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A New Spin Test for the Equivalence Principle[1]

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Abstract

The existing impressive tests for the strong equivalence principle are reviewed and their classical nature is emphasized. The possibility is raised here that intrinsic quantum spins may behave differently from orbital angular momentum in gravitational fields. The techniques developed to measure the electric dipole moment of the neutron are shown to offer hopes of testing this hypothesis. Einstein's theory predicts a null result for this experiment. This would constitute the first quantum test for the strong equivalence principle. Deviation from a null result would invalidate Einstein's theory of gravitation, as well as indicate the failure of the discrete symmetries (P, T) in gravitation.

The equivalence principle forms the logical basis for Einstein's general theory of relativity. Observed universality of the accelerations of all bodies in a given external gravitational field supports the weak form of this principle, which equates the effects produced by a local gravitational field with the purely coordinate effects associated with an accelerated frame. It is conceivable that one could distinguish gravity from coordinate effects if other aspects of the behavior of matter in the local field were studied. Einstein, however, urged a stronger interpretation of the equivalence principle, assuming that the two situations are the same in every physical respect. The all-encompassing nature of the strong equivalence principle places severe restrictions on potential theories of gravitation and should therefore be subjected to as severe observational tests as current experimental techniques permit.

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The Eötvos-Dicke-Braginsky (EDB) experiments have confirmed the weak equivalence principle to the accuracy of 1 in 10^{12}. When the sizes of the gravitating objects in the EDB are increased to astronomical dimensions, one is also able to test the relative contributions of the gravitational binding energy to inertial and gravitational masses. While this experiment is able to distinguish between the Brans-Dicke and Einstein theories, the EDB is not. The most recent measurements [1] agree with general relativity to 8 in 10^3. Since the strong equivalence principle is a dynamical principle, experimental confirmations of non-Newtonian effects predicted by it constitute additional tests. The most notable of these, the motion of the perihelion of Mercury, the bending of light, and the recent radar echo delay experiments, have confirmed the predictions of Einstein's general relativity to a fairly high degree of accuracy.

The recently discovered pulsar in the binary system [2] PSR 1913 + 16 may offer a valuable laboratory for studying gravitational theories, since we have two objects of perhaps comparable masses bound in a strong gravitational field.

A hitherto untested class of predictions are the spin tests for the equivalence principle. These tests are concerned with the precession of gyroscopes in gravitational fields. For the proposed orbiting gyroscope experiment by Everitt, Fairbank, and co-workers [3], the magnitude of the effects are 8″/yr for geodetic precession, and 0.05″/yr for the Lense-Thirring precession. They hope to verify the predictions of general relativity to 1 in 10^3. The precession of the rotation axis of the pulsar PSR 1913 + 16 is expected to be [4] 1°-2°/yr and this effect may be manifested by its influence on the pulse profile. It should be stressed, however, that the confirmation of general relativistic predictions for light phenomena such as radar echo delay experiments, combined with consistency into Newtonian limit, would make it almost impossible for the gyroscope-type experiments to deviate from the predictions of general relativity. This can be understood if a macroscopically rotating object is thought of as a large collection of classical point particles in motion. As the interaction of the point particle is entirely determined from the above experiments, it follows that the gyroscope would follow the motion prescribed by general relativity, at least in the weak-field limit.

By contrast, quantum phenomena provide systems of definite angular momentum that cannot be pictured as rotating assemblies of matter. The nature of the coupling of this "intrinsic" quantum spin to gravitational phenomena is of great importance to our understanding of the gravitational interactions of elementary particles. Since in most physical systems the fraction of the total angular momentum that is intrinsic is extremely small, the existence of these quantum spin-dependent gravitational interactions will have no effect on macroscopic gravitational phenomena and hence does not contradict the previously mentioned tests. In fact, the quantum effects, if measurable, constitute fundamentally new information on gravitation complementing general relativity as perhaps the proper classical description. We devote the rest of this essay to a discussion of these new spin experiments [5].

