# The Einstein Equivalence Principle and the Search for New Physics

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**Abstract.** One strategy for searching for effects due to quantum gravity (QG) is to employ the Einstein Equivalence Principle (EEP) as an operational tool. The EEP, consisting of the Universality of Free Fall, the Universality of the Gravitational Redshift, and Local Lorentz Invariance, implies on the one hand that gravity has to be described by a space-time metric (pseudo-Riemannian Geometry) and on the other hand that the equations of motion for particles or quantum fields possess a specific structure compatible with the metrical structure of gravity. Therefore, any deviation from that structure of the dynamical equations of matter is related to a deviation from the metrical structure of gravity. As one consequence, any "new physics" will be accompanied by a breakdown of the validity of the EEP. We now take the EEP as guiding principle for the search for possible QG effects. Various proposed QG effects are then classified according to the various violations of the EEP, which is then used for a comparison with the present experimental status.

## 1 Introduction

There are many reasons that suggest that gravity has to be quantized [1]. One reason is that if all matter fields are quantized then all fields that are produced by these matter fields have to be quantized, too. Otherwise, at least within the existing theoretical schemes, inconsistencies may occur. Further reasons are that within purely classical General Relativity (GR) under very general and physically plausible circumstances singularities will occur. One way to avoid singularities may be a quantum description of gravity. For completeness one should mention that despite of these reasons there is also the option that gravity and the quantum domain remain completely disconnected so that gravity does not need to be quantized. However, the general accepted opinion is that gravity and quantum theory should go through some synthesis.

There are two main directions along which the quantization of gravity is looked for: The canonical quantization of gravity and string theory. In a sense, both approaches are complementary. The canonical quantization scheme starts from the geometric view of gravity and tries to quantize the gravitational field in form of the space-time metric or other related variables. During this process, the specific properties of matter are completely ignored, as is the case in Einstein's field equations, where matter is summarized in an energy-momentum tensor without specifying the nature of the existing matter. String theory, on the other side, starts from a specific unified concept of particles and interactions in flat

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space-time, and adds the gravitational interaction which then may depend on the particles it acts on.

It has also been discussed whether a quantization of the gravitational field is sufficient. Perhaps also the notion of an event, that is, a point in the differentiable manifold, should be quantized [2,3]. One approach in this direction is the ansatz of a non-commutative geometry.

However, as far as the experimental search for possible quantum gravity effects is concerned, the effect one is searching for can be described within the ordinary conceptual framework of fields/particles on a differential manifold. Any "new physics" then will appear as a deviation from the usual physical laws. This is obvious because in a low energy limit the usual physical concepts should come out, as, for example, for small velocities the Galilei group results from the Lorentz group or Newtonian gravity is the weak field and low velocity limit of Einstein's theory of gravity. Consequently, any new effect in these examples comes in first by small deviations from the Galilei group or from Newton's gravitational theory.

It has been shown that the low energy limit of QG theories always lead to small deviations from standard physics, mainly due to the appearance of extra scalar fields that are dynamical. These fields couple to the "bare" coupling constants like Newton's gravitational constant $G$ or the fine structure constant $\alpha$. This makes these constants effectively dynamical, that is, time and position dependent. Furthermore, Local Lorentz Invariance is found to be violated.

After having recognized that QG predicts some deviations from standard physics the next question is how to characterize 'standard physics'. In our context, standard physics is characterized by the Einstein Equivalence Principle (EEP). The EEP first implies that gravity has to be described by a pseudo-Riemannian geometry (gravity is a metric theory) and second gives a formal frame for the description all matter fields and interactions. By specifying a matter field, the structure of the corresponding field equation follows from the EEP. Furthermore, the EEP is stated in terms that are directly related to experimental experience. Therefore, the EEP is a tool to derive essential features of standard physics and serves as an *operational* guiding principle in the experimental search for new physics.

We also want to emphasize that the EEP, though being very fundamental for the general construction of theories, also bears importance for daily life, Fig. 1. The validity of the EEP is directly related to metrology, that is, for example, to the uniqueness of time-keeping or the uniqueness of the definition of other physical units. Also for the Global Positioning System GPS, neglecting GR or Special Relativity (SR) might give daily errors of the order of 10 km. And the high precision observation of the motion of the surface of the Earth with an accuracy of cm, which is at the limit of the present confirmation of SR and GR, can help in the modelling of the Earth and in predicting e.g. Earthquakes.

In this contribution we first describe the EEP, then show most of its implications, present models that lead to violations of the EEP, and give a list of tests of the EEP. At the end we expand a bit the importance of the EEP for metrology, that is, for the task to prepare, reproduce and compare physical units.

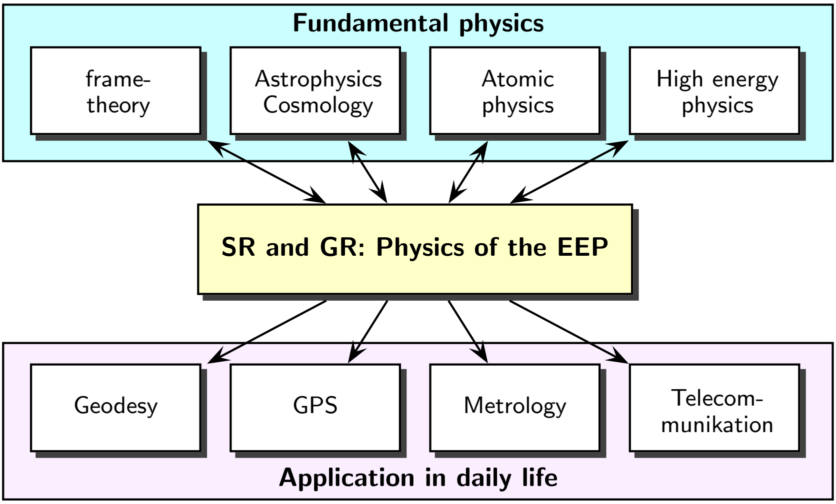



**Fig. 1.** The importance of GR and SR in fundamental physics and in daily life or, equivalently, the possible influence of "new physics"

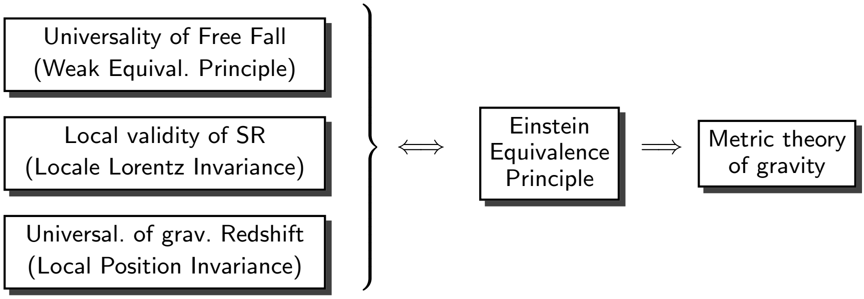
## 2 The Einstein Equivalence Principle

The EEP is a collection of principles that results in the present day formulation of relativistic physics including SR and GR as well as the Maxwell and the Dirac equation. That means, the validity of the EEP implies the validity of SR, the metrical structure of GR and the form of the equations for the electromagnetic field and for spin-$\frac{1}{2}$ quantum particles. However, the EEP is not enough to derive Einstein's field equations. For that, more input is needed, like the Strong Equivalence Principle. A scheme of how to arrive at Einstein's field equations within a metrical framework is provided by the PPN-formalism [4].

To be more precise, the EEP consists of [4]

1. The principle of the Universality of Free Fall (UFF), which means that all pointlike, structureless particles fall in a gravitational field along the same path,
2. the principle of Local Lorentz Invariance (LLI) which means that in small regions (the region must be small enough so that tidal effects can be neglected with respect to the effects under consideration) SR is valid[1], and
3. the principle of the Universality of the Gravitational Redshift (UGR) which means that all experiments, prepared with the same initial and boundary conditions, give the same results irrespective of where and when they are carried out.

[1] There is a huge set of publications on a precise mathematical and operational meaning of this point, see e.g. [5] for an early reference.




The important point of the EEP is, that it is expressed directly in terms of experimentally testable statements. Thus, it is an operational principle. That means, only if a certain set of experiments yields specific results, then gravity has to be a metric theory, the equations for the electromagnetic field must be of Maxwellian form, the Dirac equation must have their standard form, etc.

The tests of the EEP, and their corresponding meaning, are:

1. Tests of the UFF:
   By testing the UFF one explores whether all constituents of a macroscopic body, that is protons, electrons, neutrons (or the underlying quarks), that is, all forms of rest masses, behave in a gravitational field in the same way. It is certainly an astonishing physical fact that all particles "know" how other particles behave in the gravitational field. In principle, these tests must be carried through for all materials. Since, due to $E = mc^2$, the electromagnetic, weak, and strong interactions also contribute to the rest mass, UFF also controls the behaviour of these interactions in gravitational fields. In a certain theoretical frame these contributions are in general smaller than "pure" violations of the UFF due to e.g. an anomalous gravitational masses.
2. Tests of the UGR:
   These tests explore whether all kinds of clocks based on non-gravitational physics (gravitationally driven periodic systems like the motion of planets around the sun are excluded) possess constant mutual frequency ratios irrespective of their position and time. Since the gravitational field may be different for different space-time points, this means again that all interaction between particles behave in the same way under changes of the gravitational fields. Because a violation of the UFF by the participating particles also would destroy the validity of UGR, UFF and UGR are deeply linked. However, in a first approximation, UFF is connected with the rest mass and UGR with the interactions.
   A (hypothetical) violation of the UGR would mean that the physical laws depend on the time and the position of the laboratory. As an example, assume that the strength of the electric force between two charges depends on time. Since a force is always measured by comparison with another force (or inter-

action), which usually is defined by electromagnetism and quantum mechanics, this means that the electromagnetic coupling constant, the fine structure constant, depends on time. This, in turn, then leads to time-dependent frequency ratios between various clocks, like resonators and atomic clocks. Consequently, the temporal or spatial variation of physical constants is deeply connected with a violation of UGR.
Furthermore, if a violation of the UFF is due to a position-dependent scalar function, then this can be related to a violation of the UGR [6]. That means, UGR-tests are also tests of UFF, and vice versa. General arguments then give that the precision of the determination of the gravitational redshift must be $10^{-10}$ in order to compete with current UFF tests. See also [7,8] for general considerations of connections of UFF and UGR within string theory inspired dilaton models. In [9] a connection between a varying fine structure constant and violations of UFF is outlined.

