# Equivalence Principle in Classical and Quantum Gravity

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We give a general overview of various flavors of the equivalence principle in classical and quantum physics, with special emphasis on the so-called weak equivalence principle, and contrast its validity in mechanics versus field theory. We also discuss its generalisation to a theory of quantum gravity.

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## I. INTRODUCTION

Quantum mechanics (QM) and general relativity (GR) are the two cornerstones of modern physics. Yet, merging them together in a quantum theory of gravity (QG) is still elusive despite nearly century long efforts of vast numbers of physicists and mathematicians. While the majority of the attempts were focused on trying to formulate the full theory of a quantised gravity, such as the string theory, loop quantum gravity, non-commutative geometry, and causal set theory, to name a few, a number of recent studies embraced a rather more modest approach by exploring possible consequences of basic features and principles of QM and GR, and their status, in a tentative theory of QG. Acknowledging that the superposition principle, as a defining characteristic of any quantum theory, must be featured in QG as well, led to a number of papers studying gravity-matter entanglement [1–7], genuine indefinite causal orders [8–15] and quantum reference frames [16–19], to name a few major research directions. Exploring spatial superpositions of masses, and in general of gravitational fields, led to the analysis of the status of various versions of the equivalence principle, and their exact formulations in the context of QG. In particular, in a recent work [20] it was shown that the weak equivalence principle (WEP) should generically be violated in the presence of a specific type of superpositions of gravitational fields, a result seemingly at odds with some previous studies [19, 21–24] (for details, see the discussion from Section V).

Modern formulation of WEP is given in terms of a *test particle* and its *trajectory*: it is a *theorem* within the mathematical formulation of GR stating that the trajectory of a test particle satisfies the so-called geodesic equation [25–44], while it is *violated* within the context of QG, as shown in [20]. In this paper, we present a brief overview of WEP in GR and a critical analysis of the notions of particle and trajectory in both classical and quantum mechanics, as well as in the corresponding field theories, with the aim of generalising and studying WEP in the context of QG.

The paper is organised as follows. In Section II, we give a brief historical overview of various flavours of the equivalence principle, with a focus on WEP. In Section III we analyse the notion of a trajectory in classical and quantum mechanics, while in Section IV we discuss the notion of a particle in field theory and QG. Finally, in Conclusions, we briefly review and discuss our results, and present possible future lines of research.

## II. EQUIVALENCE PRINCIPLE IN GENERAL RELATIVITY

The equivalence principle (EP) is one of the most fundamental principles in modern physics. It is one of the two cornerstone building blocks for GR, the other being the principle of general relativity. While its importance is well understood in the context of gravity, it is often underappreciated in the context of other fundamental interactions. In addition, there have been numerous studies and everlasting debates whether EP holds also in quantum physics, if it should be generalised to include quantum phenomena or not, etc. Finally, EP has been historically formulated in a vast number of different ways, which are often not mutually equivalent, leading to a lot of confusion about the actual statement of the principle and its physical content [45–48]. Given the importance of EP, and the amount of confusion around it, it is important to try and help clarify these issues.

The equivalence principle is best introduced by stating its purpose — in its traditional sense, the purpose of EP is to *prescribe the interaction between gravity and all other fields in nature, collectively called matter*. This is important to state explicitly, since EP is often mistakenly portrayed as a property of gravity alone, without any reference to matter. In a more general, less traditional, and often not appreciated sense, the purpose of EP is to prescribe the interaction between *any gauge field* and all other fields in nature, which we will reflect on briefly in the case of electrodynamics below.

Given such a purpose, let us for the moment concentrate on the gravitational version of EP, and provide its modern formulation, as it is known and understood today. The statement of the equivalence principle is the following:

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> *The equations of motion for matter coupled to gravity remain locally identical to the equations of motion for matter in the absence of gravity.*

This kind of statement requires some unpacking and comments.