In weak fields, one can characterize the most general spin-dependent gravitational interactions between an elementary particle of mass m and a static object M as

$$U(r) = \alpha_1 \left(\frac{GM}{cr^3}\right) \mathbf{s} \cdot \mathbf{r} + \alpha_2 \left(\frac{GM}{c^2 r^2}\right) \mathbf{s} \cdot \mathbf{v} + \alpha_3 \left(\frac{GM}{c^2 r^3}\right) \mathbf{s} \cdot (\mathbf{r} \times \mathbf{v}) \tag{1}$$

where $\mathbf{s}(t) = \langle t | \mathbf{s} | t \rangle$ is the expectation value of the spin operators. For earth-bound test probes, the second and third terms are very small (for similar α-values) compared to the first term.

If one interprets the strong equivalence principle to include quantum systems also, then only the last term with $\alpha_3 = 2$, which produces (apart from Thomas precession) the geodetic precession, can survive. Thus a measurement of the motion of quantum spin in the earth's gravitational field would constitute an entirely new test for the principle of equivalence. If (α_1, α_2) are different from 0, and α_3 different from 2, one would have a breakdown of the equivalence principle. Our calculations indicate that probing $\alpha_1 \sim 1$ is already near present experimental capabilities.

The breakdown of the equivalence principle implied by the existence of α_1 can be understood as follows: If one goes to what used to be called the local inertial frame of general relativity (the Einstein elevator, for example), the spin of the neutron does not precess if and only if $\alpha_1 = \alpha_2 = 0, \alpha_3 = 2$. In other words, even in the so-called freely falling frame the spin vector of the elementary particle precesses with rates proportional to $\alpha_1, \alpha_2, \alpha_3 - 2$ and would enable one to detect the local gravitational field. It is precisely this possibility of detecting the field that could invalidate the principle of equivalence.

Before discussing the actual experimental setup we briefly review the existing limits on the parameters α_i. As mentioned before, α_1 terms contribute more than α_2, α_3 terms if the α_i are of the same magnitude (0-1). The fractional difference in the accelerations of spin-up and spin-down neutrons in the earth's gravitational field due to (1) is roughly $\alpha_1 \times 10^{-22}$, which is clearly too small to be seen in any EDB-type experiments. One can also use hyperfine structure data to put limits on α_1. The most accurately measured value of the fine-structure constant $\alpha^{-1} = 137.03612(15)$ agrees with the value of α^{-1} deduced from hyperfine measurements to 2 parts in 10^6. If one interprets this as entirely due to a nonelectromagnetic possibly gravitational contribution, to the hyperfine structure, this corresponds to an energy difference of 2×10^{-12} eV between spin-up and spin-down protons. The splitting introduced by the α_1 term is $\sim 2\alpha_1 \times 10^{-22}$ eV, and we see that the limit imposed by hyperfine structure on α_1 is $\alpha_1 \lessapprox 10^{10}$, which is not very useful.

For an earthbound elementary particle, the motion of the spin due to the α_1 term of (1) is described by [5]

$$\Delta s = \alpha_1 \left(\frac{GM}{RC^2}\right) \frac{C}{R\omega} \left[(\Delta\hat{R} \times \hat{n}) \times s_0 - \omega \Delta t \hat{R} \cdot \hat{n}\,(s_0 \times \hat{n})\right] \tag{2}$$

where $\Delta\hat{R}$ is the change in the radius vector during Δt, $\hat{n}$ is the rotation axis of the earth, and R, M, and ω are, respectively, the radius, mass, and rotation frequency of the earth.

The first term of (2) causes a *noncumulative* precession of $\sim 1.5\, \alpha_1 \times 10^{-3}$ rad/12 hr. This effect averages to zero after each day, and is an important signal for α_1-type effects, because conventional gravitational effects do not produce such precessions. The second term of (2) produces a cumulative precession rate of $\sim \alpha_1 \times 10^{-7}$ rad/sec. The expected general relativistic motion of the spin also produces a qualitatively similar cumulative precession rate of $\sim 10^{-11}$ rad/sec and hence should easily be distinguishable from an effect produced by $\alpha_1 \sim 1$.