3. Tests of LLI
   In order to experimentally verify SR one has to carry through the following set of experiments:
   (a) Test of the universality of the limiting speed of all particles. This includes that all particles possess as maximum speed the speed of light. Only if all phenomena possess the same limiting velocity, causality can be geometrized. That this is the case is again highly non-trivial. As for the UFF and UGR, all particles "know" about a specific property of all other particles.
   (b) Test of the independence of the speed of light from the velocity of the source. As a consequence, this is then also valid for the limiting velocities of all particles.
   (c) Test of the isotropy of the speed of light.
   (d) Tests of the independence of speed of light from the velocity of the laboratory.
   (e) Test of the time dilation given by the Lorentz factor $\gamma = 1/\sqrt{1-v^2}$.

All the above described tests are either direct comparisons between two particles or between two clocks: Tests of the UFF, of the universality of the speed of light, and of the independence of the speed of light from the velocity of the source just compare the velocity of different particles. Since these comparisons are carried through at same space-time events, there is no need for synchronization or for transporting some physical units. These comparisons are null tests and are completely *independent* from any theoretical model or framework.

The second set of tests, namely test of the UGR, of the isotropy of the speed of light, of the independence of the speed of light from the velocity of the laboratory, and of the relativistic time dilation are comparisons between different clocks. The first three of these tests compare two clocks of different physical nature in the same state of motion at the same position, see Fig. 2. The last compares two identical clocks possessing different velocities.

Though the comparison of clocks means that one just measures the ratio of two frequencies that is independent of any time unit, the description and

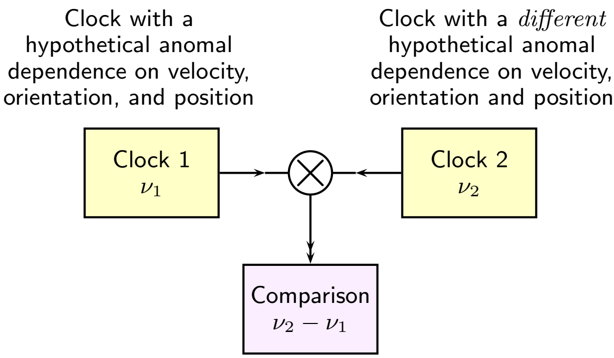



**Fig. 2.** General scheme for testing parts of SR and GR with clocks, namely the UGR, isotropy and velocity independence of the speed of light

experimental maintenance of clocks needs theoretical and experimental effort. Clocks based on resonators, for example, need, if high precision is required, very careful thermal and mechanical stabilization. In the case of atomic clocks, external stray fields have to be under very precise control. That means, that one has to use already some laws of physics in order to prepare these clocks. Only with a careful order-of-magnitude analysis of the physics needed for establishing the clocks compared to the laws one is testing, one can avoid logical inconsistencies. Furthermore, if clocks are used for searching for anomalous effects violating the EEP, the result can be interpreted consistently only if also the clock is described as a whole within this EEP-violating model.

Furthermore, from the available high-precision clocks one can infer the kind of information one may get from the corresponding clock-comparison experiments. Such clocks are

1. Atomic clocks based on electronic hyperfine transitions. They are characterized by energy levels of the form $E = \alpha^2 f(\alpha)$ where $\alpha$ is the fine structure constant and where $f(\alpha)$ is a Casimir correction factor and depends on the corresponding transition. An external field needed in order to split energy levels.
2. Atomic clocks based on nuclear transitions. These "clocks" are not used in practice as primary clocks. However, these transitions are used e.g. in Hughes–Drever experiments in order to search for LLI violations. The transitions are characterized by a nuclear fine structure constant $\alpha_{\rm n}$. Again one has to apply an external magnetic field.
3. Resonators. Here the frequency is defined by microwave or optical frequencies in a resonator. Since the length of the resonator is given by Bohr's radius, it scales linearly with the fine structure constant. Therefore, the frequencies possess a different $\alpha$-dependence than atomic clocks. A direct comparison gives information about a hypothetical time-dependence of $\alpha$ which violates UGR [75].

4. Molecular clocks. The rotation or vibration of molecules also define a frequency which depends on the fine structure constant but also on the ratio of the electron-to-proton-mass: $E \sim f(\alpha, m_e/m_p)$.

Consequently, by comparison of these various clocks one may get information about the constancy of $\alpha$, $\alpha_{\mathrm{n}}$ and $m_e/m_p$. Furthermore, since in all these clocks characteristic directions are involved (atomic clocks need some external electric or magnetic field, the geometry of resonators in general possess a symmetry axis, and molecules possess some axis of rotation or direction of vibration) a comparison of clocks with different intrinsic directions represent also tests of the rotation invariance of physics (tests of the isotropy of space, of which Michelson–Morley tests are the realization in the electromagnetic domain).

## 3 Implications of the Einstein Equivalence Principle

All interactions are discovered and characterized by the influence on matter. This is also true for the gravitational interaction. The EEP consists of statements about the properties of matter in gravitational fields from which we can conclude the properties of that field. That means, the EEP first very strictly restricts the equations for the electromagnetic field and for point particles or quantum fields. It is only through the specific properties of the dynamics of matter (point particles or fields) that gravity can be restricted to be describable solely by means of a space-time metric. Therefore, we first have to analyze the dynamics of particles and fields prescribed by the EEP and then we can define the gravitational field as that field which couples universally to matter and deduce the properties of this gravitational field.

Roughly speaking, the UFF implies the geometrization of the gravitational interaction since no particle properties influence the dynamics, LLI implies the existence of a metric tensor at each space-time point, and, finally, UGR implies that there are no scalar or other fields leading to different metrics at different space-time points.

### 3.1 Matter

As already stated, the EEP not only restricts the structure of the gravitational field but in a first step the structure of dynamical equations like the equation of motion for point particles, for quantum fields or of the electromagnetic field. For example no coupling to the curvature is allowed because curvature terms will not vanish when restricting the experiment to small regions where the dynamical equations are assumed to acquire their SR form. It should be emphasized that this holds only for the equations governing *observed* quantities, like the electromagnetic field. It is well known from Maxwell equations minimally coupled to gravity that the equations governing the vector potential couples to the curvature. The same also appears in field theory: Here the requirement that the fundamental solution of a scalar field equation has the same form as in SR leads to a conformal coupling, that is, to a curvature term in the field equation [10].

**Point Particles.** If we accept that in any situation the position and velocity is enough to determine the path of a structureless particle, then the equation of motion is, in general, given by $\ddot{x}^\mu + \widetilde{H}^\mu(p, x, \dot{x}) = 0$, where $p$ denotes all effective parameters (charge-to-mass ratio $q/m$, deviation of the ratio of the gravitational to inertial mass from unity $m_\mathrm{g}/m_\mathrm{i} - 1$, etc.) characterizing the particle under consideration[2]. By taking $p \to 0$ (either in a continuous or discrete way and leaving the masses finite) we may define that part of $\widetilde{H}$ which is independent of $p$, $H^\mu(x, \dot{x}) = \lim_{p\to 0} \widetilde{H}^\mu(p, x, \dot{x})$. Then the equation of motion may be written as $\ddot{x}^\mu + H^\mu(x, \dot{x}) + \widehat{H}^\mu(p, x, \dot{x}) = 0$ with $\widehat{H}^\mu = \widetilde{H}^\mu - H^\mu$. That part which is independent of any parameters $p$ we identify as *gravitational interaction*. $\widehat{H}^\mu$ is identified with nongravitational interactions. These terms are not present either if the corresponding charges are zero (neutral particles) or if there is no non-gravitational field. In this case we have, per definition, UFF. We used the UFF as a means to identify the gravitational interaction.

In a second step, LLI introduces at each point a frame with a Minkowskian metric. This is physically represented by the motion of light rays (up to a conformal factor).

Now we further consider structureless particles which are either neutral or which move in an interaction-free region. The equation of motion then must have the form $\ddot{x}^\mu = H^\mu(x, \dot{x})$ with no parameter noticing any property of the particle. According to LLI, there is a frame so that the equation of motion acquires the SR form $\ddot{x}^\mu \stackrel{*}{=} 0$. In this frame we also have the Minkowski metric. There cannot be any coupling of the particles to curvature like $R^\mu{}_\nu \dot{x}^\nu$ because such terms are present in any frame. The transformation of $\ddot{x}^\mu = 0$ to an arbitrary frame then yields an autoparallel equation $v^\nu \partial_\nu v^\mu + \Gamma^\mu_{\nu\rho} v^\nu v^\rho = \alpha v^\mu$ where $\alpha$ is an undetermined function. Furthermore, the Minkowski metric will transform to a metrical tensor $g_{\mu\nu}$. The compatibility of this autoparalell with SR then leads to a Weylian connection [11,12]. Furthermore, the condition of UGR in the form of a uniqueness of a transport of light clocks then reduces the Weylian connection to a Riemannian one.