- When comparing the equations of motion in the presence and in the absence of gravity, the claim that they remain identical may naively suggest that gravity does not influence the motion of matter in any way whatsoever. But on closer inspection, the statement is that the two sets of equations remain *locally* identical, emphasizing that the notion of locality is a crucial feature of the EP. While equations of motion are already local in nature (since they are usually expressed as partial differential equations of finite order), the actual interaction between matter and gravity enters only when *integrating* those equations to find a solution. In other words, the statement of EP is that interaction between matter and gravity appears in a specifically nonlocal sense.

- In order to compare the equations of motion for matter in the presence of gravity to those in its absence, the equations themselves need to be put in a suitable form (typically expressed in general curvilinear coordinates, as tensor equations). The statement of EP relies on a theorem that this can always be done, first noted by Erich Kretschmann [49].

- Despite being dominantly a statement about the interaction between matter and gravity, EP also implicitly suggests that the best way to describe the gravitational field is as a property of geometry of spacetime, such as its metric [50]. In that setup, EP can be reformulated as a statement of *minimal coupling* between gravity and matter, stating that equations of motion for matter may depend on the spacetime metric and its first derivatives, but not on its (antisymmetrised) second derivatives, i.e., the *spacetime curvature does not explicitly appear in the equations of motion for matter.*

- The generalisation of EP to other gauge fields is completely straightforward, by replacing the role of gravity with some other gauge field, and suitably redefining what matter is. For example, in electrodynamics, the EP can be formulated as follows:

  > *The equations of motion for matter coupled to the electromagnetic field remain locally identical to the equations of motion for matter in the absence of the electromagnetic field.*

  This statement can also be suitably reformulated as the minimal coupling between the EM field and matter, i.e., coupling matter to the electromagnetic potential $A_\mu$ but not to the corresponding field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, which is in fact the standard way EM field is coupled to matter. Even more generally, the gauge field sector of the whole Standard Model of elementary particles (SM) is built using the minimal coupling prescription, meaning that the suitably generalised version of the EP actually prescribes the interaction between matter and all fundamental interactions in nature, namely strong, weak, electromagnetic and gravitational. In this sense, EP is a cornerstone principle for the whole fundamental physics, as we understand it today.

Of course, much more can be said about the statement of EP, its consequences, and various other details. However, in this work our attention will focus on the so-called *weak equivalence principle* (WEP), which is a reformulation of EP applied to matter which consists of mechanical particles. To that end, it is important to understand various flavors and reformulations of EP that have appeared through history.

As any deep concept in physics, EP has been expressed historically through painstaking cycle of formulating it in a precise way, understanding the formulation, understanding the drawbacks of that formulation, coming up with a better formulation, and repeating. In this sense, EP, as quoted above, is a modern product of long and meticulous refinement over several generations of scientists. Needless to say, each step in that process made its way into contemporary physics textbooks, leading to a plethora of different formulations of EP that have accumulated in the literature over the years. This can bring about a lot of confusion about what EP actually states [45–48], since various formulations from old and new literature may often be not merely phrased differently, but in fact substantively inequivalent. To that end, let us comment on several most common historical statements of EP (for a more detailed historical overview and classification, see [51, 52]), and their relationship with the modern version:

- *Equality of gravitational and inertial mass.* This is one of the oldest variants of EP, going back to Newton's law of universal gravitation. The statement claims that the "gravitational charge" of a body is the same as body's resistance to acceleration, in the sense that the mass appearing on the left-hand side of Newton's second law of motion exactly cancels the mass appearing in the Newton's gravitational force law on the right-hand side. This allows one to relate it to the modern version of EP, in the sense that a suitably accelerated observer could rewrite the Newton's law of motion as the equation for a free particle, exploiting the cancellation of the "intertial force" and the gravitational force on the right-hand side of the equation. Unfortunately, this version of EP is intrinsically nonrelativistic, and applicable only in the context of Newtonian gravity, since already in GR the source of gravity becomes the full stress-energy tensor of matter fields, rather than just the total mass. Finally, this principle obviously fails when applied to photons, as demonstrated by the gravitational bending of light.

