons) can be either observed in beams under free fall conditions, or trapped within a combination of magnetic and electric fields, as in Penning traps [471]. In 1967 Witteborn and Fairbank observed the time-of-flight distribution of electrons and positrons in free fall inside a drift tube. Two decades later, an experiment was performed at the CERN Low Energy Antiproton Ring to measure the gravitational acceleration of antiprotons [472]. \bar{p} were collected and cooled in a Penning trap, then released in a vertical drift tube. However both freely falling and trap-based systems of charged antiparticles are affected by errors from residual stray electric and magnetic fields [473] which make any gravitational measurement extremely difficult. The static electric field mg/q required to compensate gravity acceleration is only 56 pV/m for positrons, and about 100 nV/m for antiprotons. Even in case of perfect shielding from stray fields, the gravitational sag of electrons in a drift tube produces charge density anisotropies resulting in a electric forces of the same order of gravity [474]. WEP tests with electrons under weightless conditions have been proposed to get rid of gravity-induced electric fields [231]. On the other hand, current experiments to test WEP on anti-matter are focusing on neutral systems such as positronium [475], muonium [340, 476], antihydrogen [417, 477–480], and on particle-antiparticle pairs [481], for which the effect of stray fields is strongly suppressed.

The first evidence of antihydrogen was achieved at the European Organization for Nuclear Research (CERN) in 1995 [482] and confirmed two years later at Fermilab [483]. The recombination of electron-positron pairs produced from the collision of relativistic antiprotons with Xenon targets formed \bar{H} atoms at relativistic energies, unsuitable for precision measurements. A breakthrough occurred when \bar{H} at thermal energies (few hundreds K) was first produced in 2002 by the ATHENA experiment [484], via three-body reaction by mixing trapped antiprotons (\bar{p}) with positrons (e^+) at low energies. This result, shortly followed by a similar achievement from the ATRAP experiment [485] opened the possibility to test WEP on neutral antimatter.

However the gravitational force is so weak that a WEP experiment requires further cooling of antimatter down to cryogenic temperatures. Several second-generation experiments with antimatter have been then developed at CERN, where the only intense source of low energy antiprotons is available worldwide, i.e. the Antiproton Decelerator (AD) currently under upgrade to Extra Low ENergy Antiproton ring (ELENA). Such experiments must face several challenges. Neutral antimatter particles are produced from their charged constituents. This requires complex experiments combining advanced methods from high-energy physics for particle beams optics and detectors, with advanced methods for ion trapping and atom optics. Moreover neutral antiparticles are produced at much lower rates and at much higher temperatures than in the typical quantum sensors described in section 3.3. The various experiments at AD developed different methods to produce \bar{H} at rates and temperatures suited for precision measurements.

The ALPHA experiment, designed to perform high resolution spectroscopy on \bar{H} , generates antihydrogen by three-body recombination between trapped, evaporatively cooled antiprotons and trapped positrons. The low-energy tail of the \bar{H} distribution is captured in a magnetic trap at a rate of about 10 atoms on cycles of 4 minutes [486]. ALPHA performed a preliminary measurement of the Earths

gravitational effect on magnetically trapped \bar{H} . The resulting gravitational acceleration of \bar{H} was constrained to within 100 times the g value for matter [487].

The GBAR experiment aims to generate ultracold antihydrogen through the anti-ion \bar{H}^+ [477, 478]. The \bar{H}^+ ion is produced via two cascaded charge exchange processes from the interaction of \bar{p} with a positronium target, then of the generated \bar{H} with the same target. \bar{H}^+ ion can be sympathetically cooled with laser cooled Be^+ ions down to μK temperatures. The excess positron can then be laser detached in order to recover the neutral \bar{H} with very low temperature. A high-intensity positron source has been developed for \bar{H}^+ production.

The projects Antimatter Experiment: Gravity, Interferometry, Spectroscopy (AEgIS) [417], which is operating since 2012, has been designed to measure the gravitational acceleration with matter-wave interferometry on a pulsed \bar{H} beam at sub-kelvin temperatures. In AEgIS, antihydrogen atoms are produced via a charge exchange reaction between Rydberg-excited positronium atoms and cold antiprotons within an electromagnetic trap. The resulting Rydberg antihydrogen atoms will be horizontally accelerated by an electric field gradient (Stark effect), then they will pass through a moiré deflectometer. The vertical deflection caused by the Earth’s gravitational field will provide a Weak Equivalence Principle test for antimatter. Detection will be undertaken via a position sensitive detector. Around 10^3 antihydrogen atoms are needed for the gravitational measurement to be completed. The generation of antihydrogen via charge-exchange process was already demonstrated in the ATRAP experiment [488], where Ps were excited toward a Rydberg state by collisions with laser-excited cesium atoms. AEgIS rather plans to directly laser excite positronium.