Now a precession rate of 10^{-7} rad/sec might seem like a ridiculously small effect to measure, but the situation is not so hopeless. In an effort to measure the electric dipole moment of the neutron, which is predicted to be zero if CP-invariance holds, Ramsey and co-workers [6] have developed extremely accurate techniques to measure small spin-dependent interactions. The latest measurement puts an upper limit of $e \times 10^{-24}$ cm for the EDM of neutrons, where e is the electronic charge. Translating this to the details of their equipment results in an equivalent accuracy of 10^{-6} rad in the angle measurement of the neutron spin. Neutron storage times of 30 sec [7] have already been claimed in the literature, and thus one could expect to measure the gravitationally induced spin precession for 30 sec $\sim \alpha_1 \times 10^{-6}$ rad setting limits on $\alpha_1 \sim 1$. If the recently developed methods of neutron spin echo by Mezei [8] could be utilized, the EDM sensitivities can be expected to be improved by three to four orders of magnitude, making an accurate determination of α_1 feasible in the near future. One particular technical difficulty faced by these experiments has to do with disturbances due to spurious magnetic fields, but it may be possible to eliminate such interference.

The implications of the existence of α_1 terms are enormous. Most importantly Einstein's theory of gravitation would have to be modified. On the other hand, an experimental verification of $\alpha_1 = 0$ would strengthen the support for Einstein's theory to quantum regimes. The α_1 terms also violate time reversal and space-reflection symmetries. While these symmetries are not of such relevance in classical phenomena, they play very important dynamical roles in the world of elementary particles. Further, the existence of α_1 terms implies that it would be impossible to transform away gravitational effects by appropriate coordinate transformations. On the astrophysical side, the existence of α_1 terms implies that gravitation does not couple to the energy-momentum tensor alone, and, an important ingredient in most of the singularity theorems leading to ultimate collapse may be in question. It is conceivable that other interactions violating CP may simulate these effects, but their contribution to α_1 is expected to be only 10^{-8}.

In conclusion, it should be stressed that while these proposed tests may at present be difficult to perform because of technical problems, they nevertheless offer hopes of seeing intrinsic quantum effects in gravitation within a reasonable time from now. In spite of the difficulty in providing a clear formal foundation for these effects, (see however, [9]), they should be taken in their own right as another independent class of tests for gravitational theories.

References

1. Williams, J. G., Dicke, R. H., Bender, P. L., Alley, C. O., Carter, W. E., Currie, D. G., Eckhardt, D. H., Faller, J. E., Kaula, W. M., Mulholland, J. D., Plotkin, M. H., Poultney, S. K., Shelio, P. J., Silverberg, E. C., Sinclair, W. S., Slade, M. A., and Wilkinson, D. T. (1976). *Phys. Rev. Lett.*, **36**, 551; Schapiro, I. I., Counselman, C. C., and King, R. W. (1976). *Phys. Rev. Lett.*, **36**, 555.
2. Hulse, R. A., and Taylor, J. H., (1975). *Ap. J. Lett.*, **195**, L51.
3. Bertoti, B., ed. (1974). *Experimental Graviation, Proceedings of the International School of Physics, "Enrico Fermi"*, Course LV (Academic, New York).
4. Hari Dass, N. D., and Radhakrishnan, V. (1975). *Astrophys. Lett.*, **16**, 135.
5. Hari Dass, N. D. (1976). *Phys. Rev. Lett.*, **36**, 393.
6. Baird, J. K., Dress, W. B., Miller, P. D., and Ramsay, N. F. (1973). *Phys. Rev. D*, **7**, 3147; Ramsey, N. F. (private communication).
7. Golub, R., and Pendleburry, J. M. (1972). *Contemp. Phys.*, **13**, 519; also private communications.
8. Mezei, F. (1972) *Z. Phys.*, **255**, 146.
9. Hari Dass, N. D. "Experimental tests for some quantum effects in gravitation", Niels Bohr preprint, No. NBI-HE-9-76 (submitted to *Ann. Phys, N. Y.*).