- *Universality of free fall.* Going back all the way to Galileo, this statement claims that the interaction between matter and gravity does not depend on any intrinsic property of matter itself, such as its mass, angular momentum, chemical composition, temperature, or any other property, leading to the idea that gravity couples universally (i.e., in the same way) to all matter. Formulated from experimental observations by Galileo, its validity is related to the quality of experiments used to verify it. As we shall see below, in a precise enough setting, one can experimentally observe direct coupling between the angular momentum of a body and spacetime curvature [25–44], invalidating the statement.

- *Local equality between gravity and inertia.* Often called Einstein's equivalence principle, the statement claims that a local and suitably isolated observer cannot distinguish between accelerating and being at rest in a uniform gravitational field. While this statement is much closer in spirit to the modern formulation of EP, it obscures the crucial aspect of the principle — coupling of matter to gravity. Namely, in this formulation it is merely implicit that the only way an observer can *attempt to distinguish* gravity from inertia is by making local experiments using some form of *matter*, i.e., studying the equations of motion of matter in the two situations and trying to distinguish them by observing whether or not matter behaves differently. Moreover, the statement is often discussed in the context of mechanics, arguing that any given particle does not distinguish between gravity and inertia. This has two main pitfalls — first, the reliance on particles is very misleading (as we will discuss below in much more detail), and second, it implicitly suggests that gravity and inertia are the same phenomenon, which is completely false. Namely, inertia can be understood as a specific form of gravity, but a general gravitational field cannot be simulated by inertia, since inertia cannot account for tidal effects of inhomogeneous configurations of gravity.

- *Weak equivalence principle.* Stating that the equations of motion of particles do not depend on spacetime curvature, or equivalently, that the motion of a free particle is always a geodesic trajectory in spacetime, WEP is in fact an application of modern EP to mechanical point-like particles (i.e., test particles). One can argue that, as far as the notion of a point-like particle is a well defined concept in physics, WEP is a good principle. Nevertheless, as we will discuss below in detail, the notion of a point-like particle is an idealisation that does not actually have any counterpart in reality, in either classical or quantum physics. Regarding a realistic particle (with nonzero size), WEP *never holds*, due to the explicit effect of gravitational tidal forces across the particle's size. In this sense, WEP can be considered at best an *approximate* principle, which can be assumed to hold only in situations where particle size can be approximated to zero.

- *Strong equivalence principle (SEP).* This version of the principle states that the equations of motion of all fundamental fields in nature do not depend on spacetime curvature. To the best of our knowledge so far, fields are indeed the most fundamental building blocks in modern physics (such as SM), while the strength of the gravitational field is indeed described by spacetime curvature (as in GR). In this sense, the statement of SEP is actually an instance of EP applied to field theory, and as such equivalent to the modern statement of EP. So far, all our knowledge of natural phenomena is consistent with the validity of SEP.

As can be seen from the above review, various formulations of EP are both mutually inequivalent and have different domains of applicability. In the remainder of the paper, we focus on the study of WEP, since recently it gained a lot of attention in the literature [19, 21–24], primarily in the context of its generalisation to a "quantum WEP", and in the context of a related question of particle motion in a quantum superposition of different gravitational configurations, the latter being a scenario that naturally arises in QG. Since WEP is stated in terms of a test particle and its trajectory, in order to try and generalise it to the scope of QG one should first analyse these two notions in classical and quantum mechanics and field theory in more detail.

## III. THE NOTION OF TRAJECTORY IN CLASSICAL AND QUANTUM MECHANICS

A trajectory of a physical system in three-dimensional space is a set of points that form a line, usually parametrised by time. More formally, a trajectory is a set $\{(x(t), y(t), z(t)) \in \mathbb{R}^3 | t \in [t_i, t_f] \subset \mathbb{R} \wedge t_i < t_f\}$, given by three smooth functions $x, y, z : \mathbb{R} \mapsto \mathbb{R}$. Depending on the nature of the system, the choice of points that form its trajectory may vary.