Consequently, for point particles the EEP reduces all possible gravitational interactions to the one described by a Riemannian geometry.

**Matter Fields.** Also the standard Dirac equation can be derived with the help of the EEP. Assuming the conservation of probability, the requirements of LLI leads to a system of first order partial differential equations, which have the form of a slightly generalized Dirac equation. Adding the principle of UGR one then arrives at the usual Dirac equation in pseudo-Riemannian space-time [13].

**The Maxwell Field.** It has been shown by Ni [14] that only a modest generalization of Maxwell's equations is compatible with the UFF. The modification

[2] We exclude non-scalar properties of particles because this will considerably complicate the procedure because then one has to take the dynamics of these properties into account which increases the equations of motion under consideration.

consists of an addition of $\chi\epsilon^{abcd}F_{ab}F_{cd}$ to the usual Lagrangian for the Maxwell field where $\chi$ is a pseudoscalar field and $\epsilon^{abcd}$ the totally antisymmetric Levi–Civita tensor[3]. However, this term violates LLI because it induces a precession of the polarization of plane waves. This indeed represents a violation of LLI because corresponding propagation phenomena induced by the propagation of light do not show such a precession. Requiring LLI then forbids such a precession and thus this extra term. Therefore, EEP implies the ordinary Maxwell equations.

### 3.2 The Gravitational Field

The gravitational field is now defined through the equations of motion of the various matter fields. The only gravitational interaction, which remain in the dynamical equations of point particles, the Maxwell and the Dirac field, is given by a space-time metric. This metric then is called the gravitational field. Gravity then is described mathematically by a pseudo-Riemannian geometry [4].

## 4 Models Which Violate the Einstein Equivalence Principle

### 4.1 Quantum Gravity Induced Violations of the EEP

**String Theory Induced Violations of the UFF.** The prediction [17,18] is that the UFF, in terms of the Eötvös parameter (20) below, might be violated at the order $10^{-15}$ or even at the order $10^{-13}$ [19]. This is very well in the range of the space mission MICROSCOPE [20] scheduled for 2006.

**Loop Gravity Induced Violations of LLI.**

*Modifications of the Maxwell Equations.* In loop gravity, averaging over some quasiclassical quantum state, a so-called "weave"-state, which includes the state of the geometry as well as of the electromagnetic field, gives the effective Maxwell equations [21] (see also [22])

$$0 = \boldsymbol{\nabla} \times \boldsymbol{B} - \partial_t \boldsymbol{E} + \vartheta_1 \boldsymbol{\nabla} \times \boldsymbol{B} + \vartheta_2 \Delta(\boldsymbol{\nabla} \times \boldsymbol{B}) + \vartheta_3 \Delta \boldsymbol{B} + \vartheta_4 \boldsymbol{\nabla} \times (B^2 \boldsymbol{B}) + \dots \quad (1)$$

$$0 = \boldsymbol{\nabla} \times \boldsymbol{E} + \partial_t \boldsymbol{B} + \vartheta_1 \boldsymbol{\nabla} \times \boldsymbol{E} + \vartheta_2 \Delta(\boldsymbol{\nabla} \times \boldsymbol{E}) + \vartheta_3 \Delta \boldsymbol{E} + \dots, \quad (2)$$

[3] One should distinguish between Lorentz invariance and Lorentz covariance: Lorentz covariance means the formal covariance of all expressions under Lorentz transformation, Lorentz invariance means that the result of all experiments are the same in all frames provided all initial and boundary conditions in each frame are identical. Therefore, a Lorentz covariant theory may break Lorentz invariance. As an example, applying an equivalence principle of the form that e.g. the Dirac equation should acquire its special relativistic form in a particular frame, allows a coupling to space-time torsion [15,16] what clearly introduces distinguished space-time directions.

where the $\vartheta_i$ are coefficients depending on ratios of the Planck length and a length characterizing the quasiclassical gravitational quantum state. If one introduces the plane wave ansatz $\boldsymbol{E} = \boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$ and $\boldsymbol{B} = \boldsymbol{B}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$ and neglects the nonlinearities, then one gets the dispersion relations

$$\omega = |\boldsymbol{k}| \left(1 + \tilde{\theta}_1 + \tilde{\theta}_2 |\boldsymbol{k}|^2 \pm \tilde{\theta}_3 |\boldsymbol{k}|\right) \tag{3}$$

with other $\tilde{\theta}$s. The $\pm$ corresponds to different polarization states. Moreover, dispersion occurs so that from this dispersion relation one can e.g. derive the group velocity of photons and analyze the time of arrival of signals from distant stars according to frequency or polarization.

There are two points which need to be discussed: First, with (2) also a homogeneous Maxwell equation is modified and, second, the appearance of higher order derivatives means that there are photons that propagate with infinite velocity.

1. The deviation from the homogeneous Maxwell equations has the important consequence that the unique description of charged particle interference is no longer true. If quantum mechanical equations are minimally coupled to the electromagnetic potential, that is by the replacement $\partial \to \partial - \frac{ie}{\hbar c} A$, then the phase shift in charged particle interferometry is $\delta\phi = \frac{ie}{\hbar c} \oint_C A$ for a closed path $C$. If one applies Stokes' law in the case of a trivial space-time topology in that region, then this is equivalent to $\delta\phi = \frac{ie}{\hbar c} \int F$ with $F = dA$ where integration is over some 2-dimensional surface bounded by the closed path $C$. The fact that the result should not depend on the chosen surface is secured by the homogeneous Maxwell equations $dF = 0$.
   Since the form of (2) is incompatiple with dF=0, one must consider nonminimal couplings in order to provide unique predictions for charged particle interference.
2. The appearance of higher order spatial derivatives implies that in the corresponding frame of reference there are photons propagating with infinite velocity. As a consequence, Lorentz-invariance is violated.
3. It is clear that the appearance of higher order spatial derivatives also implies the appearance of higher order time derivatives in other frames of reference.

*Modifications of the Dirac Equation.* In the same manner as for the Maxwell equation, one can derive the modified Dirac equation [23,24]. The effective Dirac equation has the form

$$i\widetilde{\gamma}^a \partial_a \psi - \widetilde{m}\psi - \widetilde{\gamma}^{ab} \partial_a \partial_b \psi = 0 \, , \tag{4}$$

where $\widetilde{\gamma}^a = \gamma^a + \kappa_1 G_1 (L_{\mathrm{PL}}/L) + \kappa_2 G_2 (L_{\mathrm{PL}}/L)^2 + \ldots$ are the usual Dirac matrices $\gamma^a$ with QG corrections ($\kappa_i$ are coefficients of order 1, $G_i$ are some matrices, and $L_{\mathrm{Pl}}$ and $L$ are the Planck length and a length characterizing the semiclassical gravitational quantum state, respectively), and $\widetilde{m} = m\big(1 + \mu_1 m (L_{\mathrm{PL}}/L) + \mu_2 (L_{\mathrm{PL}}/L)^2 + \ldots\big)$, and $\widetilde{\gamma}^{ab} = \gamma^{ab} \left(\lambda_1 (L_{\mathrm{PL}}/L) + \lambda_2 (L_{\mathrm{PL}}/L)^2 + \ldots\right)$ where again $\mu_i$ and $\lambda_i$ are parameters of order unity, and $\gamma^{ab}$ is some set of matrices.

This kind of equation can be used in order to discuss the time-of-arrival of neutrons [23,25] or can be confronted with Hughes–Derever experiments [26,27]. The result of the latter paper is that modifications linear in the Planck length are questionably.

**String Theory Induced Violations of LLI.**

*Modifications of Maxwell Equations.* In string theory the gravitational field is given by $D$-branes which interact with propagating photons via an effective recoil velocity $\bar{\boldsymbol{u}}$ [28]. This recoil effect appears as a modified space-time metric, which influences the Maxwell equations. These equations are then given by

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} + \bar{\boldsymbol{u}} \cdot \partial_t \boldsymbol{E} = 0 \qquad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \tag{5}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} - (1 - \bar{u}^2)\partial_t \boldsymbol{E} + \bar{\boldsymbol{u}} \times \partial_t \boldsymbol{B} + (\bar{\boldsymbol{u}} \cdot \boldsymbol{\nabla})\boldsymbol{E} = 0 \qquad \boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B} \,. \tag{6}$$

In this approach the homogeneous equations remain unmodified. The resulting wave equations are

$$0 = \Box \boldsymbol{E} - 2(\bar{\boldsymbol{u}} \cdot \boldsymbol{\nabla})\partial_t \boldsymbol{E} \tag{7}$$

$$0 = \Box \boldsymbol{B} - 2(\bar{\boldsymbol{u}} \cdot \boldsymbol{\nabla})\partial_t \boldsymbol{B} \tag{8}$$

which leads to the dispersion relation

$$\omega = \pm k + (\bar{\boldsymbol{u}} \cdot \boldsymbol{k}) + \mathcal{O}(\bar{u}^2) \,. \tag{9}$$

The corresponding group velocity for light is

$$\boldsymbol{c} = \pm \frac{\boldsymbol{k}}{k} + \bar{\boldsymbol{u}} \,. \tag{10}$$

From string theoretical considerations, the recoil velocity $\bar{\boldsymbol{u}}$ can be shown to depend linearly on the energy $\omega$ of the photon, $\bar{\boldsymbol{u}} \sim \omega$. Therefore, in this case we obtain an energy dependent group velocity of the photons.