An alternative to testing UFF on bound antimatter systems is the search for mass differences on particle-antiparticle pairs. In particular, neutral kaon is the only system where particle-antiparticle differences are detected; this is explained as arising from CP-violating terms in the $K^0 - \bar{K}^0$ mass matrix. In [481] upper limits on possible $K^0 - \bar{K}^0$ mass difference were determined from the analysis of data on tagged K^0 and \bar{K}^0 decays into $\pi^+\pi^-$ from the CPLEAR experiment over three years. The results are in agreement with the Equivalence Principle to a level of 6.5, 4.3 and 1.8×10^{-9} respectively, for scalar, vector and tensor potentials originating from the Sun. Such determination of mass difference for kaon is ten orders of magnitude more precise than for $p - \bar{p}$ [489].

High precision gravity measurements will require the application of interferometric methods. While the most precise quantum sensors are based on light pulse matter-wave interferometry, see section 3.3, such method is not readily applied on antimatter systems. This is mostly due to the comparably high temperatures currently achievable and to the extremely short wavelength of resonance optical transitions in \bar{H} and in ps . Inertial sensing with Talbot-Lau interferometry [490] allows to work with low-intensity, weakly coherent beams. This method has been recently demonstrated on a beam of low energy positrons [491].

4 Conclusions and outlook

General Relativity and metric theories of gravity are based on the validity of the Equivalence Principle, according to which the gravitational acceleration is (locally) indistinguishable from acceleration caused by mechanical (apparent) forces. The consequence of the Equivalence Principle is that gravitational mass is equal to inertial mass, $m_g = m_I$. This identity was already pointed out by Galileo and Newton, but Einstein recognizes it as a fundamental aspect involving also accelerations and forces and then elevating it to a principle. Equivalence Principle allowed Einstein to construct a theory capable of explaining gravity and acceleration under the same physical standard. Based on this assumption, he stated the following fundamental postulated: in a free falling frame, all non-gravitational laws of physics (hence not only the mechanical ones) behave as if gravity was absent. More generally, Equivalence Principle asserts that objects with different (internal) composition are subject to the same

acceleration when moving in a gravitational field. This new principle of nature led Einstein to the revolutionary interpretation of gravitation: gravity can be described as a curvature effect of space-time. As a consequence, the Einstein Equivalence Principle plays a crucial role in all metric theories of gravity, as well as in the Standard Model of particle physics which is not in conflict with GR.

As we have seen in this review, the Equivalence Principle essentially encodes the local Lorentz invariance (clock rates are independent of the clocks velocities), the local position invariance (the universality of red-shift) and the universality of free fall (all free falling point particles follow the same trajectories independently of their internal structure and composition). The first two principles, i.e. the Lorentz and position invariance, are hence related to the local properties of physics, so that they can be tested by using atomic clocks and measurements of spectroscopy, while the third one, the universality of free falling point-like particles, can be tested by tracking trajectories, hence freely falling test masses as discussed in the experimental part of this Review paper.

The Equivalence Principle can be formulated in two different forms: the weak and the strong form.

The weak form of the Equivalence Principle states that the gravitational properties of the interaction of particle physics of the Standard Model, hence the strong and electro-weak interactions, obey the EP. As we have pointed out, the equality $m_g = m_i$ implies that, in an external gravitational field, different (and neutral) test particles undergo to the same free fall acceleration, and in a free falling inertial frame only tidal forces may appear (apart the latter, free falling bodies behave as if the gravity is absent). However, it is worth noticing that in many extensions of the Standard Model, new interactions (quantum exchange forces) are introduced, and, in general, they may violate the weak equivalence principle owing to the coupling with generalized charges, rather than mass/energy as happens in gravity.

The second form, the strong Equivalence Principle, is such that it extends the weak one including the gravitational energy. In GR, the strong Equivalence Principle is fulfilled thanks to the gravitational stress-energy tensor, while it can be violated in some extension of GR (as, for example, scalar tensor theories discussed above where a scalar field is present in the gravitational interaction and can be non-minimally coupled with geometry).