In classical mechanics, one often considers an ideal "point-like particle" localised in one spatial point $(x(t), y(t), z(t))$ at each moment of time $t$, in which case the choice of the points forming the system's trajectory is obvious. In case of systems continuously spread over certain volume ("rigid bodies", or "objects"), or composite systems consisting of several point-like particles or bodies, it is natural to consider their centres of mass as points that form the trajectory. While this definition is natural, widely accepted, and formally applicable to any classical mechanical system, there are cases in which the very notion of a trajectory looses its intuitive, as well as useful, meaning.

Consider for example an electrical dipole, i.e., a system of two point-like particles with equal masses and opposite electrical charges, separated by the distance $\ell(t)$. As long as this distance stays "small" and does not vary significantly with time, the notion of a trajectory of a dipole, defined as the set of centres of mass of the two particles, does meet our intuition, and can be useful. Informally, if the trajectories of each of the two particles are "close" to each other, they can be approximated, and consequently represented, by the trajectory of the system's centre of mass. But if the separate trajectories of the two particles diverge, one going to the "left", and another to the "right", one could hardy talk of a trajectory of such a composite system, although the set of locations of its centres of mass is still well defined.

Moving to the realm of quantum mechanics, due to the superposition principle, even the ideal point-like particles do not have a well defined position, which is further quantified by the famous Heisenberg uncertainty relations. Thus, the trajectory of point-like particles (and any system that in a given regime can be approximated to be point-like) is defined as a set of expectation values of the position operator. Like in the case of composite classical systems, here as well the definition of a trajectory of a point-like particle is mathematically always well defined, yet for the very similar reason is applicable only to certain cases. Namely, in order to give a useful meaning to the above definition of trajectory, the system considered must be *well localised*. Consider for example the double-slit experiment, in which the point-like particle is highly delocalised, so that we say that *its trajectory is not well defined*, even though the set of the expectation values of the position operator is.

We see that, while in mechanics both the notions of a particle and its trajectory are rather straightforward and always well defined, the latter make sense only if our system is well localised in space.

## IV. THE NOTION OF A PARTICLE IN FIELD THEORY

While in classical mechanics a point-like particle is always well localised, we have seen that in the quantum case one must introduce an additional constraint in order for it to be considered localised — the particle should be represented by a wave-packet. The source for this requirement lies in the fact that quantum particles, although mechanical, are represented by a *wavefunction*. Thus, it is only to be expected that when moving to the realm of the field ontology, the notion of a particle becomes even more involved and technical.

In field theory, the fundamental concept is the *field*, rather than a particle. The notion of a particle is considered a derived concept, and in fact in QFT one can distinguish two vastly different phenomena that are called "particles".

The first notion of a particle is an elementary excitation of a free field. For example, the state

$$|\Psi\rangle = \hat{a}^\dagger(\vec{k})|0\rangle \,,$$

is called a *single particle state* of the field, or a *plane-wave particle*. It has the following properties:

- It is an eigenstate of the *particle number operator* $\hat{N} = \int d^3\vec{k}\; \hat{a}^\dagger(\vec{k})\hat{a}(\vec{k})$, for the eigenvalue 1.
- It has a sharp value of the momentum $\vec{k}$, and corresponds to a completely delocalised plane wave configuration of the field.
- It has no centre of mass, and no concept of "position" in space, since the "position operator" is not a well defined concept for the field.
- States of this kind are said to describe *elementary particles*. For a real scalar field, an example would be the *Higgs particle*. For fields of other types (Dirac fields, vector fields, etc.) examples would be an *electron*, a *photon*, a *neutrino*, an asymptotically free *quark*, and so on. Essentially, all particles tabulated in the Standard Model of elementary particles are of this type.