*Modifications of the Dirac Equation.* String theory induced modifications of the effective Dirac equation [29] have the form

$$i\gamma^a \partial_a \psi - \bar{u}^a \gamma^0 i \partial_a \psi - m\psi = 0 \,. \tag{11}$$

Also this equation can be confronted with spectroscopic results [30] with the result that first order corrections coming out from (11) are unlikely to be present.

## 4.2 LLI Violations from Non-commutative Geometry

It has been shown [31] that non-commuative geometry in general leads to violations of LLI described in general by an extension of the standard model (see below). In a non-commutative framework the commutator of coordinates $x^\mu$ is

$[x^\mu, x^\nu] = i\theta^{\mu\nu}$ where $\theta^{\mu\nu}$ is real and antisymmetric. The main argument employed in [31] is that due to the Seiberg–Witten map stating that there is a correspondence between a non-commutative gauge theory and a conventional gauge theory, non-commutative models must lie within an extension of the standard model. They applied the Seiberg–Witten map to the model of non-commutative QED and, after restricting to quadratic terms, arrived at an effective Lagrangian, which is within the model given by sum of (16) and (19) together with a coupling term that we will describe below. The violations of LLI are directly related to the coefficients $\theta^{\mu\nu}$.

**The General Structure of Modified Dispersion Relations.** In both cases, that is in (3) and (9), the general structure of dispersion relation for the propagation in one direction reads [32]

$$\boldsymbol{k}^2 c^2 = E^2 \left(1 + \xi \left(\frac{E}{E_{\mathsf{QG}}}\right) + \mathcal{O}\left(\frac{E}{E_{\mathsf{QG}}}\right)^2\right) . \tag{12}$$

where $E$ is the energy of the photon. The parameter $\xi$ depends on the underlying theory and can be derived to be $\sim 3/2$ for string theory [33] and $\sim 4$ for loop gravity [22]. The quantum gravity energy scale $E_{\mathsf{QG}}$ is, of course, of the order of the Planck energy $E_P$ so that for ordinary light which possesses an energy of the order 1 eV the correction term $E/E_{\mathrm{QG}}$ is of the order $10^{-28}$. From the above dispersion relation we derive the velocity of light

$$c_{\mathsf{QG}} = c\left(1 - \xi \frac{E}{E_{\mathsf{QG}}}\right) . \tag{13}$$

Therefore, the difference of the velocity of light for high energy photons and low energy photons, $\varDelta c = c(E) - c(E \to 0)$ is given by

$$\frac{\varDelta c}{c} = \xi \frac{E}{E_{\mathsf{QG}}} . \tag{14}$$

Exactly this quantity has been suggested to be observed for astrophysical events.

**String Theory Induced Violation of the UGR.** Violations of the UGR induced by string theory have been considered within a dilaton model by Damour [7,8].

### 4.3 General Models Violating EEP

**Bi-metric Models.** Bi-metric models describe the possibility that the limiting velocities of different kinds of particles may differ. Thus they are test theories for the universality of the velocity of light. If the speed of light is not universal then is also has as consequence that the isotropy of the speed of light and its independence from the velocity of the laboratory also will be violated.

The most elaborate model of this kind is the TH$\epsilon\mu$-formalism [34,35,4]. It is based on the Lagrangian

$$S = m_0 \int \sqrt{T - H\dot{\boldsymbol{x}}^2} dt + \frac{1}{8\pi} \int \left( \epsilon \boldsymbol{E}^2 - \frac{1}{\mu} \boldsymbol{B}^2 \right) d^4x + q \int \boldsymbol{A} \cdot \dot{\boldsymbol{x}} dt + q \int A_0 dt \,, \tag{15}$$

where $\dot{\boldsymbol{x}}$ is the coordinate velocity of a point particle with mass $m$ and charge $q$, $\boldsymbol{E}$ and $\boldsymbol{B}$ the electric and magnetic field, $A_0$ and $\boldsymbol{A}$ the scalar and vector potential. $T$, $H$, $\epsilon$, and $\mu$ are the parameters of this theory which are all unity in standard physics.

From a quick sight at this equations it is clear that the limiting velocity of the point particles is $c_{\rm p} = \sqrt{T/H}$ while the velocity of photons is $c_{\rm em} = \sqrt{\epsilon\mu}$. Only if the mechanical parameters $T$ and $H$ are related to the electromagnetic parameters $\epsilon$ and $\mu$, both velocities are equal. It is also clear that the Lagrangian (15) is written in a preferred frame characterized by the isotropy of both limiting velocities. In moving frames the difference between these two velocities depends on the direction.

For constant parameters $T$, $H$, $\epsilon$, and $\mu$ this is a one-parameter test theory for describing tests of SR. This parameter is $\delta = c_{\rm p}/c_{\rm em} = \sqrt{T/(H\epsilon\mu)}$. By replacing the point particle part in the Lagrangian by a corresponding Dirac–Lagrangian a lot of experiments including tests of UGR and LLI can be described [36,35,37,38,4].

**Ni–Haugan–Kostelecky-Formalism.** A huge generalization of the $TH\epsilon\mu$-ansatz consists in the consideration of general constitutive laws between the electromagnetic field strengths $\boldsymbol{E}$ and $\boldsymbol{B}$ and the electromagnetic excitations $\boldsymbol{D}$ and $\boldsymbol{H}$. To the knowledge of the author, Ni [39,14,40] was the first who considered this as a general starting point for the confrontation of the consequences with observations. The starting point is to replace the Lagrange density of the electromagnetic field according to

$$\mathcal{L} = -\frac{1}{16\pi} \eta^{ac}\eta^{bd} F_{ab}F_{cd} \quad \longrightarrow \quad \mathcal{L} = -\frac{1}{16\pi} \left( \eta^{ac}\eta^{bd} + \lambda^{abcd} \right) F_{ab}F_{cd} \,, \tag{16}$$

where $\lambda^{abcd}$ is an additional tensor possessing the symmetries of the Riemann-tensor. This tensor is assumed to describe the properties of the vacuum. The homogeneous Maxwell equations are still valid. Due to this, the totally anti-symmetric part of $\lambda^{abcd}$ transforms into total divergence, and the double trace amounts to a rescaling of the charge in the case that there is a coupling to matter, so that there are 19 parameters related to a violation of LLI.

With the general constitutive law $G^{ab} = (\eta^{ac}\eta^{bd} + \lambda^{abcd})F_{cd}$ the inhomogeneous Maxwell equations are

$$\partial_b G^{ab} = 4\pi j^a \,. \tag{17}$$

These equations have been used by Ni [39,14] to derive general conditions for the validity of the UFF for electromagnetic bound systems. Haugan and Kauffmann [41] used this in order to derive constraints on these coefficients from astrophysical observations. The most general analysis of consequences of these LLI-violating terms is due to Kostelecky and Mewes [42,43] who analyzed astrophysical data leading to estimates $\lambda^{abcd} \leq 10^{-30}$ for 10 of the 19 components, and showed how to treat data from laboratory experiments like Michelson–Morley experiments. First analyses of recent laboratory experiments in this test theory have been carried through by Lipa *et al* [44] and Müller and coworkers [45]. While the first paper gives estimates $\lambda^{abcd} \leq 10^{-13}$ resp. $10^{-9}$ for four linear combinations of the other 9 coefficients, the latter achieved $\lambda^{abcd} \leq 10^{-13}$ resp. $10^{-9}$ for all individual coefficients but one linear combination.

In more general models one starts with the equation of motion instead of using a variation principle. The most general Maxwell equation linear in the field and the derivative is given by [46,47]

$$(\eta^{ac}\eta^{bd} + \chi^{abcd})\partial_b F_{cd} + \chi^{acd} F_{cd} = 4\pi j^a \,. \tag{18}$$

Here, $\chi^{abcd} = \chi^{ab[cd]}$ possesses 96 and $\chi^{acd} = \chi^{a[cd]}$ 24 degrees of freedom. A first consequence of this general ansatz is that charge conservation is no longer true which has a severe impact in the standard formalism of physics. However, there seems to be no really good test of charge conservation (see below) and, furthermore, recently some papers discussed charge non-conservation models originating in higher dimensional brane theories where the charge may escape from our 4-dimensional world through higher dimensions [48]. Also its relation to the EEP has been discussed [49]. Charge non-conservation is encoded in the coefficients $\chi^{(ab)cd}$ and $\overset{0}{\chi}{}^{acd}$ where in $\overset{0}{\chi}$ the totally antisymmetric part has been removed.

**Generalized Dirac Equation.** The above-mentioned generalization of the $TH\epsilon\mu$-formalism has also a counterpart on the side of the particles, namely the generalized Dirac equation. Such generalized Dirac equations have the form

$$i\gamma^\mu \partial_\mu \psi - M\psi = 0 \,, \tag{19}$$

where $\gamma^\mu$ are *not* assumed to fulfill a Clifford algebra. A substantial part of LLI-violating effects of this kind of equations is due to the non-Clifford parts of the $\gamma$-matrices. Together with the generalized Maxwell equation (17) this is also called the 'extended standard model'. In GR, the matrix $M$ consists of the mass scalar and the spinor connection. In our generalized Dirac equation additional terms will spoil LLI and also UGR.