As a final remark, an important issue has to be discussed. It is related to the parameterized post-Newtonian (PPN) formalism [1, 492] and the Equivalence Principle. In a nutshell, the PPN approximation is a method for obtaining the motion of the system in terms of higher powers of the small parameters (gravitational potential and velocity square) with respect to the ones given by Newtonian mechanics. The relevant aspect is that this formalism allows to describe the motion of M_q celestial bodies that is common to many theories of gravity. The acceleration of a body can be written in the form $\ddot{\mathbf{r}}_p = \ddot{\mathbf{r}}_p^{GR} + \delta\ddot{\mathbf{r}}_p^{PPN}$, where $\ddot{\mathbf{r}}_p^{GR}$ is the usual acceleration derived in GR, while $\delta\ddot{\mathbf{r}}_p^{PPN}$ is the PPN corrections [1, 492]

$$\begin{aligned} \delta\ddot{\mathbf{r}}_p^{PPN} = & \sum_{q \neq p} \frac{GM_q(\mathbf{r}_q - \mathbf{r}_p)}{|\mathbf{r}_q - \mathbf{r}_p|^3} \left\{ \left(\left[\frac{M_g}{M_I} \right]_q - 1 \right) - \frac{2(\beta + \gamma - 2)}{c^2} \sum_{r \neq p} \frac{GM_r}{|\mathbf{r}_r - \mathbf{r}_p|} - \frac{2(\beta - 1)}{c^2} \sum_{s \neq q} \frac{GM_s}{|\mathbf{r}_s - \mathbf{r}_q|} \right. \\ & \left. + \frac{\gamma - 1}{c^2} |\dot{\mathbf{r}}_q - \dot{\mathbf{r}}_p|^2 + \frac{\dot{G}}{G}(t - t_0) \right\} + \frac{2(\gamma - 1)}{c^2} \sum_{q \neq p} \frac{GM_q}{|\mathbf{r}_p - \mathbf{r}_q|} \left[\ddot{\mathbf{r}}_q + \frac{[(\mathbf{r}_p - \mathbf{r}_q) \cdot (\dot{\mathbf{r}}_p - \dot{\mathbf{r}}_q)](\dot{\mathbf{r}}_p - \dot{\mathbf{r}}_q)}{|\mathbf{r}_p - \mathbf{r}_q|^2} \right] \end{aligned}$$

Some comments are in order: 1) The correction $\delta\ddot{\mathbf{r}}_p^{PPN}$ vanishes in the case of GR, that is when the PPN parameter γ and β assume the values: $\gamma = 1 = \beta$. 2) The expression for $\delta\ddot{\mathbf{r}}_p^{PPN}$ does contain the variation of the gravitational constant G , the \dot{G}/G -term, typical of scalar tensor theories of gravity; 3) In $\delta\ddot{\mathbf{r}}_p^{PPN}$ a term related to the strong equivalence principle appears, i.e. $\left[\frac{M_g}{M_I} \right] - 1$, which is typically expressed in the form $\left[\frac{M_g}{M_I} \right] = 1 + \eta \left(\frac{E}{mc^2} \right)$, where the parameter η , depending on PPN parameters as $\eta = 4\beta - \gamma - 3$, encodes deviations from GR, and it is therefore related to the violation of the strong Equivalence Principle ($\eta = 0$ in GR), while m and E are the mass and gravitational self-energy of the

body, respectively. Taking, for example, a uniform sphere of radius R one gets

$$\left[\frac{E}{mc^2} \right] = -\frac{G}{2mc^2} \int d^3x d^3x' \frac{\rho(x)\rho(x')}{|\mathbf{x} - \mathbf{x}'|} = -\frac{3GM}{5c^2R}. \quad (100)$$

This relation shows that, for Solar System it is $\left[\frac{E}{mc^2} \right]_{\odot} \sim 10^{-6}$, while for bodies of lab test one gets $\left[\frac{E}{mc^2} \right]_{lab} \sim 10^{-25}$. This suggests that planet-size bodies are required for testing the strong Equivalence Principle with a certain confidence level.

Finally, we discussed the possibility to violate the Equivalence Principle considering systems at finite temperature. Although the Equivalence Principle asserts (in the weak form) that the gravitational acceleration is identical for all bodies, i.e. $m_I = m_g$, the latter equality can be violated in quantum field theory considering a finite temperature framework. In fact, as shown in a series of seminal papers [232], *a fraction of the mass of a particle arises through the finite-temperature component of the radiative corrections*. This result is a consequence of the Lorentz non-invariance of the finite temperature vacuum. According to this result, theories at finite temperature could be the straightforward way to test Equivalence Principle at fundamental quantum level.

In conclusion, various issues in modern physics, both from gravitational and particle physics sectors, predict violations of the Equivalence Principle. Given the importance of this question, the experimental challenge is to look for frameworks where possible violations could manifest. Besides, one has to improve the limits of experimental tests. As we have shown in this review, matching experiments from laboratory and satellites is an important complement to probing fundamental physics at very high precision level, and, in turn, possible results could open new and unexpected scenarios.

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