The second notion of the particle in field theory is a bound state of fields, also called a *kink solution*. This requires an interacting theory, since interactions are necessary to form bound states. This kind of configuration of fields has the following properties:

- It is not an eigenstate of the particle number operator, and the expectation value of this operator is typically different from 1.
- It is usually well localised in space, and does not have a sharp value of momentum.
- As long as the kink maintains a stable configuration (i.e., as long as it does not decay), one can in principle assign to it the concept of *size*, and as a consequence also the concepts of *centre of mass*, *position in space*, and *trajectory*. In this sense, a kink can play the role of a test particle.
- States of this kind are said to describe *composite particles*. Given an interacting theory such as the Standard Model, under certain circumstances quarks and gluons form bound states called a *proton* and a *neutron*. Moreover, protons and neutrons further form bound states called *atomic nuclei*, which together with electrons and photons form *atoms*, *molecules*, and so on.

For a kink, the notions of centre of mass, position in space and size are described only as classical concepts, i.e., as expectation values of certain field operators, such as the stress-energy tensor. Moreover, given the nonzero size of the kink, its centre of mass and position are not uniquely defined, even classically, since in relativity different observers would assign different points as the centre of mass.

Given the two notions of particles in QFT, one can describe two different corresponding notions of WEP. In principle, one first needs to apply SEP in order to couple the matter fields to gravity, at the fundamental level. Assuming this is done, the motions of both the plane-wave particles and kink particles can be derived from the combined set of Einstein's equations and matter field equations, without any appeal to any notion of WEP. In this sense, once the trajectory of the particle in the background gravitational field has been determined from the field equations, one can verify *as a theorem* whether the particle satisfies WEP or not.

Specifically, in the case of a matter field coupled to general relativity such that it locally resembles a plane wave, one can apply the WKB approximation to demonstrate that the wave 4-vector $k^\mu(x)$, orthogonal to the wavefront at its every point $x \in \mathbb{R}^4$, will satisfy a geodesic equation,

$$k^\mu(x)\nabla_\mu k^\lambda(x) = 0\,. \tag{1}$$

However, given that the plane-wave particle is completely delocalised in space, the fact that the wave 4-vector satisfies the geodesic equation could hardly be interpreted as "the particle following a geodesic trajectory", and thus obeying WEP. Indeed, identifying the vector field orthogonal to the wavefront to the notion of "particle's trajectory" is at best an abuse of terminology.

Next, in the case of the kink particle coupled to general relativity, one assumes the configuration of the background gravitational field is such that the particle maintains its structure and that its size can be completely neglected. One can then apply the procedure given in [20, 25–44] to demonstrate that the 4-vector $u^\mu(\tau)$, tangent to the kink's world line (i.e., its trajectory), will satisfy a geodesic equation ($\tau \in \mathbb{R}$ represents kink's proper time),

$$u^\mu(\tau)\nabla_\mu u^\lambda(\tau) = 0\,. \tag{2}$$

Thus, one concludes that the kink obeys WEP as a *theorem* in field theory, without the necessity to actually postulate it.

Note the crucial difference between equations (1) and (2) — while the former features 4-dimensional variable $x$, the latter is given in terms of only 1-dimensional proper time $\tau$. This reflects the fact that the plane-wave particle is a highly delocalised object, with no well defined position and trajectory, while the kink is a highly localised object, with well defined position and trajectory. As a consequence, WEP can be formulated only for the kink, and not for the plane-wave particle.

In the case of the kink it is also important to emphasise that the zero size approximation of the kink is crucial. Namely, without this assumption, the particle will feel the tidal forces of gravity across its size, effectively coupling its angular momentum $J^{\mu\nu}(\tau)$ to the curvature of the background gravitational field [25–44]. This will give rise to an equation of motion for the kink of the form

$$u^\mu(\tau)\nabla_\mu u^\lambda(\tau) = R^\mu{}_{\lambda\rho\sigma}u^\lambda(\tau)J^{\rho\sigma}(\tau)\,,$$

which features explicit coupling to curvature (absent from (2)) and thus fails to obey WEP. In this sense, for realistic kink solutions WEP is *always violated*, and can be considered to hold only as an approximation, when the size of the particle can be completely neglected compared to the radius of curvature of the background gravitational field. If in addition the kink has negligible total energy, it can be used as a point-like test particle.