To the knowledge of the author, generalized Dirac equations and their consequences for particle dynamics have first been discussed by Liebscher [50], also [51]. An early discussion of the generalized Dirac equation with respect to tests of hypothetical violations of LLI is [52], recent discussion are due to Kostelecky and coworkers [53–58].

# 5 Experimental Tests of the Einstein Equivalence Principle

According to the principles underlying the EEP, the tests of it or, equivalently, the search for new physics can be classified along the following lines:

- Tests of the UFF
- Tests of the UGR
- Tests of LLI
  - Test of the universality of $c$
  - Test of the independence of the $c$ from the velocity of the source
  - Test of the isotropy of $c$
  - Test of the independence of $c$ from the velocity of the laboratory
  - Test of time dilation

## 5.1 Test of the Universality of Free Fall

Usually, tests of the UFF are described within a Newtonian framework: In the system where the gravitating body is at rest, the force acting on a test body, $m_\mathrm{i}\ddot{\boldsymbol{x}}$, where $m_\mathrm{i}$ is the inertial mass, is given by the gravitational force $-m_\mathrm{g}\boldsymbol{\nabla}U$, where $m_\mathrm{g}$ is the gravitational mass of the test body and $U$ the Newtonian potential. The path of the test body can be determined from the acceleration $-\ddot{\boldsymbol{x}} = (m_\mathrm{g}/m_\mathrm{i})\boldsymbol{\nabla}U$. If the ratio $m_\mathrm{g}/m_\mathrm{i}$ is the same for all test bodies, then the path will also be the same. By renormalizing the Newtonian potential by a constant, we then have $\ddot{\boldsymbol{x}} = -\boldsymbol{\nabla}U$. A hypothetical violation of the UFF is encoded in the Eövös ratio $\eta$ defined as

$$\eta = 2\frac{\ddot{\boldsymbol{x}}_2 - \ddot{\boldsymbol{x}}_1}{\ddot{\boldsymbol{x}}_2 + \ddot{\boldsymbol{x}}_1} = 2\frac{(m_\mathrm{g}/m_\mathrm{i})_2 - (m_\mathrm{g}/m_\mathrm{i})_1}{(m_\mathrm{g}/m_\mathrm{i})_2 + (m_\mathrm{g}/m_\mathrm{i})_1}\,, \tag{20}$$

where the indices 1 and 2 denote two different test bodies. The UFF implies $\eta = 0$.

The best test gives $\eta \leq 10^{-12}$ [59]. There are two space mission under way, the French MICROSCOPE mission [20] that is scheduled for 2005 but may be delayed for a year due to financial reductions in space programs, and the STEP project [60]. These missions want to test the UFF to a precision of $10^{-15}$ and $10^{-18}$, respectively. In principle, UFF should be tested with *all* pairs of test bodies. Due to new predictions [61] the UFF has recently been probed for small distances [62,63].

## 5.2 Test of the Universality of the Gravitational Redshift

**Absolute Measurement.** In GR the gravitational red shift is given by

$$\nu(x_1) = \left(1 - \frac{U(x_1) - U(x_0)}{c^2}\right)\nu(x_0)\,, \tag{21}$$

where $\nu(x_0)$ is the frequency of a clock at position $x_0$ and $\nu(x_1)$ is the frequency of this clock measured by an identical clock at position $x_1$. This relation has been tested with a space-borne Hydrogen maser compared with a ground H-maser with an accuracy of $7 \cdot 10^{-5}$ [64,65].

**Clock Comparison.** In the above formula no reference is made to the used clock. In the case that the gravitational red shift is not universal, the frequencies of the various types of clocks at different positions in the gravitational field will depend on the *type* of the clock:

$$\nu(x_1) = \left(1 - (1 + \alpha_{\rm clock}) \frac{U(x_1) - U(x_0)}{c^2}\right) \nu(x_0)\,. \tag{22}$$

In the framework of GR, $\alpha_{\rm clock} = 0$ for all clocks, such as atomic clocks, optical and microwave resonators, H-maser, quartz crystal, etc. If the redshift depends on the type of clock and we move two different clocks together in the gravitational field, then the ratio of the frequencies of these two clocks is

$$\frac{\nu_{\rm clock1}(x_1)}{\nu_{\rm clock2}(x_1)} \approx \left(1 - (\alpha_{\rm clock2} - \alpha_{\rm clock1}) \frac{U(x_1) - U(x_0)}{c^2}\right) \frac{\nu_{\rm clock1}(x_0)}{\nu_{\rm clock2}(x_0)}\,. \tag{23}$$

For a violation of the UGR we get a *position dependent* frequency ratio which is proportional to the difference of the gravitational potential difference $U(x_1) - U(x_0)$. If $\alpha_{\rm clock2} = \alpha_{\rm clock1}$, then this frequency ratio is independent of the position of the two clocks, and the constant factor $\alpha_{\rm clock}$ can be absorbed into the Newtonian potential, leading to (21).

The best tests of the UGR have been carried through by comparing an H-maser and a Cs atomic fountain clock over one year. Both are located at the same position on the surface of the Earth and experience, due to the annual elliptical motion of the Earth, the varying gravitational potential of the sun which is of the order $\Delta U/c^2 \sim 7 \cdot 10^{-10}$. The result is $|\alpha_{\rm H-maser} - \alpha_{\rm fountain}| \leq 2.1 \cdot 10^{-5}$ [66]. Other tests compare the frequency of a Cesium atomic clock and that defined by a microwave resonator which leads to $|\alpha_{\rm Cs} - \alpha_{\rm cavity}| \leq 2 \cdot 10^{-2}$ [67]. A comparison between electronic Iodine states and a cavity yields $|\alpha_{\rm Iod} - \alpha_{\rm cavity}| \leq 2 \cdot 10^{-2}$ [68]. The space mission ACES [69] comparing an H-maser and an atomic fountain clock on the ISS in the varying gravitational potential of the sun, aims to improve the presently best test by at least one order what is a consequence of the fact that the free fall condition in space will considerably improve the working conditions of the atomic fountain clock (see below p. 388). Furthermore, since the above results depend on the value of the experienced difference of the gravitational potential, space missions like SPACETIME [70] and OPTIS [71] will give huge improvements. While OPTIS compares an H-maser, atomic ion clocks and clocks based on optical resonators in an high elliptic orbit around the Earth, SPACETIME uses three ion clocks in an identical environment and aims at exploiting the huge potential difference of $\Delta U/c^2 = 3 \cdot 10^{-7}$ when approaching the sun up to 5 solar radii. As a result, UGR may be tested to an accuracy of $10^{-10}$.

Since a violation of UGR can be related to charge non-conservation, we report about its status of experimental verification.

*Charge Non-conservation.* One way to treat charge non-conservation experimentally is to look for the probability that an electron, carrying a charge $e$, may just disappear or be created during scattering processes in high energy accelerators. The underlying model is that an electron decays into a neutral particle and a photon $e \to n + \gamma$. For these kind of processes the probability was found to be less than $5.3 \cdot 10^{-21}$ y$^{-1}$ [72].

Another aspect of charge conservation is the equality of charges of the electron and the proton. This can be proven with great accuracy by testing the neutrality of atoms or molecules. The corresponding estimates state that the relative difference between the electron and proton charge $|(q_e - q_p)/q_e|$ is less than $2 \cdot 10^{-19}$ [73]. It is clear that even for equal electron and proton charges the absolute charge may vary in time. In principle, this may be proven by observing a spring connecting two, say, equally charged bodies. For a varying, non-conserved charge the spring should expand with time, if the charge decreases. However, since also the physics of the spring heavily depends on the electromagnetic properties of matter, this requires a very thorough analysis. A more simpler version of this can be found in atomic physics where obviously charge non-conservation influences the binding energy of electron in the field of the nucleus what results in a time-dependence of the fine structure constant $\alpha = e^2/\hbar c$.

If we assume that charge is not conserved, then the charge of all particles depends on time so that especially for the charge of the electron and the proton $de/dt \neq 0$ which then implies for the fine structure constant $\dot{\alpha}/\alpha = 2\alpha(1/e)(de/dt)$. If we assume a specific time-dependence of the form $de/dt = \zeta e$, then $d\alpha/\alpha = 2\zeta$. Thus experiments on the time-dependence on the fine-structure constant give estimates on $\zeta$ or, in terms of the general model (18), on components of $\chi^{(ab)cd}$ and $\overset{0}{\chi}{}^{abc}$.

A measurement of a hypothetical time dependence of the fine structure constant can be obtained by comparing different time or length standards which depend in a different way on $\alpha$ (see page 372). For example, the in the recent experiment [74] a Cs and a Rb atomic fountain clock, both based on hyperfine transitions with different $\alpha$-dependence, were compared over five years. The comparison resulted in $\dot{\alpha}/\alpha \leq 1.6 \cdot 10^{-15}$ y$^{-1}$, so that $\zeta \leq 8 \cdot 10^{-16}$. In two new proposals [75,76] using resonators, more specific, monolithic resonators for optical modes and whispering gallery modes in a single resonator, respectively, it is claimed that it might be possible to test the time-independence of $\alpha$ below the $10^{-15}$ level. Together with the tests of the equality of the electron and proton charge, tests of the constancy of the fine structure constant provide the best direct experimental proof of charge conservation.