In the above discussion, while matter fields are described as quantum, using QFT, the background gravitational field is considered to be completely classical. It should therefore not be surprising that WEP may fail to hold if one allows the gravitatational field to be quantum, like matter fields, and one needs to revisit all steps of the above analysis from the perspective of QG. In fact, the case of the kink particle has been studied in precisely this scenario [20], and it has been shown that if the background gravitational field is in a specific type of quantum superposition of different configurations, the kink will fail to obey WEP even in the zero size approximation. Simply put, the equation of motion for the kink will contain extra terms due to the interference effects between superposed configurations of gravity, giving rise to an effective force that pushes the kink off the geodesic trajectory. And of course, similar to the case of the classical gravity, the resulting conclusion is a *theorem*, which follows from the fundamental field equations of the theory.

For the case of the plane-wave particle travelling through the superposed background of two gravitational field configurations, the analysis has not been done so far (to the best of our knowledge). But in principle one can expect a similar interference term to appear in the WKB analysis, and give rise to a non-geodesic equation for the wave 4-vector field $k^\mu(x)$ as well. In this sense, it is to be expected that generically even the wavefronts of such plane-wave particles would fail to obey WEP.

## V. CONCLUSIONS AND DISCUSSION

In this paper, we give an overview of the equivalence principle and its various flavors formulated historically, with a special emphasis on the weak equivalence principle. In order to generalise WEP to QG, we performed a critical analysis of the notions of particle and trajectory in various frameworks of physics, showing that the notion of point-like particle and its trajectory are not always well defined. This in turn suggests that WEP is not a good starting point for generalisation to QG, as we argue in more detail below.

As discussed in Section IV, in [20] it was shown that if superpositions of states of gravity and matter are allowed, WEP can be violated. It is important to note that the cases considered in [20] feature a specific type of superpositions of three groups of states: the first consists of a single so-called dominant state — a classical state whose expectation values of the metric and the stress-energy tensors satisfy Einstein field equations; the second consists of states similar to the dominant one, with arbitrary coefficients; and the third consists of states quasi-orthogonal to the dominant one, but with negligible coefficients. Only then one can talk of a (well localised) trajectory of the test particle in the overall superposed state and consequently about the straightforward generalisation of the classical WEP to the realm of QG. Considering that for the dominant state, being classical, the trajectory of the test particle follows the corresponding geodesic, we see that in the superposed state its trajectory would deviate from the geodesic of the dominant state, thus violating WEP. Note that, as discussed in Section IV, this deviation, in addition to classically weighted trajectories of the individual branches, also features purely quantum (i.e., off-diagonal) interference terms.

On the other hand, in recent studies [19, 21–24] it was proposed that WEP is *not violated* in QG, thus being at odds with our result. The authors of the mentioned studies considered superpositions of an arbitrary number of classical quasi-orthogonal states with arbitrary coefficients. Their argument goes as follows: since WEP is valid in each classical branch, it is valid in their superposition as well. If taken as a *definition* of what does it mean that a certain principle is satisfied in a superposition of different quantum states, then the above statement is manifestly true. But, as such, being a definition, it tells little about physics — it merely rephrases one statement ("principle A is separately satisfied in all branches of a superposition") with another, simpler ("principle A is satisfied in a superposition"). Moreover, it can be somewhat misleading, for the following three reasons:

1. When applied to the case studied in [20] and in this paper, the reasoning from [19, 21–24] would lead to the conclusion that even in our case WEP is not violated. Which, as long as understood in terms of the above mentioned definition, is a valid point of view. But it is misleading, as we have seen that in our case one can talk of particle's trajectory and its deflection from the dominant geodesic, precisely in the same way one talks about WEP in GR, which is a way that contains non-trivial information about physics. In other words, in our approach the violation of WEP is a theorem, rather than a definition, and its consequences can in principle be experimetally tested.

2. Second, the authors of [19, 21–24] often focus on approximately balanced superpositions of macroscopically distinguishable quasi-orthogonal configurations of gravity and matter. Such branches correspond to different "worlds" of the Many Worlds Interpretation, which manifestly cannot interfere in the future. Even within the standard quantum theory, it becomes increasingly difficult to discriminate such coherent superpositions from their classical mixtures, and it was suggested to be for all practical purposes impossible to do so [53]. In light of this, one could argue that in such scenarios the mentioned statement regarding WEP says little about quantum theory.

3. Moreover, the above definition is not in the spirit of quantum mechanics. Recall that, as discussed above and in [20], the modern formulation of WEP is given precisely in terms of a particular property of a system — its position, and consequently trajectory. To say that whenever the weak equivalence principle is satisfied in a superposed state if it is so in each of the branches, would then mean that the position and the trajectory of a test particle would be well defined not only in each separate branch, but in a superposed state as well. When applied to the case of the double-slit experiment, this would mean that the position of a particle and its trajectory are well defined even in superposition. Such a statement is very difficult to find in literature, to say the least, while its exact opposite can be found in almost any textbook on QM.

Finally, we would like to argue that the very notion of WEP is ill-defined in such "extreme" scenarios of superposed quasi-orthogonal classical states, and should not be even discussed. After all, we have seen that what we call a principle is nothing but a theorem in GR about the approximate trajectory of a point-like test particle, both of which are not always possible to define. Consequently, studying generalisations of WEP might not be very useful for inferring the features of QG. In our opinion, it would be better to instead study other principles and their possible generalisations to QG, such as SEP (see Section 4.2 in [20]), background independence, quantum nonlocality, and so on.

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[1] B. S. Kay, *Class. Quant. Grav.* **15**, L89 (1998), arXiv:hep-th/9810077.
[2] T. Oniga and C. H. T. Wang, *Phys. Rev. D* **93**, 044027 (2016), arXiv:1511.06678.
[3] D. E. Bruschi, *Phys. Lett. B* **754**, 182 (2016), arXiv:1412.4007.
[4] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A. A. Geraci, P. F. Barker, M. S. Kim and G. Milburn, *Phys. Rev. Lett.* **119**, 240401 (2017), arXiv:1707.06050.
[5] C. Marletto and V. Vedral, *Phys. Rev. Lett.* **119**, 240402 (2017).
[6] C. Marletto and V. Vedral, *Phys. Rev. D* **98**, 046001 (2018).
[7] N. Paunković and M. Vojinović, *Class. Quant. Grav.* **35**, 185015 (2018), arXiv:1702.07744.
[8] O. Oreshkov, F. Costa and Č. Brukner, *Nature Communications* **3**, 1092 (2012).
[9] M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi and Č. Brukner, *New Journal of Physics* **17**, 102001 (2015).
[10] V. Vilasini, *An introduction to causality in quantum theory (and beyond)*, Master's thesis, ETH, Zürich, Switzerland (2017).
[11] O. Oreshkov, *Quantum* **3**, 206 (2019).
[12] N. Paunković and M. Vojinović, *Quantum* **4**, 275 (2020), arXiv:1905.09682.
[13] V. Vilasini and R. Colbeck, *Phys. Rev. A* **106**, 032204 (2022), arXiv:2109.12128.
[14] V. Vilasini and R. Renner, `arXiv:2203.11245` .
[15] N. Ormrod, A. Vanrietvelde and J. Barrett, `arXiv:2204.10273` .
[16] A. Vanrietvelde, P. A. Höhn, F. Giacomini and E. Castro-Ruiz, *Quantum* **4**, 225 (2020), arXiv:1809.00556.