### 5.3 Test of Local Lorentz Invariance

While it is clear that for a description of tests of UFF and UGR one has to modify the equations of motion of the test particles and test clocks, the situation is different for SR. Here one may choose between kinematical and dynamical test theories. Kinematical test theories, based on the analysis of Robertson [77] and Mansouri and Sexl [78–80] (see also the review [81]), compare the physics in two differently moving and differently oriented laboratories. Dynamical test theories examine the structure of the laws of physics. Kinematical test theories are more basic in the sense that they describe the behaviour of distinguished physical phenomena (e.g. light) with respect to *given* rods and clocks which, in a constructive approach to a physical theory, at the beginning are given by definition and thus cannot be analyzed using physical theories (simply because they not yet exist at that stage)[4]. In a later stage, after having explored physical laws, one may analyze these rods and clocks with the help of these laws. Then one may ask how these objects behave if these laws are modified. This is the task of the dynamical test theories. In the end, dynamical test theories are superior to kinematical test theories because these theories describe all objects, even the measuring apparatus.

Within the kinematical framework of Robertson [77] and Mansouri and Sexl [78–80] the velocity of light is given by

$$c(v, \vartheta) = c_0 \left( 1 + A \frac{v^2}{c_0^2} + B \cos^2 \vartheta \frac{v^2}{c_0^2} + \mathcal{O}\left(\frac{v^4}{c_0^4}\right) \right) , \tag{24}$$

where $v$ is the velocity of the laboratory with respect to a preferred frame which one chooses as the frame in which the cosmic microwave background radiation appears to be isotropic. $c_0$ is the velocity of light in the preferred frame. $A$ and $B$ are two parameters which vanish in SR. In addition, the time dilation is described as

$$\gamma(v) = \frac{1}{\sqrt{1 - v'^2/c_0^2}} \left( 1 + \frac{1}{2} \alpha \frac{(\boldsymbol{v} + \boldsymbol{v}')^2}{c_0^2} + \mathcal{O}\left(\frac{v^4}{c_0^4}\right) \right) , \tag{25}$$

where $\boldsymbol{v}'$ is the velocity of the clock with respect to the laboratory. Again, $\alpha$ vanishes in SR. Equations (24) and (25) require three independent test in order to fix $A$, $B$, and $\alpha$. As a prerequisite, the Robertson-Mansouri-Sexl test theory requires the independence of the speed of light from the velocity of the source.

In dynamical test theories of the Ni–Haugan–Kostelecky type the velocity of light again depends on the orientation and a velocity. The velocity of light can be calculated from the dispersion relation resulting from the modified Maxwell

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[4] However, at least for the existing kinematical test theories one nevertheless needs some physical information "from the outside", namely the velocity with respect to some preferred frame. What we take as preferred frame depends on our knowledge about the universe. These test theories are not intrinsically complete.

equations (17)

$$\omega = \left(1 - \frac{1}{2}\tilde{k}_\alpha{}^\alpha \pm \sqrt{\frac{1}{2}\tilde{k}_{\alpha\beta}\tilde{k}_{\alpha\beta} - \frac{1}{4}(\tilde{k}_\alpha{}^\alpha)^2}\right) |\boldsymbol{k}| + \mathcal{O}(\lambda^2) \,, \tag{26}$$

where $\tilde{k}^{\alpha\beta} = \lambda^{\alpha\beta\gamma\delta} k_\alpha k_\beta / |\boldsymbol{k}|^2|$ [43]. The velocity of light then is given by

$$c(v, \vartheta) = c_0 \left(1 + A \sin\vartheta + B \cos\vartheta + C \sin(2\vartheta) + D \cos(2\vartheta)\right) \,, \tag{27}$$

where the coefficients $A$, ..., $D$ depend on the velocity with respect to an underlying coordinate system which in this case is chosen as the coordinate system of the solar system [43] (see also [44] for an application of this to the analysis of a recent experiment). The more complicated orientation dependence is due to the tensorial character of the LLI violation $\lambda^{abcd}$. For the time dilation one needs also to calculate the influence of the modified Maxwell equations on the atomic spectra. In this test theory, too, the independence of the speed of light from the velocity of the source is a prerequisite.

**Test of Universality of *c*.** The universality of the maximum velocity of particles has been tested for various pairs of particles. For a comparison of electrons with photons and of photons with different frequencies in the laboratory one achieves the $10^{-6}$ level for the relative velocity difference $|(v_1 - v_2)/v_1|$ [82–84]. Astrophysical observations of neutrinos and photons from supernovae gives a level of $10^{-8}$ [85–87]. High energy cosmic rays are discussed in [88–90]. Another aspect of the universality of $c$ is the non-occurence of birefringence: in vacuum the velocity of light should be independent of the polarization. This has been confirmed by astrophysical observation with high accuracy [43] from which 10 of the 19 components $\lambda^{abcd}$ could be estimated to be smaller than $2 \cdot 10^{-32}$ (the other 9 components will be constrained by Michelson–Morley experiments). More indirect nuclear spectroscopical (Hughes–Drever) tests give a $10^{-22}$ level for the maximum difference of photon and proton speed [91,92].

**Test of Independence of *c* from the Velocity of the Source.** One of the most distinctive and contra-intuitive statements of SR is that the velocity of light is independent from the velocity of the source. A possible violation may be expressed as $c' = c + \kappa v$ where $\kappa$ is a parameter which has to be determined experimentally. In a Galilean framework $\kappa = 1$, in SR $\kappa = 0$. The most impressive experiment demonstrating this is from Alväger *et al.* [93] where the source of photons possesses a velocity of 99.975 % of the velocity of light. The emitted photons still propagate with the velocity of light within $\kappa \leq 10^{-6}$. Better estimates can be achieved from astrophysical observations of binary systems [94] leading to a $\kappa \leq 10^{-9}$.

**Test of Isotropy of *c*.** The isotropy of the velocity of light is subject to the famous Michelson–Morley experiments. The presently most precise test has been performed by Müller *et al.* [45] and gives $|\delta_\vartheta c/c| \leq 4 \cdot 10^{-15}$. In terms of the Robertson–Mansouri–Sexl parameters this means $|B| \leq 4 \cdot 10^{-9}$ and for remaining 9 parameters of the extended standard model $|\lambda^{abcd}| \leq 10^{-15}$. Other recent experiments are [95,44].

Since light, which properties are tested in these experiments, is a consequence of Maxwell's equations, any modification in the speed of light must be related to a modification of Maxwell's equation. Since the properties of the interferometer arms or the resonators are also strongly influenced by electrodynamics, their porperties may be modified, too. Indeed, it has been shown in two models [47,45] of modified Maxwell equations that an accompanying anomalous behaviour of the length of the interferometer arm or of the cavity may compensate or enhance the signal indicating an hypothetical anisotropy of the speed of light. Furthermore, also LLI-violations of the Dirac equation may contribute to an anomalous behaviour of the interferometer or cavity [96]

**Test of Independence of *c* from the Velocity of the Laboratory.** This part of the relativity principle has been tested first by Kennedy and Thorndike. The presently most precise test is due to Wolf *et al.* [95] and gives $|A| \leq 3.1 \cdot 10^{-7}$. Another recent experiment is [68][5].

**Test of Time Dilation.** Time dilation is the only non-null test of SR because one has to determine the Lorentz factor $1/\sqrt{1-v^2}$ experimentally. Deviations from this factor in terms of a parameter $\alpha$ in $\gamma(v) = (1 + \frac{1}{2}\alpha v^2)/\sqrt{1-v^2}$ have ben limited to $|\alpha| \leq 2 \cdot 10^{-7}$ recently [98].

# 6 New Experimental Devices and Developments

Here we describe a few experimental devices that have been developed in the last years and possess the capability to contribute a lot to improvements of experiments searching for new physics.

## 6.1 Atom Interferometry

Though atomic iterferometry has been implemented only a bit more than ten years ago, it already provides e.g. the best gyroscopes. High precision atomic interferometry is based on an effective laser cooling of atoms down to temperatures

[5] It should be noted that in terms of the variation of the velocity of light for varying velocities of the laboratory, $\delta_v c/c$, the old 1990 experiment by Hils and Hall [97] is better than the present tests. The difference is that Hils and Hall were able to measure for a few days only, while the measurements of Braxmaier *et al* [68] took approximately one year. Consequently, the change in the velocity which is essential in estimating the parameter $A$ in (24), could be chosen as twice the velocity of the Earth around the Sun while Hils and Hall were restricted to twice the rotational velocity of the Earth's surface around its own axis.

in the $\mu$K domain corresponding to velocities of the order of cm/s. These low velocities are necessary for long interaction times thereby increasing the accuracy. The use of laser beams as beam splitters has the advantage of being not influenced by the gravitational or inertial field, as it is the case for the beam splitters in neutron interferometry. Within an Newtonian framework, acceleration and rotation gives as phase shift in an atom interferometer

$$\delta\phi = \boldsymbol{k} \cdot \boldsymbol{g}\, T^2 + \boldsymbol{k} \cdot (\boldsymbol{\Omega} \times \langle \boldsymbol{v} \rangle) T^2 \,, \tag{28}$$

where $\boldsymbol{k}$ is the wave vector of the laser field, $\boldsymbol{g}$ the (gravitational or inertial) acceleration, $\boldsymbol{\Omega}$ the angular velocity, $\langle \boldsymbol{v} \rangle$ the expectation value of the velocity of atoms entering the interferometer and $T$ the interaction time. Therefore, the UFF is clearly represented by this phase shift [99]. If the inertial and gravitational mass are different, then this first term will be modified to $(m_{\rm g}/m_{\rm i})\boldsymbol{k} \cdot \boldsymbol{g}\; T^2$. Atomic interferometry has confirmed the UFF in the quantum domain to the order of $10^{-9}$. It is astonishing that (28) is an exact quantum result though there appears no $\hbar$ in it.