[17] M. Krumm, P. A. Höhn and M. P. Müller, *Quantum* **5**, 530 (2021), arXiv:2011.01951.
[18] S. A. Ahmad, T. D. Galley, P. A. Höhn, M. P. E. Lock and A. R. H. Smith, *Phys. Rev. Lett.* **128**, 170401 (2022), arXiv:2103.01232.
[19] A.-C. de la Hamette, V. Kabel, E. Castro-Ruiz and Č. Brukner, `arXiv:2112.11473` .
[20] F. Pipa, N. Paunković and M. Vojinović, *Journal of Cosmology and Astroparticle Physics* **2019**, 057 (2019), arXiv:1801.03207.
[21] F. Giacomini and Č. Brukner, `arXiv:2012.13754` .
[22] F. Giacomini and Č. Brukner, *AVS Quantum Science* **4**, 015601 (2022), arXiv:2109.01405.
[23] C. Marletto and V. Vedral, *Frontiers in Physics* **8**, 176 (2020), arXiv:2004.11616.
[24] C. Marletto and V. Vedral, *AVS Quantum Science* **4**, 015603 (2022), arXiv:2005.00138.
[25] A. Einstein, L. Infeld and B. Hoffmann, *Ann. Math.* **39**, 65 (1938).
[26] M. Mathisson, *Acta Phys. Pol.* **6**, 163 (1937).
[27] A. Papapetrou, *Proc. R. Soc. A* **209**, 248 (1951).
[28] W. Tulczyjev, *Acta Phys. Pol.* **18**, 393 (1959).
[29] A. H. Taub, *Jour. Math. Phys.* **5**, 112 (1964).
[30] G. Dixon, *Nuovo Cim.* **34**, 317 (1964).
[31] G. Dixon, *Nuovo Cim.* **38**, 1616 (1965).
[32] G. Dixon, *Proc. R. Soc. A* **314**, 499 (1970).
[33] G. Dixon, *Proc. R. Soc. A* **319**, 509 (1970).
[34] G. Dixon, *Gen. Relativ. Gravit.* **4**, 199 (1973).
[35] P. B. Yasskin and W. R. Stoeger, *Phys. Rev. D* **21**, 2081 (1980).
[36] K. Nomura, T. Shirafuji and K. Hayashi, *Prog. Theor. Phys.* **86**, 1239 (1991).
[37] K. Nomura, T. Shirafuji and K. Hayashi, *Prog. Theor. Phys.* **87**, 1275 (1992).
[38] M. Vasilić and M. Vojinović, *JHEP* **07**, 028 (2007), arXiv:0707.3395.
[39] M. Vasilić and M. Vojinović, *JHEP* **08**, 104 (2008), arXiv:0807.0596.
[40] M. Vasilić and M. Vojinović, *Phys. Rev. D* **73**, 124013 (2006), arXiv:gr-qc/0610014.
[41] M. Vasilić and M. Vojinović, *SIGMA* **4**, 019 (2008), arXiv:0802.1655.
[42] M. Vasilić and M. Vojinović, *Fortschr. Phys.* **56**, 542 (2008), arXiv:1010.1852.
[43] M. Vasilić and M. Vojinović, *Phys. Rev. D* **78**, 104002 (2008), arXiv:1010.1861.
[44] M. Vasilić and M. Vojinović, *Phys. Rev. D* **81**, 024025 (2010), arXiv:0812.4694.
[45] A. Accioly and R. Paszko, *Adv. Studies Theor. Phys.* **3**, 65 (2009).
[46] S. Longhi, *Opt. Lett.* **43**, 226 (2018), arXiv:1712.02054.
[47] P. Chowdhury, D. Home, A. S. Majumdar, S. V. Mousavi, M. R. Mozaffari and S. Sinha, *Class. Quant. Grav.* **29**, 025010 (2012), arXiv:1107.5159.
[48] G. Rosi, G. D'Amico, L. Cacciapuoti, F. Sorrentino, M. Prevedelli, M. Zych, Č. Brukner and G. M. Tino, *Nat. Comm.* **8**, 15529 (2017), arXiv:1704.02296.
[49] E. Kretschmann, *Annalen der Physik* **358**, 575 (1918).
[50] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, W. H. Freeman and Co., San Francisco (1973).
[51] E. Okon and C. Callender, *Eur. Jour. Phil. Sci.* **1**, 133 (2011), arXiv:1008.5192.
[52] E. D. Casola, S. Liberati and S. Sonego, *Am. J. Phys.* **83**, 39 (2015), arXiv:1310.7426.
[53] A. Peres and N. Rosen, *Phys. Rev.* **135**, B1486 (1964).