Further improvements are expected by using Bose–Einstein condensates as a coherent source for atoms.

## 6.2 Atomic Clocks

There are various atomic clocks available: H-maser, Rb- and Cs-clocks, and ion clocks based on $Hg^+$, $Yb^+$, or $Cd^+$, see e.g. [100,101] for reviews. The accuracy of a clock is based essentially on the line-width of the atomic transition and on the time of interaction with an external oscillator which reads out the frequency. A narrow line-width is related to long-living atomic states that are provided by hyperfine states. These transitions possess frequencies in the microwave range (several 10 GHz). The interaction time should be 1 ms or longer.

**Atomic Clocks.** Clocks like the conventional Cs atomic clock consist of an atomic beam which, during its flight, is interrogated by some microwaves for a certain interaction time. Due to gravity, a relatively high beam velocity must be chosen, so the interaction time is limited to about 1 ms. Due to this limitation and further unwanted effects like stray fields, Doppler broadening, etc., the accuracy of such clocks is of the order $10^{-14}$. For a detailed discussion, see [101]. The today's definition of the second is based on the Cs clock.

**H-Maser.** H-Masers are based on the coupling of the hyperfine transition of the ground state of the Hydrogen atom which has a lifetime of about 1 second, to the radiation of a resonator. The frequency is 1.420 405 751 Hz and the instability of this clock is of the order $10^{-15}$. H-Masers are used worldwide for the definition of time and have been used in the first gravity space mission GP-A [64,65].

**Ion Clocks.** Ion clocks are also based on hyperfine transitions of ions which are stored in traps and which are therefore isolated from many disturbing influences. The instability of this kind of clocks approaches the $10^{-16}$-level [102]. This means that within 1 billion years the clock may be wrong by 1 s. Ion clocks may be used in space missions like SPACETIME [70,69] or OPTIS [71].

**Atomic Fountain Clock.** Atomic fountain clocks use laser-cooled atoms. During a ballistic flight these atoms interact with separated fields in a Ramsey set-up. Due to the controlled flight of the atoms interaction times of up to 1 s on Earth can be obtained. This can be increased considerably in space where the atoms do not fall out of the apparatus. The corresponding project PHARAO on the ISS [103,69] is near completion.

### 6.3 Ultrastable Cavities

Cavities are made of stable materials and define a length standard. This lenght is read out by a laser frequency stabilized ("locked") to a resonance of the cavity. With $\nu = nc/L$ the information of the length $L$ of the cavity is transformed to a frequency which can be measured with higher accuracy than lengths. Consequently, resonators are a realization of light clocks. Here, the velocity of light $c$ plays an important role. It is clear that it is crucial to prohibit the cavity from any thermal expansion or distortions due to external forces like acceleration, rotation or gravity gradients. Furthermore, precise control of the laser frequency to the resonance frequency of the cavity is required. This can be controlled by means of the so-called Pound-Drever-Hall technique where one measures the modulation of the reflected or transmitted beam that.

For cryogenic optical resonators [104] the stability which can be achieved is $\delta L/L \leq 6 \cdot 10^{-16}$ [105] what, for a resonator of typical length of 5 cm, is about 1/100 of the radius of the proton.

### 6.4 Frequency Comb

Since most of the tests of the principles of relativity depend on clock comparison, a high precision technique for comparing frequencies of various ranges is mandatory. For a comparison of microwave and optical frequencies, which differ by up to 6 orders of magnitude, the recently invented frequency comb is the appropriate technique, see [106] for an overview. This technique, being simpler, cheaper, and more accurate than previous methods, can be used e.g. in Kennedy–Thorndike tests and test of the UGR. Corresponding tests are under development at the University of Düsseldorf.

## 7 EEP and Modern Metrology

Metrology is the definition, preparation, transport, and comparison of physical units like the second, the meter and the kilogram. It can be viewed as the basis for all modern physics: without the possibility to make very precise measurements no progress in physics is conceivable. For example, in order to prove the predicted dynamics of a certain physical phenomenon, a precise time-keeping is required.

Since all physical units are represented themselves by some physical phenomena, a measurement always consists of the comparison of two physical phenomena of the same kind, like e.g. the measurement of the dynamics of the Earth compared with the dynamics of an atomic clock. The important point is therefore the *reproducibility* and *stability* of physical units. The reproducibility of units based on quantum effects is arbitrarily good while the reproducibility of a macroscopic meter stick or of a unit of mass is of the order $10^{-6}$ and therefore not applicable for really high precision measurements. In fact, the uncertainty in the definition of the mass unit (and in the homogeneity of the used masses) is the main obstruction for precise measurements of the gravitational constant. However, basing units on quantum effects also means that one relies on a certain structure of quantum mechanics. As we have seen, this structure is deeply related to the validity of the EEP.

## 7.1 Ideal Rods and Clocks

In order to explore the laws of physics and to perform tests of foundations of theories one has to measure or prepare certain quantities like time and length. The first step always consists of the definition of certain quantities like the second or the meter at a certain instant and at some position. The next step then is to transport this unit to other places. A complete theory always defines a way to transport these units. For example, within SR and GR one can design a transport of length and time standards with light rays and freely falling particles only. It has been shown in the axiomatic approach to GR (and thus also to SR) using light rays and freely falling particles (those obeying the UFF) only [11], that by means of Schild's ladder [11,107] or Perlick's construction [108] it is possible to uniquely transport a length or the eigentime along a path. For more general theories the uniqueness will be lost. In a Weylian model of gravity, for example, the transport of a length scale depends on the path and thus on the history of, e.g., the meter stick, see e.g. [108]. As a consequence, modern metrology with its task of unique definition, reproduction, and transport of physical units is deeply connected with SR and GR.

Another point is that though the above constructions need no other objects than those given by the theory, these transport prescriptions of time and length standards are not always practical procedures because they may not be realizable with the accuracy needed today. The length and time standards, which are realized today with the highest internal accuracy, are provided by atomic clocks and solids. In order to describe these standards, one needs more than just light rays and particles, namely the equations of motion for electromagnetic and quantum fields.

Another definition and transport of a certain length scale is provided by quantum equations for massive particles, e.g. the Dirac equation. The Dirac equation introduces the Compton–wavelength and its transport along quasiclas-

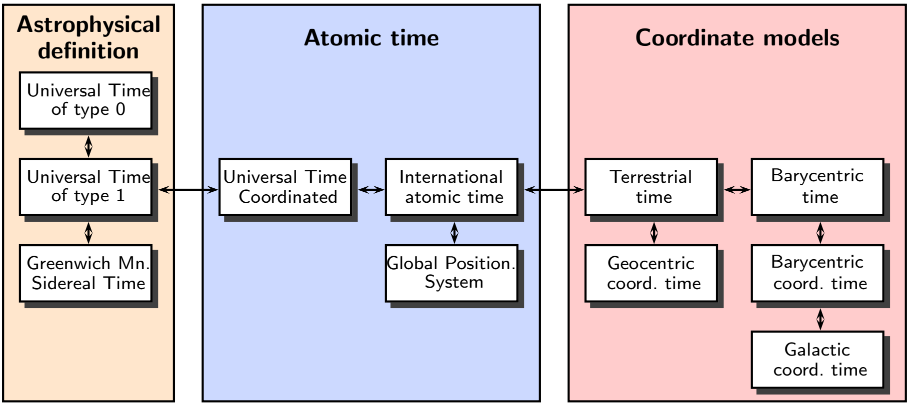


**Fig. 3.** Time scales and their relations. For all relations between the various time scales but the astrophysical ones, one needs SR and GR. See [109,110]

sical trajectories. Though the reproducibility of this length standard is very high, the Compton wavelength cannot be related to frequencies with high accuracy. Therefore, also this ideal standard is again not a practical one.

From these examples it is also clear that the unique availability of physical units is a matter of the physical *dynamics*. This is the case, because all physical standards are more or less defined by complicated physical objects that evolve in time according to the underlying physical laws.

## 7.2 The System of Units

The first worldwide accepted units of time, length and mass were provided by the revolution of the Earth around its axis resulting in the *Universal Time* scales (of various types according to the kind of averaging), by a meter stick realized as a metallic bar having the length defined as one ten-millionth of the distance between the pole and the equator, and a mass unit made, like the meter, of platinum-iridium. The definition of time suffers from the irregularities of the motion of the Earth (the length of the day, for example, increases by around 2 seconds every century compared with a time-scale defined by more stable Quartz oscillators). In 1956, the Ephemeris Time, based on the Earth's orbital motion around the Sun, was chosen as definition for time: The second was the 1/31 556 925.9747 part of the year 1900. The problems with the definitions of length and mass were that a direct comparison of these macroscopic prototypes is nor very accurate, and that the intrinsic stability of these prototypes are not known. Each material, for example, experiences some ageing. Indeed, there is an unexplained drift of the various mass prototypes during the last decades.

A first step in improving this system of units was to replace the Universal Time by atomic time. This was a natural development since atomic clocks were much more precise than the astrophysically defined time unit. Therefore, during the 17th General Conference of Weights and Measures in 1983 one defined the

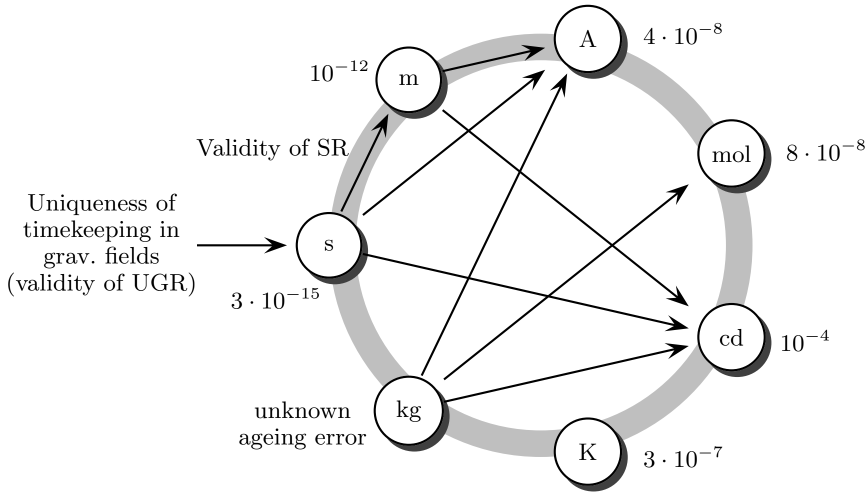


**Fig. 4.** The SI-units (s = second, m = meter, A = Ampere, mol = mole, cd = candelas, K = Kelvin, kg = kilogram) and their interdependences [111]. The numbers indicate the stability of the corresponding unit. The uniqueness of the transport of the definition of the second and of the meter depends on the validity of the EEP. A replacement of the mechanical definition of the Ampere or other quantities also requires conventional quantum theory and Maxwell theory and thus the validity of the EEP

second as the time that is needed by 9192631770 periods of the hyperfine transition of the ground level of $^{133}$Cs. Therefore time was defined in terms of a highly reproducible quantum phenomenon. However, in order to combine all the atomic clocks around the Earth (which is advantageous since by enlarging the number of clocks, the precision of time-keeping will increase, and since astrophysical observations with distant telescopes, for example, need synchronization) Special and General Relativity is needed: We need Special Relativity for the synchronization procedure on a rotating reference frame (Sagnac effect), and we need General Relativity in order to account for different counting rates at different gravitational potentials (gravitational redshift). The result is the *terrestrial coordinate time* which is a model extracted from the reading of all the clocks on the surface of the Earth and which now represents the time in a non-rotating observer located at the center of the Earth. It is well known that time provided by the GPS also needs relativistic corrections.

The next step was to replace the unit of length by a much more reproducible phenomenon. In a first step this was done in 1960 by defining the meter as 1 650 763.73 wavelengths of the red $2p_{10} - 5d_5$ transition of Krypton. This length could be reproduced with an accuracy of $3 \cdot 10^{-10}$. The disadvantage was that the coherence length of that radiation was smaller than one meter, which made it difficult to be compared with the old standard. Later on, during the mentioned conference in 1983, the constancy of the speed of light was used in order to base the meter on time. Accordingly, the meter is now given as the distance light travels within the 299792458th part of a second. This can be realized very precisely using interferometry of laser beams. Therefore, with the help of Special Relativity, the length unit was replaced by a quantum phenomenon. Obviously, this definition breaks down if Special Relativity will be proven to be wrong, i.e., if the velocity of light depends on the orientation of the velocity of the laboratory or if the usual dispersion relation for light is modified.

The replacement of the definition of the kilogram by some quantum procedure is under way in various groups. One idea is to use a fixed number of atoms in a Si one-crystal. In principle, this is a well-defined, exactly reproducible procedure. However, counting the number of crystal lattices is not very practical so that one has to use optical techniques in order to measure the geometry of e.g. a sphere of a Si-crystal. That means that the kg will be based on the second. Another idea is to use the Watt balance which connects a mass unit and Planck's constant $\hbar$, and to replace the definition of mass by a definition of $\hbar$. Then $\hbar$ receives a *defined* value and the kilogram will be derived from it. Since the Watt balance relates mechanical power to electrical power with the help of the quantum Hall and the Josephson effect, again the Maxwell equations and the laws underlying quantum mechanics are involved.

A general task of modern metrology is to base all units on quantum mechanically defined and thus highly reproducible quantities. Beside the second and the meter, this has been done already for the electric resistance (measured in Ohm) and the electric potential difference (measured in Volt), based on the von-Klitzing and the Josephson-constant, respectively. Furthermore, the current can be based on the electronic charge and the second. Again, the validity of the standard Maxwell and quantum equations is a prerequisite for the consistent realization of this task.

## 7.3 Consequences of a Violation of the EEP

From this outline one can see immediately that the full system of units in its present (and proposed) form is compatible only if the present physical theories are correct, that is, if the EEP is valid. If, for example, SR is violated and the velocity of light is not isotropic, then the definition of length in general will not be unique in the following sense: For a given unit of time we may prepare two different length units in different directions. If we rotate these units of length, then they will in general not coincide if the velocity of light depends on the direction, that is, the unit of length prepared in the direction in which the velocity of light is larger will be smaller than the other one. One may think of an effect that internal forces of the material realization of the length standard may compensate for this effect. But one cannot expect this to happen for all materials in the same manner.

Another example is the uniqueness of the velocity of light: Let us assume that the velocity of light is different from the characteristic velocity appearing in the Dirac equation. Then the fine structure constant $\alpha$ as derived from spectroscopy by $\alpha = \sqrt{2\mathrm{Ry}\lambda_\mathrm{C}}$, where Ry is the Rydberg constant and $\lambda_\mathrm{C}$ the Compton

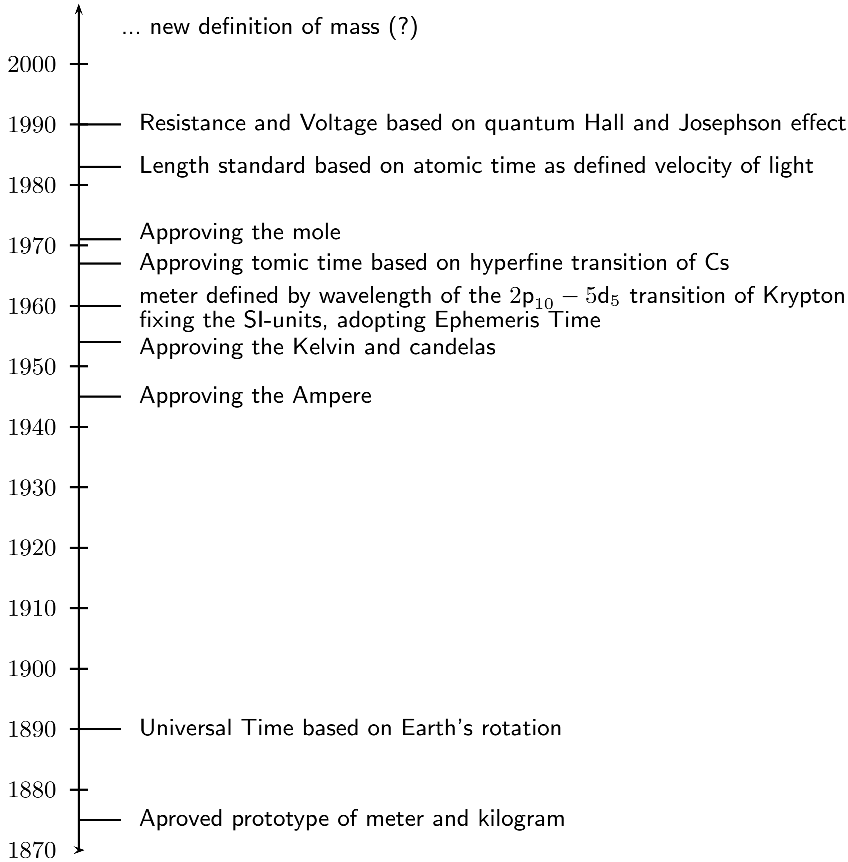


**Fig. 5.** Highlights of modern metrology

wavelength of the electron, will be different from the fine structure constant which can be derived from the quantum Hall effect by $\alpha = c/(2R_K)$ where $R_K$ is the von Klitzing constant [112]. This again amounts to a non-uniqueness in the definition of length.

## 8 Conclusion

We showed that the implications of the Einstein Equivalence Principle are two-fold: First, the EEP strongly restricts the structure of the equations of motion for all types of matter fields, as for example the Dirac equation, and non-gravitational interactions like the Maxwell equations, and it fixes the structure of the gravitational field to be described by a metric field or a related quantity. The EEP is not sufficient to derive Einstein's field equation. Second, since the EEP determines the structure of standard physics, any deviation from standard physics should show up in violations of the EEP. Consequently, any search for violations of the EEP or any experimental improvement of the validity of the principles underlying EEP is very important for any theoretical scheme trying to go beyond the standard physics. Furthermore, the EEP also has very practical consequences in the sense that only for standard physics the today's scheme of metrology will give a consistent set of physical units needed to measure physical effects and compare theoretical predictions with experiments.

QG effects that fall outside the scope of the EEP and thus are not treated here, are

- Modifications of Newton's potential (see I. Antoniadis's lecture on page 337 and H. Abele's lecture on page 355).
- Time-dependent Newton's gravitational constant $G$, see e.g. [113].
- QG induced modifications of the dispersion relation leading to the GZK-cutoff presently very much discussed in astrophysics, see e.g. [114,115].
- QG induced noise in interferometers [116].
- QG induced decoherence in quantum matter, e.g. [117–119].
- QG induced fluctuation of the light cone [32].

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